A Primary Teacher's Developing Understanding of Mathematical Reasoning

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To support teachers in their quest to incorporate reasoning as a mathematical proficiency as espoused in the Australian Curriculum: Mathematics, a professional learning research project using demonstration lessons was carried out. This paper reports on the impact of demonstration lessons on one participating teacher's pedagogical knowledge about reasoning. The growth in this teacher's knowledge was analysed using a phenomenographic framework established to evaluate teachers' development in mathematical reasoning. The results show that demonstration and subsequent trial lessons contributed to her growth.

With the implementation of the Australian Curriculum: Mathematics (ACARA, 2012) Victorian teachers have to grapple with understanding and enacting the newly articulated proficiencies of Understanding, Fluency, Problem Solving and Reasoning in their classrooms. A study by Clarke, Clarke and Sullivan (2012) revealed teachers' limited understanding of the reasoning proficiency. *The Mathematical Reasoning Professional Learning Research Program* (MRPLRP) was set up in response to calls to assist schools in addressing this issue. Baseline data regarding teachers' knowledge and understanding of the reasoning proficiency at the commencement of the project confirmed Clarke et al.'s (2012) findings (Loong, Vale, Bragg and Herbert, 2013). Demonstration lessons (Clarke, Roche, Wilkie, Wright, Brown, Downton et al. 2013) were used as the primary mode for the professional learning program. Post-lesson observation discussions were carried out to discuss student responses and what teachers noticed. Teachers individually or in collaboration with a colleague trialled the lesson or a varied version of the lesson with their own students. This case study reports one teacher's response to the professional learning program and the impact it had on her understanding of reasoning and her enactment of the reasoning proficiency in her classroom.

Theoretical framework

Learning is seen as the adaptation, or addition to a learner's existing cognitive structure or schema (Piaget, 1970). It is deemed to have taken place when there is a change in the way a learner conceives an object of learning (Ramsden, 1988). Learning is usually demonstrated by a change in the expression of the learner's conceptions of the content of instructional sequence. It is often characterised by the learner becoming aware of additional features or aspects of a concept or skill not previously discerned (Bowden & Marton, 2004). The amount of attention given to aspects of the learning intention accounts for differences in how people learn (Marton, Runesson, & Tsui, 2004). Sometimes it is necessary to explicitly guide the learner to notice features of the learning intention to increase awareness (Kaput, 1992). In phenomenographic research, learning can be said to have occurred if the learner views the phenomenon differently from when they first started (Bowden & Green, 2005). In this case study, a phenomenographic approach has been used to analyse the teacher's learning.

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Method

Faye (pseudonym), the focus of this case study, was one of seven teachers in a school which participated in the MRPLRP in Victoria. She had been teaching composite classes of Year 3/4 (8-9 year olds) each year for three years. Like all participating teachers in this project, she went through the following processes in the project: *Interview 1 → observed demonstration lesson 1 → collaborative post-lesson discussion → teacher trial lesson → Interview 2 → observed demonstration lesson 2 → collaborative post-lesson discussion → teacher trial lesson → Interview 3.*

Demonstration lessons were taught by a member of the research team and teachers were provided with a copy of the lesson plan prior to the demonstration lesson. The first demonstration lesson entitled 'What Else Belongs?' was aimed at asking children why a sequence of three numbers belonged or did not belong and to provide justification for their conjectures. The second demonstration lesson entitled 'Magic V' asked children to provide reasons for how and why the numbers 1 to 5 can be arranged in a way to make it a Magic 'V' and to explain and justify why it was impossible to make a 'Magic V' with even numbers positioned at the vertex of the V. Detailed lesson plans were provided in advance and observation sheets and seating plans were provided on the day for teachers to note down what they saw and heard. After each demonstration lesson, a post-lesson discussion was conducted to give opportunities for teachers to share what they had observed. Teachers were asked to trial the demonstration lessons in their classroom. Suggested modifications were provided to suit other year levels. In this case study, data were drawn from the three interviews conducted with this teacher. Analysis of the data led to a framework of teachers' perceptions of mathematical reasoning developed by the research team (Herbert, Vale, Bragg, Loong & Widjaja, submitted). This framework provided a way of assessing teachers' growth in their understanding of reasoning. The hierarchy of categories based on expanding awareness of reasoning can be seen in Table 1. Figure 1 illustrates the elements defining Category D: Reasoning is perceived to be validating thinking.

Table 1:
Outcome Space of Primary Teachers' Perceptions of Mathematical Reasoning.

Category	Perception of mathematical reasoning
Category A	Reasoning is perceived to be thinking.
Category B	Reasoning is perceived to be communicating thinking
Category C	Reasoning is perceived to be problem solving
Category D	Reasoning is perceived to be validating thinking
Category E	Reasoning is perceived to be forming conjectures
Category F	Reasoning is perceived to be using logical arguments for validating conjectures
Category G	Reasoning is perceived to be connecting aspects of mathematics

Findings

In the first interview, when asked what reasoning meant to her, Faye said 'I don't know' but when probed further added:

> …but I think reasoning would have a lot to do with the understanding behind the mathematical concepts…– so not the process of the mathematical concept but more the thinking that goes on during a process *maybe*.

This quote indicates that Faye was not confident about what reasoning meant but tentatively perceived reasoning as the thinking that takes place in a mathematical process. This indicated that Faye's perception was Category A at the commencement of the project. This perception however, shifted in the second interview. While she claimed "…I still don't feel like I've got the full umbrella of reasoning", when she was questioned about the main ideas she had learnt about teaching reasoning she replied:

> … So I might put up a few 2 digit numbers and then one 3 digit number and say, "What doesn't belong and let's talk about it." But that's with an agenda because I'm looking at them recognising 2 digits versus 3 digits. Whereas this takes sort of a more-worldly view of mathematics and gets them to think a bit deeper. And again, using their convincing and having reasons and then giving examples. So that's definitely something that needs to be in my classroom to make sure my students have a well-rounded view of mathematics.

Faye was now able to perceive reasoning as more than just oneself thinking through a problem in a mathematical process. Her new goal was to ensure that her students were able to convince others by giving reasons and examples to justify their answers in different mathematical contexts. Her perception of reasoning in the second interview was now Category D which involves explaining, articulating reasons and justifying verbally or diagrammatically.

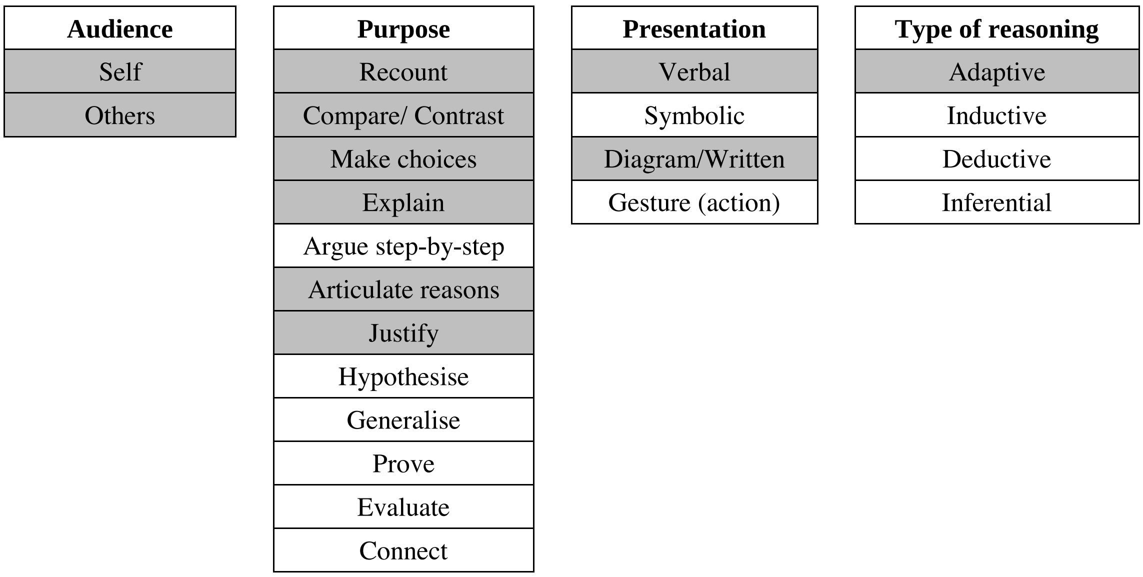

Fig. 1 Category D: Reasoning is perceived to be validating thinking

While Faye's perception of reasoning did not progress beyond Category D, she certainly seemed more confident of what it meant in the third interview:

> …Well the kids were constantly having to explain, because they work in partners which meant they could make their thinking out loud, you could always hear them justifying, thinking about other reasons why things won't work, or the reasons why things do work…

As there was no mention about students making conjectures about what would happen if the numbers in the 'Magic V' problem were changed from 1-5 to 2-6, she had not yet progressed to Category E.

Faye attributed her knowledge and understanding of reasoning to having watched the two demonstration lessons and then trialling the lessons herself.

> …I can see that just from doing these two demonstration lessons and then taking them into my classroom, and with the focus being on reasoning for those lessons, I can see the effect that it's had on the kids and the deeper knowledge that they have that without encouraging them to use their reasoning skills you wouldn't see.

She acknowledged that input from other teachers during the post-lesson discussions was instrumental in helping her become aware of the importance of teaching mathematical language to enable students to build their reasoning proficiency.

> …To have the feedback from others to work out, perhaps, what areas may have been lacking… an area we noticed was language? The students didn't have the language to say, "A 2 digit number." They weren't using the word "digit" at all…I didn't pick up the use of language and students being unable to use reasoning…. that's something that another teacher in the discussion picked up that I didn't. So that when I went to deliver my lesson that was something I was conscious of, seeing if the students were able to use the language.

Changes in the way Faye expressed herself and her awareness of aspects of the reasoning proficiency over the course of the MRPLRP indicated learning had occurred.

Conclusion

Demonstration lessons with built-in collaborative post-lesson discussions and an expectation of teachers trialling the lessons in their own classrooms seem to have the potential to impact teachers' understanding of reasoning. Further tracking of other teacher participants will reveal if this is actually the case. Deeper analysis of teachers' responses might also reveal whether having a mathematical content goal along with the reasoning proficiency made the learning design of the demonstration lesson more impactful.

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