Linking GeoGebra to Explorations of Linear Relationships

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Increasing pressure is mounting from all areas of society to maximise technology use within educational domains. Whilst curriculum documents call for the utilisation of technology as a teaching tool in the mathematics classroom, the benefits of exploring forms of dynamic mathematical software, such as GeoGebra, are often introduced in the senior years. This study investigates the challenges and understandings of a Year 9 Mathematics class who were using GeoGebra for the first time.

Numerous studies (Brown, 2010; Hopper, 2009; Kissane, 2007) have provided substantial evidence on the benefits of technology when learning mathematical concepts. Despite this evidence, various factors inhibit successful implementation and navigation of technology. As a result, when students require technology in senior years to further develop their understanding of conceptual ideas which will be assessed in final examinations, they often fall short. This is due to a lack of consolidated skills that would enable them to competently advance with technology and adequately provide time to facilitate exploration to improve and develop these skills further.

One strand of mathematics that forms an essential *building block* from which more complex analysis of change is built is the development of an understanding of the concepts of linear relationships. Notoriously, many students manage to complete closed assessment tasks by providing responses that merely follow a set of instructions as opposed to finding their own way with meaning to the solution of a problem. The proposed research study is unique as it explores characteristics of conceptual development alongside a targeted investigation of the nature of the student engagement whilst they proceed through a unit of work (four-week sequence), making use of the GeoGebra learning environment.

Background

Technology is recognised as a changing medium, through which, students can explore interactive tasks in varied contexts (Hopper, 2009). Studies involving technology in mathematics are constantly emerging, however, a majority of them deal with graphics or Computer Algebra System calculators (Brown, 2010; Cavanagh, 2006). Other studies include those that have been centred around the benefits of using technology (Kissane, 2007), teacher professional development or pre-service teacher training (Goos, 2005; Obara, 2010) and discussion about introducing technology into assessments (Brown, 2010). Recent studies have also reported on the factors inhibiting teachers' use of technology in mathematics classrooms (Goos & Bennison, 2008). However, few studies were found that reported specifically on students' difficulties (Mitchelmore & Cavanagh, 2000). Teaching with technology requires a change in role of the teacher and the respective pedagogy

2014. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). Curriculum in focus: Research guided practice *(Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia)* pp. 79–86. Sydney: MERGA.

(Ertmer, 2005). Learning becomes more student-orientated as opposed to concept-orientated (Allan, 2006). The open-source software, GeoGebra, was released in 2004. One of GeoGebra's strengths is that it can facilitate the mathematical learning of students of every level ranging from primary to university (Hohenwarter, Hohenwarter, Kreis, & Lavicza, 2008; Kllogjeri & Shyti, 2010).

The four-week teaching unit, that is central to this study, applied Van Hiele's teaching phases as a five phase pedagogical framework to sequence the student activities. Van Hiele's levels of thinking and learning phases are widely documented in studies, as a framework on which to base student understanding (Serow, 2008). The effectiveness of the phases is not presented in this paper and will be a focus of subsequent research. The questions which form the basis for this research are:

- What are the characteristics of students' responses when exploring concepts of linear relationships using dynamic mathematics software?
- What is the nature of student interaction when using GeoGebra as an exploration tool?

Method

The study uses a pre-experimental design with one class from a systemic Catholic secondary school within the Wagga Wagga Diocese during 2013. The design used a pre-test, teaching intervention and end of topic test with a delayed post-test (6 weeks delay) to ascertain any change in the nature of students' responses. The unit of work was programmed according to the new Australian Curriculum for Year 9 Stage 5.3 (New South Wales Board of Studies, 2012). It involved a four-week teaching sequence (fifteen lessons of sixty-three minutes) for a Year 9 (14-15 years) mathematics class. The researcher and classroom teacher collaboratively implemented team teaching strategies for the teaching sequence (Conderman, 2011). The focus content of the teaching sequence was the sub-strand Linear Relationships. The outcomes addressed were "uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures", "generalises mathematical ideas and techniques to analyse an solve problems efficiently", "uses deductive reasoning in presenting arguments and formal proofs", and "uses formulas to find midpoint, gradient and distance on the Cartesian plane, and applies standard forms of the equation of a straight line" (New South Wales Board of Studies, 2012) .

The mathematics class consisted of 26 students with an equal number of boys and girls. From this class, 25 students agreed to participate in the study. Purposive arrangements were implemented by the daily classroom teacher to pair students together according to similar mathematical ability and social connections to ensure that students would confidently discuss the tasks for the duration of the unit sequence. The pre-test, end of topic test and delayed post-test were delivered using an online Google Form. The students were required to navigate these tests just as they would approach a new app or game. Each student had individual access to a laptop with GeoGebra and screen-capture software. The students were required to complete the task using GeoGebra first, with its tools and functions, and then without GeoGebra using only pen and paper techniques.

The teaching sequence is summarised below in Table 1. On completion of the teaching sequence, students completed the end of topic test online using Google Forms with an extension question provided on paper.

Table 1
Lesson Teaching Sequence

Lesson Title	Content Description
Real Life Relationships	Investigated simple geometric patterns and plot graphs, creating a rule in algebraic terms and repeat using GeoGebra
	Algebra Walk activity
	Concepts of y-intercept, gradient and resulting equation investigated using GeoGebra
	Derive formula for gradient
	Determine equation of lines with/without GeoGebra
Midpoint	Concrete paper folding activity to identify midpoint
	Linking the midpoint to the coordinates using GeoGebra
	Deriving a formula for finding the midpoint without drawing checking using GeoGebra
Distance	Pythagoras' theorem
	Linking the theorem to coordinates on a graph
	Deriving a formula and using to calculate the distance checking solutions with GeoGebra
	Mixed problems involving both concepts using GeoGebra
Further Graphing – Parallel & Perpendicular Lines	Drawing graphs and linking the properties of graphs to their equations using GeoGebra
	Establishing criteria for graphs to be parallel and perpendicular
Geometric Problems	Investigate properties of the shapes looking at features through using concepts learnt using GeoGebra

The focus of this paper centres on students' attempts and pair dialogue when asked to draw a straight-line graph. The students were asked: How do you draw the line $y=4x+8$ using GeoGebra? Without GeoGebra? What changes are noticed when the graph is moved within the GeoGebra environment?

Analysis of data occurred following a content analysis of each response. This involved categorising the raw qualitative data into specific characteristics or patterns of commonality indicative of inductive coding. The aim to understand the context with which the themes occur, such that, meaning and inferences are recognised that make the data intelligible and relevant to the research questions (Fetterman, 1989; Hammersley, 1995).

Results

A summary of the results is presented under headings corresponding to the questions from the study with the responses coded into common response types. The pre-test and end of topic test recorded the results of 12 and 10 student pairs respectively. For the delayed post-test 19 students submitted their results individually. The number of each response characterised to each response type can be found in Tables 2, 3, and 4 at the end of each sub-section. The pairs are referred to as colours for researcher identification between tests.

Question 6. Using GeoGebra can you draw the graph of $y=4x+8$*?*

Type A Response. This type of response indicated a limited to no understanding of how to input an equation into GeoGebra. Typical responses answered whether or not the graph could be drawn and incorrectly described procedures for doing this. For example, Team Brown indicated, "It is too hard to extend the numbers" and three student pairs left the comment blank. Whilst no student pairs provided this type of response in the post-test, the delayed post-test saw four students coded as Type A.

Type B Response. This type of response indicated recognition that the equation needed to be typed into GeoGebra but no specific direction as to how to do this. For the pre-test one student pair, Team Orange, coded a Type B response stating, "By typing in the equation GeoGebra automatically does it for you." In the end of topic Test four student pairs recorded this type of response. A typical Type B response is: "Type in the equation", "Write it in the tool bar", or, "I put the equation into GeoGebra as the question says."

Type C Response. This type of response correctly indicated that the equation needed to be typed into the input bar of GeoGebra. Explanation included key-words such as *bottom of the page*, *input bar* and *bottom bar* to provide clarification on how to perform the required operation. A typical response provided by Team Lemon being, "Yes, I wrote the equation in the input bar." Team Lime responded with, "We wrote in the bottom bar of GeoGebra $y=4x+8$ and it plotted it for us." In the delayed post-test Team Maroon A clearly articulated that, "To create the graph of $y=4x+8$, I wrote the equation into the input bar which then automatically created the graph."

Table 2
Drawing the Graph of $y=4x+8$ *using GeoGebra*

Response Type	A	B	C	Total
Pre-Test	5 (42)	1 (8)	6 (50)	12 pairs (100)
Test	0 (0)	4 (40)	6 (60)	10 pairs (100)
Delayed Post-Test	4 (21)	5 (26)	10 (53)	19 students (100)

Percentages of the sample for each response type in each test are included in brackets.

a. How would you do this without GeoGebra using pen and paper?

Type A Response. This type of response indicated no knowledge of how to draw a graph in their workbooks. This was demonstrated using a blank comment or words reflecting limited understanding. Comments included, "I don't know" or "No idea." For the end of topic test only one of the ten student pairs were coded to this type of response. For the Delayed Post Test, six students were coded as a Type A response, two students admitting they did not know the remaining four leaving blank comments.

Type B Response. This type of response indicated a basic understanding of what was required to draw the graph. No specific instructions were offered in the response and it would be difficult to draw a graph using the information provided. In the pre-test one student pair provided a Type B response, Team Red stated, "Use a ruler." Two student pairs claimed to "Use a formula" but failed to provide any description of what formula to use. In the delayed post-test five students gave mixed results on how to draw the graph: Team Orange A stated, "Work out the sum" with no clarification as to what "sum"; Team

Red A stated, "By using a calculator and a ruler" without mentioning what calculation should be performed; Team Lemon stated, "Using a certain formula."

Type C Response. This type of response builds on Type B indicating that whilst understanding was present, specific elements were either incorrect or not enough information was provided to enable the reader to follow the instructions to draw the graph. For example, Team Indigo stated, "Draw a Cartesian plane and plot the y-intercept on the y-axis and put the gradient on the x-axis", incorrectly describing the function of the gradient; Team Cream stated, "Start at the y-intercept which is 8"; and Team Lime who stated, "$y=4x+8$, rise = 4, run = 1, y-intercept = 8", both requiring more information.

Type D Response. This type of response indicates an understanding of how to draw a graph; however, they lacked details relating to the graph in question. In the pre-test Team Brown was coded a Type D response when they submitted, "Draw a table with x and y." For the end of topic test two student pairs were coded this type of response with Team Brown again "$y=mx+b$, m = gradient, b = y-intercept. To find m use rise over run. To find y use the interval and it will pass through a point on the y-axis." The student did not link the information with the specific graph and how to use it to draw the line. Another team responded with "Plot down the y-intercept number and work out the rise over run."

Type E Response. This type of response builds on the Type D response and provided clear instruction of how to draw a graph specific to the graph $y=4x+8$. A typical response being, "(1) from the point 0 move upwards 8 points along the y-axis and plot a point there. (2) move downwards 4 points and to the left one point and plot the second point (3) join the points with a continuous line" and another being, "8 in the y-intercept, then it's 4 over 1 so you rise by 4 and run by 1."

Table 3
Drawing the Graph $y=4x+8$ without GeoGebra

Response Type	A	B	C	D	E	Total
Pre-Test	10 (83)	1 (8)	0 (0)	1 (8)	0 (0)	12 pairs (100)
End of Topic Test	1 (10)	2 (20)	3 (30)	2 (20)	2 (20)	10 pairs (100)
Delayed Post-Test	6 (32)	4 (21)	5 (26)	3 (16)	1 (5)	19 students (100)

Percentages of the sample for each response type in each test are included in brackets.

b. Can you move the graph using GeoGebra? Notice what changes on the graph and what changes in the equation. Explain these changes below.

Type A Response. This type of response indicated that students were unable to work out how to move the graph. For example, Team White claimed, "Nothing changes." For the end of topic test three student pairs presented blank comments and were coded to this type of response. In the delayed post-test eight of the nineteen students were coded a Type A response. Team Indigo obtained a Type A response for all three tests.

Type B Response. This type of response indicated that whilst a change was noticed, the change could not be correctly articulated. Demonstrating a lack of understanding of the concepts related to linear relationships. For example, responses in the end of topic test included, “Yes you can move it. The points move and the numbers change.” Team Brown A stated, “The gradient and y intercept changes while moving the graph” and Team Maroon stated, “The line increases according to the *y*-intercept.”

Type C Response. This type of response suggested the change that occurred without precise terminology. Only one student pair was coded a type C response in the pre-test, Team Cream, who stated, “Yes, the first part of the equation stayed the same but after the + the number changed.” Typical responses included, “The number changes at the end” from Team Orange A, “The equation remains y=4x but the thing we add changes” from Team Red A, “The slope stays the same but the interval changes” from Team White A and, “In the equation the *y*-intercept changes but the *x* stays the same” from Team Yellow A2.

Type D Response. This type of response indicated a sound understanding of what was happening. Descriptions make reference to the change in the *y*-intercept and the constant gradient. Responses used correct terminology including “slope”, “gradient” and “*y*-intercept.” For example, Team Orange stated, “Yes the slope stays the same and the *y*-intercept changes.” Other responses included, “The 4*x* stays the same and the +8 changes” Team Purple A and Team Blue B who teamed up for this task also stated, “To move the graph all you need to do is change the y-intercept and keep the same gradient.”

Table 4
Identifying Function of y-intercept

Response Type	A	B	C	D	Total
Pre-Test	10 (83)	1 (8)	1 (8)	0 (0)	12 pairs (100)
End of Topic Test	3 (30)	4 (40)	0 (0)	3 (30)	10 pairs (100)
Delayed Post-Test	8 (42)	4 (21)	5 (26)	2 (11)	19 students (100)

Percentages of the sample for each response type in each test are included in brackets.

Relating to the first research question posed, the findings from this study indicate that GeoGebra, an example of dynamic mathematics software, assisted students in increasing the complexity of their responses when exploring linear relationships. At the completion of the teaching sequence, the use of correct terminology was evident with students linking the concepts of *y*-intercept and gradient to form the equation of a line. Team Blue A1 and Team Yellow A2, who initially left a blank comment when asked how to draw a graph with the use of GeoGebra in the pre-test, combined for the end of topic test to successfully provide one of only two type E responses as provided in the tables of results.

Once the teaching sequence was completed, changes in responses were observed with the question relating to finding the equation of the line. Three student pairs, Team Maroon, Team Black and Team Cream all provided Type A responses for the pre-test and then submitted Type E responses for the end of topic test. Unfortunately, the delayed post-test demonstrated that retention was not as successful, with all three teams returning Type A responses. It appeared that those students who made a more gradual movement in types of responses were those who retained their knowledge the most. Team Red and Team White, who both returned Type B responses in the pre-test for the same question and then combined to provide a Type E response in the end of topic test, maintained a Type C response for the delayed post-test. Team Brown who submitted a Type D response in the pre-test and Type E response in the end of topic test maintained their level of understanding with a Type E response in the delayed post-test.

The most consistent results occurred with the question asking students to explain how they would draw the graph $y=4x+8$ without GeoGebra. With the exception of one student pair, all other student pairs recorded an improvement in response from the pre-test to the end of topic test. By the end of the teaching sequence 60% of students could articulate exactly how to draw the graph without the assistance of software.

In response to the second research question, students appeared to enjoy the dynamic nature of GeoGebra, as they were able to explore tasks quickly. Initially, students were reluctant to engage with the various tools GeoGebra offered, and were constantly seeking information about how to accomplish tasks in the pre-test. As the teaching sequence progressed, students became familiar with using GeoGebra as a tool for learning mathematics. It intrigued them as they attempted to investigate different properties of a graph that could not be achieved using pen and paper as evident below.

Team Yellow A2: It has a zoom.
Team Yellow B: How long does it go for? … billions and billions.
Team Yellow A2: Yep.
Team Yellow B: A spider web.

GeoGebra provided different obstacles, thus prompting students to interpret and manipulate information in order to obtain the required outcome. For instance, by default GeoGebra presents the equation of a line in general form. Hence, students were exposed to this format, which had not been presented in the teaching sequence. They were required to work out how to change the format to gradient-intercept form using GeoGebra and also convert it without GeoGebra. Another default within GeoGebra settings presents the gradient as a decimal. The concept of gradient was initially presented to the students as rise over run leading to the gradient formula with the final result being a simplified fraction. Whilst most students confidently converted basic fractions to decimals the more difficult decimal gradients, such as 0.67, confused students.

Team White A: But that means what I did is just wrong here.
Teacher B: No, that's right ... look at 2 over 3 on the calculator.
Team White A: Ah … oh well, we are right.

Students were able to use GeoGebra as a tool to assist in checking their solutions. They were encouraged to solve problems in their workbook and test their solution by completing the problem on GeoGebra. GeoGebra assisted with recall of terminology and notation in a dynamic environment. Not only could students work to find a solution, but they could also commence working the solution to see if it presented them with the same question. Students were learning without the intention to learn. The students commented:

Team Maroon B: Have you noticed we haven't worked off the textbook?
Team Maroon A: Yeah I have, it's great. I love it. It is so much easier.
Team Maroon B: Yeah it is.

Through the use of GeoGebra, they could approach questions in a different way. This was unlike their usual teaching sequence, which relied heavily on the use of textbooks.

Conclusion

This study aimed to identify the nature of student interactions and responses to linear relationship tasks whilst completing interactive tasks using GeoGebra as an exploration tool. GeoGebra is often used by teachers in one-off lessons; however, many teachers have difficulty designing a unit of work using GeoGebra at different stages in the lesson sequence. This study is a preliminary investigation into student thinking that captures student-to-student dialogue to identify some of the catalysts for change. The study aimed to examine students' exchange of ideas when using dynamic mathematics software as a tool for exploring linear relationships. Results indicate changes in the nature of the students' responses and a shift to more formal language use. Students were not immediately comfortable with the dynamic environment and this was an important component of the learning environment. Findings suggest benefits for GeoGebra in the Linear Relationships strand in the middle years and that it is essential to build familiarity of the tools so students can focus on the exploration at hand, rather than the tools they are using to explore it.

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