Projects, Puzzles and other Pedagogies: Working with Kids to Solve Local Problems

Margaret Marshman
University of the Sunshine Coast
<mmarshma@usc.edu.au>

Engaging and extending middle years students in mathematics is a continual challenge. One of the aims of the *Australian Curriculum: Mathematics* is to ensure that students are "confident, creative users and communicators of mathematics" (ACARA, 2011). Use of mathematical models and/or problems has been suggested as methods of achieving this aim, and mathematical investigations have been shown to improve student engagement. This paper looks to build on these ideas and combine them with the framework of Knowledge Producing Schools (KPS) (Bigum & Rowan, 2009) to determine whether, when students are working on a community based project of their choice, students become "confident, creative users and communicators of mathematics" (ACARA, 2011).

Introduction

One of the stated aims of the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2011): is that "mathematics aims to ensure that students are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens". Another is that students "recognise connections between the areas of mathematics and other disciplines."

This paper argues that one way to achieve those aims is through the use of mathematical problem solving and modelling, especially through the use of a variation of project-based learning. The proposed research, outlined below, draws on a highly successful approach to project-based learning in which students work on problems that matter to the local community. This is known as Knowledge Producing Schools (KPS) (Bigum, 2002, 2004; Bigum & Rowan 2009; Rowan & Bigum, 2010). Using this approach students are able to produce products and outcomes that are valued by, and have value in the local community. Support is provided by experts, at appropriate moments in the project. A significant outcome of this approach is that students, who were previously disengaged with their work in schools, become engaged. The focus of KPS is on student formulation and finding solutions to community based problems or issues. Arguably, real problems provide students with more opportunities for meaning making, which results in deeper understandings of the discipline (Schoenfeld, 1992; Romberg, 1994).

This paper is based on a current research project where a group of identified gifted and talented students at a regional primary and secondary school in Queensland are undertaking a community project on their local park. This project involves students redesigning the park to include a multicultural theme inclusive of art work that reflects the different cultures from the local community. It will require students to liaise with different community groups and the local council. The students want to include seating and BBQs to encourage a wide range of people to use the park as a meeting place and to make an actual difference in their local community. Despite the categorisation of gifted and talented, I am confident that there is a diversity of talent and expertise (Hong & Page, 2004).

Data collection for this research will include students' video documentation of their progress, their contributions to a collaborative Wiki and written journal reflections.

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Current thinking in mathematics education research is to get students working as mathematicians (Boaler, 2002). Burton (1998-1999) found that mathematicians collaboratively negotiate by exploring problems and actively seeking connections between the real world and mathematics. They also seek connections between the different areas of mathematics and value intuition, insight and the aesthetics of mathematical solutions. “The ability to look at a problem from different angles is crucial.” (Burton, 2001, p. 597).

This study has been broadly located and informed by the use of mathematical investigations, problem solving and mathematical modeling as well as problem-based learning and knowledge producing schools. These approaches and how to learn in mathematics, is considered below.

Mathematics Learning

Mathematics teachers want students to ‘do’ mathematics and ‘to make sense of mathematics’. The question is how this can be achieved while also trying to conform to the aims of ACARA. Schoenfeld (2002) and others maintain that problem solving is the goal of mathematics learning while communication, that is teachers being aware of the students’ thinking, is and always has been an important part. So how can we get our students “to make sense of a real-world use of mathematics, to get them involved in ‘problem formulation, problem solving, and mathematical reasoning?’” (Battista, 1994, p. 463). Problem solving “is not only a goal of learning mathematics but also a major means of doing so” (National Council of Teachers of Mathematics (NCTM), 2000, p. 4). Problem solving tasks also build the discourse in the classroom as students “conjecture, test, and build arguments about a conjecture’s validity … and ... are encouraged to explore, guess and even make errors “(Battista, 1994, p. 463).

Identity formation is an essential part of learning to be a mathematician, “because learning transforms who we are and what we can do, it is an experience of identity. It is not just an accumulation of skills and information, but a process of becoming …’ (Wenger, 1998, p. 215). To be a mathematician means belonging to the discourse community (Gee, 2011) so that one can engage in specific ways of thinking, acting and perceiving (Gee, 2011). Burton (1998-1999) identified being a mathematician as participating in collaborative exploration. Boaler, William and Zevenbergen, (2007) discuss the importance of helping secondary students develop an identity that includes being successful at mathematics.

It is also about engaging in a ‘dance of agency’ (Pickering (1995 p. 116) where mathematicians constantly move between their personal or human agency; their initial thoughts and ideas or their extension of established ideas, and the ‘agency of the discipline’ when it is necessary to follow standard mathematical procedures. For Boaler (2002) developing one’s human agency, by using and applying mathematics and participating in mathematical discourse, enables the learner to participate in the dance of agency, develop identities as mathematics learners and a “relationship with the discipline of mathematics” (Boaler, 2002, p. 10).

While problem solving, mathematicians use different representations; (66%) used predominantly visual thinking, (47%) conceptual thinking and (37%) used analytical thinking (Burton, 2001). Ernie, LeDocq, Serros, and Tong (2009) support this by arguing that signature pedagogies for mathematics should focus on “. . . teaching students to use multiple representations to reason about interesting and challenging real-world problems in a student-centred environment” (p. 264). Students need to be assisted to see connections between multiple representations of mathematical ideas, to persevere, to collaborate with their peers so as to co-construction understanding. They also need to be provided with a dynamic environment where students can learn by doing and experimenting.

Problem Solving

Schoenfeld (1992) observed high school and college students working with unfamiliar problems and noted that “roughly sixty percent of the solution attempts are of the ‘read, make a decision quickly, and pursue that direction come hell or high water’ variety” (p. 61). The students do not change tack even when the method is not working. On the other hand, mathematicians when solving an unfamiliar problem spend time making sense of the problem, then pursue leads, abandon attempts that are not getting anywhere and then solve the problem (Schoenfeld, 1992). To develop this ability Schoenfeld (1992) believes that students need to be taught the “metacognitive aspects of mathematical thinking” (p. 63). Metacognitive ability takes time to develop and includes “assessing one’s own knowledge, formulating a plan of attack, selecting strategies, and monitoring and evaluating progress” (Schoenfeld, 1985 cited in Yimer & Ellerton, 2006, p. 575) It is this metacognitive ability rather than mathematical knowledge that influences a student’s ability to problem solve (Carlson, 1999).

For students to successfully work with these tasks, they may initially need a significant amount of thought and/or mathematics before they can start mathematising. Romberg (1994) describes how to do mathematics and solve problems:

- initially students need to make sense of the task
- formulate the problem and decide on the important variables and relationships between variables;
- determine a model;
- substitute numbers into the variables to find a solution of a numerical model; and finally
- consider the validity of the solution – does it make sense? What effect will minor changes make? This may mean that it is necessary to go around either part or the whole cycle again.

Schoenfeld (1999, 1992) discusses the need for productive beliefs and mathematical dispositions to become good problem solvers. As Boaler (2002) argues, when a mathematician has a problem to solve for which she does not have the necessary knowledge, she will have the mathematical practices previously learnt and have productive relationships with the discipline, meaning she will try a variety of methods to solve the problem.

Problems in the real world are ‘ill-structured’ and so it is necessary to initially formulate them in a well-structured way (Heylighten, 1988). Problem formulation is commonly carried out by the teacher which leaves the student with the task of applying an appropriate algorithm that may be able to be calculated by machine. Taking problem formulation away from students removes a key opportunity for students to engage in mathematical sense making (Battista, 1994).

This can also add extra pressure on the teacher. For the teacher, working with problems or modelling to help students make sense of their world can be much harder than teaching factual information. A focus on pedagogy rather than content is a major shift that needs to occur. As Burkhardt (1988) explains, teachers:

- need to consider the different approaches taken by the students;
- need to decide when to support students with suggestions or questions that will help whilst still allowing the students to be responsible for finding their own solution and this is for each student or group of students in the class; and
- may be put in the potentially uncomfortable position of not knowing all the answers.

Mathematical Investigations

A mathematical investigation is real-life or life-like learning which is: open ended and provides opportunities for students to use multiple pathways to investigate the situation/problem. It may be framed as: a problem to be solved, a question to be answered, a significant task to be completed or an issue to be explored.

A good investigation has multiple entry points allowing students to start at their own level and to design their own pathway (or pathways) through it. Indeed, investigations allow students to undertake activities and thinking that resemble that of the practice of mathematicians, and so they can be viewed as authentic mathematical tasks (Burton, 1998-1999). In this way, investigations allow for the alignment of teaching, learning and assessment.

Boaler (2008) demonstrated that it is possible to engage students in deep mathematical learning using an investigative pedagogy, particularly those students who have been alienated by traditional approaches to mathematics education. Investigations are open-ended questions or problems that are set in a range of contexts. By using investigations that are directly related to the students' lives, mathematics becomes no longer 'useless'. To achieve this, teachers need to provide a socially supportive and intellectually challenging environment in the classroom (Fredricks, Blumenfeld & Paris, 2004) so that students are able to develop strong mathematical identities. When the task is relevant and meaningful most students enjoy a challenge (The Centre for Collaborative Education 2000 cited in Hilton & Hilton, 2005).

When students were asked to design and make a three dimensional model of a swimming pool using a minimum of two different shapes (Marshman & Grootenboer, 2012), many students chose to make creative designs including a heart shaped pool with sloping base, arrangements of three dimensional shapes and pools with spas and tiered seating. This was despite the fact that maintaining the same scale throughout for many was a challenge that meant it took a number of attempts to get their models correct, and the calculation of the area to be tiled was very detailed.

To encourage students to formulate problems the pedagogy needs to change to mathematical activities without one right answer. Increased engagement occurs when students have the opportunity for more ownership of the investigation by having to formulate the problem themselves rather than just being given the necessary algorithms to calculate answers. An example of this is reported by Marshman, Pendergast and Brimmer, (2011) where students were asked to design a middle years' area outside their classroom. At a professional development session prior to beginning the investigation, the teacher had expressed frustration with her class, describing them as "impossible". She felt that they were disengaged, of low ability, badly behaved and could not cope with the work. However, when the researchers visited, the students were excited about the task and could explain what they had to do, how they were going to approach the task and what they were going to put into their middle years' area.

Problem–Based Learning

"Originated in the 1960s at McMaster University Medical School, Canada, Problem Based Learning (PBL) is essentially a collaborative, constructivist, and contextualized learning and teaching approach that uses real-life problems to initiate, motivate and focus knowledge construction" (Ribeiro, 2011). It has been argued that engineering graduates are often unable to work effectively in multi-disciplinary teams and lack the skills to transfer basic knowledge to real-life situations, solve unusual problems in situations, and think critically and creatively (Ribeiro, 2011). To address this concern, engineering faculties use problem-learning to develop discipline specific knowledge as well as gaining these other skills. PBL engages the learner in authentic activities that simulate professional practice. The tutors support the learning through discourse and scaffold the students' self-directed study.

Modelling as Real World Problem Solving

Galbraith (2011a) argues that when students solve problems in their world they are developing and using mathematical modelling skills. He argues that this empowers students to attempt future problems. The process is messy and cyclical as indicated below.

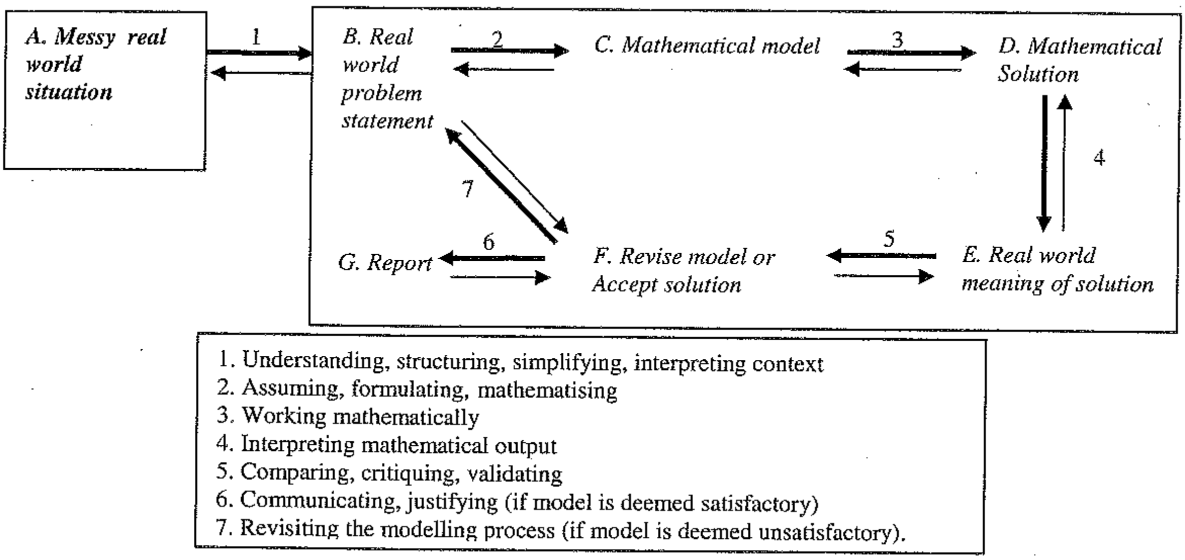

Figure 1. The modelling process (Stillman, Galbraith, Brown & Edwards, 2007)

When comparing "mathematical outcomes with reality" the students must return to the world where the problem is. The important message here is that the process cannot remain solely in the classroom. The biggest challenge students face is "learning how to identify problems, and to formulate related mathematical questions that can be addressed with existing mathematical knowledge" (Galbraith, 2011a).

Knowledge Producing Schools

Scardamalia and Bereiter (1999) argue that schools should be knowledge building organisations,

> Community knowledge building, by contrast, is aimed at producing something of value to the community – theories, explanations, problem formulations, interpretations, and so on, which become public property that is helpful in understanding the world and functioning intelligently in it. …within that group students are contributors to the common good. … The job of an elementary class that adopts a knowledge-building approach is to construct an understanding of the world as the students know it. (p. 276-7)

Students can function as mathematicians once they are able to participate in the social practices and mathematical discourses. Unlike professional research groups, who are attempting to solve new and specific problems, students in schools are attempting to solve the problems that will help them understand their world. These students when actively attempting to solve a 'knowledge problem' are "trying to negotiate a fit between their own ideas and information obtained from an authoritative source" (Scardamalia & Bereiter, 1999, p. 278). They are trying to construct their knowledge by connecting the new information to what they currently understand.

There is strong evidence that when students are engaged in project work that matters to them, excellent engagement is achieved and if timely access to expertise is provided then the quality of the work is cutting edge. In Knowledge Producing Schools students create products and/or productions for the local community who in turn provide 'experts' to work with the children (Bigum & Rowan, 2009).

Conclusion

The proposed research is radically different from all previous attempts to address a problem. It draws on a highly successful approach to project-based learning in which students work on problems that matter in the local community (Bigum, 2002, 2004; Bigum & Rowan 2009; Rowan & Bigum, 2010). With the support of experts at appropriate moments in the conduct of a project, students are able to produce products and outcomes that are valued by and have value in the local community. One of the significant outcomes of this approach is re-engagement with schooling by the students who do such work.

In this research, mathematics will *not* be an upfront focus for students. However, in undertaking this community project there will be instances where mathematical thinking will contribute. These opportunities will be identified by an expert mathematician who will mathematically support the students. Students will invite the mathematician to work with them in the same manner that other experts will be drawn upon to support the project.

While problem solving, mathematical investigations, problem-based learning and mathematical modelling have their place in mathematics classrooms, I argue that they do not support students to develop expertise in "formulating a mathematical problem from a messy real world context" which is unfamiliar (Galbraith, 2011b, p. 931). Arguably, much of the mathematical modelling carried out in schools skims over problem formulation. While some argue that this is accomplished when students interpret their results and evaluate their solution, I believe that this amounts to "stunt mathematics", that is, giving students kudos for doing the least demanding part of working on a real world problem.

Solving problems in the real world is more than crunching numbers that have been generated by a model developed by someone else. To initially formulate the problem one needs to have a comprehensive understanding of the context in which the problem is set. This may be achieved by visiting the site of the problem, acting out the problem or carefully reading materials associated with the problem (Galbraith, 2011b). Problem based learning is one way to achieve these goals but I argue that a key element in any problem-based approach is genuinely considering the interests and needs of a local community. This occurs in a Knowledge Producing School approach. This approach to working mathematically while resembling what happens in the world outside schooling is untested in the Australian school context and is unique to this research project.

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