From Arithmetic to Algebra: Sequences and Patterns as an Introductory Lesson in Seventh Grade Mathematics

Diana Grace Aniban
De La Salle University – Manila
<dianneaniban@gmail.com>

Von Christopher Chua
De La Salle University – Manila
<von_christopher_chua@dlsu.edu.ph>

Jellen Garcia
De La Salle University – Manila
<garcia.jellen@yahoo.com.ph>

Levi Esteban Elipane
University of the Philippines – Diliman
De La Salle University - Manila
<levielipane@yahoo.com>

Guided by the principles of lesson study as applied to microteaching, this paper discusses the results and conclusions of a series of activities done by some graduate students of De La Salle University, Philippines, in an attempt to test the applicability of the lesson – Sequence and Patterns – to facilitate the transition of seventh graders from arithmetic to algebra. The post-lesson discussion and *a posteriori* analysis proved the lesson to be a practical means to address the issue as it was able to put forward discourses and elaborations on possible students' understanding of variables generated through attempts to describe patterns occurring in sequences presented.

Algebra is an important area of mathematics used in generalising arithmetic through letters, symbols and signs—the use of which makes it an abstract subject (Ali Samo, 2009). Furthermore, the abstractness of algebra is one reason for students' problems (Lee & Wheeler, 1989). It has been the practice of mathematics education to start with the more basic knowledge of arithmetic and use this as the basis for the more abstract field of algebra. So, when students start to learn the more complex mathematics, they are expected to be proficient in the concepts of arithmetic.

The shift from arithmetic to algebra is considered to be a difficult but an essential step for mathematical progress. As many teachers would attest, students may find it difficult to accept the notion of letters being used to represent numbers. The contradiction of how they defined numbers to be specific quantities with exact values and how letters may represent general numbers oftentimes leads the students into confusion. It is therefore necessary that teachers examine the feasibility of probable lessons that may aid in the smooth transition between the two branches of mathematics giving students the opportunity to connect previous and new knowledge. These lessons present mathematics as coherent, systematic and cohesive.

Steedly, Dragoo, Arafeh, and Luke (2008) argued that a systematic and explicit instruction guides students through a defined instructional sequence. In this sense, we see that mathematics education should be done with a defined sequence of topics that are not entirely isolated from each other. Mathematics educators are faced with the need to establish linkages that let students see how one develops from the other. The transition between these two may just be a part of the whole mathematics curriculum but those lessons that culminate arithmetic and commence algebra may just mean the difference between students' understanding of how mathematics evolves from one concept to another and increased mathematics anxiety.

2014. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). Curriculum in focus: Research guided practice *(Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia)* pp. 63–70. Sydney: MERGA.

In fact, several researchers have investigated cases on the transition from arithmetic to algebra. For instance, Warren (2003) said that the majority of students are unable to abstract the relationships and principles in arithmetic that are needed for algebra. This should urge researchers to probe appropriate learning experiences that would assist students to move successfully from arithmetic thinking to algebraic thinking. Darley (2009) also pointed out the necessity for a teacher to neither assume that students are aware of the 'arithmetic to algebra' connections nor assume that there is a deep understanding of underlying arithmetic concepts. Without the aid of an effective topic that clearly links the two disciplines the student is left unknowledgeable about how mathematics has evolved from constants into variables.

This paper discusses the conduct of a series of activities that tested the feasibility of such a lesson described above. The topic on sequences and patterns was used as an introductory lesson in algebra. It examined whether or not this practice would be able to properly foster students' understanding of basic concepts in arithmetic to the introduction of the ideas of variables, therefore eradicating the possible students' perception that the two are entirely different, unrelated branches of knowledge. Specifically, the research activity described in this paper was done with the following objectives: (1) to assess the feasibility and suitability of the lesson on sequence and patterns as a transition lesson from arithmetic to elementary algebra; and (2) to employ some of the basic principles of lesson study in microteaching and evaluate the strategy as a means of improving instruction.

The study was conducted by three graduate students of De La Salle University (DLSU), Philippines, together with their Theories of Teaching and Learning professor during the second term of the 2013-2014 academic year. The fundamental principles of lesson study are encapsulated in the doctoral dissertation of the professor (Elipane, 2012).

Theoretical Background

Based on ideas of Vygotsky's theory of constructivism, the authors planned a problem-based lesson on the topic taking into consideration students' previous knowledge of arithmetic and set theory to present definitions of constants and variables. Vygotsky's theory emphasises that learners gain knowledge through interaction with their environment in order to solve problems. A ten year study by Crouch and Mazur (2001) showed that most students learn more from group learning activities than they do studying alone or listening to the teacher dispense information.

Moreover, two principles of constructivism developed collectively from the ideas of prominent constructivists support the paper's aim to investigate transition lessons. First, learning is contextual, and second, knowledge is needed to learn (Hein, 1996). These principles point out the idea that human beings learn through associating the idea presented to them to their prior knowledge. Any effort to teach must direct the learner into the subject with a basis on that learner's previous understanding. In the case of learning algebra, one can infer that students will only begin to appreciate variables if they see the essentiality of using them—something that is based on their knowledge of numbers and how their values are considered constant.

Teachers are typically acutely aware of this role of prior knowledge in students' learning. Some overlook the fact that students bring with them a rich array of prior experiences, knowledge and beliefs that they use in constructing new understandings (Jones & Brader-Araje, 2002). These preconceptions work to both the advantage and disadvantage of learning but the teacher should find a means to make use of the essential ideas as a good means to properly institutionalise new concepts.

Such is the case when introducing variables in algebra to students who think that knowledge on arithmetic seems weakly relevant. Understanding the concept of a variable has a vital importance in constituting a strong basis for other algebraic concepts related to it (Balyta, 1999). If the concept of a variable is not thoroughly grasped, the student would encounter the succeeding topics with inceasing difficulty. Guided by these principles, the researchers were led to think that transition lessons developed and tested through research would facilitate the introduction of algbra more effectively than the usual exposition.

In order to measure how efficient the transition lesson discussed in this paper is, the authors adapted the core principles of lesson study in microteaching. Although there were limitations, such as the learning environment was not typical to public schools in the country or the inauthenticity of the supposed students, the concept of research through practice was evident in the whole process.

Method

It is through the Theories of Teaching and Learning class that the authors came across the principles of lesson study. The class was required to carry out activities similar to those used in lesson study. The authors cooperatively identified the problem and planned for an activity that would test a solution and there was a post-lesson discussion that focused on evaluating the process and results. In general, the practice may be analysed through three different phases: (a) *a priori* analysis; (b) data gathering done through the actual microteaching; and (c) *a posteriori* analysis through post-lesson discussion.

A Priori Analysis

The initial phase focused on identifying the specific goal of the group with the main intention of presenting an innovation to a practice in mathematics education through research. Considering major educational reforms done in the country during the conduct of the activity, an analysis of a problem arising from this transition seemed relevant. Specifically, the modification of linear to spiral progression in the Philippine mathematics curriculum prompted us to question how instruction transcends from the elementary school to the seventh grade where algebra is formally introduced. The country has recently decided to modify basic education to meet the standards of the international community.

Upon inspection, it was found that there was no specific lesson listed in the curriculum (Department of Education, 2010; 2012) that aimed to introduce the concept of variables using prior knowledge as the basis. To verify this, seven public school teachers who teach Grade 7 mathematics where interviewed informally regarding the method they use to present variables in class as part of introcuding algebra. Five use deduction while the others use exposition. They have continuouly encountered students having difficulties in the lesson. Two teachers said their students found the lesson to be disconnected from what they learned in their past mathematics lessons making the idea seem extremely new to them. This supports the need for a lesson that would help students establish linkages between their prior knowledge and algebra but this further raises the question of what lesson may be used to serve this purpose.

In the study conducted by Darley (2009) the number line was used as the transition lesson for students to ‘travel back and forth’ from arithmetic to algebra. The author cited the practice as an aid to promote a solid foundation for learning algebra. For this paper, the lesson on sequence of patterns was tested for its feasibility of addressing the same objective. Selection was based on the following reasons: (1) simple integer sequences are developed as an application of fundamental operations in arithmetic—a prerequisite to learning algebra; (2) knowledge of arithmetic is essential and is put to the test when students solve for the succeeding terms; and (3) generating patterns can only be done by careful examination of mathematical sentences prompting the students to observe how numbers may vary from one sentence to another.

Constructivism led to the decision of utilising a problem-based lesson to generalise the students' knowledge through inquiry. Moreover, students should know how to apply conceptual knowledge and use the tools available in their environment (Driscoll, 2005). This, on the other hand, impelled the use of manipulatives in the activity.

Data Gathering Through Microteaching

After planning, the second phase was done wherein one of the authors was assigned as teacher and was then observed by a class of master's degree students specialising in mathematics education in DLSU, Manila. Six graduate students specialising in physics and biology education acted as grade seven students. The group tried to make the situation as realistic as possible. This included conditioning the supposed students by taking the mindset of typical seventh graders and using instructional materials that are typical to the Filipino classroom.

A Posteriori Analysis

The final phase of the activity was an overall assessment with a focus on how effective the selected topic is for introducing variables and the efficiency of lesson study based microteaching as a method of obtaining and analysing data. The group reflected on the outcome, narrating the salient points, and the others gave comments making the activity reflective. This is elaborated in the next part of the paper.

Statements, comments and actions were taken to address the problem posted. Data were gathered through video recording, the contents of which was analysed after it was transcribed. The success of the activity was not solely determined by how effective the microteaching was, but it also took into consideration the problems which rose during pre- and post-microteaching, giving the group the opportunity to conduct similar, more efficient lesson study based activities in the future.

Results and Discussion

For this unit of the paper, the results of the research practice are discussed through three parts. The first gives a comprehensive narrative of what happened during microteaching and the responses of students to the questions raised. Under the second part, salient points in the post-lesson discussion are used to explain what occurred in the lesson proper with emphasis on the areas needing improvement. Finally, reccommendations are raised based on the outcome of the whole set of activities.

Microteaching

With the lesson being anchored by the principles of constructivism, the lesson started with the following problem which the class had to solve in pairs.

> You are given a set of 25 unit triangles of the same sizes. Your first task is to find how many unit triangles you need to form the smallest possible triangle. Then, identify how many unit triangles you should add to have the next biggest triangle. Do this repeatedly until it is impossible to make any bigger triangle from the last. Can you predict how many more unit triangles of the same size you would need to create the next biggest triangle? How about the one after that?

While doing the activity, a pair of students got confused with the directions presented to them. Specifically, with what the teacher meant by a "triangle" which they had to form from the unit triangles given. This led to the pair asking the teacher if one unit triangle is enough to construct the triangle they are after. In response, the teacher asked the students what they thought about it and they responded with affirmation reasoning that the unit triangle is in itself a triangle so they can consider it as the smallest.

All three pairs of students were able to form the following figures out of the 25 unit triangles.

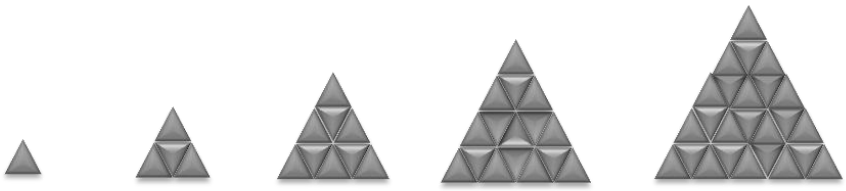

Figure 1. Triangles formed by students during the activity.

During discussion, the students answered the guide questions. Their actual responses are as follows.

> "One unit triangle was needed to form the smallest triangle."
>
> "Three more were added to form the next, then five, seven and nine."
>
> "Eleven will be needed to form the one after that and thirteen for the next."

In between these questions, students were asked to react to the responses their classmates raised. The class generally agreed to the ideas presented. For the last two questions, students explained how they attained the answers. A part of the class discussion is presented below.

Teacher:	So we have the set of numbers: 1, 3, 5, 7, 9, 11, 13… How did you know that you needed eleven more unit triangles to form the next triangle after the last?
Student:	We noticed that when you add two to one number, you get the next one.
Teacher:	How would you prove that?
Student:	For example, to get three, we add one plus two. To get five we add three and two.

While students are giving explanations, the teacher translates these verbal answers into mathematical sentences on the board as follows:

$1 + 2 = 3$ $\quad$ $3 + 2 = 5$ $\quad$ $5 + 2 = 7$ $\quad$ $7 + 2 = 9$ $\quad$ $9 + 2 = 11$

From here, they verified if they have common processes. These responses led the teacher into inquiry to help students define sequences. Then a formal definition was provided thereafter. Three other examples were then given and the students predicted the next three numbers in each sequence with ease. The teacher then proceeded as follows:

Teacher:	Can we present the process you all used in a different form? Is it possible to use only the first number and the number you continuously added?
Student:	I think five can be expressed as 1 + 2 + 2.
Teacher:	Why did you say that?
Student:	Since we got three from 1 + 2 and we added another two to get five.
Teacher:	Can we do the same for the other numbers?
Student 2:	Yes sir. To get seven, that is 1+2+2+2. To get nine, we add 1+2+2+2+2.

Similar with what has been done in the previous responses, these were also written on the board beside the original mathematical sentences. The teacher then asked what they noticed about the equations they have formed. To this they explained that the quantity of two's added increases but the number 1 does not. This prompted the teacher to ask if they can make use of multiplication instead of addition. The following equations are the results of this question:

$1+2(1)=3$ $1+2(2)=5$ $1+2(3)=7$ $1+2(4)=9$ $1+2(5)=11$

This showed that students were gaining ideas of constants and variables. The next conversation was very crucial to how variables were introduced; however, this is also where the teacher seemed to encounter difficulty in directing the class towards what he wanted them to infer.

Teacher:	What do you notice about the equations you have formed? What makes them similar? What is different about them?
Student:	All of them have the number 1 and 2 but the other numbers are increasing.
Teacher:	Which number is increasing?
Student:	The one multiplied to 2.
Teacher:	So if we want to know a number which is part of this set, how do we find out that number?
Student:	We add 2 to that number?
Teacher:	You have a point, but what if we don't know what the previous number is? Like how do we know what is the 100^{th} number in our set of numbers?

The class did not directly respond to this question so the teacher asked them how the term in the sequence is related to the "changing numbers" they noticed in the last set of equations. They were then able to identify that the number is one less than the term. Consequently, one student pointed out that to get the 100^{th} term of the sequence, the term should be multiplied by two and then subtracted by one. This was verified by solutions shown on the board. Finally, he presented the concept of variables as the term in the sequence discussed reasoning out that the value changed depending on the term. The pattern therefore was presented as $2n$-1, where n is the nth term of the sequence.

Post-Lesson Discussion

In the post-lesson discussion the whole graduate class was given the opportunity to evaluate the outcome of the microteaching with an emphasis on whether or not sequence and patterns would be a good choice for a transtion lesson to algebra. Some of the more important points brought up are enumerated as follows:

1. Would a different problem on sequences aside from the unit triangles activity be more effective? This question was based on two claims. First, teachers think that problems based from students' real life experiences tend to be more effective than

those that are purely mathematical. Second, those who acted as students initially found the instruction a bit confusing. Part of this was due to the fact that the teacher's definition of a "unit triangle" was not elaborately presented.

2. How important is the formulation of questions in this kind of a lesson? The class agreed that the teacher should be able to guide the students into the knowledge targeted by the lesson. In this case, it is crucial for the students to discover the necessity of variables by themselves. Asking the right questions at the right time and in relation to the ideas the students presented themselves would establish a coherent generalisation. This is further supported by Rahimi and Ebrahimi (2011) who said that the nature of questions presented to students greatly influences the depth to which the students search for answers.

3. Are the use of manipulatives and other visual materials necessary? The students generally found working with the unit triangles easy. The manipulatives presented sequences as not entirely being abstract but the teacher may choose to be more creative with the kind of materials presented to the class.

In general, the class agreed that sequence and patterns as a transition lesson can potentially present variables in a less abstract form, making them more relatable to the students. Its success, though, heavily depends on the teacher's ability to guide the learners without dominating the class. The less teacher intervention there is, the more students will appreciate the need for the existence of variables in mathematics. Careful planning is required to meet the expectations of both students and teachers with a high regard for details such as asking the right questions at the right time, the sequence of examples to be presented and the method of guiding the students into forming their generalisations.

Recommendations

Sequence and patterns as a transition lesson can potentially introduce variables and algebra in connection to and with consideration of students' prior mathematical knowledge. Yet, the teacher should consider the following recommendations to improve such practice.

1. Formal definitions of sequence and patterns do not need to be presented as they can only complicate the lesson. The idea behind the topic should only act as the means to introduce variables and should not therefore be the focus of the discussion.
2. The teacher should deliberately list a series of guided questions to ask the class during the discussion and prepare for possible responses that might lead the class into ideas not entirely related. This would help the class stay on track of the discussion with the objective as end. This may also help avoid the instance of giving away the answer instead of the students expressing freely what they think and then generalising afterwards.

3. A lesson study dealing with the same concept and objective should be done with actual students. This is to test whether the responses expressed by the conditioned students in the activity would be similar in the proposed case.

4. An examination of other potential transition lessons for arithmetic-algebra transition through lesson study could help in finding more efficient means of transcending from arithmetic to algebra

Final Remarks

The authors believe that this paper has brought about two important ideas. First, the use of sequence and patterns as a lesson to aid transition from arithmetic to algebra, specifically in placing significance on variables is very promising if developed further. Considering it shows a clear link of generalising a set of numbers into an algebraic expression makes it suitable for that objective. It cannot be denied though that there is a need to do similar microteaching activities dealing with the same topic but based on the results that this paper narrates. Second, lesson study is a good avenue for the educational community to build lessons based on practice and not on general theories. It gives the teacher an idea of what works and what does not, given a specific kind of learner or class. It also promotes a healthy and productive relationship amongst teachers in the school community by fostering constructive criticism and the need for professional development.

References

Ali Samo, M. (2009). *Students' perceptions about the symbols, letters and signs in algebra and how these affect their learning of algebra: A case study in a government girls' secondary school.* (Unpublished master's thesis). Aga Khan University, Karachi, Pakistan.

Balyta, P., (1999). *The effects of using motion detector technology to develop conceptual understanding of functions through dynamic representation in grade 6 students.* (Unpublished master's thesis). Concordia University, Montreal, Quebec, Canada.

Crouch, C. H., & Mazur, E. (2001). Peer instruction: Ten years of experience and results. *American Journal of Physics*, *69,* 970-977.

Darley, J. (2009). Traveling from arithmetic to algebra. *Mathematics Teaching in the Middle School, 14*(8), 458-464.

Department of Education. (2010). M*inimum learning competencies – Basic Education Curriculum*. Manila: Author.

Department of Education. (2012). *Minimum learning competencies – K-12 Curriculum.* Manila: Author.

Driscoll, M.P. (2005). *Psychology of learning for instruction (3rd Ed.).* Boston: Allyn and Bacon.

Elipane, L.E. (2012). Integrating the essential elements of lesson study in pre-service mathematics teacher education, IND skriftserie no. 27. Copenhagen: Department of Science Education.

Hein, G. (1996). *Constructivist learning theory.* Retrieved December 18, 2013, from http://www.exploratorium.edu/ifi/resources/constructivistlearning.html.

Jones, G., & Brader-Araje, L. (2002). The impact of constructivism on education: Language, discourse, and meaning. *American Communication Journal, 5(3)*, 25-35.

Lee, L., & Wheeler, D. (1989). The arithmetic connection. *Educational Studies in Mathematics, 20*(1), 41-54.

Rahimi, A., & Ebrahimi, N. (2011). Constructivist vs. objectivist learning environments. *Contemporary Online Language Education Journal, 1*, 89-103.

Steedly, K., Dragoo, K., Arafeh, S., & Luke, S. (2008). Effective mathematics instruction. *Evidence for Education National Dissemination Centre for Children with Disabilities, 3(1),* 1-12.

Warren, E. (2003). The role of arithmetic structure in the transition from arithmetic to algebra. *Mathematics Education Research Journal, 15*(2), 122-137.