Using Coaching to Improve the Teaching of Problem Solving to Year 8 Students in Mathematics

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This study investigated how to improve the teaching of problem solving in a large Melbourne secondary school. Coaching was used to support and equip five teachers, some with limited experiences in teaching problem solving, with knowledge and strategies to build up students' problem solving and reasoning skills. The results showed increased confidence by all teachers in the range and use of problem solving strategies, and for students increased use of strategies and improved reasoning skills to solve problems.

The Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority, 2010) has Problem Solving as one of its four Proficiency strands where students are expected to formulate and solve problems when they: use mathematics to represent unfamiliar or meaningful situations; plan their approaches; or apply their existing strategies to seek solutions. From informal discussions with Year 8 teachers at the first author's school it was clear that many had not taught problem solving strategies unless they had participated in programs like Maths Olympiad or Gateways. Some teachers also reported not having learnt strategies for problem solving at all. This study looks at how coaching can be used to assist teachers to improve their pedagogy of problem solving in order to develop students' interpretation, investigation and communication of solutions.

Literature Review

Teaching Problem Solving

In this study, problem solving means mathematical questions for "which the solution method is not known in advance" (National Council of Teachers of Mathematics, 2000, p. 52). For Schoenfeld (1992), problem solving is a problem which has no immediate answer and no algorithm that can be used directly to solve it. He claims that teaching problem solving is difficult, with teachers needing additional mathematical content as well as methods. It requires students to analyse the question then draw on prior knowledge to devise a strategy and find a solution.

Tripathi (2008) argues that the role of the teacher in this process is to act as a facilitator "by asking questions that help students to review their knowledge and construct new connections" (p.168). For Cobb, Wood and Yackel (1991), an important aspect for teachers is accepting right or wrong answers in a non-evaluative way, knowing when to intervene and when to allow the students to find their own way.

Lester, Masingila, Mau, Lambdin, Santon, and Raymond (1994) argue that teachers need to encourage students to make use of strategies and explain their mathematical reasoning, thus "helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, exploring, testing, and verifying" (p.154).

The National Numeracy Review Report (2008) distinguishes teaching problem solving within a single context and teaching across several contexts. Teaching problem solving within one context may constrict students' creativity, thinking of recently taught concepts

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instead of linking to their previous knowledge. Teaching across several contexts may extend opportunities for teachers to help students to make links to previous knowledge.

The National Council of Teachers of Mathematics (2000) and Woodward et al, (2012) recommend teaching multiple strategies to students The effectiveness of explicitly teaching students problem solving strategies is shown, for example, in a study by Jitendra et al. (2009) involving 148 students in 7th grade from the United States. It used a randomised controlled trial involving instruction in problem specific multiple strategies versus traditional instruction with an effect size for the post-test yielding 0.33. While these and other authors such as Rigelman (2007) give specific attention to the teacher's role in fostering mathematical thinking and problem solving, it is assumed that teachers themselves have the skills to be able to problem solve and can teach these skills to students. There has been little attention to the specific role of coaching in this regard.

Coaching and the Teacher's Role

Coaching literature tends to focus on general features. For example, Joyce and Showers (1995) argue that coaching can move a teacher much further than a communal professional development activity that may not address the specific needs of the teacher. In a typical cycle, a coach works with a teacher to plan a lesson or a key aspect of a lesson focussing on some aspect of teaching and/or student learning that the coach and the teacher have agreed to work on. The coach may demonstrate a part of a lesson or observe a teacher doing that part of the lesson. After the lesson, the coach and the teacher de-brief, attending to evidence of changed teaching and/or student learning (Stephens, 2011). They will then use this evidence to plan the next stage of the coaching cycle to ensure that newly attained skills are refined (Feger, Woleck & Hickman, 2004; and Kise, 2006).

Coaching Teachers in Problem Solving

Lester, Masingila, Mau, Lambdin, Santon, and Raymond (1994) recommend teaching students to make use of strategies and explain their mathematical reasoning, thus "helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, exploring, testing, and verifying" (p.154). Woodward et al. (2012) also recommend teaching multiple strategies. The National Numeracy Review Report Panel (2008) identifies two approaches to teaching problem solving: teaching within a context and across different contexts. Problem solving within a context may constrict students' creativity, thinking of recently taught concepts instead of drawing upon previous knowledge. Teaching problem solving involving several different contexts may extend opportunities for students to generalise problem solving skills.

Polya's (1945) four-step problem solving process comprises understanding the problem, making a plan, carrying out the plan, and looking back. Goldman (1989) has a similar four-phase model which includes: 1) read and become familiar with the problem, 2) find the necessary information, 3) set up the problem with numbers and symbols and solve, and then 4) see if the solution makes sense. The steps put forward by Polya, while seeming straightforward, make no mention of prior knowledge, mathematical skills and appropriate strategy selection which are important factors that teachers (and students) need to be aware of. Lesh and Zawojewski (2007) see Poyla's four-stage model as a general picture of how to move through problem solving, whereas the strategies are different tools to help students interpret, move forward and try different options during the problem solving process.

Methodology

The study was conducted in a large, coeducational government school North East of Melbourne. It began in April and concluded in June 2013. It utilised 72-minute lessons. A Teacher Coach introduced five Year 8 teachers, aged between 27 and 55 years old, to a range of problem solving strategies to help improve both their and their students' ability to solve mathematical problems.

The coach (the first author) prepared a 34-page problem solving resource booklet for teachers using Polya’s four-step problem solving process. The booklet provided multiple examples with worked solutions for the seven strategies. These were: *Draw a diagram*, *Look for a pattern*, *Work Backwards*, *Make a table*, *Act it out*, *Use logical reasoning*, and *Use simpler numbers*. The resource booklet also contained a four-point teaching strategy to “unpack” Polya’s four steps. The first teaching strategy was to help students to understand the problem by asking: *What do we know?* and *What do we need to find out?* A second strategy was to help students to *Plan a Solution* and to *Communicate their Findings*. This teaching strategy was intended to help students to identify the most appropriate of several possible starting points for each problem. A third strategy focussed on *Reflecting and Generalising*; that is, looking at whether the solution makes sense, and identifying any patterns. The fourth and final strategy was *Extending*, intended to assist students to modify a problem, for example by changing its conditions. Providing teachers with several problems and worked solutions for each of the seven strategies was intended to extend opportunities for teachers and students to generalise problem solving skills.

The coach had a school-wide responsibility for teaching and learning and had taught mathematics at all levels for 15 years. Using modelling, trialling, team teaching and one-on-one discussions with Year 8 mathematics teachers, the goal was to help them to refine methods of instruction that directly influence students’ learning of how to solve problems.

Five Year 8 mathematics teachers agreed to participate in the project: Teacher A – (Beginning teacher, 3 years’ experience); Teacher B – (Expert teacher, 20 years); Teacher C – (Accomplished teacher, 10 years); Teacher D – (Expert teacher, 30+ years); and Teacher E – (Beginning teacher, 2 years). Fifty-three Year 8 students, aged between 13 and 14 years from five classes, agreed to participate in the project. The sample comprised of 20 boys and 33 girls, including students of low, medium and high ability in Mathematics (as rated by their teacher). Table 1 shows how many students participated from each class. Teacher D’s 16 students were from an accelerated learning program.

Table 1
Number of Participating Students from each Class

Teacher’s class	
Teacher A Class	12
Teacher B Class	8
Teacher C Class	8
Teacher D Class	16
Teacher E Class	9
	Total 53

A pre-project survey aimed to identify current teaching of problem solving. Survey items included important aspects of teaching problem solving such as group work (Tripathi, 2008), promoting and discussing different strategies (Woodward et al., 2012), connecting ideas to other subject areas, and writing reflections. This snapshot of current practice guided the coach in moving individual teachers forward.

A pre-test was administered to students, and the results were used to assign the students into ability groups within each class and also give each teacher and the coach an understanding of students' prior knowledge. The pre-test consisted of four questions which could be solved using four problem solving strategies that teachers identified as the most commonly used in their current practice, such as *Draw a diagram*, *Find a pattern*, *Logical reasoning*, and *Make a table*. The pre-test required students to explain their reasoning. The test was marked out of eight; four marks were awarded for a correct answer and four for evidence of explanation and reasoning.

After this initial testing and the teacher survey, the coach discussed results with each teacher as part of one-on-one coaching sessions to equip them with strategies to teach problem solving, using Polya's model and the seven specific problem solving strategies. All teachers had a minimum of two one-on-one planning sessions with the coach. Each session ranged from 50 to 72 minutes.

Coaching involved explicitly teaching each strategy by drawing on examples from the resource booklet. Teacher A was completely unfamiliar with the strategies. The coach needed to show Teacher A both how to teach each strategy and also how to complete the individual problems. Teacher E needed similar support. Team teaching with the coach was utilised by Teacher B and Teacher C on two occasions. With all teachers, the coach demonstrated parts of lessons, especially helping teachers to identify the kind of support students typically need when problem solving.

Teachers A, B and C completed the project as a unit, taking between two and three weeks, whereas Teachers D and E included a problem solving component in their normal program over three to four weeks. Teachers completed a survey about what they had learned and how they will incorporate problem solving strategies into their teaching, and about the effectiveness of coaching.

Students of Teachers A, B and E were encouraged to re-write problems in their own words before proceeding. This step was useful in making sure that students understood what was being asked. All students were encouraged to work through each problem using steps based on Polya's method, whilst also selecting a particular strategy as part of their plan. A last step was to reflect on their answer and their process. Finally, all students were asked to see if other strategies could be used to solve the problem.

Students completed a post-test consisting of four new questions which could be solved using any of the seven strategies the students had learnt. Each question also required an explanation of their reasoning to find a solution (as in the pre-test) and to identify and discuss the strategy used. The marking scheme used was the same as the pre-test: four marks were awarded for correct answers and four marks for evidence of explanation and reasoning.

Results and Discussion

Teacher Survey (Pre-Project)

The five teachers were asked to rank how much emphasis was placed on the following aspects of teaching problem solving. Table 2 shows those aspects of problem solving that the teachers rated low. For example, all five teachers gave a low ranking to helping students to reflect on their solutions. These results indicated where specific coaching was needed.

Table 2
Teachers' Ranking of Different Components of Problem Solving

	Number of teachers who ranked this aspect low
Learning how to solve problems	4
Learning how to justify answers	4
Real life situations	2
Connections to other subjects	5
Using different approaches to problem solving	4
Reflections on solutions	5

Teachers were also asked how often they problem solved in their classes: four teachers indicated very rarely; one teacher every couple of months; and Teacher D almost every lesson. Teachers were also asked to nominate problem solving strategies they knew best:

- Teacher A (Beginning Teacher) – I don't know them
- Teacher B (Expert teacher) – *Draw a diagram*, *Guess and check*, *Make a table*
- Teacher C (Accomplished teacher) – *Trial and error, Find a pattern*
- Teacher D (Expert teacher) – *Guess and check*, *Draw a diagram*, *Modelling*
- Teacher E (Beginning Teacher) – No answer

The survey put to rest any assumption that teachers were familiar with teaching problem solving. Only the two Expert teachers were able to list three distinct strategies. Two teachers could not list any. *Guess and check* seemed to be the most familiar. Teachers B and D (Expert teachers) were able to link problem solving to real life situations and other mathematical topics or concepts. By contrast, Beginning and Accomplished teachers (Teachers A, E and C respectively) needed more basic help to teach problem solving.

Coaching Tailored to Teachers' Needs

Teachers A and E (Beginning teachers) needed to have worked solutions. Although many questions could be solved using algebraic methods, few students had been exposed to these methods. A key goal was to demonstrate non-algebraic solution methods to all teachers. Teacher C (Accomplished teacher) also completed her own worked solutions to the questions. Comparing her answers with the coach's boosted her confidence level. During group discussion, the teachers decided that *Elimination* should be explicitly taught. Teachers also included *Making an organised list* as one of the problem solving strategies. Although this strategy is similar to *Make a table*, students saw them as two separate strategies. *Make a model strategy* was not taught explicitly. *Trial and error* also came up as a strategy. However, *Guess and check* was deemed to be similar so it was not included on the list. Students were made aware that these processes were the same.

Teacher Survey (Post-Project)

All teachers acknowledged coaching as being essential in helping them to focus on the 'how' of teaching as well as the 'what'. All five teachers indicated that they feel more confident and now have the necessary skills to teach problem solving.

Teacher A (Beginning teacher) is now comfortable teaching the strategies and acknowledges that every student has different ways of approaching the questions. She is also more comfortable in directing discussion to show alternative ways of completing questions. Teacher A reported that coaching was important in learning to teach problem solving techniques and obtaining the students' interest. Teacher A also found the resource booklet invaluable. Teacher B (Expert teacher) acknowledged that problem solving is important, and that students are now "more systematic in their approach to maths problems"; and said that the coach helped move him from a "common sense" approach to a more systematic approach of teaching problem solving.

Teacher C (Accomplished teacher) reported that better relationships were built between students in her class because of the nature of the activity and groupings. The problems were challenging for all students and allowed multiple entry points even for the weaker students. Students were given 5-6 questions at a time and could choose which order to complete them in their group. Teacher C referred to support provided, including teaching materials, questions to be used in class, and feedback as a result of coaching. Teacher C now explicitly refers to problem solving strategies by name.

Teacher D (Expert teacher) now makes links to using algebra through problem solving. She said that the students found the problems interesting and that they worked productively, acknowledging feedback provided to the teacher by the coach about different ways to engage students in the problems. Teacher D now uses the names of the different problem solving strategies as hints/prompts in class to help students, and now expects her students to be able to justify or explain how they arrived at their solution to a problem and to be able to tell if their answer is logical or not.

Teacher E (Beginning teacher) mentioned the one-on-one sessions as being important in understanding how to teach problem solving. The worked solutions helped Teacher E explain answers and allowed him to see responses in a different way. Students now have many strategies and different ways they can confidently try. Students liked the challenge and even the weaker students often thought of different ways to help those around them.

Impact of Teaching on Students' Problem Solving

Gains in scores were evident across all classes in the post-test on problem solving, and are important as a means of corroborating the success of coaching. The post-test consisting of different problems was scored as for the pre-test with credit given for correct solutions, as well as for reasoning and explanation. Gains, as shown in Table 3, were strong for the classes of Teachers B and D, the two Expert teachers. For the class of Teacher E (Beginning teacher) strong gains were also evident. When asked after the study if they were more confident with problem solving, almost all students reported that they were more confident. Six students reported "No change" – two from Class B, one from Class C, one from Class D, and two from Class E. They may have been confident to start with.

Table 3
Analysis of Student Results from Post-test Compared with Pre-Test

	Higher score	Same score	Lower score
Class A N=12	8	3	1
Class B N = 9	8	1	0
Class C N=8	5	3	0
Class D N=16	13	3	0
Class E N=9	8	1	0

Further corroboration of the impact of coaching is the fact that many students after the study were able to identify and recall specific problem solving strategies, as shown in Table 4. More than 70% of students could recall three or more strategies, with 30% of students able to recall more than five strategies. These results add substance to the comments made by teachers in the post project survey.

Teacher D's class had the best gains in terms of increased reasoning skills and recall of strategies (six students were able to recall more than 5 strategies). This may be due to Teacher D's experience or to the students' participation in an accelerated program.

Table 4
Students' Capacity to Recall Specific Problem Solving Strategies

No. of Strategies	0	1-2	3-5	5+
Class A N=12	0	5	4	3
Class B N=9	1	3	4	1
Class C N=8	0	2	3	3
Class D N=16	0	2	8	6
Class E N=9	1	1	4	3

Qualitative differences between students in classes where teachers received more coaching were hard to ascertain, as all the teachers received one-to-one coaching in some form. The pre-test and post-test could have included questions which may have provided more insight into which strategies were more familiar to students.

The findings in this relatively small local project, where students' post-test results showed nearly 80% scoring higher compared with their pre-test, were consistent with previous studies involving explicit instruction of multiple strategies in problem solving, for example Jitendra et al. (2009). Students' reasoning skills and explanations both improved. No conclusion could be drawn regarding which class improved the most. All classes showed progress using the strategies and improved reasoning skills.

Conclusion

How was Coaching a key to the success of this study? The five teachers and the coach shared a common commitment to improving students' capacity to undertake problem solving. Polya's four-step process and the seven specific strategies, as exemplified in the resource booklet, also provided a common focus. However, each teacher needed different support from the coach, especially to help students to apply the Polya model. Some needed help to understand the different problem solving strategies and to become confident to teach them. Most needed to be shown how to teach problem solving, including using the four teaching strategies, described earlier, to assist students to make a start, to monitor their own progress, to explain their thinking, and, finally, to reflect, generalise and extend their solutions. These strategies allowed the teachers to appreciate more clearly what they needed to do to in the classroom. The less experienced teachers needed to see the strategies modelled by the coach with their students. The three more experienced teachers worked more confidently with the coach to build their students' capacity to use multiple strategies for problem solving and to justify their own thinking. It was also essential to be able to show how a particular strategy might be applied to different contexts, and how a given context could be extended. As a direct result of the coaching, it was agreed that a unit of work on problem solving would be built into the Year 8 course next year.

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