Primary School Teachers' Perceptions of Mathematical Reasoning

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Little is known about how Australian teachers interpret, enact and assess reasoning. This paper reports on primary teachers' perceptions of reasoning prior to observation and subsequent trialling of demonstration lessons in a primary school. The findings indicate that while some teachers were able to articulate what reasoning means, others were unsure. It is argued that to facilitate curricular change and reform, teachers need support in understanding mathematical reasoning and how they can further develop this proficiency in their primary classrooms.

Reasoning is a proficiency that underpins mathematical thinking and is one in which, most if not all, mathematics curricula around the world would charge teachers to facilitate in the classroom (see for example, National Council of Teachers of Mathematics, 2013; Ministry of Education, Singapore, 2012). In Victoria Australia, reasoning has been emphasised in one form or another in past curriculum documents and in 2012 the new Australian mathematics curriculum made explicit that reasoning is one of four proficiencies to be developed by students (Australian Curriculum, Assessment and Reporting Authority [ACARA], n.d.). While past documents had advocated that mathematical reasoning is a proficiency to be taught and learnt in classrooms, its enactment is often unclear and seldom emphasised (Stacey, 2003). Little is known about the depth of teachers' knowledge and understanding of reasoning or how comfortable and confident they are with planning and implementing tasks that promote the development of this proficiency. Kilpatrick (2009) reminded us that

> We need to understand that curriculum change is not a technical matter. Instead, it is a personal journey for mathematics teachers. Any attempts to change the curriculum…need to take teachers where they are and invite them to join the process of reflection and mutual encouragement. (p. 119)

This research group seeks to investigate what teachers understand and recognise in terms of reasoning and how they might be engaged in change that enable students to develop this proficiency. This paper reports on baseline data about the knowledge and understanding of reasoning derived from individual interviews with seven Victorian primary teachers that was aimed at finding answers to the following research questions:

a) What are primary teachers' current knowledge and understanding about the reasoning proficiency?
b) What examples can primary teachers give to demonstrate their understanding of the meaning of reasoning?

Our theoretical framework is based on Carpenter and Lehrer's (1999) proposal that articulating what one knows is one of the forms of mental activity from which understanding emerges. To further frame our analysis, we reviewed the mathematical knowledge and understandings that teachers require for effective teaching. A brief description of the overall project is provided. The paper concludes with a call for teachers to be supported in their endeavour to reform teaching and learning.

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Background

Reasoning is widely recognised as central to the practice of mathematicians and regarded as significant for engagement and success in mathematics learning (Nunes, Bryant, Barros & Sylva, 2012; Polya, 1990). Kilpatrick, Swafford and Findell (2001) claimed that reasoning is the "glue that holds everything together, the lodestar that guides learning" (p.129). The development of this proficiency had always been recognised as important and critical in the mathematics curriculum in Victoria, Australia. In 2002, the Curriculum Standards Framework II (CSFII) (Victorian Curriculum and Assessment Authority [VCAA], 2002) set out the *Reasoning and Strategies Strand*. Then in 2007, the Victorian Essential Learning Standards (VELS), which replaced the CSFII, organised the Standards according to five domains, namely, *Number*, *Space*, *Measurement, chance and data*, *Structure* and, *Working mathematically* (VCAA, 2007). In this curriculum, reasoning was subsumed in the *Working mathematically* domain and students were to be "involved in the application of principled reasoning in mathematics using natural and symbolic language, through the mathematical processes of conjecture" (para.16). In 2012, the reasoning proficiency was made more explicit in the Australian Curriculum: Mathematics (AC: M). This curriculum is organised around the interaction of the content strands of *Number and Algebra*, *Measurement and Geometry*, and *Statistics and Probability,* and the four proficiency strands: *Understanding*, *Fluency*, *Problem Solving*, and *Reasoning*. Here, students are deemed to have developed reasoning proficiency when they

> …develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices." (ACARA, n.d., p. 5)

It can be seen from the above that the new Australian mathematics curriculum sets out clearly what teachers should be anticipating from students when they plan for reasoning to occur in the mathematics classroom. Whereas in the past, students' actions for reasoning had been relatively obscure, students are now expected to be able to explain, deduce and justify, adapt, transfer learning, prove, and, compare and contrast ideas to demonstrate their reasoning proficiency. The use of these action verbs suggests an intention on the part of curriculum writers to draw teachers' attention to aspects of reasoning that they could purposefully incorporate in their planning. For teachers to be able to teach, enact and assess reasoning, it is imperative that they are clear about its nature. Unless this is evident, we are in danger of a syndrome of shallow teaching where low complexity problems abound and mathematical reasoning is absent in classroom discourse (Stacey, 2003).

Mathematical reasoning has been defined in various ways. According to Carpenter, Franke and Levi (2003) reasoning involves "being able to explain why a procedure works or why a particular statement is true" (p. 5). This is often seen as explaining, justifying or proving. Brodie (2010) claimed that mathematical reasoning assumes the requirement to communicate "lines of thinking or argument" (p. 7). In mathematics, this communication may take the form of pictures, symbols, diagrams or models.

In an attempt to understand what teachers think and know about reasoning, Clarke, Clarke and Sullivan (2012) surveyed 104 primary teachers in a recent project. They found that there was a wide range of views on the extent to which the *Reasoning* proficiency statement in the new AC: M was comprehensible to teachers. "On a scale of 1 to 10 with 1 being 'difficult to understand' and 10 being 'easy to understand' the overall mean of the

responses was 5.7" (Clarke et al., 2012, p. 31), indicating a low to moderate understanding of the statement. Teachers were asked how regularly they used the following reasoning adverbs: *explaining*, *justifying*, *proving*, *reasoning*, *evaluating*, *analysing*, *generalising*, *inferring*, *deducing*, *adapting*, *transferring* and *contrasting*. They found that reasoning terms such as *explaining* were used by all teachers and *proving* by three quarters of the teachers but terms such as *contrasting*, *transferring* or *deducing* were less commonly used. This indicated that teachers integrate only some aspects of reasoning in their teaching but it was less clear which aspects. Their study however, does not reveal the depth of teachers' understanding of reasoning, what examples of reasoning are evident in their classrooms, and whether teachers are aware of the differences and relations between the four proficiencies: understanding, fluency, reasoning and problem solving.

According to Carpenter and Lehrer (1999), "the ability to communicate or articulate one's idea is an important goal of education and is a benchmark of understanding" (p.22). They noted that students initially may have difficulty articulating their ideas about an unfamiliar topic or task, but by struggling to articulate them, students eventually develop the ability to reflect on and articulate their thinking. We argue that just as students are encouraged to articulate their understandings of mathematical concepts, teachers also need to be encouraged to articulate their understandings of the curriculum documents to raise awareness of the mathematical knowledge they need to have for teaching. Interviewing teachers about what reasoning means and how they have incorporated it into their lessons provided opportunities for teachers to reflect on and articulate their understanding.

Prior to the inclusion of the reasoning proficiency in the new Australian mathematics curriculum, Stacey (2010) had called for a firmer place for reasoning in classrooms. She recommended the addition of reasoning instruction in classroom tasks, classroom discourse, unit planning and curriculum description. This would require (a) selecting and developing worthwhile tasks which have the potential to immerse students in significant mathematics content, and (b) orchestrating classroom discourse focused on mathematical thinking, reasoning, and communication (Peressini, Borko, Romagnano, Knuth, and Willis, 2004; Stein, Engle, Smith, and Hughes, 2008). We wondered whether in the process of teaching mathematics teachers are utilising tasks that engage students in developing mathematical reasoning through productive discourses. In this paper we examine the reasoning examples teachers offer to support their articulation of the meaning of reasoning and their views about classroom discourse in promoting mathematical thinking.

Method

This paper arises from the first phase of a larger study on r easoning in primary mathematics classrooms. In this phase we investigated teachers' current understandings about and perceptions of reasoning. We present the findings drawn from semi-structured interviews with seven teachers from one of the four Victorian primary schools participating in the broader study. The teachers were female with mathematics teaching experience ranging from three years to twenty-five years. Three had experience teaching in the lower primary years (Foundation /Preparatory Year to Year 3) while the other four had teaching experience in Foundation Years through to Year 6. Three of the teachers held leadership positions in the school (Assistant Principal, Numeracy Coach and Mathematics Coordinator). The teachers were reminded that the AC: M includes reasoning as one of the proficiencies that students are expected to learn and develop throughout the primary and secondary years of schooling. Teachers were made aware that the aim of our project was to investigate teachers' reasoning capacity, their knowledge of reasoning and its relationship

to learning mathematics and other mathematics proficiencies, their approach to teaching reasoning, and how their knowledge develops through participation in a program of demonstration lessons conducted by the research team. Semi-structured interviews were conducted with individual teachers in the week prior to the teachers observing the first demonstration lesson. The interview focused on their ideas about various aspects of their teaching practice and their knowledge, interpretation and implementation of reasoning in the previous Victorian curricula and in the new Australian curriculum for mathematics. The interviews were audio-taped and transcribed. Analysis of the data was undertaken employing NVivo, a computer software program which enables answers from each teacher to be grouped under specific codes. Emergent themes from this analysis were grouped together for discussion. The findings from this initial teacher interview are reported in this paper. Pseudonyms are used to differentiate teachers.

Findings and Discussion

In order to understand the depth of teachers' knowledge and their interpretation of reasoning, we examined how fluently teachers were able to express their understanding of reasoning and the similarities and differences between *Working mathematically* (as stated in the VELS) and *Reasoning* as defined in the AC: M.

Teachers' explanations of reasoning

When asked what they thought reasoning means, two teachers were able to articulate what they thought succinctly:

> …Whether it's a practical way of doing something, whether it's logical, whether it's going to work. Whether – and the ability to explain that thinking… (Ally)

> …I think if they're able to explain their responses, if they're able to justify, if they're able to make good estimates, if they can do that sort of stuff then they're doing well… (Brianna)

These two teachers were confident in articulating what they thought reasoning meant. Words like *explain* and *justify* were offered without hesitation. This is consistent with the findings of Clarke et al. (2012) who found that these two terms were used readily by most teachers in their survey. The two teachers were in senior leadership positions and had more than 20 years teaching experience. Hence they were likely to have lived through at least two major curriculum changes, CSFII and VELS, which afforded them more opportunity to be informed about curriculum change and development.

Two other teachers (Darlene and Carol) attempted to articulate what reasoning means but struggled with their explanation.

> …I think it's when you need to – I'm trying to work out how to say it, like test …and providing evidence that they can understand it. (Darlene)

> … I honestly don't know too much about it, just sort of learning still. I guess it's being able to use things like estimation skills to figure out whether the answer that you've got is in the ball park, or if it's way off. (Carol)

These two teachers struggled to articulate their understanding but had captured, at least partially, elements of reasoning. Upon further reflection, Darlene had responded that reasoning was 'providing evidence'. She was using other terms to describe what Clarke et al. (2012) referred to as *justifying* and *proving* (p.31). While Carol indicated she was still learning, her difficulty in articulating her understanding of reasoning led her to reflect that reasoning is about being able to tell if an answer is logical or not. This is similar to what

Clarke and colleagues term *evaluating* (p. 31). Consistent with Carpenter and Lehrer (2008), the process of struggling to articulate seems to have helped these teachers to reflect and find terms that enabled them to express their understanding and thinking.

Three of the seven teachers interviewed responded that they did not know or were unsure how to define reasoning. For example one teacher said "I don't know… a lot to do with the understanding behind the mathematical concepts" (Faye). Another teacher said, "I think reasoning is more about having some of the information and, yeah I'm not sure" (Elizabeth). A third teacher said "…mathematical thinking and reasoning – in my head, they mean the same thing… I really don't know how to explain that" (Grace).

The three teachers appeared to lack the vocabulary to express what they understood reasoning to be and this could be due to their lack of familiarity with the curriculum documents. As Grace said, "First of all, I have not looked at the national curriculum in the maths areas…" and Faye had said, "I know I need to do some more research on exactly what reasoning is". Amongst those who were not able to articulate readily what reasoning meant, there was acknowledgement of their limited understanding and a need to do more reading to better understand reasoning terminology.

Comparing and contrasting Working mathematically and Reasoning

When asked about the similarities and differences between *Working mathematically* and *Reasoning*, most teachers said the two curriculum areas were similar and noted that there are slight differences between them. There was, however, a certain amount of uncertainty as to what these differences were. Five of the teachers could not articulate the differences or saw little difference between the two.

> …I think it's similar in the way that it's not teaching – it is skills but skills in a different way. (Faye)
>
> …At this stage, I don't see it any different. Having not read it in the national curriculum, I would not expect it to be that different. (Grace)
>
> …Well they are, I think they're quite similar …but I'm not really sure at this stage I guess, I'm still trying to work that out, yeah the difference. (Elizabeth)
>
> …They're sort of similar. But I think, they're mathematical, its understanding them, the mathematics, whereas, reasoning is providing examples of how that mathematics is conducted, I think. (Darlene)
>
> …looking at your patterns and things like that. And maybe, reasoning is being able to determine whether those strategies are effective or not, so, yeah, through estimation skills and things like that. (Carol)

It is evident that Faye, Grace and Elizabeth were unable to articulate the difference between the two terms. This may be because these teachers, as indicated earlier, had not read the AC: M deeply enough to comment. Another reason could be that two of these teachers did not have any prior experience of adapting to changes in curriculum documents which requires reflection on and analysis of curricula (Kilpatrick, 2009). Darlene and Carol, on the other hand, attempted to articulate the differences. After some reflection, they succeeded in providing some information about where they thought the differences lay. This shows once again that their grappling to communicate their thinking about the differences had caused them to reflect more deeply on their own understanding.

One more experienced teacher was able to differentiate between the two curricula commenting that the proficiencies are much more specific and focused whereas *Working mathematically* had an overarching focus which was too broad for teachers to work on.

> …It's how you go about – it's working like a mathematician. So looking at the problem from a whole lot of different angles. And identifying your thinking and identifying the strategy that you might use to solve the problem.…I think it's perhaps the proficiencies are being a bit more specific…working mathematically was a bit broad for a lot of teachers. (Ally)

Another experienced teacher said *Working mathematically* is

> …around making the right choices for the task that's in front of you and knowing which choice to make...so it's using better strategies, it's common sense maths and knowing when to use the appropriate strategy. (Brianna)

Ally distinguished *Working mathematically* to be looking at the problem from a variety of angles and utilising a variety of strategies to solve a problem. However, she saw reasoning as a process of determining whether a particular strategy is logical and workable and whether one has the ability to articulate that thinking. Brianna similarly saw *Working mathematically* as being much broader encompassing making choices about appropriate strategies but reasoning as being able to explain and justify those choices. Ally and Brianna have demonstrated a clearer overview of thinking skills within *Reasoning,* and *Working mathematically* (than some other teachers), and were also able to identify differences between these curriculum terms. This raises the question: Did these two teachers' greater number of years of experience in teaching, and exposure to debates and discussion about curriculum through their leadership roles contribute to this, and if so how?

Examples of reasoning from their classrooms

When asked how reasoning is incorporated in their lessons and teaching, most teachers gave examples of activities they thought would promote reasoning. A number of teachers offered examples of children's reasoning as children's ability to articulate their thinking and the strategies they use to solve a particular problem.

> …I have a big emphasis in my class on getting the kids to think about the strategies and being able to articulate the strategies that, or how they've worked out a problem, rather than just saying, right that's the answer. (Carol)

> …With my own lessons, it's always having the children talk about their strategies and how they went about solving the problems. (Ally)

These teachers see children's ability to reason as the ability to explain and justify their answers or their working. Classroom discourse was regarded as valuable by these teachers. Student responses were emphasised, anticipated and monitored (Stein et al., 2008).

When asked for examples of children's reasoning, very few teachers offered distinct examples that illustrated children's reasoning. However, one Preparatory teacher reported listening to her students to see whether they explained and justified their answers:

> ...how many things do you think are in this jar? Oh 100. I'd be going …mm possibly not working very well mathematically, not good reasoning here. Whereas if another prep came in and said, 'Well I think it's about 25 and I think it's that because there's probably 5 on the bottom and 5 in the next layer and da, da, da' then I'd be saying 'Wow!'. So it's that ability to actually talk their way through a problem. (Brianna)

In the examples above, the teachers variously demonstrated their awareness of what it means for students' to display mathematical reasoning as stated in the curriculum documents, that is, "…students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached…"

(ACARA, 2012). These teachers perceived themselves as encouraging discourse that focused on mathematical thinking, reasoning, and communication (Peressini et al., 2004).

Two teachers offered examples of activities where they felt children were reasoning:

> … So there's a type of activity where you're using those hands on activities to estimate and then problem solve by checking and does it, is it the same as that one or so measuring the different things. (Darlene)

> … so if we were going to work out the area of a shape or something, we would, reasoning comes into the estimating part and having a go at how many counters might cover that shape … rather, and trial and error in that kind of situation. (Elizabeth)

In these activities children *were* estimating using reasoning but the teachers were not able to identify that reasoning (e.g., using logical thinking, or adapting the known to the unknown such as using a measurement referent to carry out the estimation). Elizabeth, instead, referred to problem solving strategies such as trial and error as reasoning. In the examples cited, these children may have displayed their reasoning capability when they compared and contrasted the size of the counters with the given shapes and gave an appropriate estimate using the measurement referent or visual logical reasoning (Barbosa, Palhares, & Vale, 2007). The illustrations provided suggest a lack of certainty in the teachers' minds as to distinctions between reasoning and problem solving. Confounding these terms is consistent with teachers' limited use of reasoning verbs (Clarke et al., 2012).

Conclusion

The findings revealed that primary teachers may not have a clear understanding of reasoning. Some teachers in this study could not articulate their understanding of reasoning or confused reasoning with problem solving. This confusion is to be expected since reasoning is entwined with other proficiencies (Kilpatrick et al., 2001). Professional learning is required to develop inter-connections and distinctions between these proficiencies so teachers can better attend to developing these proficiencies in their students. Teachers need to be clear about these two proficiencies so that they are able to plan worthwhile tasks that immerse students in significant mathematical content that elicits reasoning that is made public through group and whole class discussion. The gradual development of reasoning capability using age appropriate tasks as advocated by Stacey (2010) may be slow to materialise if teachers confuse problem solving with reasoning.

There is a need to provide teachers with more explicit examples of reasoning opportunities for the classroom as most of the examples cited by the teachers were limited to students explaining and justifying their choices. Teachers should have in their repertoire of reasoning tasks, activities that require reasoning to occur because of the need to: adapt the known to the unknown, transfer learning from one context to another, generalise, prove that something is true or false, compare and contrast related ideas and explain choices as recommended in the AC: M (ACARA, 2012). An ambiguous understanding of reasoning is likely to perpetuate a lack of definitive action in the classroom. There is a need to support teachers on their journey to understanding and implementing curricular change. Such support is in the interest of students' mathematical learning.

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