Accelerating the Mathematics Learning of Low Socio-Economic Status Junior Secondary Students: An Early Report

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The *Accelerating the Mathematics Learning of Low Socio-Economic Status Junior Secondary Students* project aims to address the issues faced by very underperforming mathematics students as they enter high school. Its aim is to accelerate learning of mathematics through a vertical curriculum to enable students to access Year 10 mathematics subjects, thus improving life chances. This paper reports upon the theory underpinning this project and illustrates it with examples of the curriculum that has been designed to achieve acceleration.

An inability to use mathematics to effectively meet the general demands of life has serious impacts upon an individual's employment opportunities and life chances. Quantitative and anecdotal evidence suggest that many Australian students entering junior high school, especially in low socio-economic status (SES) areas, do not have the requisite level of numeracy to engage and continue in learning. The *Accelerating the Mathematics Learning of low Socio-Economic Status Junior Secondary Students* project, or more simply XLR8, will build theory to inform practices that scaffold the accelerated learning of mathematics by underperforming junior secondary students.[1] It is anticipated that a successful program of accelerated learning will advance underperforming students to levels normally associated with their year level in a compressed amount of time, and so prepare these students to successfully undertake Year 10 and post-compulsory mathematics study at elementary (and above) levels with the view to entering tertiary education and/or apprenticeships/traineeships.

The XLR8 project is only in its early stages; it began in late 2012, with the first classroom teaching of the XLR8 curriculum beginning in the first term of 2013. Achieving the goal of increased employment potential and life chances is complex. This paper focuses on the conceptual framework for an accelerated learning curriculum. This theoretical positioning is augmented with an overview of the project's research design and an example learning sequence taken from the XLR8 curriculum.

A Conceptual Framework for Acceleration

The project's conceptual framework brings together the theoretical lineage of cognitivist ideas (Cooper & Warren, 2011; Warren, 2008; Warren & Cooper, 2009) with the culturally-based ontological viewpoint of Matthews (Matthews, 2009; Matthews, Cooper, & Baturo, 2007) and a complementary pedagogical framework, with ideas related to professional learning that supports the development of teaching-learning trajectories

[1] The XLR8 project is an extension of an earlier project for Aboriginal and Torres Strait Islander students called Accelerated Indigenous Mathematics.

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(Baturo, Warren, & Cooper, 2004; Guskey, 2002). These are summarily discussed in the following paragraphs.

Structured Sequences

The longitudinal Early Algebraic Thinking Project (EATP) (Warren & Cooper, 2009) followed the development of algebraic thinking amongst primary-aged children. Conceptually, the project adopted a structural, cognitivist perspective, framed by such works as Sfard (1991), English and Halford (1995), and Hiebert and Carpenter (1992). Warren and Cooper described mathematical understanding to be of the connectedness of a learner's internal mental models (or mathematical ideas). In turn, the development of such a connected schema is via cognitive processes that determine the structural similarities and differences between mental models which in turn lead to the construction of more abstract mathematical ideas. Central to Warren and Cooper's work was the cognitive interplay between what they identified as models and representations. In their words, "models are ways of thinking about abstract concepts" and "representations are the various forms of the models" (Warren & Cooper, 2009, p. 78). To augment their conceptual framework, Warren and Cooper drew upon relevant theory regarding the use of representations in mathematics. This included Bruner's (1966) enactive-iconic-symbolic representation sequence, Dreyfus' (1991) sequencing of representation use and Duval's (1999) notions regarding the importance of the coordinated use of representations, including verbal language. While not explicitly identified, Payne and Rathmell's (1975) assertions regarding the significance of verbal language when coordinating the use of representations were also evident in Warren and Cooper's conceptual framework. Related to the use of representations, Warren and Cooper also drew upon Filloy and Sutherland's (1996) notions of translation (the use of increasingly abstract representations) and abstraction (the activity-based construction of higher level mathematical constructs), noting that without intentional teacher intervention often the construction of more abstract, or formal, mathematical ideas may not occur.

Warren and Cooper (2009) re-examined the EATP dataset and drew nine conjectures regarding the use of models and representations in relation to algebraic thinking. From these, six interrelated general hypotheses were made regarding the use of models and representations, which have been paraphrased as:

1. The processes leading to the construction of an abstract concept occur across models and representations and follow a structured sequence.
2. Effective models and representations highlight the mathematical concept to be learnt and are easily extended to include new components or to be applied to new situations.
3. An effective structured sequence uses models and representations in increasingly flexible ways, has decreased overt structure, provides increased coverage and has a form that is related to real-world instances.
4. An effective structured sequence ensures that the mathematical concepts are nested, that is, latter ideas fit 'within' earlier ideas.
5. Complex procedures that involve the coordination of several parts will give rise to the need for a superstructure – a mathematical concept that integrates the coordinated parts.
6. A mathematical concept is abstracted through the comparison of its various representations.

Warren and Cooper's identification and description of the cognitive interplay between concepts (both more informal models and more formal mathematical principles) is consistent with Sfard's (1991) reification theory, Gray and Tall's (1994) notion of procept and, more fundamentally, Piaget's (1977/2001) theory of reflective abstraction and the processes of interiorisation, coordination, encapsulation, generalisation, and reversal.

While based upon cognitive theories regarding an individual's sense-making, Warren and Cooper's hypotheses also incorporated a social-constructivist perspective that recognised the trajectory of learning mathematics cannot be considered without also considering the intertwined teaching trajectory. The interplay and effect between teacher action and student learning (the teaching-learning trajectory) is evident in each of the six hypotheses. These hypotheses and their implications for the design of instruction form the first pillar of the XLR8 project's conceptual framework.

RAMR Cycle and Pedagogy

Matthews (2009), an applied mathematician, provided an account of his own personal epistemology of mathematics. Matthews focused on the critical role that a mathematician's own reality has upon their mathematical activity, stating that mathematical activity begins with a particular part of the mathematician's reality – a real-life situation – from which, through a process of abstraction, they create a representation using a range of mathematical symbols. Using the representation, the mathematician is then able to explore particular attributes and behaviours of the real-life situation. Matthews stressed the importance of critical reflection to ensure that the mathematical representation and the discoveries made fit within the observed reality. Such a cycle of mathematisation is differentiated from popularly accepted views of mathematical activity by the importance placed upon the mathematician's personal reality, which encompasses not only their extant mathematical knowledge but also their social and cultural background. Based upon Matthews' prior work in Indigenous Mathematics Education (Matthews et al., 2007), Matthews claimed that pedagogy which is based upon reality-based mathematisation may lead to authentic mathematical literacy and a high standard of achievement.

Matthews' call for an innovative, reality-based pedagogy has given rise to what is referred to as the Reality–Abstraction–Mathematics–Reflection, or RAMR, cycle and accompanying pedagogical model. RAMR proposes: (a) working from reality and local culture (ensuring prerequisite knowledge and including everyday kinaesthetic activities); (b) abstracting mathematics concepts from everyday instances to mathematical forms through an active pedagogy (kinaesthetic, physical, virtual, pictorial, language, and symbolic representations, i.e., body → hand → mind); (c) consolidating the new concepts as mathematics through practice and subsequent building of connections; and (d) reflecting these new concepts back to reality through a focus on problem solving and laying the foundation for acceleration by constructing abstract, generalising concepts. The RAMR pedagogy, which underpins much of the current work of QUT's YuMi Deadly Centre, forms the second pillar of the project's conceptual framework.

Professional Learning

In the preceding sections it was claimed that a learner's trajectory of cognitive development cannot be considered without also considering the trajectory of teacher activity (i.e., the teaching-learning trajectory). The success of the XLR8 project will be based upon the implementation of a mathematics program that may require major changes to teachers' and students' attitudes and beliefs towards the learning of mathematics.

Guskey (2002) suggested that teachers are ill-inclined to change their attitudes and beliefs until they have experienced practices that positively impact student learning outcomes. Guskey's definition of learning outcomes is broad and includes academic achievement and affective gains, all of which teachers use to judge the effectiveness of their teaching and hence shape their attitudes and beliefs. To achieve significant and sustainable teacher change, and hence impact upon student learning, Guskey made three recommendations: (a) recognise that change is a gradual and difficult process; (b) ensure that teachers receive regular feedback on student learning progress; and (c) provide continued follow-up, support and pressure. These recommendations are echoed in the findings of Baturo, Warren, and Cooper (2004) who devised a set of principles and procedures for encouraging teacher change, including: the importance of expert involvement to guide a teacher's professional growth; authentic 'in situ' consideration of the idiosyncratic needs of each teacher in their classroom; provision of adequate time for teachers to construct detailed teaching plans; and the provision of just-in-time support to teachers to address problems in a timely manner.

A model of effective teacher professional learning and support that is inextricably linked with mathematical learning objectives forms the third pillar of the XLR8 project's conceptual framework.

The XLR8 Conceptual Framework

In summary, the conceptual framework of the XLR8 project has the following three pillars.

Structured sequence. The acceleration of mathematics learning will be based upon a curriculum that follows a structured sequence: a carefully selected sequence of focal mathematical concepts and a complementary set of representations that have strong isomorphism to the focal concepts and which scaffold the construction of understanding highly connected schema. The structured sequence will develop a student's understanding within a particular strand of mathematics, beginning at the student's extant understanding and extending their understanding as far as reasonably possible within that strand. This will give rise to a vertical, rather than horizontal, curriculum. Unlike more traditional horizontal curricula in which all strands of mathematics are visited each year, the XLR8 curriculum will focus intently upon a strand for an extended period of time (thus allowing the development of structural thinking and anticipated gestalt jumps in understanding).

RAMR pedagogy. To realise the structured-sequence-based vertical curriculum in the classroom, the RAMR pedagogical framework will be adopted as the basis for planning modules of study. In each module several iterations of the RAMR cycle will be employed to develop learners' reality-based understanding. Each iteration of the RAMR cycle will develop a focal concept within the strand, linking it to existing knowledge structures and creating new, super-structural knowledge that spans between the concepts of RAMR cycles.

Professional learning. While the XLR8 research team will develop curriculum materials that describe the structural sequence and its development using the RAMR-based pedagogy, the implementation of the curriculum requires the close entwinement of student learning and teaching practice. To this end, significant professional learning will be provided to the teachers. This will take the form of: face-to-face discussions regarding the XLR8 curriculum (before, during, and after the delivery of each XLR8 module); the timely analysis and reporting of student pre/post test data, such that teachers can use this data to inform their teaching; and the provision of in-class support (observation and critical discussion/reflection, model teaching).

Based upon this three-pillared conceptual framework, the content of the XLR8 curriculum to accelerate mathematics learning has been proposed and is the subject of refinement. In the following sections, the research methodology used to iteratively develop the curriculum is described and then an example of the curriculum, in particular the use of the RAMR cycle as the organiser of classroom activity, is presented.

XLR8 Research Methodology

The participants in this project are junior secondary students at five low SES state high schools, their teachers and other teaching staff. Two cohorts of students will be involved: those entering Year 8 in 2013 and those entering Year 8 in 2014. Each cohort will participate in the program for two years. The project is longitudinal because over the course of the project each participant's development of mathematical ability and affective change in response to the series of interventions will be tracked. The project is iterative because after the application of an intervention to one cohort of students, the intervention and the theory upon which it is based will be refined and re-applied.

The design of the project will be mixed method (Burns, 2000), integrating decolonising approaches (Smith, 1999) with predominantly qualitative methodologies and some quantitative methodologies. The qualitative aspect will be based upon principles of action research (Kemmis & McTaggart, 2000), and will use design experiments (Cobb, Yackel, & Mclain, 2000; Lesh & Kelly, 2000) to propose, apply, and refine the theory-based XLR8 curriculum and the program of professional learning and support. This qualitative approach will allow the research to respond flexibly to the anticipated cognitive, social, and cultural differences in each school community. The quantitative methodology will involve regular pre/post testing of students and accompanying analysis to measure mathematical growth, in particular the acceleration of mathematical ability from lower primary to lower secondary levels. The results of testing will be made available to teachers as soon as possible so that this data can inform their teaching practice. Overall, the research will lie within the empowering outcomes approach to decolonising research where research is designed to benefit the researched.

The XLR8 Curriculum

The two-year vertical XLR8 curriculum is broken into 16 modules, each nominally 5 weeks in length. The organisation of these modules is shown in
Figure 2.

	Term 1	Term 2	Term 3	Term 4
Year 8	Whole-number numeration	Pattern and variable	Arithmetic and algebraic structure	Metric measurement
	Fraction and decimal numeration	Operation concepts and strategies	Shape	Measurement of time, money, and other quantities
Year 9	Coordinate systems	Applications of linear relationships	Equations and functions	Statistical representation
	Flips, slides, and turns	Projections, topology, and trigonometry	Probability	Statistical inference

Figure 2. XLR8 curriculum scope and sequence.

Within each of the 16 modules the curriculum is presented as a series of RAMR cycles that progressively explore the content, beginning firstly with foundational concepts and then progressing to more complex concepts.

Example RAMR Cycle

The first module in the XLR8 curriculum is titled *Whole-Number Numeration* and develops learners' ability to flexibly represent the cardinality of sets, beginning initially with 3-digit numbers but then progressing to very large numbers. To exemplify the RAMR-based approach to learning mathematics in a structurally sequenced manner, the first RAMR iteration of the module is described in the following paragraphs. This first iteration of the RAMR cycle is in a unit titled *The unit, place-value and reading and writing of 3-digit whole numbers*.

In this example RAMR cycle, the focal concept to be developed is the notion that 'one' is the foundation of the number system. In turn, this big idea of mathematics is in two parts: (a) perceiving number requires flexibility in changing the unit that is counted (singles, groups and groups of groups can all be counted); and (b) the notion of place-value is developed by continuously changing the perception of unit. Across all early number activities, the learner's ability to read and write 3-digit numbers will develop as they flexibly perceive and describe the counted unit. During the RAMR cycle, representations are selected that are isomorphic with the place-value concept and which can, in later cycles, be extended to express larger numbers.

In the *reality* phase, students are firstly asked to identify as many situations as possible that involve the use of whole numbers and to describe what the numbers mean. The discussion is guided towards the realisation that some numbers represent position, some numbers are simply unique labels, and others (which are of immediate interest) represent the cardinality of a set (or, in simple terms 'how many?'). Students are then asked to identify situations in which objects are treated as singles, as groups and as groups of groups. Examples are provided such as 'student, class, school' and 'lolly, bag of lollies, box of bags of lollies'. At this point, students have already begun to use the focal concept (that the counting unit can be changed), albeit expressed informally and perhaps even without numbers.

In the *abstraction* phase the teaching-learning trajectory moves from the concrete representation of quantity to the symbolic (numbers-based) representation of quantity. The abstraction sequence begins with a kinaesthetic body activity. One suggestion is to arrange the class into variable-sized groups (all of size less than ten) and to get the students to identify, for each case, how many groups and how many are 'left-over' (i.e., the ones). The description of 'groups' and 'ones' can be scaffolded using a large, floor-based place-value chart (identifying the 'groups' and 'ones' columns). The sequence then moves to hand-based activities. One suggested activity is the manipulation of bundling sticks on a place-value chart – synchronously adding sticks to the ones column and as appropriate trading them for bundles in the tens column (and, removing sticks requiring the splitting of bundles into single sticks). Instead of bundling sticks, paper money ($10 notes and $1 coins) could be used. Throughout these activities, verbal language is consistently used and, as appropriate, the digit symbols are introduced as replacements for the manipulatives on the place-value chart. The previously mentioned activities may be done as a whole-of-class activity, or, student autonomy and peer-peer interaction could be encouraged by playing simple games that involve the addition/subtraction of small numbers. As proficiency with 2-digit numbers increases, 3-digit numbers can be introduced using a similar range of activities. The abstraction phase closes with mind activities which require the students to picture in their mind similar actions as those they previously encountered using the concrete materials and place-value charts.

In the *mathematics* phase, learners continue to practise the flexible counting of different place-values and the reading and writing of those numbers. In this practice, emphasis should be placed upon the interpretation and creation of symbolic representations of numbers, and should only revert to the more informal, concrete representations as required (i.e., when students are having difficulty working with the abstract symbols and so need the scaffolding of the concrete materials). During the mathematics phase attention should also be paid to reversing the stimulus – students should be able to flexibly interpret and create verbal, word, and symbolic representations of numbers.

In the *reflection* phase the teaching trajectory encourages the learners to identify the 'singles, groups, groups-of-groups' pattern in a wider range of situations, including time (seconds, minutes, hours), fractions (quarters and wholes) and measurement (millimetres, metres, kilometres). Also, students could be encouraged to create their own decimal-like number systems which use different number names.

This brief description of one RAMR cycle in the XLR8 curriculum has served to illustrate the hypotheses proposed by Warren and Cooper (2009) and their implementation using the RAMR pedagogical framework. The focal concept – the notion of unit and place-value – is expressed using a range of representations that vary in abstraction but which are isomorphic with one another and all highlight 'singles, groups, groups-of-groups'. Unit and place-value concepts emerge through the comparison of the various representations and situations. Initial activities were general in nature and provided an experientially constructed 'nest' in which to locate more specific and more formal representations of unit and place-value concepts. While this cycle is limited to 3-digit numbers, the representations are extendable to larger numbers and also decimal numbers. Ultimately, the representations will scaffold students' construction of super-structural concepts that organise the entire number system.

Conclusion

The XLR8 project has a solid, empirically tested basis. The proposal features a curriculum based upon carefully sequenced instruction that leverages the interplay between concept and representation, the use of a reality-based pedagogy and the support of teachers with timely, comprehensive, and tailored professional learning. Through the intervention implemented by partnering teachers and careful observation and analysis of student learning, the project aims to extend the theory related to the structured-sequence-based design of instruction and its implementation using a reality-based pedagogy that will ultimately lead to accelerated learning, high student achievement and affective change.

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