When Practice Doesn't Lead To Retrieval: An Analysis of Children's Errors With Simple Addition

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Counting strategies initially used by young children to perform simple addition are often replaced by more efficient counting strategies, decomposition strategies and rule-based strategies until most answers are encoded in memory and can be directly retrieved. Practice is thought to be the key to developing fluent retrieval of addition facts. This study examines the errors made by five children in Year 3 as they perform simple addition and illustrates why practice does not assist all children to develop retrieval strategies.

Developing proficiency with simple addition

Simple addition refers to adding together single digit numbers. Proficiency with simple addition is evidenced by performance dominated by the accurate use of direct retrieval and decomposition strategies (Cowan et al., 2011). Direct retrieval refers to retrieving the answer from memory and is sometimes referred to as recall in curriculum documents. Decomposition strategies are strategies that make use of a directly retrieved fact to derive an answer to a different problem.

The term 'simple' is somewhat misleading as attaining proficiency with single digit addition is not a straight forward process and can become a major obstacle towards developing mathematical competencies. Research suggests that a lack of proficiency with basic facts is predictive of subsequent problems in higher level mathematics skills and the inability to retrieve answers creates fluency problems for many students (Geary, 2004).

Before developing proficiency, children typically use different *back-up strategies* to perform simple addition. A back-up strategy is defined as any strategy other than direct retrieval (Shrager & Siegler, 1998). Back-up strategies often encompass counting strategies including counting-all strategies, where the count is started at one, and counting-on strategies, where one addend is counted on from the other. The most efficient counting-on strategy is the min-counting strategy, where the smaller (minimum) addend is counted on from the larger addend. As children learn to recall some addition facts as a result of practice with backup strategies, they can then apply these facts to find answers to other problems using decomposition strategies (e.g., 4+5=4+4+1=9). With further practice they may come to recall these answers (i.e., 4+5=9).

Different strategies are considered appropriate for children at different stages of development. Guidelines set out in the Australian curriculum [Australian Curriculum Assessment and Reporting Authority (ACARA), 2013] indicate that around Year 2, children should be encouraged to perform simple addition and subtraction calculations using a range of strategies including counting on strategies, recall, decomposition strategies and rule-based strategies. By around Year 3, children should be able to recall addition facts for single-digit numbers and go on to develop increasingly efficient mental strategies for computation with multi-digit numbers. This pattern of transition is not always evident as some children still rely on counting strategies up to and beyond Year 7 (Hopkins & Lawson, 2006; Ostad, 2007). The impact of a longer processing time to execute simple addition is thought to become a barrier to learning as children are expected to solve increasingly more complex tasks (Kirby & Becker, 1988).

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It is often thought that the provision of sufficient practice is the key to fluent retrieval of addition facts. This assumption is captured in a concern expressed by the US National Mathematics Advisory Panel, (2008, p. 27) where it is stated that “few curricula in the United States provide sufficient practice to ensure fast and efficient solving of basic fact combinations.” While practice is important, research suggests that not all children benefit equally from practice with simple addition (Hopkins & Egeberg, 2009; Hopkins & Lawson, 2006; Lin, Podell & Tournaki-Rein, 1984).

A well-established theoretical model known as the distributions-of-associations model (Shrager & Siegler, 1998; Siegler & Jenkins, 1989; Siegler & Shrager, 1984) explains that associations between problems and answers strengthen in long-term memory as a result of correct practice with a back-up strategy. For simple addition, back-up strategies include counting strategies as well as decomposition strategies. This means that for practice to lead to direct retrieval and thus benefit children, children must be using a back-up strategy that consistently results in the correct answer. When this occurs, correct problem-answer associations are formed in memory, increasing the likelihood of direct retrieval. Furthermore, each time an answer is correctly retrieved, the association is strengthened further, making retrieval likely to dominate performance. If children make inconsistent errors using back-up strategies then the correct problem-answer association in memory will not strengthen but will weaken as incorrect associations form and compete with it - resulting in the continued use of back-up strategies. Alternatively, if the back-up strategy produces an incorrect but consistent error, an incorrect problem-answer association will strengthen in memory leading to a retrieval error.

According to the distributions-of-associations model, the most likely factor inhibiting the benefits of practice is the inaccurate use of back-up strategies. Whilst it has been well documented that students with a mathematics learning difficulty do not use direct retrieval to the same extent as their typically developing peers and produce more counting errors (Jordan & Hanich, 2000) and retrieval errors (Geary, 1990), few studies have examined in any detail the types of errors made by children who exhibit difficulty making the transition from counting to retrieval.

The investigation involved five children from a Year 3 cohort of children (n=60) at one school who showed signs of having difficulty learning mathematics and who were identified by their teachers as being reliant on counting for simple addition. Each child worked through a set of 36 simple addition problems each day, for eight days. The strategies they used to perform each problem, on each occasion, were recorded along with the errors they made. Four research questions were addressed:

1. Was there evidence to suggest that participants were not benefitting from practice with simple addition?
2. Did the number of errors across time intervals increase, decrease, or remain the same?
3. Were the same errors repeated or was a more inconsistent pattern of errors evident?
4. Could children’s errors be classified?

Method

Participants

Children were initially selected based on the criteria that they were low-achieving, as indicated by performance on a standardised test of achievement in mathematics (Wood & Lowther, 1999) and were identified by their Year 3 teacher as often counting to perform simple addition. A third criterion was later applied; that participants were using a counting on strategy rather than a counting all strategy – the latter being a particularly inefficient counting strategy for this age group. Children who were using a counting-all strategy received support to learn counting-on strategies and did not participate further in the study.

Participants were withdrawn from their class to participate in eight sessions to practice their simple addition skills. Each session lasted approximately 30 minutes and sessions were conducted in the morning on consecutive school days.

Instrument and procedure

Instruments used in the course of this study included a computer program designed to randomly display a simple addition problem from a set of problems, record the strategy used by children to solve each problem, measure the time taken to solve the problem (in milliseconds) and record the answer given by the child. The problem set comprised all 36 problems with addends greater than 1 (from 2+2 through 9+9) written in the form 'x + y =' where $x \leq y$. The smaller addend was written as the augend (e.g., 2 + 6 =) to enable the researcher to distinguish between the use of a min-counting strategy (where two is counted on) and a less mature count-from-first strategy (where six is counted on). Problems with an addend of one were omitted from the problem set due to difficulties in distinguishing between a retrieval strategy and a count of one.

When using the program, the researcher (first author) pressed the space bar and this activated the timer and a bell was sounded to alert the child that a problem was being presented. The child was not aware of the timer. The bell coincided with the presentation of a single-digit addition problem on the computer screen. Immediately after the child had calculated the answer the researcher pressed the space bar for a second time, which stopped the timer and removed the problem from the screen. The child was then instructed to type their answer in the computer and push the enter button. Following this, a screen appeared that prompted the child to explain the strategy they had used for calculating the solution to the problem just completed. The researcher also asked the child: "How did you do it?" After describing the strategy they had used to the researcher, the child selected the corresponding option on the screen: 'I counted' (for a min-counting strategy), 'I just knew it' (for a retrieval strategy), 'I did something else' (for a decomposition strategy), or 'I don't know' (if the child was unable to explain his/her thinking during the calculation of the problem). When this procedure had been completed, the process started again for the next problem when the researcher again pressed the spacebar and the next problem was displayed on the screen. No feedback was given to the child about the correctness of his or her answers during data collection.

Data collection and analysis

Children's self-reports of strategy use are considered the most suitable approach for documenting the strategies used by children to perform simple addition when they are collected on a problem-by-problem basis with an observer present (Siegler, 1987). It is important that a child is asked to describe the strategy they used immediately after performing a problem since children in a transition stage will use different strategies for different problems and different strategies for the same problem on different occasions (Siegler & Jenkins, 1989, Siegler, 1995). Thus, it is also important that data are not collected on just one occasion but on different occasions. The validity of self-reports for strategy use have been confirmed by Siegler (1987) and LeFevre, Sadesky and Bisanz, (1996), and have been subsequently used by several researchers (e.g., Canobi, Reeve and Pattison, 1998; Geary, Hamson, & Hoard, 2000). A case study approach was taken to analyse the data. Each child's performance (representing a case study) was scrutinized and the findings were synthesised and summarised.

Results

Area graphs were produced to illustrate the strategies each child used to correctly perform the set of 36 problems over eight time intervals (see Figure 1). Due to reasons unrelated to the study, Jane only participated in seven practice sessions. Strategies that produced errors are not included in Figure 1 but are analysed separately below.

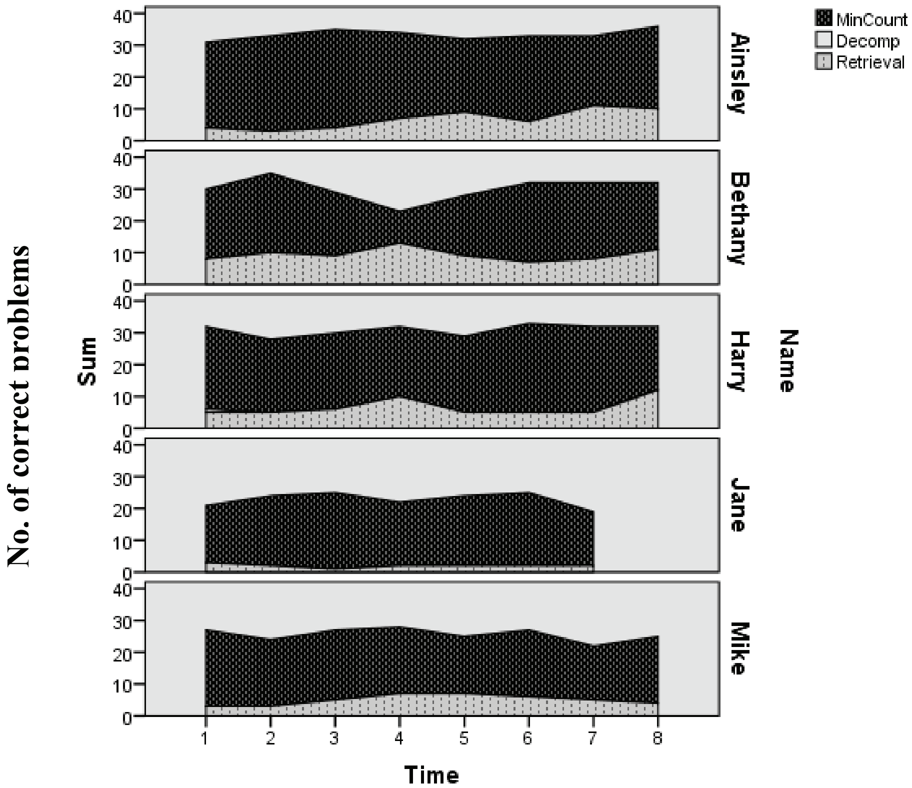

Figure 1. Area graphs depicting accurate strategy use only across eight time intervals. All names used are pseudonyms.

The graphs shown in Figure 1 suggest that practice, by itself, was not a particularly effective approach for increasing the likelihood of direct retrieval for these five children. Ainsley demonstrated the greatest increase in her use of retrieval, from 4 problems in time 1 to 11 in time 7. A closer inspection of the different strategies used to perform each problem on different occasions showed that Jane was not consistently retrieving any problem in the set but the other four children were consistently retrieving at least some of the lower tie problems. These four children switched between min-counting and direct retrieval to solve problems with a minimum addend of two.

The number of errors made by each child across time intervals was illustrated using bar graphs (see Figure 2). These graphs were examined to see if the number of errors decreased over time, (suggesting errors were self-corrected).

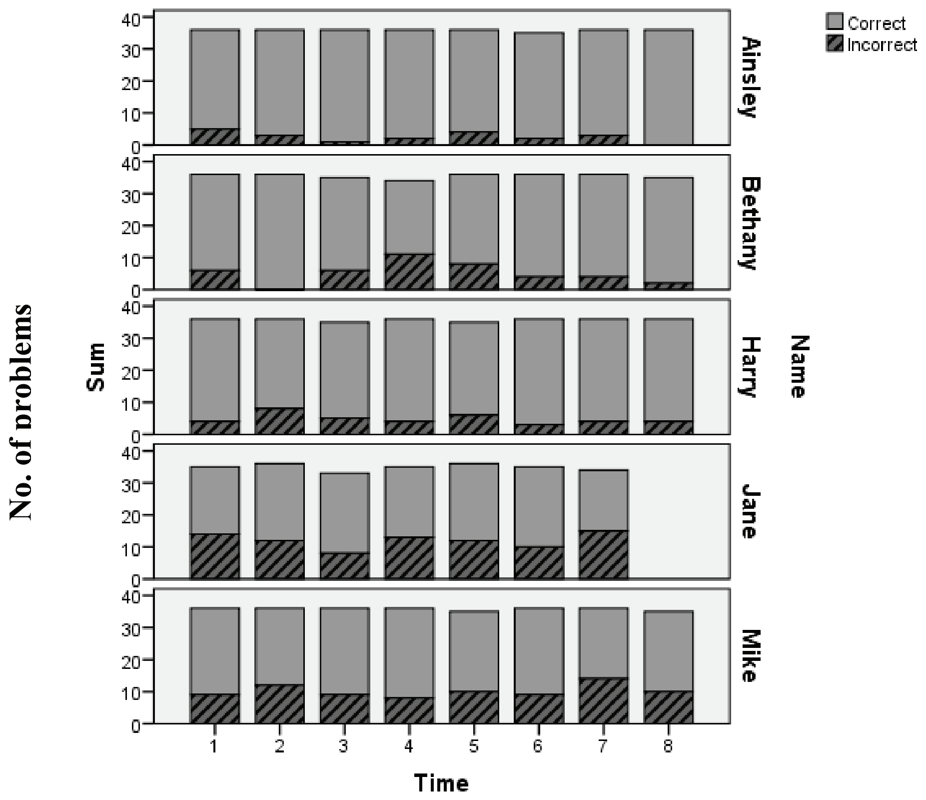

Figure 2. Bar graphs depicting the number of incorrect and correct problems across eight time intervals.

There was no obvious pattern of decrease in the number of errors made over time. Although the bars in Figure 2 suggest that the number of errors were similar for each child across time intervals, this does not necessarily mean that each child was making the same errors each time. A closer examination was made to identify where errors had been repeated.

A repeated error is one where the same strategy produces the same incorrect answer on more than one occasion. Repeated min-counting errors and decomposition errors are suggestive of a procedural "bug" where a strategy is misapplied because of a misconception or faulty rule. According the distribution-of-associations model, repeated min-counting errors and decomposition errors are likely to result in repeated retrieval errors. Min-counting and decomposition errors that are not repeated are suggestive of a difficulty keeping track of the count and/or monitoring the processing episode.

Table 1
The number of repeated (R) and non-repeated (NR) errors by each participant

	Min-counting			Retrieval			Decomp	DK	Total
	NR	R	total	NR	R	total			
Ainsley	10	8	18	2	-	2	-	-	20
Bethany	18	12	30	7	3	10	-	1	41
Harry	10	12	22	1	13	14	2*	-	38
Jane	50	32	82	2	-	2	-	-	84
Mike	29	14	43	8	30	38	-	-	81

*These decomposition errors were not the same.

The majority of min-counting errors were not repeated. Those that were repeated were scrutinised to uncover a possible pattern. A pattern was identified for Jane who often confused a 9 with a 6 (calculating, for example, that 5+9=11 on more than one occasion). Overall, retrieval errors were not nearly as common as min-counting errors. Mike was an exception as he made 30 repeated retrieval errors. These repeated errors were scrutinised and an obvious pattern emerged: Mike was applying a faulty rule to tie facts stating that 4+4=14, 6+6=16, 7+7=17, 8+8=18, 9+9=19. Also, both Bethany and Harry consistently retrieved the answer 14 when shown the problem 8+8. It was thought they had confused the tie fact (with 7+7). It was also noted that each child made min-counting errors on problems with small minimum addends (2 and 3) and well as problems with larger minimum addends.

Min-counting errors were then classified. Stating one more than the count, one less than the count, or starting at one addend and counting on the same addend (producing an incorrect answer that is double one of the addends), is indicative of losing track whilst applying a counting strategy. These errors were classified as being *one over*, *one under* or *double addend* respectively (see Table 2). For problems where there is a difference of one between addends, an error of one more (or one less) than the correct answer is equivalent to a doubled addend (e.g., 4+5=10 or 4+5=8). To avoid double coding these errors, a separate code was used, namely *difference of one*. Min-counting errors resulting from confusing a 9 for a 6 were coded as *9-6 mix*. Any min-counting errors that could not be classified using these five categories were coded as *other* (these represented errors that were out by more than one and did not appear to follow a pattern).

As shown in Table 2, the majority of min-counting errors could be classified: 89% (16/18) of Ainsley's min-counting errors were classified, 87% of Bethany's errors, 68% of Harry's errors, 76% of Jane's errors and 81% of Mike's errors. Apart from Jane's errors caused by confusing 9 and 6, all classified min-counting errors were indicative of children losing track of the count during the counting procedure.

Table 2
Classifying min-counting errors

	One over	One under	Double addend	Differen ce of one	9-6 mix	Other	Total
Ainsley	4	2	5	5	-	2	18
Bethany	7	13	2	4	-	4	30
Harry	8	6	-	1	-	7	22
Jane	2	12	9	6	33	20	82
Mike	7	20	4	4	-	8	43

Retrieval errors were similarly classified (see Table 3). Errors resulting from applying an incorrect rule-based strategy for tie problems was labeled *faulty rule* and retrieval of the wrong tie fact was labeled *double fault*.

Table 3
Classifying retrieval errors

	One over	One under	Double addend	Difference of one	Double fault	Rul e	Other	Total
Ainsley	1	-	-	-	-	-	1	2
Bethany	3	1	-	-	3	-	3	10
Harry	1	3	-	-	4	-	6	14
Jane	-	1	-	-	-	-	1	2
Mike	-	7	-	-	-	25	6	38

While there were fewer retrieval errors to classify, most could be classified. Retrieval errors included errors that were one off the correct answer. This finding is consistent with the fact that children were often over counting or under counting answers by one.

Discussion

A group of five children were closely monitored as they practiced their skills solving simple addition problems each day, for eight days. Their infrequent use of direct retrieval confirmed that they had made little progress towards becoming proficient with simple addition. The considerable number of counting errors made over this time suggested that inaccurate performance was a significant factor contributing to their lack of progress. While teachers need to monitor practice to make sure children are performing back-up strategies accurately, a noteworthy finding was that the majority of each child's counting errors were not repeated but were indicative of the child losing track of the steps during the counting procedure. For teachers, this makes recognising and addressing children's inaccurate performance difficult since errors are more random than consistent.

Further research is needed to explore how to best improve children's accuracy with counting. Few empirical studies have been published but recommendations have been made. Jordan, Kaplan, Ramineni, and Locuniak (2008) recommended that children, who want to use manipulatives or their fingers to help them keep track of the count, should be encouraged to do so. Ostad and Sorensen (2007) suggested that systematic training to promote private speech internalisation could be effective. There is also a lack of empirical research that has used robust assessments of strategy use to explore the advantages of teaching children an alternative to counting as the principle back-up strategy for learning to retrieve simple addition facts, in particular for children who tend to lose track of the count. These are important directions for future research.

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