The Classic Word Problem: The Influence of Direct Teaching

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Australian and Singaporean students have been exposed to different forms of teaching due to cultural differences in education. In each country, varying degrees of importance has been placed on explicitly teaching problem-solving heuristics. This paper highlights the different strategies employed by students from each country when solving a word problem and the role direct teaching has played in the development of these strategies. Implications for classroom practice are also discussed.

Teaching Successful Strategy use in Problem Solving

Studies have shown that students who are more successful at solving mathematics tasks use different types of strategies to those students who are less successful (Pape, 2004; Pearn, 2009). Nevertheless, there is much debate with regard to how teachers should develop students' proficient strategy use. There is a view that explicit teaching of particular problem-solving strategies is an ineffective way of improving problem-solving skills in children (Muir, Beswick, & Williamson, 2008). However, in Confucian-heritage culture countries, such as Singapore, there is emphasis on repetitive learning (continuous practice with increasing variation) in order to understand and internalise a mathematics concept. Teachers tend to promote and value specific strategies that equip students to be successful across a range of tasks and especially in assessment situations. Indeed, Singapore has structured its mathematics program around problem solving with the explicit teaching of processes and heuristics highly prominent (Ho & Hedberg, 2005). Similarly, Sweller, Clark and Kirschner (2011) argued for more direct, explicit teaching of particular strategies to be undertaken through the study of worked examples, where the problem solving steps and solutions are provided for students. They suggested that to be a successful mathematical problem solver, one must have a large repertoire of specific strategies appropriate to particular problems and they contest the usefulness of general problem-solving strategies. Regardless of where you stand on the debate, evidence highlights that Confucian-heritage countries are outperforming their Western counterparts on international tests such as TIMSS (Thomson et al., 2012). This paper draws direct comparisons of the types of problem-solving strategies students, from two culturally different countries, employed when solving a word-based mathematics task.

Research Design and Methods

This pilot project draws attention to cross-cultural dimensions of students' learning by analysing students' mathematics assessment performance and sense-making in high performing but culturally different contexts—Australia and Singapore. The aim of this paper was to identify the problem-solving strategies employed by the different cohorts when solving a non-graphic, word-based problem.

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Participants

1,187 Grade 6 students (aged 11-12) from 5 Singapore (three government and two government-aided) (n=607) and 12 Australian schools (all non-government) (n=580) took part in the study. There were 600 boys and 587 girls in the sample.

Instrument and Procedure

Students' ability to decode mathematics tasks were investigated using a *Mathematics Processing Instrument* (MPI) developed and designed by the research team. The instrument comprised 24 items sourced from both Australia's National Assessment program Literacy and Numeracy (NAPLAN) and Singapore's Primary School Leaving Examination (PSLE), of which there were: (a) 6 graphic and 6 non-graphic items from the NAPLAN; and (b) 6 graphic and 6 non-graphic items from the past-years PSLE items. A graphic task has a graphic (e.g., diagram, chart, graph or map) embedded within the task, where the graphic contains information essential for task solution. A non-graphic task has only text (such as a word problem). Two staff from the research team attended specified schools during their morning classes. The classroom teachers were asked to help administer the MPI along with the research staff. The administration of the MPI was in two parts. Firstly, students answered the 24 mathematics items as a pencil and paper test. The duration of the paper-and-pencil test was 1 hour. After a short break, students then filled out the MPI. This paper reports on one of the Singaporean non-graphic mathematics items—The Chairs Task (Figure 1).

> The chairs in a hall were arranged in rows. Each row had the same number of chairs. Weiming sat on one of the chairs. There were 5 chairs to his right and 5 chairs to his left. There were 7 rows of chairs in front of him and 7 rows of chairs behind him.
> How many chairs were there in the hall?

Figure 1. The chairs task.

Analysis of data

Student's responses were coded as either visual or non-visual according to the processes they described within the MPI. A non-visual method was defined as one that *did not utilise* imagery of any kind and tended to consist of computations. A visual method was categorised as one that utilised diagrams, visualisation and gestures. This criterion was also matched with correct and incorrect responses.

Results and Discussion

The first level of analysis focused on the performance of students on The Chairs Task. An Analysis of Variance (ANOVA) (country x mean score correct) revealed a statistically significant difference between the performance of the Singaporean and Australian students [$F(1,1186) = 246.3$, $p<.001$], with the Singaporean students (mean = .77) outperforming the Australian students (mean = .32) on the task. The second level of analysis considered the performance of the Singaporean and Australian students along with the types of strategies utilised for The Chairs Task.

The entire cohort solved the tasks with similar proportions of both non-visual and visual strategies (Table 1). However, given the overwhelming performance difference in favour of Singaporean students on this item, it was not surprising that cross tab analysis revealed a higher proportion of Singaporean students answering correctly using both types of strategies compared to the Australian students.

Table 1
Cross tab Analysis of the Chairs Task by Country

Item		Singaporean strategies (n=566)		Australian strategies (n=568)	
		Non-visual %	Visual %	Non visual %	Visual %
Chairs	Incorrect	28.8	25.3	67.2	68.0
	Correct	71.2	74.7	32.8	32.0
	Total	49.1	50.9	50.5	49.5

It was evident from the way the students understood and represented the task that the Singaporean students had been taught a set of appropriate heuristics to solve these types of tasks and these strategies were effective irrespective of whether the students solved the task in a visual or non-visual manner (Figures 2 and 3). The Singaporean students strategy use was more structured and regulated—suggesting that the Singaporean students had been exposed to explicit teaching that demonstrated specific strategy use. As Pape (2004) suggested, these successful problem solvers had "a well-formed base of conceptual knowledge" (p. 213) and hence developed a logical, consistent mental model of the situation using both visual and non-visual methods.

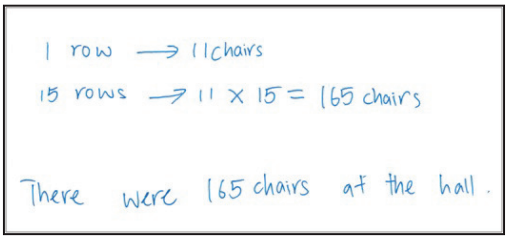

Figure 2. Singaporean non-visual.

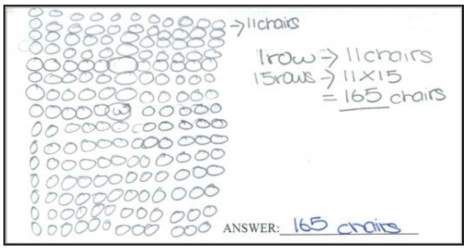

Figure 3. Singaporean visual.

By contrast, the Australian students represented the task with a diverse number of visual and non-visual approaches. It was evident that they did not possess strong mental models of how to situate, represent and solve the task (Figures 4 and 5). Muir et al. (2008) would identify these students as naive problem solvers and, in line with Pape's (2004) suggestion, these unsuccessful problem solvers selected key words, phrases and numbers from the problem and attempted arithmetic operations based on their visual or non-visual methods. Figure 4 indicates a lack of comprehension of the task, with little mathematics sense making. Despite the diagrams being represented correctly in the visual solutions (Figure 5), in that they contained the essential information, a lack of understanding of the subsequent structures required to decode their own representations coupled with inadequacies with the actual mathematical concepts and procedures produced an incorrect solution. The Australian and Singaporean visual solutions were very similar apart from the final calculations; it is here that explicit teaching could provide students with the structures to finish off the strategy in a successful manner.

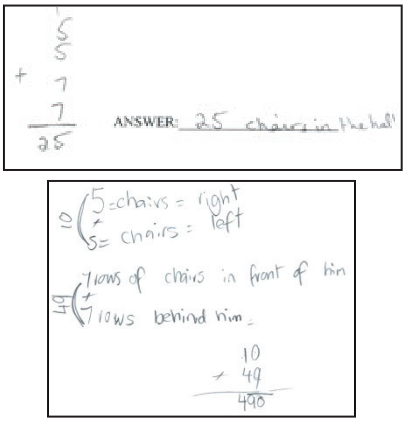

Figure 4. Australian non-visual.

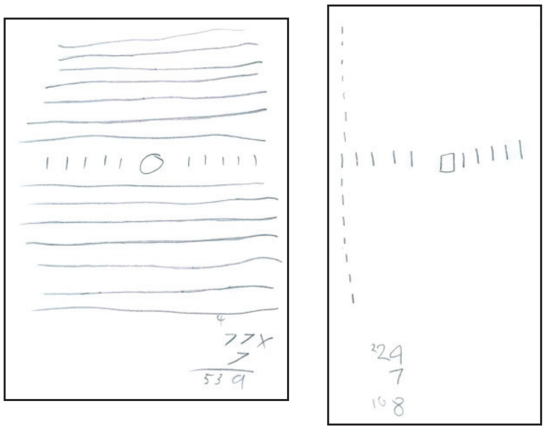

Figure 5. Australian visual.

Conclusion

The success of the Singaporean students and their efficient use of different strategies on this word problem highlight how effective explicit teaching can be. The challenge in Australian classrooms is framing specific strategy use within problem-solving contexts, given the predominant environment is to promote constructivist thinking, as opposed to the controlled Confucian-heritage ideology. An approach to counter this is to provide students with a small number of effective heuristics when they are encountering novel or complex problems. Explicit teaching through worked examples can provide students with the appropriate knowledge to apply to such problems and through this, students develop a repertoire of reliable strategies. The teacher could subsequently engage students with creative and open-ended problems, resulting in flexibility and utility when students' problem solve. Indeed, the aim for teachers is to provide scope for students to develop mental models they identify with, draw on, and can appropriately apply in any situation.

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