Changing our Perspective on Space: Place Mathematics as a Human Endeavour

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This paper collates some of the systematic ways that different cultural groups refer to space. In some cases, space is more strongly identified in terms of place than in school Indo-European mathematics approaches. The affinity to place does not reduce the efficient, abstract, mathematical system behind the reference but it does strengthen its connection to the real world of place. This review of research uses a critical approach to develop an ecocultural perspective on spatial referencing. It refers to studies on the Polynesian Wayfinders; the Garma project at Yirrakala in the Northern Territory of Australia identifying Yolgnu sense of position; original field data mainly from Papua New Guinea; and on the work of linguists who have recorded and analysed the ancient languages of the Pacific region. The paper provides the mathematics educator with a richer perspective on position and a way of understanding the thinking of students with language and cultural backgrounds other than English. The Indigenous languages of Australia form a basis for ways of thinking spatially for specific groups of Indigenous students.

For many years the strand of space and geometry has begun with a focus on language. Early childhood experiences emphasise the use of prepositions like "in", "on", "inside" and words like "left" and "right". This is followed with giving position in space in terms of orthogonal dimensions with an origin from which to mark off segments or to delineate points. Location is usually considered in terms of an orthogonal coordinate system (Uttal, Fisher, & Taylor, 2006, p. 2). Students may begin with locating on one dimension before using two and three orthogonal dimensions. The use of x and y axes numbered from zero with positive integers are later developed for positions described in terms of negative to positive numbers. This system is a basis for the visual representations of algebraic relationships.

The early prepositional world of identifying place (e.g., on the table in the house) and referring to constructions with blocks is followed by mapping familiar places. In primary schools mapping tends to move from more pictorial representations with some indication of direction to those showing relative lengths and greater accuracy in terms of angles formed by intersecting lines representing roads or paths (Owens, 2000; Owens & Geoghegan, 1998). Mapping is introduced as a plan view, but there may be few links to maps used everyday or to those with contour lines. Furthermore, the way places are described and represented in big space using landmarks is generally not linked to descriptions of spaces that can be within a person's immediate space such as on a piece of paper or in a room.

The use of polar coordinates referencing by direction (e.g., an angle from north) and distance from the reference point is often left to higher school levels. The use of direction by turning through an angle and moving a certain distance was central to LOGO geometry and this geometry has returned to early childhood education with the availability of programmable toys (Highfield, Mulligan, & Hedberg, 2008).

However, what school systems currently fail to realise is the diversity of ways in which location in space can be described systematically by different cultural groups. One way of exploring this is through language.

L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia.* Fremantle: MERGA.

Methodology

This study consists primarily of a literature review of the ways that different people refer to a place or position in space. It draws mostly on linguistic data presented in original research papers but also in some secondary sources (e.g., Tuan, 1977; Worsley, 1997). In addition, the study draws upon original data obtained by both semi-structured interviews and story telling by people who speak one of the 800 languages in Papua New Guinea (PNG). The critical approach taken in this review is based on an ecocultural perspective that considers place when referencing space. The spatial references relate to people's place, cultural activities, and language. Much of the data comes from Indigenous communities whose relationships with place ensure spatial referencing is closely linked to their holistic world view.

There are many different Indigenous groups within Australia. Elders are the knowledge keepers and respected people of the community. Indigenous people identify with their language group and continue their cultural knowledge as best they can despite the varying degrees of loss due to poor colonising practices. For example, people use song-lines to maintain the connection with the route that is taken when traversing their land. The Galma study comes from the Yolgnu people at Yirrakala who have two clans, the Yirritja and Dhuwa.

The languages of the Pacific especially those of PNG are central to this study. The early waves of languages of PNG are grouped together as Papuan although there are several quite distinct Families and Isolates. Oral and recorded data were obtained on the following languages: Abau, in the Sandaun Province in the west of the mainland; Alekano in the Eastern Highlands Province; and Yupno in the Madang Province towards the east. Later migrations of people to PNG have resulted in languages classified as Austronesian. Kilivila, an Oceanic variant, is spoken by people who live on the Trobriand Islands in the far south east of PNG. Oral data also comes from an Australian language, Wiradjuri (in New South Wales), which is being revived.

Central to traversing space and linking places is navigation, a mathematical activity. Studies that form a basis for the discussion in this paper are by Akerblom (1968) on the Caroline, Marshall, and Gilbert Islands; Lewis (1973) on the Caroline Islands; Bryan (1938), Davenport (1960), and Spennemann (1998) on the Marshall Islands; as well as Davis (2010), Worsley (1997), and the University of Pennsylvania Museum (1997) whose website provides dynamic images to assist understanding.

Results

Issues Related to Spatial Reference Systems

In western mathematics, position is generally referred to in terms of two orthogonal dimensions with the speaker at the centre. The spatial frame denoted by the directions—north, south, east, and west—was also common across Asia and Africa but not elsewhere. However, this static frame was an issue by the 6th century BC for the Greeks viewing the world as a sphere, so they divided the heavens into zones and the earth into five latitudinal zones.

Reference to space that is far distant is problematic. Saulteaux Indians of Manitoba know of distant landmarks referred to by the points determined by the course of the North Star, the sun, and the home of the four winds (Tuan, 1977). Pacific navigators divide the plane of the earth into roughly 32 equal parts by marking the rising and setting of star constellations. The Australian and PNG Indigenous peoples know of trading partners far

away, and in some cases, dance routines represent the directions of these places. Even European folklore linked people to their environment; for example, the north were hardy, the south easygoing (Tuan, 1977).

Space as Place

One reason for the diversity of spatial frames of reference is the connection between the space and the holistic view of people's lives. The denoted space bears other connotations. For the Chinese the four sides of the rectangle were represented by animals. Ancient Greece used planetary gods – east connotes light, white, sky and up while west is darkness, earth and down. For the Pacific navigation there are star songs. For Europe zodiac star signs link with patterns of farm work like the coming of rain, breeding flock, harvest, mowing, raking (Tuan, 1977). Space is closely linked with time, and distance may be associated bodily with time taken to cover the distance. The time factor may also link to the stories of the past. For example, to walk a trek in Kaveve village (Eastern Highlands Province, PNG) will be to walk the story of the half-man who lived in that place. A community project in the Blue Mountains (on the outskirts of Sydney) involves maps, pathways and "song-lines" (Cameron, 2003). There are many such stories across PNG and Australia. Thus a place-based view of mathematics links position to the understanding of place and people's relationship with the place.

Space as place may have special personal or relational characteristics, and the spatial system may represent forces of nature and society. For the Yolgnu (Northern Territory, Australia) every person and every other thing is either Yirritja or Dhuwa. The division of land is dependent on the sacred sites with the nature of Yirritja or Dhuwa diminishing with distance so there are indistinguishable or grey areas too. (This referencing of space is not by an orthogonal grid.) The creation of the sites comes from the dreaming creatures who created the clans, are responsible for the sacred sites, and who maintain the power by observing appropriate ceremonies and through painting, dance and song (Thornton & Watson-Verran, 1996). "There is a metaphorical force essential for their way of life and sustaining their world" (Watson-Verran & Turnbull, 1995).

Kinship patterns and relational concepts are central to Yolgnu thought. A balance is achieved by using both Yirritja and Dhuwa knowledge for they have different knowledges. In discussing and drawing the various places, Larry (a clan Elder) represents the connectivity of the water flow, thus a line is not a Cartesian mapping but a topological mapping in western mathematical terms. However, such a description (topological mapping) does not present the fullness of the representation. Each place has connections with activities carried out by ancestors such as a place for camping when visiting, or a place for washing cycad nuts to remove poisonous chemicals. A walking track was Larry's responsibility and he would maintain it in song-lines, ceremonies and practices (Thornton & Watson-Verran, 1996).

The place where negotiations occur is roughly the shape of a stingray which buries its tail in the sand just as Larry and the ancestors before him buried their spears in the sand when negotiating a peaceful solution for revenge. Thus the shape and the place are metaphors and powerful images for complex ideas. Activity is set in kin relations, land rights and responsibilities, and sacred understandings. The land is constituted by living it. The conventions of the map representation are interpreted in terms of systematic relationships. The Yolgnu system of spatial knowledge (*Djalkiri*) is detailed and provides a means by which a person can find his/her way anywhere across the land. The structures of

the various forms of representation in ceremonies and everyday living and as found in the land itself locate space appropriately in the footsteps of the ancestors.

Another example of referencing space in maps without orthogonal axes is that of the Sámi (Indigenous people of Scandinavia). These maps provide a view from the North Pole and the map is demarcated by winds, rivers, and the routes of the reindeer (Jannok Nutti, 2009). In a similar way, the referential system for the Abau of PNG comprises the rivers, and the points on the river where certain activities take place. Different groups wash at different places in the river and so they mark these as fixed places. Any point on land can be given by reference to the distance from one of these points. The side of the river is indicated by stating the left or the right from the line of flow.

By contrast, the Yupno divide their valley area into a quadrant and can refer to these areas in terms of uphill and downhill. However, to describe a route meant reference to the landmarks of villages, resting-places and rivers. Wassman (1997) noted that the descriptions and even more the map drawing (both of which are not generally required in everyday communication except with people from outside) that some sense of walking the route was involved. For example, a slightly longer line represented a difficult time-consuming stretch of the track. Left and right in the language refer only to the body and extensions (e.g. a spear) so would not be used in a route description. In other areas of PNG, distance was marked by a stone's throw or a day's walk but short distances may be measured by body parts, sticks or ropes.

Locating and Communicating

> One of the main features of natural language is its 'contextuality' – and it is in this context-boundness that language, perception, and cognition meet. …Space, our perception of space, and our orientation in space are basic for human action and interaction in a number of domains- Konrad Lorenz even regards our spatial cognitive capacities as one of the roots for human thinking (Senft, 1997, p. 2)

From a perspective based on Indo-European languages we might consider that space is initially referred to in terms of the planes associated with the body. These are the central vertical planes providing (a) left and right and (b) front and back. The third plane may be at our feet as the plane of the ground providing a height dimension. Such a way of referring to space is consistent with a three-dimensional orthogonal Euclidean approach that provides for pathways, areas and volumes. The natural symmetry of the left-right plane and the expectation that one is standing in a vertical position underlie these expectations. The speaker's position and orientation are important referentials.

In many Indo-European referencing systems the speaker or the listener is distinguished but it is also possible to locate in terms of the third object or to allow the context to provide the meaning (Senft, 1997). For example, a ball in front of the tree may mean in front of the tree in alignment with the front of the house, not between the speaker and the tree. Static configurations may use the way one faces but a dynamic configuration may be more about alignment or parallel relationships. Furthermore, metaphoric and extended use of words may be linked by visuospatial reasoning (Lakoff, 1987). For example, "over" is used in a number of ways associated with position and action on a hill or other object. Such words and oppositional concepts such as "here" and "there" are very much determined by sociocultural experiences. English also changes when "it is cold here" is reported as "it is cold there" (Ehrich, 1991, cited in Senft, 1997). Finally the words may be symbolic. "Over the top" refers to a person's exaggerated expression, and "behind" and "front" may be used for denoting people's status in different ways in different languages.

Some words have an emphatic purpose (like the *su* in Turkish, Ozyürek, 1998, cited in Senft, 2004b). Similar emphatics are evident in Papuan languages (Tupper, 2007). Such emphatics can indicate value or other meanings required to understand the reference to place. For example, the same or similar word is used in some Arabic dialects for "right", "south" (once flourishing Yemen) and "plenty".

Linguistic Diversity in Referencing Space

Position depends on frames of reference and the semantics of the language. In general, space is referred to by local and directional prepositions or postpositions (e.g. denoted in English by "at, on, in, behind, in front of"); locatives - local or place adverbs ("here, there"); dimensional or spatial adjectives ("high, low, wide"), demonstratives ("this, that"), static and dynamic motion verbs ("to stand, to come, to go, to bring, to take"); directionals (e.g., "to, into"); and presentatives ("there is") (Senft, 1997, p. 8). These are called deictic systems and there is a large variety of these systems across languages. In addition, languages have gestures such as pointing or raising the eyebrows to indicate position.

The number of terms used in any one language may vary. Senft (1997) presents an argument made by others that the more man-made spaces in a society, the smaller the size of the spatial deictic system. He gives as examples the fact that English has two terms ("here, there") but Yup'ik Eskimo has 30 terms, East Eskimo has 88 terms. This is partly attributed to the man-made function given to the object associated with the position. For example, "the key is in the door" or "the satellite is in space". The locative markers of a language impose an implicit classification on spatial configurations. Indo-European categories are topological relationships (e.g. proximity, inclusion, surface, contact), Euclidean notions, and functional notions concerning typical uses.

Prepositions or postpositions generally provide a connection that is obvious by the expected relationship between objects e.g. the book on the table. For this reason, prepositions are frequently minimal and may or may not impact on word order. The Alekano language (from an area in the Eastern Highlands, PNG) has up to 15 slots or positions for different types of words and relationships between words in their sentences (Tami, 2007) presenting a more complex way of connecting objects than that presented in English. The Wiradjuri language has one positional suffix for being "next to a person", another for "coming to a person" and another for "going away from a person". These suffixes are attached constantly to nouns, a feature which Codrington (an early contact linguist) (1885) noted by saying of Melanesians and Polynesians that they continually introduce adverbs of place and of direction such as "up and down, hither and hence, seaward and landward".

For the Anindilyakwa from Groote Island, northern Australia words like "below" would be used for a canoe on the sea but not on the land, and it was the same as the word used for "in" the shelter which was only over and not surrounding the person. Interestingly some words involving motion like "come" and "go" were the same and might require the context to determine the speaker's intention (Worsley, 1997). Many alternative perspectives have been presented by linguists in the Australasian region (see papers in Senft, 2004b; 1997).

One example of a deictical system that can be found in a number of Papuan and Austronesian languages includes words which refer to a place quite distant, ones that encode medial distance and ones that imply proximity with visibility impacting on choice of words. The expectation of distance varies between languages so the Wiradjuri have a word for "here", another for "there nearby", and another "there" referring to an object or

person across the visible space. Sometimes words vary with the use of gestures (e.g. Saliba, Milne Bay, PNG, from Margetts, 2004; Wiradjuri; Yolgnu). In other examples, words vary with addressor and/or addressee or a third person or object as reference point. The number of people involved may also modify the words to be used (e.g., Samoan, from Mosel, 2004).

Local landmarks and environmental features may be used to denote places and the position of objects. For the Iaai on Uvea island (New Caledonia), the word for "down" is used for "west" and "sea" since the mountain is in the east and the land slopes down to entry into the sea in the west. Hence, the word for "east", "land", and "up" also reflects the geography and ecology of the island (Ozanne-Rivierre, 2004). In contrast, compass points or cardinal points may be denoted from the way a person is facing (Harris, 1989). In addition, dimensional axes, usually in reference to the body are used but in some cases, the position of the axes can be moved which happens in Kilivila (Senft, 2004a). Such diversity merely hints at the diverse ways of thinking spatially.

A further consideration in discussing locating and communicating is the way in which a group might reach a decision about a place. In other words, it may not be a single or small number of words that locate or describe an object or person but it might be part of a larger discussion about the position or object. It is the discussion itself that can be significant to the speakers (cf. Salzmann, 2006 on disease in Mindanao, Philippines). Similar data were recalled in discussions on measurement of pigs, land, and food in exchanges in Papua New Guinea.

Direction and Travel

Some intricate ways of describing place and space for navigating in the Pacific Ocean provide important knowledge in understanding an ecocultural perspective on space. Uses of star charts in "wayfinding" occurred in the large Polynesian routes such as from Hawaii to Tahiti (Davis, 2010; Polynesian Voyaging Society, ~2003). Islands are out of sight and without a magnetic north compass, these sailors had sophisticated and skilled ways of travelling. Some sailors travelled thousands of kilometres. In the Carolinian Islands, the tilting of the head to 45° provides a kinaesthetic means of selecting the angle of inclination to view the star constellations and note which one is at the point of rising or setting (Worsley, 1997).

Children learn the star positions on a star compass of 32 points. Stones are used to learn the main positioners first and gradually the various games become more complex. These star positions vary over the course of the year as the earth is on a tilt. One game, island hopping, requires the correct order of names of islands on a particular star direction. The sailors also have sea roads that are taken regularly which take account of the swells and currents. Routes are combined and reversed in the games which have various nautical names. Some places on a sea road are given names of sea fauna or flora. For example, from Puluwat to Eauripie (University of Pennsylvania Museum, 1997) spots called whales 1 to 6 provide a day's sail to a spot directly south of an island. Dragging is a game naming sea places from an island that is not their own. The idea of a right-angle turn (breadfruit picker symbol) or zigzag route is developed.

Implications for Mathematics Education

There are several reasons to be aware of the ecocultural and sociolinguistic aspects of mathematics education. First, mathematics education acknowledges the cultural diversity

of mathematics. The brief discussion above illustrates how rich Indigenous cultures are in referencing position in space and how limited a Eurocentric view of mathematics can be. This richness assists students to appreciate that mathematics is created by man, it is developing and changing and has a purpose. Broadening students' views of how to reference position in space provides for further developments of systems for referencing. For example, topological approaches expanded views beyond the Cartesian coordinate system and brought huge advantages to the studies of mathematics and related sciences.

Second, it provides connectivity with culture and place. Connectivity is an important aspect of all quality teaching but especially in terms of the way we teach mathematics, not in a narrow disconnected way but in a way that is rich, purposeful, interesting, and related to people's lives. When we ask children to map their classroom or their route from home to school, we are connecting to the students. However, we should extend this idea to developing a sense of place. It should be about mapping where their cubby house is, why it is significant, why their environment is significant and thus establish the importance of valuing our land (Sobel, 2008).

A third purpose relates to a study of language in a multicultural world. Some of the aspects of language related to position are relatively simple to follow. In particular, the Australian Indigenous languages can be referred to in mathematics. For example, the postpositional choices for Wiradjuri are relatively easy to understand although we must be careful not to isolate the language from the culture.

By extension, an ecocultural perspective assists the teacher of Australian Indigenous students to have a greater appreciation of the cultural heritage of their students. In areas where Aboriginal English is prevalent, the underlying local language (e.g., Wiradjuri) will influence the expressions of the students. An understanding of these can assist in bridging from Aboriginal English to Standard Australian English. Positional language is one small aspect of this because it is generally embedded in the deixis of the language. When students struggle with English prepositions or add an intonation to a word or sentence, this may reflect the deictical structure of the local language.

It was Thornton and Verran-Watson's perception that the use of a reference point could be a link to school mathematics work on grids. The differences between the two forms of mathematics could be juxtaposed to assist with school learning. In school mathematics, the wind is named by the direction it blows from, whereas in Yolgnu it is where the wind blows **to**, just as the shadow is opposite the position of the sun. For the Yolgnu, the cusped sandbank and crocodile tail can be placed in any direction unlike the school way of placing north at the top of the vertical page.

Finally, a brief study of linguistics situated around mathematics can assist teachers with English as a first language to appreciate the complexity of learning for students from a diversity of language and cultural backgrounds other than English. School mathematical concepts are not merely expressed in English but the language brings certain connotations of culture which may or may not be evident to the student.

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