Exploring Student Reflective Practice during a Mathematical Modelling Challenge

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This paper seeks to compare the reflective writings of two cohorts of students (Year 4/5 and Year 8/9) participating in mathematical modelling challenges. Whilst the reflections of the younger cohort were results oriented, the older cohort's reflections spoke more to the affective domain, group processes, the use of technology and the acquisition of mathematical knowledge. This study supports the idea that with scaffolding middle years students can engage in reflective practice to develop mathematical modelling skills.

The ability to mathematically model real world problems so they can be solved is a skill that is required in many fields: All fields of science, engineering (Gainsberg, 2006), the social sciences, economics, computer science, politics and so forth (Lesh, 2007). While mathematical modelling has been typically reserved for high school mathematics courses (Galbraith, Blum, Booker & Huntley, 1998) other researchers are finding students in primary and middle school are capable of modelling real world situations beyond simple number and space concepts such as counts and measures (English, 2006).

Lesh and English (2005) describe a mathematics and modelling (M&M) approach which moves beyond mathematics as computation to mathematics as conceptualisation, description and explanation. Modelling also differs from 'word problems', which often only require one or two steps, in that it requires the mathematisation of real world situations in ways that are meaningful to the student (English, 2006). Lesh and Doerr (2003) describe mathematisation as a cyclical process of interpreting the problem, selecting relevant quantities, identifying operations that may lead to new quantities and creating meaningful representations. Teaching from an M&M perspective is in agreement with constructivist notions of learning through requiring students to construct their knowledge and understanding of a situation.

Advocates of the teaching of mathematical modelling (e.g., English & Watters, 2005) in the middle years state a number of benefits of this approach. Boaler (2001) compared students taught traditionally (completing exercises from a text) to students taught through problem solving and mathematical modelling. After three years students taught using the later approach were able to apply their knowledge in a greater variety of situations and also outperformed students instructed traditionally in the UK national examinations on both procedural and conceptual mathematical questions. Students at the later school were also more likely to see the applicability of mathematics to their lives and future jobs.

English (2006) examined the modelling abilities of 10-year-old students in their construction of a an answer to the question 'which is the best snack chip?'. Students worked collaboratively over six 40-45 minute lessons to answer this question. Transcripts of audio and video recordings and student created artefacts were interrogated for '...the ways in which the children interpreted and reinterpreted the problem, their approaches to selecting and categorising factors, their mathematisation processes as they quantified factors and transformed quantities, their attention to issues of sampling and representation, and their establishment of group roles (English, 2006; p. 309). English concluded that students in

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primary school are capable of engaging in mathematical modelling and do so at quite sophisticated levels. This is in agreement with English and Watter (2005) who found similar results in the mathematical modelling abilities of 8-year-old students. Benefits in addition to those found by Boaler (2001) were students were engaged in social interaction, reflection, critique and higher order thinking as they participated in the activity.

Another advantage in teaching from a M&M perspective is the ability to promote interdisciplinarity (English, 2009). Due to commonalities between mathematics and science Lesh and Zawojewski (2007) propose an approach to mathematical modelling which involves model construction, evaluation and revision. This cycle is similar to that employed by professional scientists and mathematicians (Romberg, Carpenter & Kwako, 2005). This paper seeks to analyse the first day reflections of two cohorts of middle years students who participated in two Modelling Challenges and explores their use of reflective practice to enhance the collaborative aspects of mathematical modelling.

The Context

This study compares two groups of students who participated in two different Modelling Challenges, the first located on the Gold Coast, Australia and the second located in Melbourne, Australia. In both cases the students in the study had the same mentor, that is, a mathematics teacher who scaffolded student participation in the completion of an inquiry based task. The students in the Gold Coast Challenge were in Years 8 and 9 (mean age = 13 years) while the students in the Melbourne Challenge were in Years 4 and 5 (mean age = 10 years). The Years 8/9 group (24 students) was a mixture of boys and girls with students from Australia and Singapore while the Years 4/5 group (15 students) consisted entirely of boys from Australia. To aid readability the students in Years 8/9 will be referred to as the 'older students' and the Years 4/5 students as the 'younger students'. Groups in both locations were randomly assigned to groups of four. A balance of age, school and gender (for the older students) was ensured for all groups. Groups in both cohorts were assigned the same equipment and task.

Students were asked to construct a rotor for a wind powered generator that would maximise electricity generation and construct a tower to support it. All groups were provided with a dynamo; cardboard for the blades of the rotor; balsa wood; glue and pins for the construction of the tower; and a fan for testing. All students engaged in an introductory discussion on wind turbines, the affordances and constraints of wind turbines and the horizontal or vertical orientation of the rotor blades. Students also participated in a teacher led discussion of the variables that might affect the voltage generated.

Students had access to the TI-Nspire graphics calculators and voltage probes. These were used to measure, tabulate and analyse the dependent variable of voltage. Some of the older students had previous experience with the graphics calculators whilst none of the younger students had any previous experience with these calculators. However, after the mentor conducted a 30 minute session with the younger students they had sufficient working knowledge of the calculator to collect, tabulate and graph data.

Students were asked to present a proposal using a poster presentation with the aim of convincing the executives of General Electric (GE) to adopt their wind turbine design in the next wind generator. Students systematically manipulated and recorded the variables of blade area, blade shape, the number of blades and blade pitch to maximise electrical output of the generator. Students were asked to record the changes to the variables and to present them as part of their poster presentation.

Each challenge was conducted over two days with presentations at the end of the second day. After the initial introduction session where the context was introduced and elaborated by the mentor, the groups of students mostly worked independently asking questions to clarify their understanding of the task or the use of the technology. At the conclusion of the first day students were asked to reflect on their progress during that day by responding to the following questions:

1. What did you like about today?
2. What are you going to do differently tomorrow?
3. Did your group work as effectively as you would have liked? Why or why not?
4. If you could change one thing, what would it be?

In this study we analyse the students' written reflections that were provided to the mentor at the end of the first day of the Challenge. While much of the literature on reflective practice within mathematics focuses on pre-service and in-service teachers (e.g., Graham & Phelps, 2003) many of the precepts can be applied to the students learning of mathematical modelling. For example, Whitton et al., (2004) conceptualises reflective practice as three steps involving (a) Direct experience; (b) Analysis of beliefs, value or knowledge; and (c) Consideration of the options as a function of the analysis. These three steps correlate with the questions asked of students. The purpose of this study is to explore middle years students' reflections on their practice during the Challenge.

The Data

A thematic analysis was conducted using a grounded theory approach (see Table 1).

Table 1
Identified Themes in Students' Written Responses to Each Question

Question	Themes
What did you like about today?	Affective (Fun, challenging/interesting, creative)
	Working in teams
	Technology
	Learning new concepts
	Results focus
What would you do differently tomorrow?	Time management (individual and team)
	Results focus
	Thoughtful inclusion of mathematics and analysis of data
Did your group work as effectively as you had hoped?	Work effectively
	Team co-operation
	Efficiency
	Collaboration
	Communication
If you could change one thing about today what would it be?	Time management
	Data collection
	Output focus
	Affective
	Group involvement

The comments of the older students were categorised first to yield a greater variety of themes as there were more older students. Students' responses could be coded into more than one theme. A Year 8/9 student responded with the following reflection to the question 'what did you like about today?

> "I got to meet new friends and we managed to work together in solving the math modelling task. I learnt how to use the graphing calculator to collect the voltage and also learnt more about wind turbines."

This reflection was coded into the themes of *Working in Teams, Using Technology* and *Learning New Concepts*. In some instances extra information is provided to qualify the reflection provided by the student. Table 2 shows the coding for question 1.

Table 2
Coding for Question 1, 'What did you like about today?'

Theme	Older	Younger
Affective	10	0
Working in Teams	13	4
Technology (Calculations and Collecting Voltage data, calculating area)	5	2
Learning New Concepts (learning about wind turbines, optimisation)	10	3
Results Focus (designing and constructing Rotors, Tower using the generator, planning the poster)	12	11

An analysis of Table 2 reveals distinct differences in the respective foci of the two cohorts. The older students' written reflections were spread across the themes of *Affective* (fun, challenging/interesting and creative), *Working in Teams*, *Learning New Concepts* and a *Results Focus* (designing and constructing the rotors and tower for the generator, planning for the poster), whilst coding of the younger students responses revealed a *Results Focus* as being a priority. Of lesser importance to the younger students were the themes of *Working in Teams*, *Learning New Concepts* and *Technology*.

Some of the differences may be attributed to the maturity level of the students, and hence their level of cognitive development and also to the group composition of the older and younger students respectively. The younger students appear to be focused on engaging in the task and achieving results, they do not rate the theme *Working in Teams* with the same level of priority as the older students. It is interesting to note the absence of any reflections from the younger students that could be related to the *Affective theme*, a theme of considerable importance to the older students.

The coding for question 2, 'What would you do differently tomorrow?" is shown in Table 3. Both groups identified *Time Management* as a priority to be engaged with at the group level for the next day of the Challenge. The distribution of reflections though was spread between *Time Management* for the *Individual* and *Group Time management*. During the introduction phase of the challenge the mentor did provide a quasi-timeline for both groups of students but left the ultimate timing of the various activities up to the individual group. As such, each group decided on the plan of attack and the length of time spent developing the rotor, the number of iterations of the modelling process and the variables they would investigate. The time and work pattern devoted to the analysis of the data from each iteration, building the tower and the development of the poster presentations were also determined by each group.

Table 3
Coding for Question 2, 'What would you do differently tomorrow?'

Theme	Older	Younger
Time Management		
Individual (Work Harder to meet objectives/Plan actions /not waste time)	6	3
Team (Leadership)	6	7
Results Focus		
Make the Tower	4	1
Make the poster	7	
Company name and logo	1	1
Work on Rotor	1	
Thoughtful inclusion of Mathematics and analysis of data	3	2

As can be seen in Table 3, the older students identified the theme *Results Focus* as a priority for the second day of the challenge, interestingly for the younger students, while this theme was a priority for the first day of the Challenge, it was not seen as a priority for the second day. Of interest too are the reflections that indicate the need for students to be thoughtful about the data collected from the rotor and about the inclusion of suitable mathematics to support their conclusions. Students in both cohorts, albeit small in number, indicated that this should be a priority for their group's time on the following day.

The mentor in the introduction sessions to each Challenge indicated the need for the students to justify their conclusions and to provide supporting mathematics as part of that justification. The limited number of students who questioned their data and the procedures implemented during that first day suggest that in the exploration of the rotor blade the students' focus in both groups was more on building the rotor and measuring the subsequent voltage generated rather than being thoughtful about the variables and how they might be varied in order to collect reliable data. This could be interpreted that the justification of their findings using mathematics played a secondary role to what they perceived to be more pressing issues.

An analysis of Question 3 (see Table 4) suggests that the majority of students in both cohorts thought that their group did work effectively together. It was interesting to note that while a number of younger students indicated that their group was working effectively they also indicated that there were disagreements of sufficient nature to be mentioned in their reflections. However, as one younger student wrote,

> "Yes my group is work[ing] as effectively as I would have hoped even though we did have our disagreements but what's a group without disagreements."

As the members were randomly assigned to the groups, adjusting for the same school and age, prior to that morning, the students did not know each other. However, the majority of students reflected that they were able to come together for the task and make contributions to create an effective group. Not all students in the older cohort felt this way.

Four older students indicated that their team did not function effectively. All these responses were from the same group. It is of interest that while these written reflections were categorised as not working as a team, in two instances the student reflections indicated that these students, felt that the group was effective even though the group was split in two, boys and girls. A number of reflections from the older students indicated the need for the group to work faster or that the group was wasting time.

Table 4
Coding for Question 3, 'Did your group work as effectively as you had hoped?'

Themes	Older	Younger
Work effectively (divided the tasks up)	5	9a
Complexity of the task inhibited the group members	2	
Team- Co-operation		
Cooperative	8	1
Need to be More Cooperative	1	
Efficiency		
Need to Work Faster	3	1
Wasting Time	1	
Collaboration in the team	3	1
Communication	2	3
Lack of Communication	1	1
Did not work as a team	4	1

[a] three groups indicated disagreements.

Table 5 presents the categorisation of students written reflections about Question 4.

Table 5:
Coding for Question 4, 'If you could change one thing about today what would it be?'

Themes	Older	Younger
Time Management (Plan more carefully from the start. More time on rotor design)	10	9
Data Collection (More accurate data collection. Complete Data collection)	3	1
Results Focus (Building the Tower)	2	2
Affective (Change Mindset/Attitude. Not my type of Maths. Challenging and Confusing)	4	0
Group Involvement	4	2

Question 4 'If you could change one thing today, what would it be?' (see Table 5) identified the themes of *Time management* and *Group Involvement* as being the two main issues students from both cohorts would like to change. This is consistent with the reflections recorded for Question 2. There are a small number of reflections in both of the cohorts that prioritised the theme *Results Focus*. Again, this is consistent with Question 1 where a high proportion of the reflections from both cohorts (50% for the older students and 80% for the younger students) identified this theme as being a feature of the first day about which they were satisfied. Once again some students seemed to be contemplating the accuracy of the data and the method used to collect the data. It is also interesting to note that a small number of student written reflections appear to show concern about the reliability of the data and the efficacy of the method used to collect data.

These reflections also indicated that some students were concerned about their attitude and mindset. However, these students do not seem to have recorded a related reflection in Question 2 indicating that they were prepared to change their mindset for the following day. For example, one of these students wrote:

> "If I could change something, I want to change my working attitude. I was a little distracted too. It is very common that anyone can get bored, thus I won't bear grudges with anyone. I just want to make friends with them; not enemies. So, please understand that I am NOT insulting Australians, I apologise if I have any offensive words here."

This student's written reflection for Question 2 was,

> "I am going to try to be even more pro-active tomorrow and try to get the group to work more efficiently because of the tight schedule that we have."

This student's reflection of 'what they would change about today', suggests they were concerned about their attitude, about being a little distracted and wishing to make friends. However, in their reflection about their actions for tomorrow they refer to being "pro-active… and getting the group to work more efficiently". It would appear that some of their perceived poor attitude might be attributed to the function of the group. It may also have a basis in the cultural differences that may exist in the way the members of the groups from the different countries come to know and do mathematics.

Conclusion

The two cohorts of students were very different in composition. One was comprised of Years 4 and 5 Australian boys, whilst the other was mixed gender groups of Year 8 and 9 students from two different countries. Yet surprisingly their reflections at the end of the first day on the same task share a number of common themes. Both cohorts of students valued working with their peers and had a results focus when faced with a modelling problem. For a small number of students the task was too challenging, but for most of the older students they were happy to accept the challenge and in the words of one older student,

> "I liked the interaction and communication among[st] our group. Everyone chipped in and we could get along well. I think the question/task we were asked to do was quite interesting."

While the novelty of the task was not something that was rated highly by the younger students they focused on the hands-on component which allowed them to try a design and then change that design to see the effect.

> "What I like[d] about today was meeting my group and making the blades."

Mathematical Modelling is about students engaging with a task, properly scaffolded to allow them to achieve the intended aims of the activity without removing any of the complexity, freedom of choices and richness of exploration so that students are able to utilise their mathematics in meaningful ways and to operate as a mathematician could. This study identifies that both cohorts of students, through engagement with this inquiry based task, wrote common reflections that suggested that they were focused on the collaborative aspect of their mathematics.

It can be concluded from these students' reflections not only was collaboration necessary to engage with the mathematical modelling task, but that this aspect of doing mathematics was valued by the students in both cohorts. Students' reflections suggest that engagement in group modelling tasks encourages students to value diversity of thinking without devaluing the quality of the mathematics. For example, a younger student in reflecting on what they would change about the day wrote,

> "I would change how we approached the task of the model, because I thought straight away, Oh' I can do it by myself and I already had it planned in my head, but I didn't consider my group, but my groups ideas turned out great."

This paper provides some insights into how engaging students in reflective practices such as providing written responses to open questions about their doing of mathematics can be used to inform the collaborative learning and teaching of mathematics in the middle years of schooling.

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