# Diffusion of the Mathematics Practical Paradigm in the Teaching of Problem Solving: Theory and Praxis

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In this paper, we discuss the diffusion (of an innovation) and relate it to our attempt to spread our initial design of a mathematics practical paradigm in the teaching of problem solving.

Mathematics education researchers build innovations to improve mathematics teaching and learning (Roschelle, Tatar, Shechtman & Knudsen, 2008). For innovations to effect the intended changes at scale, they need to be supported by education theory as well as a theory of spreading the innovation. Yet, it was acknowledged that few mathematics education researchers are involved in research on the challenges, demands and outcomes of implementing innovations on a larger scale (Elmore, 1996).

This paper discusses issues related to the diffusion of our initial innovation for the teaching of mathematical problem solving in Singapore schools. This is part of the project MProSE (Mathematical Problem Solving for Everyone).

Briefly, the research project MProSE is an attempt to address the gap between what is enacted as teaching problem solving in Singapore mathematics schools and what is intended in the Singapore mathematics curriculum. Anecdotal evidence suggests that most teachers at the primary school levels have reduced problem solving to the teaching of heuristics, and that problem solving is usually downplayed in view of the national high-stake examinations at the Upper Secondary level (Toh, Quek, Leong, Dindyal & Tay, 2009). For details on the rationale and conceptualization of the earlier phases of the MProSE project, the reader may refer to Toh, Quek and Tay (2008) and will not be elaborated in this paper.

## Diffusion of Educational Innovations

In examining the model of diffusion, we draw heavily from Rogers (2003) who is widely acknowledged to hold a seminal position in the *theory of diffusion of innovations*. According to him, diffusion is the "process by which an innovation is communicated through certain channels over time among the members of a social system" (p. 5). Rogers further explained diffusion as one which is dictated by uncertainty reduction behaviour among potential adopters during the introduction of innovations. Innovative practices offer its potential adopters new ways of handling the problems, but the uncertainty as to whether it will be better than the existing method poses an obstacle to the adoption process. To overcome this, potential adopters must seek additional information, particularly from their peers. Rogers proposes five factors that influence the rate of adoption: observability, trialability, compatibility, complexity and relative advantage. In MProSE, observability refers to the degree in which the MProSE innovation is seen by the adopting schools as

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producing the expected results; trialability as the degree in which the schools can experiment with the innovation in a limited way; compatibility as the degree to which the schools see the innovation as being in line with their values, experiences, and needs; complexity or, simplicity and ease of use, as the degree to which the schools perceive the innovation as easy to understand and use; and lastly, relative advantage as better than their current teaching of mathematical problem solving

Diffusion occurs fast if only minimal amount of attention is needed for people to embrace the innovation (Larson & Dearing, 2008). However, the literature abounds with examples of many highly effective innovations which did not achieve widespread use. According to Fishman, Blumenfeld, and Krajcik (2004), many effective innovations in Learning Sciences have not spread widely into the classrooms. This is the problem space in which research on diffusion fills. MProSE is a part of this diffusion research that attempts to spread the mathematics practical paradigm design beyond the initial site of innovation.

## MProSE as Innovation

Rogers (2003) states that "[a]n innovation is an idea, practice, or object that is perceived as new by an individual ... If an idea seems new to an individual, it is an innovation." (p. 11) Since mathematical problem solving has been at the heart of the Singapore mathematics curriculum from the 1980s, in what way is MProSE an innovation? While MProSE does not claim to be innovative at the level of theory-generation (for we built on the theoretical cornerstones of Polya and Schoenfeld's work), the innovation is in the way that schools carry out the teaching of problem solving. The theory has been worked out, as Schoenfeld (2007) pointed out:

> That body of research—for details and summary, see Lester (1994) and Schoenfeld (1985, 1992)—was robust and has stood the test of time. It represented significant progress on issues of problem solving, but it also left some very important issues unresolved … The theory had been worked out; all that needed to be done was the (hard and unglamorous) work of following through in practical terms. (p. 539)

### *Making Mathematics Problem Solving 'Practical'*

MProSE introduces a practical paradigm for the teaching of mathematical problem solving, analogous to the practical in the teaching of science. In an attempt to 'make' the students follow through Polya's processes of problem solving[1], especially when they were clearly struggling with the problem, MProSE problem solving lessons use a worksheet similar to that used in science practical lessons. The students were instructed to treat the problem solving class as a mathematics 'practical' lesson, in the sense that they are asked to work on only one problem just as they would work on only one experiment in the science practical. For details on the complete set of materials for the specialised problem solving lessons, including the mathematics problems used, lesson plans and teacher guides, the reader may refer to Toh, Quek, Leong, Dindyal & Tay (2011a).

Our use of the word 'practical' has also a different sense: the meaning of practicability. An attempt to implement any novel curriculum—including one centring on problem solving—must take into account the complexities and realities of classroom practice (Ball, 2000; Lampert, 2001). Teachers need to balance different and sometimes competing goals

[1] Polya's model is highlighted in the Singapore mathematics curriculum.

of teaching (Wood, Cobb, & Yackel, 1995). Teaching problem solving is thus not seen as an isolated or only goal in this project but is analysed in the realistic context of other interacting and worthy instructional goals (such as keeping to time in teaching) as teachers carry out practice in the classroom (Leong, Dindyal, Toh, Quek, Tay & Lou, 2011). As such, the focus in MProSE involves not only careful framing of the roles of problem solving in the school mathematics curriculum and the ongoing development of teachers to prepare them for the enterprise but also the workability of a heavy emphasis on problem solving in actual classroom practice.

*Design Experiment*

MProSE uses design experiment as the overarching methodological approach (Brown, 1992; Collins, 1999; Doerr & Lesh, 2003; Middleton, Gorard, Taylor & Bannan-Ritland, 2006; Quek et al, 2011; Wood & Berry, 2003), which has its roots in the field of engineering. Design experiment is adopted by the education research community to address the demands of research in real-life school settings in all its complexity. It is an approach to "the development of theory and method based in the real-time, formative experience of implementing, assessing, and improving classroom practice, classroom research, and classroom learning" (Larson & Dearing, 2008, p. 512).

The methodology of design experiment argues for the application of multiple techniques to study a complex phenomenon in education. The envisaged outcome of MProSE is to produce a workable problem solving design that can be adapted to the setting of mainstream Singapore schools (Quek, Dindyal, Toh, Leong & Tay, 2011). In the process, like product refinements in engineering design experiments, education designs undergo iterative cycles of adjustment to fit local school conditions of implementation. This aspect of design evolution dovetails with a key principle in Rogers' *Diffusion of Innovations*: reinvention. The principle of reinvention refers to the degree to which the innovation evolves to meet the individual needs of demanding or risk-averse people in the population. As such, in the MProSE design experiment, school leaders and teachers have key roles in the design. The designer-researcher and teachers must collaborate in the entire process. We have identified two key sub-processes in the design experiment to develop the MProSE problem solving product: (1) *refining* in the light of research findings; and (2) *accommodation* to meet the realistic constraints faced in practice. We think it is important in design research to distinguish between the two sub-processes. Figure 1 shows the design-theory-practice troika underlying our design-experiment for MProSE. The initial design had to be adapted for the purpose of diffusion among the mainstream schools. The school has to make changes (for example, curriculum reorganisation and teacher capacity) to accommodate the problem solving design.

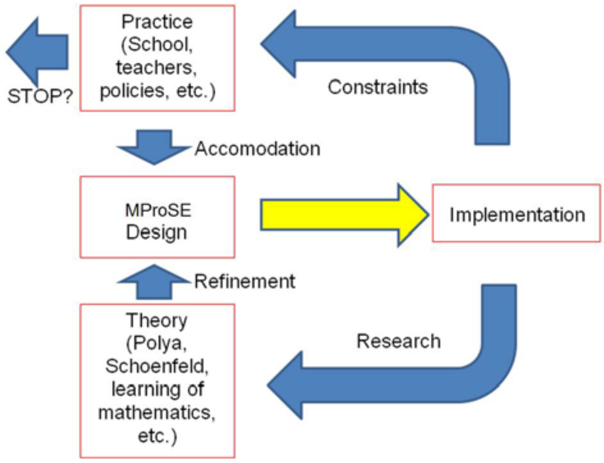


*Figure 1.* Design-theory-practice troika underlying MProSE.

## Diffusion of MProSE

MProSE first implemented the innovation in a school in Singapore which specialises in the teaching and learning of Mathematics and Science (hereafter we shall refer to as MProSE Research School). This school admits students it considers to be high achievers in mathematics or science. MProSE reasoned from a "best-case scenario" perspective that the test bed for the initialisation phase of an innovation should be in a school that would be most conducive for success.

After two years of study in the MProSE Research School, the school adopted the mathematics practical for all Year 8 (age 15 to 16) students. The relevant findings may be summarised as follows:

- the practical paradigm useful to guide them in solving mathematics problems (Toh, Quek, Leong, Dindyal & Tay, 2011b); and
- the students were able to demonstrate the use of Polya's model in solving mathematics problem . (Toh, Quek, Tay, Leong & Dindyal, 2011).

A problem solving seminar was organised with a view of inviting teachers from other mainstream schools to participate in the diffusion stage of the MProSE design experiment. The seminar disseminated the findings and shared lessons learnt from the MProSE Research School (http://math.nie.edu.sg/mprose/seminars.aspx). More than 200 teachers participated in the seminar. Generally, the feedback was positive. Teachers from thirteen schools expressed their initial interest to participate in the MProSE project. Subsequently, the MProSE team obtained written forms of commitment of participation from four schools, roughly spanning across the performance band. The original MProSE mathematics problems were modified to suit the needs of these schools.

## Focussing on First Stage of MProSE Diffusion: Adoption

According to Rogers (2003), the first stage in the diffusion process is adoption. Potential to address adoption we consider the five aspects as spelt out by Rogers.

*Relative Advantage*

Singapore teachers have been successful in preparing their students for the high-stake national examinations. They would be hesitant to adopt the MProSE design if they do not perceive its relative advantages. Through personal communication between the MProSE team and the teachers, we noted that teachers recognized the trend of increasing the number of non-routine problems occurring in the national examinations. They also acknowledged that the usual practice of "routinizing non-routine problems" might not be a sustainable mode of instruction to this end. They were looking for alternative approaches in the teaching problem solving. This is where they think MProSE could fill the gap.

*Compatibility*

The MProSE is compatibility with the existing Singapore mathematics curriculum. The heart of the Singapore mathematics curriculum is mathematical problem solving, and Polya's model is highlighted as a model to use. There is thus no conflict at the curricular level between MProSE design and the Singapore mathematics framework.

At the local level, to make the MProSE design more workable for mainstream schools, the original MProSE mathematics problems were modified to render them more compatible with the needs of the schools. For example, in the original MProSE design, Problem A was used to introduce the heuristics of "Using Suitable Numbers" and "Thinking of a Related Problem"

<u>Problem A</u>
Show that the integer $n$ always has the same last digit as its fifth power $n^5$.

While Problem A was suitable in the MProSE Research School, it was deemed not so appropriate in the mainstream schools, as they do not emphasize Number Theory in the Lower Secondary school curriculum. Keeping the same objective of introducing heuristics, Problem B was designed to replace Problem A.

<u>Problem B</u>
In 2009, Peter was given a pay rise of 5% and in 2010, he was also given a pay rise of 5%. Is his total pay rise 10% over the two years? Justify your answer.

Problem B was considered to be suitable for mainstream schools since the topic of Percentage is in the school mathematics curriculum.

*Complexity*

In acknowledging that teachers may find the MProSE design to be complex, we provided the necessary support in building teachers' capacity to understand the design as well as to implement it. We devote much of the professional development time to building the requisite content knowledge and the matching pedagogy. In particular, during the professional development courses, one of the MProSE researchers discussed explicitly the content knowledge required for the problems and modelled the pedagogy for teaching the problem solving processes. In addition, schools that needed further support in teacher implementation can tap upon the MProSE team's expertise in observing their teaching in the context of Lesson Studies. During these post-observation meetings with teachers, some of these complexities were discussed and unpacked.

*Trialability*

There was opportunity within the MProSE design for the collaborating schools to trial the design on a limited scale. For example, in one school, the Head of Department proposed the trialling of MProSE for a selective number of Year 7 classes. In the language of Rogers (2003), the early implementers could serve as early *adopters* of the innovation. Subsequently, the innovation may spread within the school to a stage where the later adopters from the remaining Year 7 classes could buy in and implement the design. This is in line with the vision of the Principal of the same school: that other teachers of the remaining Year 7 classes would implement the design after they have witnessed the tangible benefits of the initial design.

*Observability*

Observability refers to how visible the *results* of an innovation is. In the case of Singapore teachers, the *results* are often interpreted in terms of student achievement in examinations. More specifically, they may measure the usefulness of the innovation in terms of their student results in these examinations, after attending MProSE lessons. In keeping with the sub-process of accommodation within our design experiment, we recognize this need to consider examination results as an indicator of success of the innovation. In fact, the MProSE team rode on the affordances within the project to answer the following question:

> *Do the students who participate in the MProSE programme improve their performance in the mathematics achievement tests?*

The observability of such results in an educational innovation would aid in the adoption of our MProSE innovation.

## Summary

In this paper, we briefly reviewed the theoretical and practical issues regarding the attempt to spread the MProSE design innovation to other schools in the light of Rogers' (2003) theory of diffusion of innovations. In particular, we are at the stage of negotiating adoption with a number of schools which have shown initial interest. Using further Rogers' five aspects to predict adoptability it seems promising. But we anticipate the entire process of diffusion to the social system (Singapore schools) to be a long process fraught with challenges. We take comfort that this investment of time to cultivate ongoing collaborative researcher-practitioner partnership is one of the key enablers of diffusion of innovations. In fact, according to Lemke and Sabelli (2008), "[t]he development of effective partnerships takes 5 – 10 years" (p. 125). Hopefully this project could provide insight into the issues, challenges, and critical success factors in the diffusion of innovations to mainstream schools.

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