Noticing Critical Incidents in a Mathematics Classroom

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What teachers attend to, how they make sense of, and respond to critical incidents in the classroom are important for improving teaching. However, seeing and understanding important features of critical incidents can be difficult. In this paper, I propose a notion of productive noticing, which I used to analyse a case study of what teachers noticed about a critical incident that happened during a research lesson. Findings suggest productive noticing can help teachers to focus on important mathematical aspects of critical incidents, and understand how these can lead them to refine their teaching practices.

Critical incidents are events that occur during a lesson, which have the potential to deepen our understanding of students' mathematical thinking (Goodell, 2006; Yang & Ricks, 2012). These events or incidents can involve students' unexpected responses to teachers' questions (Yang & Ricks, 2012); those that raise questions about teaching approaches or students' understanding (Goodell, 2006); or events that change the direction of the lesson from what was planned (Fernandez, Cannon, & Chokshi, 2003). Reflecting productively on critical events is important for developing teaching practices that enhance students' mathematical thinking (Clea Fernandez et al., 2003; Goodell, 2006; Yang & Ricks, 2012). Therefore, developing teachers' ability to notice—to see, make sense of, and respond to—instructional events has been a recent focus of teacher preparation and training (Sherin, Jacobs, & Philipp, 2011). However, being able to see the "right" things, and respond appropriately to these incidents can be challenging (Ball, 2011; Star, Lynch, & Perova, 2011; Star & Strickland, 2007; Vondrová & Žalská, 2013). In this paper, I present a case study of what mathematics teachers notice about critical events that occurred during the research lesson of a lesson study. More specifically, a notion of productive noticing is applied to analyse what mathematics teachers notice. Key questions addressed in this study include: What do teachers notice about critical events, and how do they make sense of the mathematical features of these events? More importantly, what do they learn from critical incidents when they notice productively?

Productive Noticing of Critical Incidents

Mathematics teacher noticing—what mathematics teachers see and how they understand instructional events or details they see in classrooms (Jacobs, Lamb, & Philipp, 2010; Mason, 2002)—is considered necessary for improving teaching (Schoenfeld, 2011). The processes of noticing help teachers analyse their practice in order to learn from their teaching (Mason, 2002; Sherin et al., 2011). However, not all noticing is productive. The crux lies in what teachers attend to, and how they think about instructional events (Ball, 2011). For example, it can be difficult for teachers to notice the mathematical features of learning tasks (Star et al., 2011; Vondrová & Žalská, 2013), or teachers may be distracted by noticing features that are not useful for enhancing mathematical thinking (Ball, 2011; Star & Strickland, 2007). Furthermore, it is possible for teachers to describe the specific

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strategies that students use to solve problems, but they may be unable to relate these strategies to important characteristics of the problems (Fernandez, Llinares, & Valls, 2012).

Research has usually focussed on developing teachers' ability to notice a wide range of classroom features—classroom environment; classroom management; tasks; mathematical content; communication; mathematical thinking, etc.—without specifying what teachers should notice (Jacobs et al., 2010; Star et al., 2011). However, it is not clear whether teachers should have a specific focus, such as students' misconceptions, to make their noticing more productive (Star et al., 2011). A study by Star and Strickland (2007), shows improvement in teachers' ability to notice more instructional events, *both* mundane and important. A replication study (Star et al., 2011) was done, but neither study tested whether it would be better to provide a focus for noticing. In another study, Goldsmith and Seago (2011) found that the use of classroom artifacts, such as students' work, encourages teachers to notice specific mathematics in students' reasoning. They argue it is the "artful" use (p. 184) of classroom artifacts that directs teachers to focus on potential student thinking and provides a way for teachers to justify their claims about students' reasoning.

Even when teachers are given a focus—such as students' strategies—to notice, it is challenging for them to sieve out critical incidents amongst the 'buzz' in the classroom, and to reflect productively about them. Hence, being able to "highlight noteworthy events" (van Es, 2011, p. 139) and make connections between these events and students' thinking is an indication of noticing expertise (Yang & Ricks, 2012). To provide a focus for teachers, Yang and Ricks (2012) highlight the use of the 'Three-Point framework' by Chinese teachers in examining crucial events. According to Yang and Ricks (2012), the *key point* refers to key mathematical ideas of the lesson; the *difficult point* refers to cognitive obstacles encountered by students when they attempt to learn the key point; and the *critical* point refers to the approach that teachers take to help students overcome the difficult point. These three points can provide a useful frame for teachers to focus on the mathematical content, students' thinking, and generate teaching approaches that are associated with both content and students' thinking.

In an earlier paper (Choy, 2013), the construct of productive noticing was characterised using the 'Three-point framework' (Yang & Ricks, 2012) and notions of insight (Sternberg & Davidson, 1983). Building on these ideas, I conceptualise productive mathematical noticing as the ability to:

- attend to specific details related to the key point, difficult point or critical point that could potentially lead to new responses;
- relate these details to prior knowledge and experiences to gain new understanding for instruction (key point and difficult point); and
- combine this new understanding to decide how to respond (critical point) to instructional events.

This characterisation of productive mathematical noticing uses the 'three points' to direct teachers' attention to specific details of what they notice. More importantly, the notion views 'difficult points' as possible sites for teachers to generate approaches that target these cognitive obstacles associated with the 'key points'. Therefore, for noticing to be productive, there is a need to align the 'three points'. This alignment can enhance teachers' noticing and has the potential to encourage practices that enhance students' mathematical thinking.

Method

This study investigated how productive noticing can provide a means for teachers to reflect on and examine critical events in the classroom. Six Singapore mathematics teachers formed a lesson study group that explored the teaching of ‘Fractions of a set’ for Primary Four students (aged 10). All teachers had at least five years of teaching experience and Mr Jefferson—a teacher with five years of experience—is of particular interest in this paper. Mr Jefferson graduated with a degree in Civil Engineering before obtaining a Post-graduate Diploma in Education. Mr Jefferson taught the research lesson that was planned collaboratively by the lesson study group. The critical incident happened during the research lesson and was the focus for the post-lesson discussion.

In this study, Mason’s (2002, p. 95) practices of noticing—systematic reflection; recognising; preparing and noticing; validating with others— were incorporated into lesson study protocols. The modified protocols made teachers’ noticing more visible because they provided a way for teachers to discuss the ‘three points’. The researcher primarily took on a moderate participant observer role, shifting between purely observational to participatory during the seven lesson study sessions. During discussions, the researcher used questions to prompt and direct teachers’ attention to the ‘three points’, and provided necessary mathematical content knowledge when needed.

Data were collected and generated through voice recordings of the lesson study sessions and video recording of the lesson. The transcripts of the sessions were parsed into episodes according to what was discussed. Findings related to teachers’ noticing were developed through identifying categories, codes and themes related to the elements of productive mathematical noticing. This paper focuses on the findings drawn from one research lesson observation, which was followed by a post-lesson discussion.

Results and Discussion

This section consists of three parts. First, an overview of the lesson is presented to set the context for the critical event. Mr Jefferson’s ability to notice mathematical features of the task during the planning is discussed briefly to provide a glimpse of his thinking. Next, the critical event is described and analysed through the lens of the ‘three points’ to show how the notion of productive noticing might be used to highlight aspects of teachers’ thinking. Finally, I examine the teachers’ noticing during the post-lesson discussion, focussing on what Mr Jefferson noticed about the critical event that happened.

Overview of the lesson

The lesson spanned two 1-hour sessions on two consecutive days. The design of the lesson was based on two elements—unitising and partitioning—as discussed by the team. Unitising refers to seeing the whole set of objects or items as a ‘whole’ or ‘unit’ and partitioning refers to seeing the subset of interest as one of the equal parts of the ‘whole’. The first session focused on getting students to see a collection of discrete objects as a ‘whole’. This idea was actually based on a point raised by Mr Jefferson during the planning. He suggested that the main difficulty (difficult point) faced by students is that they still view fractions as ‘part of a whole’ and have problems seeing them in relation to a set of whole items:

> Previously, the fraction [concept] they learnt is more of part of a whole. They are very used to thinking about part out of a whole [object]. Now that we give them a lot of whole things, they cannot link that actually these fractional parts can refer to a set of whole things also. So I think, to me I feel that the connection that is missing, is that, how this fraction concept—which is part of one whole [object] which they have learnt so far—can be linked to whole things [objects].

Mr Jefferson then suggested to use the part-whole bar model (Figure 1), which students were familiar with, to help students make the link:

> I was thinking whether we can put it into … something more familiar because … eh … they have learnt models, how to represent questions in model also, so … could we box the whole thing up instead? And to them, they are familiar with the part-whole model … a whole box is a whole … so while keeping the items inside and we draw the box … and … and … yes … we tell them that this looks familiar, and it looks like the model as a whole, right? These lines can be the partitioning of the whole model. While doing that … they can still see that the four items are still inside the parts. I don't know whether that can help them to make the connection that if this 1 box is ¼ of the whole, inside that box, I have four things. And this is where the four came from?

Figure 1. Part-whole model.

He reasoned that the whole set could be represented by the box ("box the whole thing up" and "a whole box is a whole"), which constituted the unitising part of the approach (critical point). The partitions of the bar model could then be to represent the fraction (partitioning part of the approach). Mr Jefferson conjectured that students might be able to see that there were 'whole' items in each partition ("four items are still inside the part").

The second session was designed to build on what was done in the previous session. Students, working in teams, were given a set of 24 coloured cubes with different colour configurations. For example, a group of students might be given 6 blue, 8 red, 2 green and 8 yellow cubes. They were supposed to make fraction statements about the cubes (for example, ¼ of the cubes are blue), and show how they partition the 24 cubes to represent the fraction statement they made. Students then shared their answers and showed the whole class how they partitioned the 24 cubes. The critical incident of interest in this paper occurred during this session when Mr Jefferson explained the task using 12 cubes.

The Critical Event

Mr Jefferson started with a simple 'warm-up' task in which he got the students to make simple fraction statements about set of items with two distinct subsets (e.g., four buttons were shown: 1 orange and the others red). He then moved on to explain the task using 12 cubes instead of the 24 cubes. He used a colour configuration of 2 green, 4 blue, 3 red and 3 yellow:

Mr Jefferson:	What fraction of my cubes is green? OK, Gerald?
Gerald:	One out of six.
Mr Jefferson:	One out of six ... one-sixth. Let me shift it up a bit (Mr Jefferson shifts the cubes on the table so that everyone can see on the projector). Anybody disagree with Gerald? He said it's one-sixth. Hey ... Leon? No? Do you agree or disagree with Gerald?
Leon:	No.

Mr Jefferson:	Don't agree. Then what would be your answer then?
Leon:	Two out of 12.
Mr Jefferson:	Ok. We have two answers here. Two out of 12 and Gerald said one out of six. (Writes the fractions on the white board) Do you think they are related?
Students:	[Chorus] Yes …
Mr Jefferson:	Ok. First, Gerald. Can you come and show us how you got one part out of six when there are so many cubes here. (Gerald comes out and arranges the cubes. See Figure 2) Ok. Gerald, stay there ... stay there. Where's your six parts? (Gerald points to the cubes.) And the green is what? One out of? Six, is it?
Gerald:	Yeah.

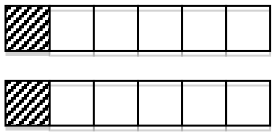

Figure 2. Gerald's first arrangement of 1/6.

Mr Jefferson:	Then what about the remaining cubes?
Gerald:	Still the same.
Mr Jefferson:	Still the same, ok? If I put it this way? (Mr Jefferson puts the two groups of cubes together. See Figure 3.) Would you all be able to see the six parts?
Students:	[Chorus] Yes ...

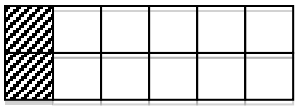

Figure 3. Mr Jefferson's arrangement of 1/6.

Mr Jefferson:	Yes... So, Gerald, where are the six parts? (Gerald points to the cubes again, and shrugs his shoulders.) Ok. Can you imagine the imaginary lines between the cubes? OK. How can you have put this better? (Gerald rearranges the cubes. See Figure 4.) How many parts can you see now? Anybody wants to give Gerald a hand? Yes, Ginny. Ok. Thank you, Gerald. (Ginny comes out to do another arrangement.) Mmm ... Something different from what Gerald did. (Ginny rearranges the cubes to be six groups of two. See Figure 5.) Ok. Let's shift this a bit. Ok. Do you see six parts now?
Students:	[Chorus] Yes …

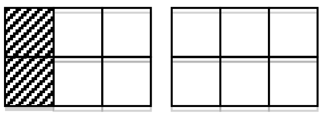

Figure 4. Gerald's second arrangement of 1/6.

Mr Jefferson:	A bit clearer?
Students:	[Chorus] Yes …

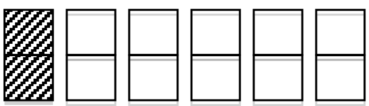

Figure 5. Ginny's arrangement of 1/6

In this episode, Mr Jefferson seemed to attend to Gerald's arrangement of the cubes to represent 1/6. He noted that Gerald's arrangement (see Figure 2) did not clearly indicate the six equal partitions that he expected. The episode was surprising to Mr Jefferson because he knew that Gerald was mathematically inclined. In a bid to help the other students see the 'partitioning' (which was the 'critical point' agreed upon during the lesson discussions), Mr Jefferson put the two rows of six cubes together (see Figure 3), and asked the class whether they could see the six groups. The chorus answer from the class did not seem to convince him that the class understood, and he decided to get Gerald show the six groups.

However, Gerald hesitated and Mr Jefferson tried to direct Gerald's attention to the "imaginary lines" between the cubes. At this point, Mr Jefferson noticed Gerald was seemingly confused and asked Gerald to rearrange the cubes so that he could probe Gerald's understanding further. However, Gerald came out with an arrangement that did not represent 1/6 (see Figure 4) and Mr Jefferson then asked for another student to show the partitioning. It seemed that Mr Jefferson did not manage to figure out why Gerald could not show the partition. Mr Jefferson could have asked Gerald to explain his answer so that he might make sense of Gerald's thinking (Burns, 2005). What made this incident critical was that subsequent groups of students also had problems showing the partitions. The inability of the students to show the partitioning, even though they were able to formulate correct fraction statements, suggests a possible gap in their understanding. The teachers picked this up during the post-lesson discussion.

Post-Lesson Discussion

The modified lesson study protocol for this study uses the 'Three-point framework' to frame post-lesson discussions. The focus on the 'three points' seemed to help teachers zero in on the mathematical features of the critical incident. The discussion generally centred on students' strategies related to the incident, and the implications for the design of the task. The first thing that Mr Jefferson brought to the attention of the teachers was students' inability to partition. During the post-lesson discussion, he hypothesised that students did not partition because they counted and simplified the fraction. For example, students might have counted two green cubes out of 12, wrote the answer as 2/12 before they simplified to 1/6:

Mr Jefferson: Maybe I share what I observed from my point of view? The glaring thing that I noticed about my pupils is that too many of them, they didn't get their fraction by partitioning ... they got it more by counting and then simplifying … so that was the easy option to them. Which was why later when I got them to explain "How did you get this fraction for example?" … "One sixth of the cubes were red" or something like that. Some of them were not able to show the six parts or to group the objects into six parts. So they were a bit lost. Because how they did it was, count the number of red cubes over the total number of cubes, then simplify. When they cannot put it in parts, right … it was very clear what their thought process was – simplify …

Mr Jefferson attended to the 'critical point'—getting students to show their understanding by representing the fractions through partitioning of the cubes—and realised that students had difficulties doing that (a new difficult point), and supported his claim using the critical incident. He reasoned that there could be a gap in students' understanding even though they might give the correct answers. Two other teachers, Ms Rachel and Ms Kirsty, supported this with their observation that students "avoided five green cubes out of 24 cubes" because 5/24 was already in its lowest terms.

Ms Rachel: For some reason, they did not want to put five out of 24 right?
Ms Kirsty: Yes … Yes …
Ms Rachel: They did not what to use the denominator 24. They wanted to simplify.

Ms Kirsty: They just simply … just go and simplify.

An interesting conversation then revolved around students "bypassing" partitioning by simplifying the fractions directly. The teachers agreed that students might not really understand the notion of fractions as part of a set even though they were able to make correct fraction statement. They argued that getting students to explain their partitioning could have given teachers insight into students' thinking. Ms Fay articulated the need to listen and referred to Gerald as an example:

Ms Fay: Gerald is very complex when he does maths. I've had him explain to me. He can get an answer just like that – without workings or anything. The boy is very complex up here (pointing to her head). And I don't fault him for doing things a bit differently – as long as I understand what he is trying to say like, I can imagine how he does things. I think it's okay. Like for him, he may arrange it that way, but he may mean it like the second way.

Mr Jefferson agreed and also highlighted that it is important to be more specific in the questioning with regard to the key point, difficult point and critical point. The emphasis on getting students to explain more specifically in order to reveal their reasoning suggests a shift from explaining to listening as a result of teachers' noticing. As Mason (2002) suggests, noticing is productive when it brings to mind the possibility of a different decision. Mr Jefferson's noticing, throughout the post-lesson discussion, sensitised his awareness and helped him think more deeply about students' thinking beyond giving the right answers:

Mr Jefferson: I was just thinking the danger of – during the design of this lesson, we didn't see that maybe they may skip the partitioning part of it ... they didn't show how the answer is found. It is something we need to recognise. It is good that we now know that if they missed the partitioning part… this may cause a problem later. Missing the partitioning part will be fine until we show them they have a problem. Even though they can do a fraction of a set, and they can solve fraction of a set problems – it will pose learning problems in future when they move on… I think we need to look at it more carefully.

The use of the 'Three-Point Framework' provided a language for teachers to direct their attention to the mathematical aspects of the critical incident as they tried to understand students' difficulties in showing the partitions. Mr Jefferson's noticing can be largely characterised as productive, especially in terms of how he saw and understood the critical incident: students' difficulties in partitioning and how that is related to understanding fractions. He recognised the 'blind spot'—that students might skip the partitioning—when they planned the lesson. What Mr Jefferson noticed helped other teachers to gain insights about students' thinking. He was able to see how this gap in students' understanding could have implications beyond the lesson, or what Yang and Ricks (2012, p. 46) termed as "finding the general meaning of such incidents". The use of specific instances to support his claims or suggestions also indicates that Mr Jefferson has begun to take on a "Researcher's lens" (Clea Fernandez et al., 2003, p. 173) when viewing critical incidents.

Concluding Remarks

Notwithstanding the limitations of a case study, this paper highlights the potential of productive mathematical noticing in two ways: as a practical means for teachers to direct their attention to the relevant mathematical details of critical incidents; and as a theoretical construct to analyse what teachers see and understand during post-lesson discussions. The

'Three-point framework' provides teachers with a way to relate what they observe about students' thinking to the mathematical ideas, and possible obstacles in understanding these ideas. Thinking about the three points also helps teachers to generate possible teaching strategies that are more targeted at students' difficulties in learning the key mathematical content. It remains to be seen whether a continual focus on productive noticing helps teachers to notice important aspects of critical incidents and more work is needed to see how productive noticing of these critical events impacts teachers' teaching practices. The findings of this study, which is part of a larger project, provide a small step towards understanding the role noticing plays in improving the teaching of mathematics.

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