Teaching Mathematics in a Project-Based Learning Context: Initial Teacher Knowledge and Perceived Needs

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This paper reports on initial data from teachers at an Australian Year 9-12 secondary school which is attempting to implement a project-based learning model across the entire curriculum. The eight teachers had diverse prior teaching experiences, including work in primary schools, special education, Year 7-10 secondary schools, and technical and further education. None had studied mathematics at tertiary level and just one listed maths as a main teaching area. Results of an initial survey indicate that most had reasonable levels of personal mathematics competence, and could identify relevant mathematical concepts among the affordances of a particular scenario, but that they struggled to articulate how they would work with students to pursue any of the relevant mathematics in depth.

Innovative models of schooling such as those described as project-based or design-based learning (Doppelt, 2009) share an emphasis on learner autonomy and choice, intrinsic motivation (Lam, Cheng, & Ma, 2009; Quek, et al., 2007), and authentic assessment. In such environments, teachers have a largely facilitative role. Benefits have been identified in terms of student engagement with their learning (Lam, et al., 2009), as well as increased opportunities to develop students' abilities to think creatively and innovatively (Lee & Breitenberg, 2010) and to work independently (Doppelt, 2009). Nevertheless, many teachers and parents share a concern that students in these schools might be disadvantaged by not having access to the entire curriculum (Toolin, 2004). To be effective in these contexts teachers need to acquire pedagogical knowledge and skills in addition to those that they have honed in traditional contexts, and this presents difficulties for many (Toolin, 2004). The school that was the site of this study had adopted project-based learning across the entire curriculum under the auspices of Big Picture Education Australia (BPEA). The model of project-based learning used in these schools includes particular emphases, described in the sections that follow that impact the nature of knowledge required by teachers.

Quantitative Reasoning

The participants in the study were interested in how students' quantitative reasoning (QR) could effectively be developed in their context. Thornton and Hogan (2003), in their discussion of cross-curriculum numeracy, link the term quantitative reasoning (QR) to quantitative literacy as used by Steen (e.g., 2004). The concept is very like that of numeracy as defined by the Australian Association of Mathematics Teachers (1997, p. 15), that is, "To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life". The term has been adopted by BPEA to describe one of five major learning goals, three of which include the word reasoning. They define reasoning as "the process of forming conclusions, judgments, or inferences from facts or premises" (Down & Hogan, 2010, p. 63). This definition is consistent with the description of the Proficiency Strand of the same name in the *Australian Curriculum: Mathematics* (Australian Curriculum Assessment and Reporting Authority

In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia). Singapore: MERGA.

[ACARA], 2012), although ACARA describe reasoning in the context of mathematics in greater detail. Down and Hogan are careful to point out that the learning goals, including QR, are not content areas and cannot be substituted for traditional curriculum areas. They acknowledge, however, that the learning goals of BPEA cannot be achieved in the absence of content and, indeed, that important content should be learned in the context of each. The relevant content for QR is largely mathematics and hence, in this paper, QR and mathematics can be used essentially interchangeably.

Teacher Knowledge

In any teaching context teachers of mathematics need not only to understand the relevant mathematical content but also how students develop understanding of specific mathematical content and how that content can be represented and made accessible for students (i.e., pedagogical content knowledge (PCK)) (Ball, Thames, & Phelps, 2008; Chick, 2007). Following Beswick, Callingham and Watson (2011), in the construct of teacher knowledge we also include teachers' relevant beliefs, for example, about their role as teacher, the nature of mathematics, and how mathematics is best taught and learned. Confidence, in both the everyday uses of mathematics and to teach various aspects of the curriculum, is also considered part of teacher knowledge. Confidence to teach is related to content knowledge and PCK, with increasing confidence in relation to specific aspects of the mathematics curriculum associated with higher level of overall knowledge for teaching mathematics (Beswick et al., 2011).

In project-based learning contexts teachers also need to be able to identify the mathematics inherent in a range of contexts and applications, draw students' attention to that mathematics, and engage them with it in authentic ways. Furthermore, they must be able to identify and structure projects that present the necessary opportunities for mathematical learning (Lee & Breitenberg, 2010). In addition, in Big Picture schools teachers encourage students to pursue their own interests and, therefore, rather than providing projects, teachers must guide students as they explore projects of their own choosing (Down & Hogan, 2010). Our hypothesis is that the development of knowledge for mathematics teaching as documented by Beswick et al. (2011), and shown in the left column of Figure 1, might be a longer process involving one or more additional stages in the context of project-based learning, as well as knowledge specific to teaching mathematics in project-based contexts.

Specifically, developing confidence to teach mathematics in project-based contexts may be a longer process because most teachers in these contexts are transitioning from more traditional contexts and must come to terms with substantial differences in their role as teacher, which are central to their beliefs about themselves as teachers, as well as learning to teach mathematics (Beswick, 2007). In particular, they need to move from seeing themselves as content and curriculum experts responsible for assigning tasks and managing students' engagement with them, to reconceptualise their role as facilitators of individual students' explorations of ideas and contexts that are of interest to them. The development of relationships is central to the teacher's task, "to know students well and to provide the right measure of challenge and support for each student in each activity to promote academic and social growth" (Down & Hogan, 2010, p. 34). The transition to project-based learning thus requires an increase in teachers' knowledge regardless of the extent of their knowledge at that time and, as shown in right column of Figure 1, in the nature of knowledge required in accordance with the differing demands of teaching in project-based learning contexts compared to traditional contexts. Understanding the knowledge required is vital given the established importance of the teacher to the effectiveness of project-based learning (Quek, et al., 2007).

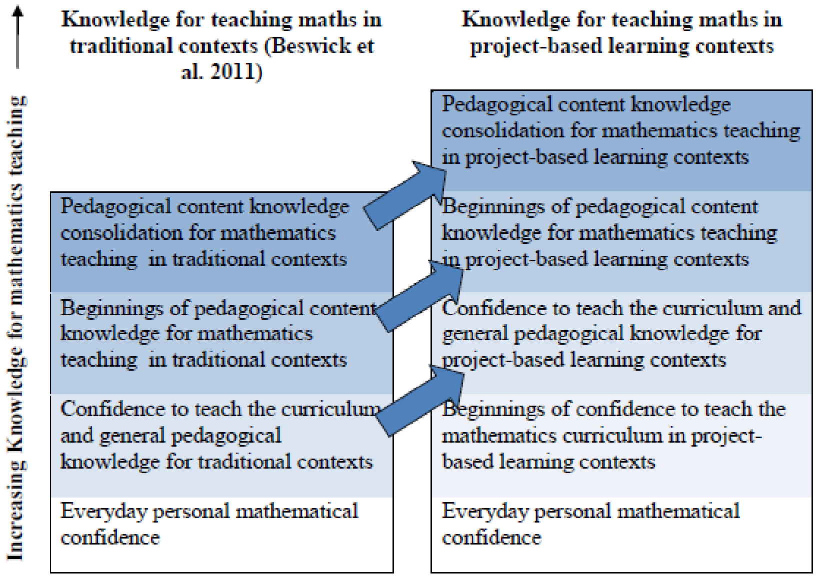

Figure 1. Hypothesised model of knowledge required in traditional and project-based contexts.

The Study

BPEA is the umbrella organisation for a group of 28 Australian schools in which project-based learning has been adopted in whole or part. The Year 9-12 school in which this study was conducted had been in operation for just one year prior to the start of this project. The school caters for the full range of student abilities including those intending to pursue vocational and academic paths after Year 12.

The study was designed to find out about the initial knowledge for mathematics teaching of teachers in the school at the start of a 1-year project investigating the development of their knowledge to teach mathematics in a project-based learning context. The project was initiated by the school principal and constructed as a genuine partnership in which, consistent with BPEA principles, all of the participants, including the researchers, are learners bringing their differing expertise to the problem of how QR can be best taught in a project-based context. The focus of this paper is the mathematics content knowledge, PCK, and confidence that the teachers brought to the project and the outcomes for which they hoped as a result of their participation.

Participants

Six of the school's eight teachers, the school principal, and another Department of Education leader with responsibility for innovative schools across the state completed the initial survey. For the purposes of this paper all are henceforth referred to as teachers. The teachers' backgrounds were diverse; six had bachelor degrees in (general) education, one had a Diploma of Teaching and Graduate Certificate in special education, and another was enrolled in a Bachelor of Education (Applied Learning) and had two trade certificates. None had tertiary mathematics qualifications. Four of the teachers reported having taught in primary schools for from 2 to 18 years, seven had secondary school teaching experience of 2 to 28 years. In addition, four reported having taught in other contexts including technical and further education, K-12 contexts, or special schools. One was in her first year of teaching. In terms of a main teaching area, English, literacy, and history were each named by two teachers, and special education, arts, science/maths, humanities, manual arts, drama, and sport were each named once. One teacher declined to nominate a main teaching area.

Instrument

In addition to asking about teachers' qualifications, teaching experience, and main teaching area, the initial teacher survey comprised 15 Likert-type items, 2 multiple-choice items, and 9 open-response items. Many were taken or adapted from similar items used by Callingham and Beswick (2011). Of the Likert-type items, five asked teachers to indicate the extent of their agreement with statements about their personal use of quantitative reasoning and ten concerned their teaching of QR. The multiple-choice items concerned only mathematical content knowledge, and eight of the open-response items concerned both this and the teachers' PCK. Some open-response items required respondents first to solve a mathematical problem and then to suggest opportunities it afforded for QR or to critique it in terms of being the basis of a project in which QR could be developed. The final item asked teachers about what they hoped to gain from their participation in the project. The results presented here relate to the open-response items that related to the teachers' content knowledge, the item that most authentically assessed their PCK in relation to project-based mathematics teaching, and the items asking about their aspirations for the project's outcomes. The relevant items or parts thereof are shown in Table 1.

Table 1
Relevant Items from the Initial Teacher Survey (pictures and formatting removed)

Item no.	Item
1a	A student is telling you about her weekend. She says, "We went cycling. After 45 minutes we had only completed 15 km and we still had 25 km to go. Guess how long it took us." Work out a mathematical answer to the problem in any way that you like.
4	An upright 1-metre stick casts a shadow that is 60 centimetres long. At the same time, a flagpole casts a shadow that is 5.4 metres long. How high is the flagpole?
6	Below are two currency conversion graphs [graphs omitted]. How many Brunei dollars are equal in value to 50 British pounds?
7	Which one of the following contains a set of three fractions that are evenly spaced on a number line? Tick one box only. [Options:] (a) 3/6, 3/5, 3/4; (b) 3/4, 19/24, 5/6; (c) 4/5, 5/6, 7/8; (d) 3/4, 19/24, 7/8
8	I think of a number, multiply it by 5 and add 7 to get an answer of 52. If my number was *x*, what equation represents this? Tick one box only. [Options:] (a) $5(x + 7) = 52$; (b) $5x + 7 = 52$; (c) $7x + 5 = 52$; (d) $7(x +5) = 52$; (e) $x = 52 \times 5 + 7$
9	A [cross-shaped] skydiving target is made from 5 squares the same size, as shown [diagram omitted]. The perimeter of the cross is 48m. What area in square metres (m^2) is covered by the cross?
10a	Write, in as much detail as you can, about the QR opportunities that could arise from this photograph [photograph omitted].
10b	Choose one of the QR possibilities you have identified and describe how you might encourage / assist a student to engage with that idea.
11	What are the main things that you would like to get out of your participation in this project?

For space and copyright reasons diagrams, graphs and photographs have been omitted. The photograph that was part of Item 10 depicted motorcycles at various stages of approaching and leaving a ramp to land on a truck mounted platform. This question was designed to provide insights into the teachers' abilities to identify QR opportunities in a context that could be of interest to students and to explain how they might use it to develop QR (i.e. aspects of their PCK in a project-based learning context).

Procedure

The survey was administered towards the end of a one-and-one-half day professional learning workshop facilitated by two of the authors. The focus of the workshop was on identifying and raising teachers' awareness of contexts in which QR might be developed, finding resources to support their own and their students' learning of relevant mathematics, and building their confidence to identify QR opportunities and facilitate their students' exploration of them. Teachers completed the survey individually, using as much time as they wished, and were able to use calculators.

Results and Discussion

Mathematics Content Knowledge: Items 1a, 4, 6, 7, 8 and 9

No teacher answered all of these items correctly. One provided only correct answers but omitted Item 8. A further five provided just one incorrect answer, although three of these also declined to answer Item 8. One teacher provided three incorrect answers and another, four.

All but one teacher answered Item 1 correctly. The teacher who answered this item incorrectly provided a correct response only to Item 6; the reasoning employed to obtain the incorrect answers was unclear and involved various choices and/or manipulations of numbers in the items. Two other teachers also provided incorrect answers to Item 4. Both of these showed evidence of attempts to use proportional reasoning but, in one case, exhibited confusion about exactly which lengths were in proportion and, in the other, revealed an apparently unsuccessful attempt to set up the standard proportions. The first of these obtained the intuitively plausible result of 8.55m whereas the second arrived at 3.24m. One of two teachers who answered Item 6 incorrectly did not provide an explanation. The other correctly identified the proportional relationships between British pounds and Australian dollars, and between Brunei and Australian dollars but appeared to have manipulated these rather than using the graphs to solve the problem. Seven teachers made the correct choice for Item 7 and of the four who attempted Item 8, three were correct. Item 9 was answered correctly by two teachers with a further two showing correct working ending with an incorrect calculation of 16×5. Another provided sound reasoning but included the statement that $4 \times 4 = 12$, while another correctly calculated the area of a single square but did not multiply by the number of squares. Two of the teachers provided no evidence of understanding the relevant concepts (perimeter and area). One of these added the statement, "I need a formula in front of me to work from."

Most of the teachers demonstrated understanding of mathematical ideas related to speed and time (Item 1), proportional reasoning (Items 1, 4, 6 and 7), reading graphs (Item 6), equivalent fractions (Item 7) and area and perimeter (Item 9). However, only one (whose only error was stating $16 \times 5 = 70$) was consistently successful with the mathematical items even though they required less sophisticated mathematics than some of their students would be studying. Some of the approaches used, such as manipulation of numbers in the problem, reflect those identified by Muir, Beswick and Williamson (2008) among Year 6 problem solvers termed "naive." Consistent with literature described by Mewborn (2001), most of the teachers lacked conceptual understanding of one or more topics, including area and perimeter, and aspects of proportional reasoning. Many of these teachers appeared to have been taught mathematics in a procedural way resulting in a reliance on ill-remembered procedures. The fact that four of the eight did not attempt the algebra question (Item 8) is also instructive. This was the only question that was not attempted by all teachers and therefore appears to have been judged the most difficult by these teachers.

Pedagogical Content Knowledge: Item 10

The teachers' responses to Item 10 are shown in full in Table 2.

Table 2
Responses to Item 10 (motor bike picture)

Teacher	Response to Item 10a	Response to Item 10b
1	Height, distance, length, speed, cost blah, blah, blah [diagram of triangle with length and height labelled]	Too many to discuss!!!
2	Acceleration, speed needed to clear the jump; velocity and measurement, horizontal, vertical [arrows]; angle of ramp; weight of bike and rider; distance of run up needed	Plotting angles on a graph; Get them to walk up road and get weight of same type of motorbike - weigh themselves; get starting info. for equation needed.
3	Plotting trajectory and estimating landing. Estimating speed required; crash helmet strength? Why angle of jump doesn't equal path of bike, what other factor at play? Wind direction of flags and calculated speed of wind assist; Added weight of bike x height - is the mattress thick enough to cushion fall	Plot the trajectory and work with them - to find other examples of little cars and jumps
4	Where is this? Why are they doing it? What is your prediction of what is happening? Trajectory, curves, speed, power of bikes? Distances; 2 stroke motors, 4 strokes	1. What makes motor bikes go faster? 2. How much are motor bikes - new? Second hand? 3. If you were buying a second hand bike what would you need to consider? Could go on and on.
5	Degree; if you alter the degree of the ramp will the bikes land safely on the truck still? Weight of riders/bike, does weight of riders and bike affect the height the bikes need (how much air can they achieve?); how much money does a bike cost? How much is it to maintain? How many litres is the tank? How much does fuel cost per litre?	Fuel tank capacity and cost of fuel e.g., 45 litres x $1.36 per L =; compare different fuel tank sizes and the types of fuel each motorbike uses - unleaded or diesel? What would be the most efficient and cheapest fuel? Money; duration fuel last for - TIME (graph and table results)
6	Bike speed, weight, height, angles	HARD!!! Speed and angle and what happens. No idea how to do this.
7	Angle, velocity, speed up and down, distance travelled, wind factor; What speed would you have to reach to be successful? What time would each rider need to wait before going? Wheel rotation, petrol consumption	Speed of leaving ramp compared to height, how could this be tested without having to actually ride? Can it be tested?
8	Speed required to get them safely to the landing ramp; speed vs. height to perform tricks; speed vs. height - trajectory; questions about power (cc's) of motorbike to accomplish.	Mass vs. power vs. speed to determine rate of fall; determine amount of force associated with stuffing up trick and landing on ground.

The responses to Item 10a show that the teachers had no difficulty identifying relevant quantitative aspects of the context depicted in the photo. However, responses 4, 5, and 7 also included less directly related aspects, including the location and reasons for the event, and costs involved in running a motor bike.

Although Item 10b asked teachers to choose just one possibility only responses 3, 6, and 7 did so. Responses 4 and 5 focused on tangential aspects involving money, running costs, and fuel tank capacity. Simulations were suggested in responses 3 and 7 but none provide clear evidence that the teachers had any knowledge of the mathematics involved in any of the QR opportunities that they identified.

In light of the difficulty posed by the elementary algebra item, the responses to the motor bike photo are unsurprising. The photo was chosen because of the opportunities we believed it afforded for exploring some of the more complex mathematics that many students in Years 9-12 are required to learn: for example quadratic relationships, rates of change, trigonometry, differential calculus, and vectors. The fact that none of the teachers mentioned any of these ideas is of concern. Nevertheless, the fact that most recognised the opportunity the context afforded to ask questions that could lead to these ideas means that there is a basis from which to work in helping the teachers to build the required mathematical knowledge.

Desired Outcomes of Participation in the Project: Item 11

Item 11 asked teachers about what they hoped to gain from the project. The desire most commonly mentioned (by 5 teachers) was for ideas for making QR engaging and interesting for their students, with four mentioning the importance of applied or real world contexts. Three expressed a desire to deepen their understanding of QR and three also mentioned improved confidence with respect to QR. Other aspects, mentioned once each, were links to the *Australian Curriculum: Mathematics* (ACARA, 2012), resources at an appropriate level to which students could be directed, appropriate language with which to talk to parents about QR, and deep understanding of students' learning of QR and appropriate pedagogy for individual students.

Conclusion

These initial data provide tentative support for the hypothesised model of knowledge development for teaching mathematics shown in Figure 1. Overall, the teachers in this study demonstrated a degree of mathematical knowledge and skill that warrants confidence in their personal use of mathematics. This is not, however, evidence of adequate knowledge to teach the subject (Beswick et al., 2011). Apart from one teacher, all had taught for no more than a single year in project-based contexts and confidence to teach mathematics (QR) in that context was lacking for most of the teachers (as evidenced by responses to Item 10b). Progress in mathematical understanding will need to take account of a parallel learning curve in relation to general pedagogy for project-based learning. It may, therefore, be more difficult and hence slower than in traditional contexts. Indeed, equipping teachers with the resources and dispositions to be able to learn new mathematics as required is likely to be a more realistic but no less challenging aim.

Completing the survey was a confronting but potentially valuable experience for some teachers. In addition to anecdotal evidence to this effect, one teacher, in response to Item 11, articulated a need to,

> ... really increase my capabilities when I do not have a formula to work from or an example to view first (I have not undertaken any maths or taught it above primary level for over 8 years) Tests make me feel overwhelmed and today I realised I have lost most of my understanding and I have identified the large emphasis I have placed on text books and learning via these.

None of the teachers in this project had any qualifications for teaching secondary mathematics and could not be expected to know or understand all of the relevant mathematics content. It is also not reasonable to expect that a 1-year project can equip teachers with all of the knowledge that they need. The teachers expressed a need to learn more about QR as well as to develop their pedagogical knowledge in relation to QR in the highly student-focussed project-based learning context. The challenge for this project is to find ways to equip them to build their own mathematics understanding while simultaneously acquiring the knowledge to teach it and attending to the entire education of the students in their care.

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