Students' Performance on a Symmetry Task

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This paper describes Singapore and Australian Grade 6 students' (n=1,187) performance on a symmetry task in a recently developed Mathematics Processing Instrument (MPI). The MPI comprised tasks sourced from Australia and Singapore's national assessments, NAPLAN and PSLE. Only half of the cohort solved the item successfully. It is possible that persistence of prototypical images of a vertical line of symmetry and reinforcements in the classrooms could have attributed to this low success rate.

The increase in use of graphics in online media has redefined the skill set one needs for communication and interaction with others. In particular, acquisition of a digital capacity has been recognised to be increasingly important in workplaces (Kerr, 2011; Ong, 2011). Acquiring digital skills provides an opportunity to evoke visuospatial reasoning to make sense of information. Given most school systems have a philosophy of preparing students for societies' future expectations, it is unsurprising that more attention is being paid to developing students' reasoning (e.g., Ministry of Education, 2006).

Visuospatial reasoning is an essential ingredient in concept acquisition (Ahsen, 1989). Research has shown that learners' conceptual understanding can be increased when visual imagery is used. Hence, concept acquisition is influenced by the way these visual images are represented. The ability to make sense of spatial information also aids in the process of problem solving (e.g., Presmeg, 1986, with problem solving playing a key role in both the Australian and Singapore primary mathematics curriculum (Ministry of Education, 2006; Board of Studies NSW, 2002). A current concern brought up by some researchers (e.g., Aspinwall, Shaw, & Presmeg, 1997; Jones, Fujita, & Kunimune, 2012) is that persistence of a prototypical image could be a hindrance in successful spatial concept acquisition and problem solving.

Research Design and Methods

The Cross-Cultural Studies in Mathematics Sense Making project investigates students' ability to process and decode mathematics tasks (see Logan & Ho, this symposium). Specifically, this paper investigates the performance of students in a multiple-choice Symmetry task (see Figure 1). This Symmetry task is a graphic item from the Space and Geometry strand of the NSW Mathematics K-6 curriculum and is sourced from the Australian NAPLAN (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2010). It is categorised as a retinal-list item (see Lowrie & Diezmann, 2009) and requires students to identify the correct reflected image of the letters RZ across a diagonal line of symmetry. The answer to this item is option (4). Note that diagonal symmetry is not covered in either Australia or Singapore's primary mathematics curriculum.

In V. Steinle, L. Ball & C. Bardini (Eds.), Mathematics education: Yesterday, today and tomorrow (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne, VIC: MERGA.

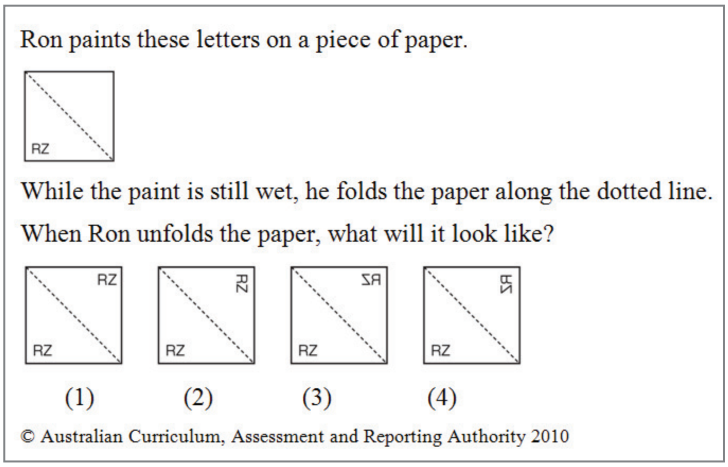

Figure 1. The Symmetry task (ACARA, 2010).

The conceptual underpinnings for data analyses were drawn from the work of Presmeg (1986). The data in the study were coded according to two criteria: (a) whether the answer was correct or incorrect; and (b) the type of method of solution used. The coding scheme incorporates Presmeg's (1986) definitions of visual imagery. A coding of '0' was assigned to students who used a non-visual method. A non-visual method is one that does not have a visual image as an essential part of the method of solution. A coding of '1' was given to a method of solution that showed use of a visual method where a diagram drawn on paper plays an essential role in the solution. A coding of '2' was given when the student used concrete (mental) imagery as a method of solution. A coding of '3' was assigned to a method of solution that involved kinaesthetic imagery. When students use kinaesthetic imagery as a method of solution, they employ physical movement, for example, turning or folding the page along a line.

Results, Discussion and Implications

Overall, only 50.1% of the students solved the Symmetry task successfully (see Table 1). The majority of the students (75.3%) used concrete imagery or visualised to solve the task. With respect to the incorrect responses, the majority of the students chose option (3) which is an answer that would have been correct had the line of symmetry been a vertical one and the letters situated in the top left corner.

Table 1
Percentage Correct and Incorrect Responses on the Symmetry Task by Solution Method

Coding	Non-visual	Diagram	Visualisation	Gesture	Total
Correct Response	0.4%	0.7%	34.2%	14.8%	50.1%
Incorrect Response	0.3%	0.9%	41.1%	7.6%	49.9%
Total	0.7%	1.6%	75.3%	22.4%	100.0%

Note: 32 students did not answer the second part of the MPI. Hence, the percentages in Table 1 were computed over the base of 1,155 students who answered both parts of the MPI.

A student, Daniel, provided the following explanation for choosing option (3) as his answer—"If you write R and Z like the other way around it would be ZR". The implication for such reasoning is that successful problem solving may have been hindered by prototype limitation of an image of the line of symmetry always being represented in a v ertical position. Another implication is the use of the non-mathematical language "the other way around". What does it mean when used in symmetry tasks?

Previous research findings have indicated that students 'fold back' (Pirie & Kieren, 1994) or revert to visual approaches (visualisation and using gestures) when faced with complex or novel tasks. Figure 2a shows how a student, Jasper, solved the task using visualisation. He visualised folding the paper along the line of symmetry. It has been documented that gesturing influences and aids in problem solving (Goldin-Meadow, 2003; Logan, 2010). 22.4% of the students indicated using gestures when solving the task. They either folded along the line of symmetry in the item itself, or drew a similar diagram of the task in the working space provided and folded along the line of symmetry in their diagram. Another student, Lily, reported that she drew a diagram and folded the paper along the line of symmetry in her diagram (see Figure 2b). Both Jasper and Lily solved the Symmetry task successfully.

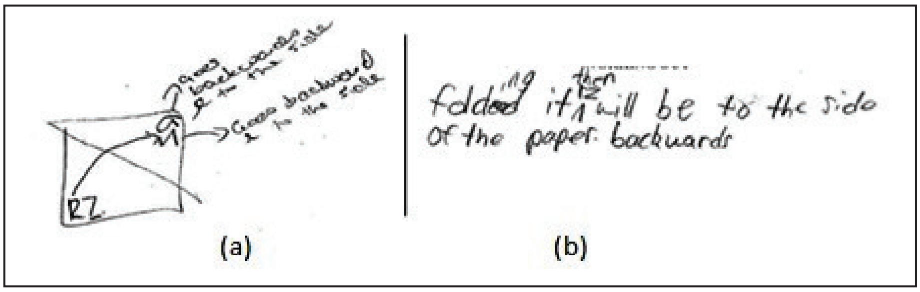

Figure 2. (a) Jasper's explanation and (b) Lily's explanation.

However, a small number of students (7.6%) used a gestural approach and obtained the incorrect answer. Patrick, for example, chose Option 3 as his answer and reported that he folded the paper along the symmetry line in his test booklet. Lowrie (2012) noted that students tended to want to "construct their own images and representations of tasks despite the fact visual and graphic representations were already contained within these tasks" (p. 3). In other words, such students "used decoding skills to interpret the graphic information whilst also using encoding techniques to produce images (either on paper or in the mind's eye) to help organise information and potentially scaffold understandings" (p. 3). Another student, Michael, also chose Option 3 as his answer. He drew his own diagram and folded along the line of symmetry in his diagram. He then took an additional step to turn the diagram 180° to see how they would look in that orientation—"I drew it then I turned the paper upside down to see what was right".

The above findings suggest that adopting a gestural strategy does not guarantee successful problem solving. Explicit teaching of gesturing by teachers when they introduce such a problem solving tool to students may be pertinent.

Conclusion

From a theoretical perspective, researchers are encouraged to consider the relationship between task correctness and the choices given in multiple-choice assessment items. Researchers are also encouraged to investigate why some students choose to first construct their own images (encode) in spite of the image being already provided in the task.

From a classroom perspective we argue that since concept acquisition is influenced by the way visual images are represented, we encourage teachers to provide students with opportunities to work on symmetry problems where the line of symmetry is not always vertical. In addition, teachers could also provide students with symmetry tasks where the letters RZ are not in their usual upright position. In this way, students are given opportunities to develop powerful visuospatial reasoning when confronted with tasks that are novel or complex.

Indeed, curriculum designers and textbook publishers need to be mindful of the role of prototypical images in spatial concept formation. We argue prototypical images will hinder the learner from understanding spatial concepts fully as such prototypical images may cause the learner to reject images which do not conform to these prototypical images, leading to unsuccessful concept acquisition and problem solving.

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