Computational Estimation in the Primary School: A Single Case Study of One Teacher's Involvement in a Professional Learning Intervention

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This paper focuses on the initial analysis of a study of a professional learning intervention. Using a case study design it was possible to describe one teacher's involvement in this research. The study revealed how the teacher's beliefs and pedagogical content knowledge of computational estimation was altered as a result of participating in the research. This development appeared to have an impact on her approaches to the teaching of computational estimation.

For this study computational estimation is defined as a process in which some or all of the numbers in an arithmetic problem are approximated to simplify the computation of the estimate (Mildenhall, 2009) and it is also being asserted in this study that computational estimation is a component of number sense (Greeno, 1991; McIntosh, 2004). This term is a relatively recent one and has been described by McIntosh, Reys, Reys, Bana and Farrell (1997) as "a person's general understanding of number and operations, along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations" (p. 3). On the basis of this definition, the ability to estimate numerical quantities is an integral component of number sense.

This study investigated the impact of a professional learning intervention that focused on developing teachers' beliefs, knowledge, and practice when teaching computational estimation. It is becoming apparent that teachers' beliefs affect their pedagogical decisions (Carpenter, Fennema, Franke, Levi, & Empson, 1999) therefore it was important to consider how changes in teachers' beliefs might affect classroom practice.

The type of knowledge that teachers require to teach effectively has been defined as pedagogical content knowledge (PCK) (Shulman, 1986). The term PCK includes subject matter knowledge, curricular knowledge and pedagogical knowledge so it was important to observe the teachers' development of these aspects. The research questions addressed in this paper were:

1. How did one teacher's participation in a professional learning intervention develop her PCK, beliefs and teaching approaches about computational estimation?

2. How did the teacher's development of beliefs and PCK about computational estimation inform her teaching approaches?

Theoretical Framework

The theoretical framework for this study was set in a social constructivist and critical paradigm (Crotty, 1998). Table 1 identifies the core component of these theories and explains why this theoretical framework underpins this study.
Table 1

Theory	Justification	Importance for the research
Social constructivism	All tenable statements about existence depend on a world view, and no world view is uniquely determined by empirical or sense data about the world (Patton, 2002, p. 97)	The different world views in the research study are acknowledged, represented, and valued
Critical theory	*In this type of inquiry spawned by the critical spirit, researchers find themselves interrogating commonly held values and assumptions, challenging conventional social structures and engaging in social action (Crotty, 1998, p. 157)*	This study involves a collaborative group participating in a cyclical process of self-reflection and action in order to bring about change in the understanding and practice of the participants

The Professional Learning Intervention

Factors of effective teacher professional learning were considered in the design of the professional learning intervention. These included, providing on-going support (Hackling, Goodrum, & Rennie, 1999), teachers working in collaboration (Bray, 2002; Keady, 2007) and time spent with students in order to reflect on how the learning in the professional learning situation can be incorporated into the classroom (Bobis et al., 2005; Fennema et al., 1996). Year 6 teachers from low fee, non-government schools were invited to join. These schools are established by religious groups and despite being privately run, still receive government funding based on the socio-economic status of the parents of the school (Independent Schools of Australia, 2007). Wendy (a pseudonym) was one teacher who participated in the year-long professional learning intervention. This paper is a description of her journey.

The initial professional learning day introduced four principles from the research literature. These were that mathematics teaching and learning is effective when it is; active (Franke, Kazemi, & Battey, 2007), metacognitive (McKeachie, Pintrich, Lin, & Smith, 1986), and contextual (Gravemeijer & Terwel, 2000). The fourth principle was that numerical estimation is an integral part of number sense (McIntosh, Reys, Reys, Bana, & Farrell, 1997). The teachers were introduced to six estimation strategies; front end loading, range, compatible numbers, rounding, intuition, and benchmarking (Mildenhall, 2009). They were presented with suggested teaching activities and it was also recommended that 'estimation as a checking device' become part of the normal expectations of teachers and students in the mathematics lessons, that is the sociomathematical norm (Yackel & Cobb, 1996).

The teacher went back to school and trialled the estimation tasks in a way that she thought was pedagogically appropriate. On each of the further three professional learning days there was time for reflection and discussion of the estimation tasks trialled in the classroom and an opportunity to reflect on the research literature. Mathematical tasks were also presented to develop content knowledge. Using an action research methodology

(Somekh, 2006), this cycle of plan, act and reflect ran through three complete cycles ending with a twilight plenary session at the end of the year.

Methodology

The methodology for this case study used purposeful sampling (Merriam, 1998; Patton, 2002) in order to select an appropriate unit of analysis. This unit was the professional learning intervention, which included the teacher participants and their students. Convenience sampling was also undertaken (Merriam, 1998) so that one teacher's journey in this professional learning could be analysed.

Data Collection

Multiple data collection methods were used to maximise the evidence available to answer the research questions and to increase the internal validity of the study (Merriam, 1998). Table 2 illustrates how the data was collected in order to capture the longitudinal nature of the study.

Data Analysis

The data, which were predominately qualitative, were entered and analysed using the computer software NVIVO 8 (QSR, 2008). Apriori coding by the researcher created the initial themes of teacher beliefs, pedagogical content knowledge, and teaching approaches of computational estimation. These were created as tree nodes in NVIVO 8. Inductive coding then took place as themes emerged from the data to identify the individual teacher's beliefs, PCK, and teaching approaches. Table 2 illustrates how the data analysis was categorised into three sections in order to capture teacher change in PCK, beliefs and teaching approaches. In this way it was also possible to interpret how these changes may have impacted their teaching approaches.

Table 2
Data Collection and Analysis

Data analysis section	Data collection instrument	Data analysis
Beginning of the study	Semi-structured 1st teacher interview	Qualitative data recorded, transcribed, and then entered and analysed in order to create themes using NVIVO
	1st professional learning day initial reflection	
	Focus group student interviews	
	Multiple choice computational estimation student pre-test	Quantitative pre-test data entered and analysed using SPSS
During the school year	Participant observation of 1st visit to classroom (work samples collected)	Qualitative data recorded and transcribed and then entered and analysed in order to creating
	Participant observation of 2nd PL day	

	Participant observation of 2nd visit to classroom (work samples collected)	themes using NVIVO
Midyear-6 months into the study	Semi-structured 2nd teacher interview	
	Participant observation of 3rd PL day	
	Participant observation of 3rd visit to classroom (work samples collected)	
End of study-final term of the school year	4th Professional learning twilight session	Qualitative data recorded, transcribed, entered and analysed to create themes using NVIVO
	Semi-structured final teacher interview	
	Focus group student interviews	
	Multiple choice computational estimation student post test	Quantitative post test data entered and analysed using SPSS

Results

Development of Wendy's Beliefs

At the beginning of the professional learning intervention Wendy had the belief that estimation was useful as a checking device when doing algorithms as shown in Table 3. Table 3 reveals how, as the study progressed, these beliefs had broadened so that Wendy now saw computational estimation as a strategic number sense activity as well as checking device.

Table 3
Wendy's Beliefs during the Study

Data collection instrument	Qualitative data
1st Interview	I find children need it [estimation when doing procedural mathematics] ... Is the answer going to be bigger or smaller. So what do we need to do?
2nd Interview	... and I think they are seeing the value of estimating gradually whereas before when it was a text book thing they don't see the point
4th Professional learning session	... what I think is so important is the number sense aspect of it [computational estimation]
	… but they [the computational estimation strategies] made so much sense to me ,that is what amazed me
3rd Interview	I think doing those problems that didn't have right answers really helped with that [broaden the students perception of mathematics]

Development of Wendy's Pedagogical Content Knowledge

Wendy did not have any knowledge of computational estimation strategies other than rounding at the beginning of the project. At the end of the professional learning intervention she was asked to summarise how she thought computational estimation should be taught. She had developed new pedagogical approaches:

Using lots of practical activities; Starting with numbers the children understand and can relate to (in the lower grades). Building to higher numbers; Games; Journaling of their understanding; Children need to understand why they are estimating.

She also developed a growing understanding of the estimation strategies. When asked how she would rate her awareness of estimation strategies at the beginning compared to the end, she said, "oh I only knew rounding really, so now I know lots".

Development of Wendy's Teaching Approaches

Before the intervention Wendy did not teach students how to undertake computational estimation using a variety of strategies and taught the estimation strategy 'rounding' procedurally as an algorithm. After the intervention Wendy spent time teaching students that computational estimation could be part of their problem solving repertoire in mathematics. In the 3rd classroom observation Wendy provided a learning task where the students had to collaboratively plan a trip to a park. There were many opportunities for the students to use the computational estimation strategies including working out how far it was from the school to the park, the cost of the barbeque items and the itinerary. The example below is taken from the start of the lesson where Wendy reminded the students how the estimation strategies could be useful:

We have to bring in the estimation strategies we have been talking about because the prices will vary from day to day. We can only have an estimate 'cos we can't really have an accurate amount

… You need to use friendly numbers, if we are looking at prices and something cost $3 .99 and you need 10 or 12 or 15 packets of them, what would you probably do with the $3.99?

Discussion

Over the period of the professional learning intervention Wendy's beliefs about computational estimation changed. She now perceived that computational estimation could be a strategic number sense activity rather than just another step for checking procedural algorithms. Wendy was able to understand the computational estimation strategies herself and she believed that they were appropriate to be taught to Year 6 students. This acceptance of the strategies is a positive sign that primary school teachers may be able and willing to incorporate the variety of computational estimation strategies into their teaching repertoire.

Wendy's enhanced beliefs and PCK appeared to inform her teaching approaches as noted in classroom observations of her teaching. She used her increased content knowledge of computational estimation strategies in her discourse with the students. She also implemented estimating problem solving tasks set in contexts, which were not part of her teaching repertoire before the study began. This suggests that she had learnt new pedagogical approaches whilst being involved in the professional learning intervention.

Conclusion and Implications

It is not possible to generalise from this single case study and it is also important to acknowledge that, as this research is set in a social constructivist theoretical framework, this research is conducted from the particular world view of the researcher. This case study showed that the intervention enabled the teacher to enhance her computational estimation PCK and this appeared to change her teaching approach of estimation. Initial analysis of student pre and post test data would appear to indicate that students benefitted from their teacher being involved in this professional learning. Further cases are being considered as part of a larger study that should provide an even deeper understanding and these findings will be reported at a later date.

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