Steps in Developing a Quality Whole Number Place Value Assessment for Years 3-6: Unmasking the “Experts”

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Upper primary school students’ difficulties with decimal place value are widely recognised due largely to the availability of assessment tools that accurately identify students’ misconceptions in this domain. Yet, many students’ decimal difficulties seem to stem from a lack of understanding of whole number place value for which there are few, if any, comprehensive assessment tools at the upper primary school level. The inevitable result of this is that teachers tend to assume upper primary students have a deeper understanding of place value than is often the case and limit their teaching to fairly routine tasks that do not expose student misconceptions. This paper describes the derivation of a research-based, multi-dimensional Hypothetical Learning Trajectory for whole number place value. It explains the process involved in identifying the key aspects of place value and includes indicative examples of the assessment items used to evaluate this.

The Problem

As a teacher in Victorian Catholic Primary Schools for over seven years, I have experienced many facets of teaching place value. However, while teaching in the upper year levels (Years 3-6) I was constantly dismayed by students’ lack of whole number place value knowledge.

Research documents the difficulties students have grasping decimal place value (Baturo, 1998; Irwin, 1996; Steinle & Stacey, 2003) and provides teachers with assessment tools to accurately identify their misconceptions. While these tools are undoubtedly valuable for this purpose, Irwin (1996), Baturo (1998) and Moloney & Stacey (1997) suggest that many students weak grasp of decimal fractions can be attributed to their shaky foundation in whole number place value.

As a consequence, I set about looking for an appropriate assessment tool to measure students’ level of whole number place value understanding. It quickly became apparent that there was a lack of comprehensive assessments addressing this area for upper primary school students. This exposed the need for a quality place value assessment tool to assist teachers to effectively monitor and scaffold their students’ whole number place value development. It was from these informal observations that the impetus for my research developed.

Whole Number Place Value Research- Unmasking the ‘Experts’

An understanding of place value underpins our capacity to work flexibly within the English written and spoken number system. Yet, despite the unchanging and recursive nature of our base-ten system, it seems some students never manage to fully unravel the hidden code that underlies place value, failing to become true “*place value experts*”. The complexities associated with place value understanding for students have been well documented (Cobb & Wheatley, 1988; Ellemor-Collins & Wright, 2009; Ross, 1989; Steffe, 1992; Thomas, 2004).

Essentially, place value refers to the conventions we use to read, say and record numbers. To be a true *place value expert*, one must comprehend all aspects of our numeration system. This system uses a positional base 10 structure, whereby the value of

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each digit is indicated by its relative position in the numeral. For example, in 3456, the position of the digit 3 indicates a count of *thousands*. In contrast, the spoken system (beyond hundreds) involves using the number word followed by the value of that number. For example, 3456 is read as *three* thousand, *four* hundred and fifty-six, therefore the "thousand" names the value of the digit three (Fuson & Briars, 1990). Perhaps one of the most important, yet least understood characteristics of our system is the "10 of these is 1 of those" concept. This relates to the "infinitely extendable" (Thomas, 2004, p. 305) concept that each place value unit is ten times the value of the unit to its right (Fuson, 1990). This is known as the "recursive multiplicative structure" (Thomas, 2004, p. 311) of the base 10 numeration system.

Much research has been conducted into the teaching and learning of place value in primary schools. This research, for the most part, has been associated with the acquisition of two-digit place value knowledge in young students (6-8 years old) (Baroody, 1990; Cobb & Wheatley, 1988; Fuson & Briars, 1990; Ross, 1989; Steffe, 1992). Studies have stemmed from observations of students attempting to make the conceptual leap from a unitary view of number where individual items are counted, to the recognition and understanding of abstract composite units (Steffe, 1992). This conceptual jump is considered to underpin place value knowledge. Yet many researchers seem to incorrectly assume that students who understand two-digit numbers can automatically apply this knowledge to the entire number system. This is disputed by Thomas (2004) and Sinclair, Garin & Tieche-Christinat (1992) who document the significant difficulties students have in comprehending and applying the recursive multiplicative structure of the number system beyond two-digits.

It is widely recognised that multiplicative thinking poses major difficulties for students, with fewer than 10% of Year 4-8 students able to work flexibly with large whole numbers and decimal fractions (Siemon, Breed, Dole, Izard, & Virgona, 2006). Yet a deep understanding of the place value system is intrinsically linked to multiplicative thinking (Thomas, 2004). Irwin's (1996) work with decimals supports this idea suggesting that "complete understanding (of decimal fractions) requires multiplicative thinking" (p. 418). Thomas' (2004) provides further evidence of the difficultly students experience, stating that even by Year 6 very few students in his study were able to generalise the multiplicative structure of the number system. This considered, the difficulty students have understanding the multiplicative numeration system beyond two-digits may be underestimated by educators.

Place value is a undoubtedly a difficult concept for students to grasp. Ross (1989) describes how many students in Year 4 and 5 have serious place value misconceptions, but "appear to understand more than they actually do"(p. 50) and Sinclair, et al., (1992) note that often students' explanations of place value are "trivial or glib and not representative of their knowledge" (p. 194). Ross (1989) explains the gaps in students' place value knowledge may be a result of teachers using routine instructional tasks and standardised tests which fail to expose students' place value misconceptions. Thus assumptions that a student is a *place value expert* may be inaccurate.

As Ross (1989) suggests, the incorrect identification of *place value experts* may be a result of the type of assessment items teachers typically rely on to measure whole number place value. One of the most widely used place value assessment tools in Australia is the *Mathematics Online Interview (MOI) (Department of Education & Early Childhood Development, 2011b).* This assessment is designed for use with "students in the first five years of school" (Department of Education & Early Childhood Development, 2011b, p. 3) . However, in the absence of a whole number place value assessment tool that specifically caters for students in year 3-6, teachers may assume that students who reach the ceiling of the MOI place value section to be *place value experts* requiring little further instruction. It is not my intention to criticise the MOI. It is a completely valid assessment tool for the cohort of students it is designed for. I merely wish to point out that an assessment tool is desperately needed to measure upper primary students' understanding of the whole number place value system, particularly the multiplicative nature of this system.

It is clear that developing a deep understanding of whole number place value is of utmost importance in mathematics education. Studies have suggested the possible implications of poor place value knowledge not only include difficulty in understanding decimal place value (Irwin, 1996), but also difficulty in achieving relational understanding in algorithms (Fuson, 1990), and general problems in developing number sense and accessing mathematics at the secondary school level (Seah & Booker, 2005). As such, the need for teachers to be able to accurately identify true *place value experts* is fundamental to improving place value teaching and learning in the upper years of schooling.

Current Study

The methodology of this study has been significantly influenced by the *Scaffolding Numeracy in the Middle Years* (SNMY) (Siemon, et al., 2006) research project that used a Hypothetical Learning Trajectory (HLT) (Simon, 1995) to develop a research based Learning Assessment Framework for Multiplicative Thinking (LAF). This project will aim to do the same for Whole Number Place Value. In contrast to the SNMY project, this research is of a much smaller scale and will use an embedded single-case design (Yin, 2009). The major aim of the project is to produce a comprehensive research-based whole number Place Value Assessment Tool (PVAT) for use in Year 3-6 classrooms across Australia.

The first phase of this research included a comprehensive review of the literature associated with place value. From this review, a HLT for whole number place value was created. The HLT was then used as a guide to source and design assessment items to create the PVAT. Once initial investigations related to PVAT item readability and difficulty were completed, a pilot case study of the PVAT took place with all year 3-6 students (approximately 280 students) at a large Catholic Primary School in metropolitan Melbourne, Victoria (School A). The results from this pilot are currently being analysed using Rasch analysis (Adams & Khoo, 1996), which is a probabilistic model that measures item difficulty and student achievement on the same logit scale(Siemon, et al., 2006). This analysis will provide invaluable insights into the strength and behaviour of each item and the test as a whole

The final phase of this research will see the PVAT trialed with all Year 3-6 students (approximately 290 students) at another large Catholic Primary School in metropolitan Melbourne (School B) during May and December 2012. A Rasch analysis will take place using this data and these results will be used to further validate the PVAT and the whole number place value HLT. At both schools, interviews will take place with 10% of the sample group in order to gain qualitative insights into the approach students take when completing the PVAT. The theoretical paradigm that underpins this research study, is informed by Cobb's (1995) emergent perspective that acknowledges the social and psychological elements at play in the construction of meaning.

Derivation of a Multi-dimensional HLT

As defined by Simon (1995), a Hypothetical Learning Trajectory (HLT) is the "the teacher's prediction as to the path by which learning might proceed" (p. 135). It is based on the idea that most student learning follows a similar path and is described as "hypothetical" because it is only a "expected tendency"(Simon, 1995, p. 135). In the years since Simon introduced the term HLT, researchers have interpreted and developed this construct in many different ways (Clements & Samara, 2004). Teaching Sequences, Learning and Assessment Frameworks and Learning Progressions have been created, and despite differing in their design and theoretical underpinning, all have the common purpose of attempting to map the path students take when coming to understand a particular domain.

In Australia major learning trajectories are exemplified by the identification of Growth Points across different areas of mathematics (Department of Education & Early Childhood Development, 2011a) and the LAF for multiplicative thinking (Siemon, et al., 2006). These frameworks assist teachers to be more informed about finding student's point of need, and better able to scaffold their mathematical learning (Clark, 2001).

Clements and Samara (2004) describe the three aspects that they consider contribute to a complete HLT- "the learning goal, developmental progressions of thinking and learning, and sequence of instructional tasks"(p. 84). The first two aspects were used to form the basis of the whole number place value HLT described in this paper.

Firstly, a learning goal for the whole number place value HLT was identified. Simon and Tzur (2004) explain a learning goal is "based on knowledge of the students' current mathematical knowledge"(p. 96). The learning goal for this HLT was to identify the broad sequence of concepts and strategies needed for students from Year 3-6 come to a full understanding of the whole number place value system.

The next aspect of creating the HLT was to investigate the "developmental progressions of thinking and learning"(Clements & Samara, 2004, p. 84) for place value. This process began with a comprehensive review of research literature associated with whole number place value. Many assessment instruments were also reviewed in order to gain an indication of the number and type of items assessing place value, particularly those involving numbers beyond three-digits.

A synthesis of the literature saw the emergence of seven major aspects which appeared to be integral to the development of whole number place value understanding. These aspects were: *Count, Make/Represent, Name/Record, Rename, Compare/Order, Calculate and Estimate*. As these aspects were quite distinct, it was decided that individual HLTs would be created for each aspect.

It was suspected that each of the seven aspects not only related to the overall "big idea" of place value, but in some way linked to each other. The interplay between these aspects would be investigated in more detail using the results of the Rasch analysis later in the project. This information would then be used to create a multi-dimensional whole number place value HLT.

The following is a brief summary of the skills, thinking and literature incorporated into each aspect of the place value HLT (No developmental order implied):

<u>Count</u>: Counting forwards and backwards in place value parts (e.g., 45, 55, 65 is counting using the unit ten). Bridging forwards and backwards over place value segments (e.g., 995 and one more ten requires bridging forwards over hundreds to thousands). (Cobb & Wheatley, 1988; Jones et al., 1996; Steffe, 1992)

<u>Make/Represent</u>: Make, represent or identify the value of a number using a range of materials or models- these may be proportional, non-proportional, canonical and non-canonical. (Baroody, 1990; Jones, et al., 1996; Thomas, 2004)

<u>Name/Record:</u> Read and write a number in words and figures (e.g., 75 is written as 'seventy-five'). Identify the value of digits in a number (e.g. The value of 3 in 345 is 3 hundreds). Rounding numbers to the nearest place value part.(e.g., Round 2456 to the

nearest thousand). (Jones, et al., 1996; Ross, 1989)
Rename: Recognise and complete partitions and regrouping of numbers. (e.g., 1260 has 126 tens). (DES, 1979; Jones, et al., 1996; Resnick, 1983)
Compare/Order: Compare numbers to determine which is larger or smaller and place them in descending or ascending order(Bednarz & Janvier, 1982).
Calculate: Apply knowledge of place value when completing calculations (e.g., 45 by 10 is 45 tens) (Fuson, 1990; Wearne & Hiebert, 1994)
Estimate: Use knowledge of magnitude of numbers when estimating (e.g., estimate how many oranges fill a classroom 10?100?100 000?) (Siegler & Booth, 2004; Thomas, 2004)

Having identified the seven aspects, empirical evidence was used to order the ideas and strategies within each aspect from least to most difficult. Each aspect was broken into hypothetical "stages". The accuracy of this hierarchy will be tested using the results from the Rasch analysis. An abridged version of the HLT for the "Rename" aspect is included below in Table One.

Table One
Abridged Version of the Hypothetical Learning Trajectory for the 'Rename' Aspect.

Stage	Hypothetical Learning Trajectory for Rename (from least to most difficult)
1	May describe 10 ones is 1 ten and 10 hundreds is 1 thousand but they cannot rename accurately. Asked how many tens in 345, they may answer 4 (Sinclair, et al., 1992).
2	Can recognize and use equivalent representations like 2 hundreds=20 tens= 200 ones (Jones, et al., 1996).
3	Students are able to complete non-standard partitions and regrouping of numbers. (Resnick, 1983).
	Students may appear to use the multiplicative structure when multiplying and dividing numbers by 10, 100,1000, but may use rote learned rule e.g., adding a zero.
4	Students recognise when it is appropriate to split and combine numbers in place value parts (Jones, et al., 1996).
	Understand the exponential and reciprocal nature of number system (Baturo, 2000) .
5	Able to apply their base 10 knowledge to problems with other bases (DES, 1979).

Illustrative Examples of Items related to the HLT

Once the seven HLTs were created, these were used as a guide to select, modify and design a range of items to address that particular aspect in the PVAT. This was considered to be parallel to the "sequence of instructional tasks" (p. 84) which Clements and Samara (2004) describe as the third part of the development of a HLT. PVAT items were designed to be in the form of a paper and pen test as this was considered to be both appropriate for the year levels being tested (Year 3-6) and the most time effective assessment for teachers to administer. Items were a combination of multiple choice or short response questions with a mix of partial and full credit questions. At least one item was developed to address each stage of the HLTs. This was to ensure all parts of the HLT could be validated using the results from the PVAT Rasch analysis. Below are two illustrative examples of trial PVAT items which were designed to address Stage One and Two of the aspect "Rename" (See Figures 1 and 2 below).

Item 43b How many tens are in 387?	Item 21b 40 tens are the same as ___ ones?
Figure 1. Stage 1 Rename	*Figure 2.* Stage 2 Rename

The *Stage 1* item requires students to display their knowledge of renaming the place value parts of a three-digit number. An incorrect response of "8" may be indicative of a student who does not recognise that there are 30 tens in 3 hundreds so 38 tens altogether.

The *Stage 2* item requires students to recognise and use equivalent representations. A student who answers 400 recognises that 40 tens is the same as 400 ones.

Finally, all items were collated into the first form of the PVAT which was trialed at School A.

Where to Next?

To date, the trials of the PVAT have been successfully conducted at School A and a Rasch analysis of the data is currently taking place. This analysis will allow items to be validated or eliminated in preparation for the pilot form of the PVAT which will take place at School B during 2012. It is anticipated that the Rasch results will be used to investigate possible links between the seven aspects and form a multi-dimensional HLT for whole number. This will allow for a greater understanding of the complexities of each aspect of place value and provide teachers with a comprehensive whole number place value teaching sequence.

It is hoped that the PVAT will also allow teachers to more accurately determine Year 3-6 students' level of whole number place value understanding and as a result determine students' readiness to move onto decimal place value.

In conclusion, the PVAT will attempt to finally "unmask" students and provide teachers with the accurate diagnostic assessment information they require to challenge their students to truly become *place value experts*.

References

Adams, R. J., & Khoo, S. T. (1996). *QUEST-Version 2.1* The interactive Test Analysis System.

Baroody, A. (1990). How and when should place-value concepts and skills be taught? *Journal for Research in Mathematics Education, 21*(4), 281-286.

Baturo, A. (1998). The implication of multiplicative structure for students' understanding of decimal-number numeration. In F. Biddulph & K. Carr (Eds.), *People in Mathematics Education (Proceedings of the 20th Annual conference of the Mathematics Education Research Group of Australasia),* pp. 90-97. Rotorua, NZ: MERGA.

Baturo, A. (2000). Construction of a numeration model: A theoretical analysis. In J. B. A. Chapman (Eds.), *Mathematics Education Beyond 2000 (Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia*, pp. 95-103*)*,. Fremantle, WA: MERGA.

Bednarz, N., & Janvier, B. (1982). The understanding of numeration in primary school. *Educational studies in Mathematics*, *13*(1), 33-57.

Clark, D. (2001). Understanding, assessing and developing young children's mathematical thinking: Research as a powerful tool for professional growth. In J. Bobis, B. Perry & M. Mitchelmore (Eds.), *24th Annual Mathematics Education Reserch Group of Australasia Conference*, pp. 9-26. Sydney, Australia: MERGA.

Clements, J., & Samara, J. (2004). Learning Trajectories in Mathematics Education. *Mathematical Thinking and Learning, 6*(2), 81-87.

Cobb, P. (1995). Constructivist, emergent and sociocultural perspectives in the context of developmental research. In D. Towens (Eds.), *17th Annual Meeting North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 3-29). The Ohio State University, Columbus Ohio, USA: ERIC

Cobb, P., & Wheatley, G. (1988). Children's initial understandings of ten. *Focus on Learning Problems in Mathematics, 10*(3), 1-28.

Department of Education & Early Childhood Development. (2011a). Early Numeracy Research Project growth points Retrieved November 12 from: http://www.education.vic.gov.au/studentlearning/teachingresources/maths/enrp/enrplaf.htm#H3N10078

Department of Education & Early Childhood Development. (2011b). Mathematics Online Interview Booklet Retrieved 22 December from: http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/onlineinterviewbklet.pdf

DES. (1979). Mathematics 5-11: A Handbook of Suggestions *HMI Matters for Discussion 9*. London: HMSO.

Ellemor-Collins, D., & Wright, R. (2009). Developing conceptual place value: Instructional design for intensive intervention. In R. Hunter, B. Bicknell & T. Burgess (Eds.), *Crossing Divides (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia,* pp. 169-176*)*. Palmerston, NZ: MERGA.

Fuson, K. (1990). Issues in Place-Value and Multidigit Addition and Subtraction Learning and Teaching. *Journal for Research in Mathematics Education, 21*(4), 273-280.

Fuson, K., & Briars, D. (1990). Using a Base-Ten Blocks learning/Teaching approach for First and Second-Grade Place-Value and Multidigit additon and Subtraction. *Journal for Research in Mathematics Education, 21*(3), 180-206.

Irwin, K. (1996). Making Sense of Decimals. In J. Mulligan & M. Mitchelmore (Eds.), *Children's Number Learning* (pp. 243-257). Adelaide: Australian Asscociation of Mathematics Teachers.

Jones, G., Thornton, C., Putt, I., Hill, K., Mogill, T., Rich, B., & Van Zoest, L. (1996). Multidigit number sense: a framework for instruction and assessment. *Journal for Research in Mathematics Education, 27*(3), 310-336.

Moloney, K., & Stacey, K. (1997). Changes with age in students' conception of decimal notation. *Mathematics Education Research Journal, 9*(1), 25-38.

Resnick, L. B. (1983). A developmental theory of number understanding. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 109-151). New York: Academic Press.

Ross, S. H. (1989). Parts, wholes and place value: a developmental view. *The Arithmetic Teacher, 36*(6), 47-51.

Seah, R. T. K., & Booker, G. (2005). Lack of Numeration and Multiplication Conceptual Knowledge in Middle School Students: A Barrier to the Development of High School Mathematics? In B. Bartlett, F. Bryer & D. Roebuck (Eds.), *Stimulating the 'Action' as Participants in Participatory Research* (Vol. 3, pp. 86-98). Nathan, Qld: Griffith University.

Siegler, R., & Booth, J. (2004). Development of Numerical Estimation in Young Children. *Child Development, 74*(2), 428-444.

Siemon, D., Breed, M., Dole, S., Izard, J., & Virgona, J. (2006). *Scaffolding Numeracy in the Middle Years-Project Findings, Material and Resources*. Final Report Retrieved 23/11/11 from: www.eduweb.vic.gov.au/edulibrary/public

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114-145.

Simon, M. A., & Tzur, R. (2004). Explicating the Role of Mathematical Tasks in Conceptual Learning: An Elaboration of the Hypothetical Learning Trajectory. *Mathematical Thinking and Learning, 6*(2), 91-104.

Sinclair, A., Garin, A., & Tieche-Christinat, C. (1992). Constructing and Understanding of Place Value in Numerical Notation. *European Journal of Psychology of Education, VII*(3), 191-207.

Steffe, L. (1992). Schemes of action and operation involving composite units. *Learning and Individual Differences, 4*(259-309).

Steinle, V., & Stacey, K. (2003). Grade-related trends in the prevalence and persistence of decimal misconceptions. In N. A. Pateman, B. J. Dougherty & J. Zilliox (Eds.), *27th Conference of the International Group for the Psychology of Mathematics Education*, pp. 259-266. Honolulu: PME.

Thomas, N. (2004). The Development of Structure in the Number System. In M. J. Hoines & A. B. Fuglestad (Eds.), *28th Conference of the International Group for the Psychology of Mathematics Education*, (pp. 305-312). Bergen, Norway: Bergen University College Press.

Wearne, D., & Hiebert, J. (1994). Place Value and Addition and Subtraction. *The Arithmetic Teacher, 41*(5), 272-274.

Yin, R. (2009). *Case Study Research Design and Methods* (4th ed.). London: Sage.