

Walking the Talk: Translation of Mathematical Content Knowledge to Practice

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Recent debates on students' learning outcomes in mathematics have shifted the focus to better understanding the types of knowledge that teachers need in order to support children. In the present study, we examined the quality of knowledge of a cohort of prospective teachers along the dimensions developed by Ball et al. (2008). We found support for the contention that beginning teachers tend to have built a body of content knowledge. However, that knowledge remains less germane to teaching children. Implications for translation of this knowledge for teaching are presented.

In the past decade or so, we have noticed a steady decline in the number of young Australians pursuing mathematics, in particular, advanced mathematics in high schools. This trend has raised alarm among teachers, tertiary institutions and other key stakeholders alike. While the problem is most visible in secondary education, the root of the problem appears to lie in the attitudes towards, and knowledge of, mathematics that children develop in years K-6. How can we start arresting and possibly reversing this trend in mathematics in Australia? One potentially fruitful area could be to take a serious look at the quality of our mathematics teacher and teaching. The education system of countries that are consistently ranked in the top five, based on Trends in Mathematics and Science Study (TIMSS) results, regard quality of their teachers as being the major contributory factor for the performance of their students. Thus, it would seem that mathematics learning, to a large extent, is dependent on teacher quality. But what aspect of teacher quality is instrumental in improving access to and participation in mathematics?

Context and Significance of Issue

Teacher quality as it relates to mathematics learning can be given many interpretations but our focus here is on the knowledge that teachers bring to support deep mathematics learning and understanding. Nationally, these knowledge categories have been gaining increasing attention in professional standards set by teacher accreditation bodies such as the New South Wales Institute of Teachers, Queensland College of Teachers, and the research community (Frid, Goos, & Sparrow, 2009). The need to ensure that our teachers of mathematics had developed an adequate knowledge base of mathematics was also echoed in the recent review of mathematics education conducted by the Group of Eight Universities that drew the attention of teacher educators to examining the quality of mathematics acquired by primary teachers (Brown, 2009).

Many studies focus on what pre-service teachers (PSTs) do *not* know (Mewborn, 2001). Others have looked at misconceptions including Afamasaga-Fuata'I (2007) who found evidence that students come to university with misconceptions. There is also a paucity of research about the PSTs who *do* have 'strong conceptual knowledge of mathematics' (Mewborn, 2001, p. 33). Our study is set against this background and is motivated by the desire to better understand the state of our prospective teachers' knowledge of mathematics and teaching as they entered and completed our course. Data on PSTs' knowledge is expected to: a) provide clear directions for design and conduct of



methods subjects and b) better inform us about the kind of professional experiences that would be needed before our PSTs commence their professional work as fully-fledged teachers.

Related Literature

The issue highlighted in our introduction has been the subject of several lines of inquiry. We examine the broader issue within the context of the types of knowledge that teachers require to engage students in a variety of ways. Shulman (1987) pioneered the area of research that focussed on the relationship between teacher knowledge and teaching by identifying two major dimensions of that knowledge: Subject Matter Knowledge and Pedagogical Content Knowledge. The positive correlation between teachers' mathematics knowledge and student performance is gaining increasing support particularly since the seminal work by Ma (1999) where it was found that superior mathematical learning in China could be ascribed to the mathematical expertise of their teachers.

Delaney, Ball, Hill, Schilling, and Zopf (2008) have been active in analysing and developing Shulman's (1987) knowledge categories that are relevant to mathematics. Their ongoing research has established a solid platform for the study of fundamental knowledge components that a teacher needs. Instead of taking into account the multiple facets of PSTs' knowledge and beliefs, there appears to be a tendency among teacher educators to view PSTs as simply lacking particular knowledge. Furthermore, although some PSTs are able to successfully solve mathematical problems, many are unable to explain the concepts and procedures they perform (Mewborn, 2001).

Significantly, Ball, Hill, and Bass (2005) found a correlation between a teacher's mathematical knowledge and student achievement. However, Ball, Hill and Bass (2005) conclude that teaching PSTs more content knowledge is not the answer, and teaching for understanding is required. As well as the content, *what of mathematics*, teachers also need to know *how to teach Mathematics* which led Delaney et al. (2008) to coin the term Mathematical Knowledge for Teaching (MKT). MKT is multilayered, interdependent, and interrelated. Hill, Ball and Schilling (2008) provide more nuanced dimensions of MKT: Common Content Knowledge (CCK), Specialised Content Knowledge (SCK), Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT).

While CCK, SCK, KCS and KCT provide a solid theoretical framework, Delaney et al. (2008) suggest that there is a need for further empirical evidence gathered from a range of contexts. We pursue this issue in the present study.

Conceptual Framework

Data analysis and interpretations were guided by the following schematic representation of teacher knowledge for teaching mathematics (MKT) (Figure 1) (Hill, Ball & Schilling, 2007, p. 174). The four dimensions are defined as follows:

Common Content Knowledge (CCK): Mathematical knowledge and skill possessed by a well educated adult.

Specialised Content Knowledge (SCK): Knowledge of how to: a) use alternatives to solve a problem; b) articulate mathematical explanations; c) demonstrate representations.

Knowledge of Content and Students (KCS): Knowledge that combines knowing about mathematics and knowing about students. Knowledge of how to: a) anticipate what

students are likely to think; b) relate mathematical ideas to developmentally appropriate language used by children.

Knowledge of Content and Teaching (KCT): Knowledge that combines knowing about mathematics and knowing about teaching. Knowledge of how to: a) sequence content for instruction; determine instructional advantages of different representations; b) pause for clarification and when to ask questions; c) analyse errors; observe and listen to a child's responses; d) prompt; e) pose questions and probe with questions; f) select appropriate tasks.

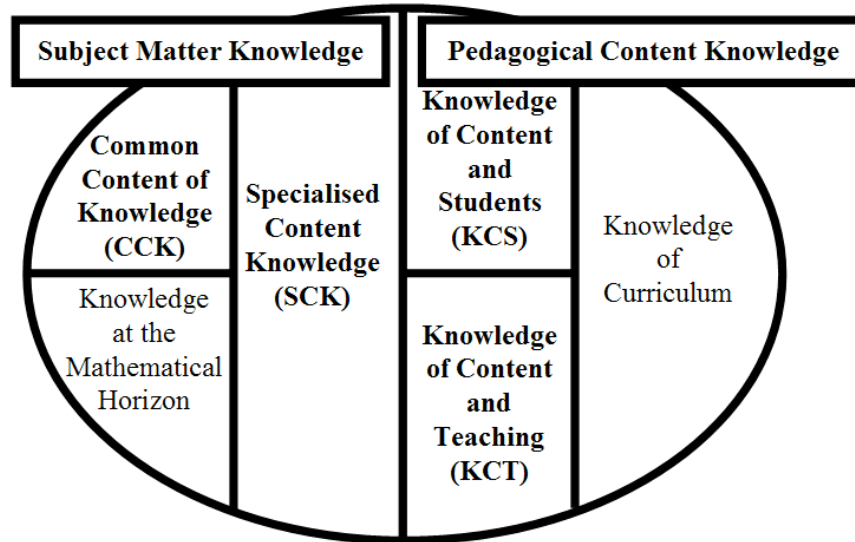


Figure 1. Schematic representation of teacher knowledge for teaching mathematics (MKT).

Research Questions

Are there differences in the activation of pre-service teachers' MKT as used in the context of solving a focus problem (Truss Bridge Problem)?

What are potential relations among the four dimensions of MKT that prospective teachers activate in the context of a focus problem (Truss Bridge Problem)?

Methodology

Design

The aim of our larger study is to examine the issue raised by Ball, Hill and Bass (2005) by investigating potential relations between the four categories of MKT before and after PSTs' professional experience in schools. We followed a design experiment which Cobb, Confrey, diSessa, Lehrer and Schauble (2003, p. 8) suggest as appropriate for 'engineering particular forms of learning and systematically studying those forms of learning within the context defined by the means of supporting them'. The study involves two phases: Phase 1 (pre-professional experience) and Phase 2 (post-professional experience). This report concerns Phase 1 of the larger project in which PSTs attended 16 hours in lectures and 8 hours in tutorials. Data about pre-service teachers' MKT reported in the present study were collected in the tutorials that followed the aforementioned sessions.

Participants:

A cohort of 40 Graduate Diploma of Education students participated in the study. The cohort included both Australian and Canadian students who have completed a variety of undergraduate degrees including psychology, social sciences and science.

Context and Procedures for Data Collection:

We provided a range of prompts and support structures in the course of 4 weeks prior to the focus activity via a series of lectures and tutorials. The teaching program included demonstrations and opportunities to use aids such as arrays, Base-10 Blocks and activities from NSW *Count Me In Too* programme. The focus of the lectures and tutorials was teaching, learning, and assessing numeracy. The activities in the lectures and tutorials were geared at fostering awareness among PSTs of the need to provide both developmental and differentiated opportunities for children to make connections in mathematics. The tutorial activities were designed to be learning opportunities that moved the students beyond talking and reading about pedagogy (Ball & Cohen, 1996). PSTs were thus involved in an authentic teaching and learning context where they could independently and collaboratively explore preconceived ideas and extend ideas. This strategy afforded explanation, representation, utilisation of language, as well as interpretation of others' thinking of the content and pedagogical issues.

Following the four-week intensive session the participants were asked to solve the Truss Bridge Problem (focus activity), designed for upper primary students. The goal of the problem was to determine the number of beams on either side of the given bridge as well generating patterns for bridges of similar design but of different lengths. PSTs were asked to think like a child and concomitantly 'put on a teacher's hat' in order to identify and scaffold potential difficulties that could occur in the process of understanding and solving this problem. We provided a coloured photograph of the Truss Bridge Problem (Figure 2), the NSW K-6 Syllabus (Board of Studies, 2002) as well as concrete aids such as coloured paddlepop sticks and grid paper, to support the activation of PSTs' mathematical knowledge for teaching. In addition, we provided four focus questions to participants (Table 1).

Table 1
Focus Questions

| | Question |
|---|--|
| A | What are some of the difficulties that might be experienced by Stage 3 children in understanding and solving this problem? |
| B | How would you help children develop skills and knowledge in solving this type of problem? |
| C | How would you weave Working Mathematically Processes in the course of doing the activity? |
| D | What are the types of connections or relations that can be supported via this activity? How can these connections be extended? |

The solution of the problem involves integration of content knowledge from a number of strands such as geometry and algebra, and use of a range of problem-solving strategies, such as reasoning and pattern identification. The richness of this problem to elicit the four

knowledge categories was assessed by comments from an experienced primary teacher. We are confident that we provided ample prospects within the context of the Truss Bridge Problem for the PSTs to access MKT, which includes CCK, SCK, KCS and KCT.



Figure 2. *Truss bridge problem.*

Data Analysis

The purpose of the study is to identify and describe four categories of pre-service teacher knowledge, namely, CCK, SCK, KCS and KCT. The participants' written responses and work samples (comments, diagrams and photographs) were examined to determine activation of the knowledge components.

Research Question 1

- 1) Are there differences in the activation of pre-service teachers' MKT as used in the context of solving a focus problem (Truss Bridge Problem)?

The means and standard deviations for the four knowledge components are presented in Table 2. The table shows that our PSTs' accessing of CCK was almost three times that of the other categories. This indicates that during the solution of the Truss Bridge Problem, our PSTs tended to solve the problem as adults despite being scaffolded with both teaching aids and focus questions that prompted attention to the other three knowledge components.

Table 2:

Component Means and Standard Deviations

| Component | Mean | Std. Deviation |
|-----------|------|----------------|
| CCK | 4.15 | 1.73 |
| SCK | 1.50 | 1.33 |
| KCS | 1.47 | 0.99 |
| KCT | 1.38 | 0.90 |

Research Question 2

What are potential relations among the four dimensions of MKT that prospective teachers activate in the context of a focus problem (Truss Bridge Problem)?

The bivariate correlations were computed for the four knowledge dimensions (Table 3). Common Content Knowledge (CCK) is significantly correlated with Specialised Content Knowledge (SCK) and Knowledge of Content and Students (KCS). Likewise, teachers' SCK was significantly related to KCS. Knowledge of Content and Students (KCS) is related to Knowledge of Content and Teaching (KCT).

Table 3
Bivariate Correlation Results

| Component | CCK | SCK | KCS | KCT |
|-----------|-----|---------|---------|---------|
| CCK | | 0.740** | 0.437** | 0.062 |
| SCK | | | 0.475** | 0.139 |
| KCS | | | | 0.518** |
| KCT | | | | |

** . Correlation is significant at the 0.01 level (2-tailed)

Three Vignettes of PSTs' Responses

In order to illustrate the interconnectivity amongst the knowledge components we provide vignettes of three pre-service teachers' actions.

Vignette 1 (PST1). PST1 stated that a child might have difficulty in seeing that the triangles share a side (KCS). In order to aid a child's visualisation and to also enable physical interaction, PST1 demonstrated how paddlepop sticks could be used (Figure 3). She commented, "The child can touch and count the sides" (KCT). This identifies an instance of the relationship between KCS and KCT. While this shows some understanding of MKT, notions of 2D shapes and one-to-one correspondence, the participant did not articulate how she would assist the child to extend the links to patterns, which is necessary to deduce lengths of different size bridges. Even though PST1 had initially solved the problem herself in adult terms, "Twice the number of triangles plus one is equal to the number of sticks" (CCK), she did not make the connections in ways that would enable children to develop the sequence (KCT).

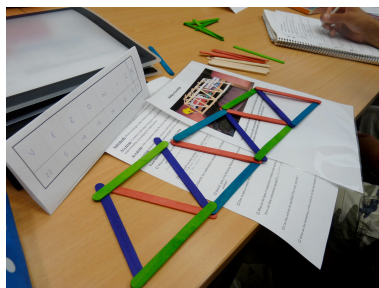


Figure 3: PST1.

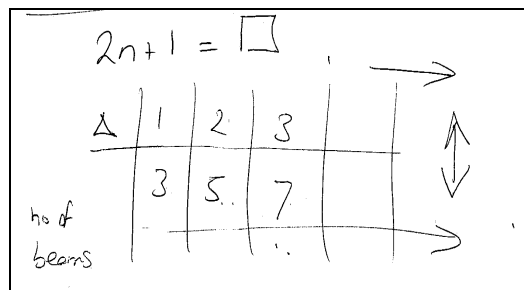


Figure 4: PST2.

Vignette 2 (PST2). PST2 wrote the equation $2n + 1 = \square$ and drew a table to show how the pattern is developed (Figure 4). Other patterns were also outlined, "... ordinal numbers along the top and uneven numbers along the bottom" (SCK). To teach this she "... would tell the patterns as I filled in the table on the whiteboard". PST2 identified the only difficulty a child might have "is they are unable to read the question" and to assist a child, she would "read the question aloud to the child". While this demonstrates a level of support, there is a need for further development of KCS and KCT.

Vignette 3 (PST3). Activation of CCK, involving geometry and algebra, is identified with PST3 stating, "The problem requires breaking down from 3D to 2D and then it is $2n + 1 = \square$ ". PST3 noted to assist a child he would, "teach the 3 times tables and also

encourage physical engagement (hands on) with objects”. These comments indicate that PST3 needs further development of KCT in order to translate CCK. The only evidence of any difficulties a child might face in understanding and solving this problem is, “Children might play with the sticks”, indicating a need for further development of KCS.

Discussion and Implications

Our current work is part of a larger study with an aim to capture change in the quality of the four dimensions of teacher knowledge that Ball, Thames and Phelps (2008) suggest is required for effective pedagogical mathematical practice. We examined the knowledge of a cohort of pre-service teachers before their professional experience. Our research was structured around two key research questions.

Analysis of data relevant to Research Question 1 suggests that PSTs activated a higher proportion of common mathematical content knowledge (CCK) in relation to the other three knowledge components, namely Specialised Content Knowledge (SCK), Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT). The examination of links (Research Question 2) indicated strong positive correlations amongst all the elements with the exception that KCT was found to be related to KCS and not the others.

Given that all our participants had completed a variety of undergraduate degrees we expected a reasonable level of CCK in their MKT indicative of a strong mathematical knowledge base and some degree of understanding of how children learn mathematics. This expectation was supported by the results of the study as our teachers produced lower knowledge in three of the four constituents of the knowledge complex proposed by Ball, Thames and Phelps (2008). We are conscious that our data collection was conducted in the first semester of the students’ teacher education program and the results would seem to be consistent with their level of understanding of mathematical pedagogy one could expect at this stage of the development as professionals.

The draft of National Curriculum for K-10 mathematics (ACARA, 2010, p. 1), suggests that teaching ought to support children’s ability to ‘recognise connections between the areas of Mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study’. The data from this study indicates that while the beginning teacher did show evidence of acquisition of the four knowledge clusters, this knowledge of the PSTs is not quite robust enough to buttress the development of the above relations. However, we anticipate that this cohort of student teachers would have built a wider knowledge base after their professional experiences and further completion of other courses. The second phase of the study will examine if this is indeed the case.

Our measurement of pre-service teacher’s knowledge within the category of KCT was somewhat constrained due to the limitation of not having access to classroom children. In their responses, the participants had to ‘work’ within a hypothetical situation. While pre-service teachers’ CCK was evident within the context of the Truss Bridge Problem, it is possible that given a wider range of problems, additional evidence of knowledge within CCK and the other categories could be obtained. This issue could be the subject of further investigation.

While our prospective teachers’ CCK seems to be relatively strong, a further development of this knowledge into SCK is a critical aspect of teacher knowledge development as argued by Ball et al. (2008). Current debates on teacher mathematical knowledge required for teaching seem largely driven by concerns with CCK. But this knowledge has to be tempered so that it is germane to making connections with children’s

prior understanding of mathematics and numeracy. We contend this process and resulting children-friendly mathematical knowledge have not received sufficient attention in current debates and reform statements. The professional experience of our teachers is expected to engage them in ways that would bring a higher degree of focus to SCK.

The implications for translating the content matter of mathematics into effective pedagogical practice are paramount in raising the profile of mathematics. Assisting pre-service teachers to 'walk the talk' could lay the necessary foundations in our primary schools.

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