A Reflective Approach to NAPLAN: Exploring the Implications of Students' Responses to an "Adding Fractions" Item

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Large-scale numeracy assessments are intended to facilitate the improvement of educational outcomes; however, it is not clear exactly how this is to be achieved. To move towards the goal of numeracy for all, it is necessary to systematically address issues that are known to be difficult, pervasive and persistent. This paper includes an analysis of an `addition of fractions' item from the Australian 2008 Year 7 NAPLAN assessment and draws insights that may be generalised to improve overall numeracy.

The Role of Large-Scale Assessments in the Education System

The importance of numeracy skills in modern society is clear. People who lack sufficient numeracy skills to evaluate whether a financial statement or contract is reasonable are vulnerable. Large-scale assessments (LSA's) function as a means of evaluating the performance of the education system as a whole against pre-defined criteria and therefore play a role in the national development of numeracy as a matter of governance. In the Executive Summary of the Australian Government-initiated Review for School Funding, Gonski (2011) states: "… no student in Australia should leave school without the basic skills and competencies needed to participate in the workforce and lead successful and productive lives" (p. xv). LSA's might be considered as a governmental response to the evolving needs of modern society by obtaining data on which to base policy decisions while recognising, as Gonski (2011) notes, that these criteria are contained within the broader goals of "young Australians becoming successful learners, confident and creative individuals, and active and informed citizens" (p. 217). The problem that this paper seeks to address is how the data from large-scale assessments can be used to attain educational improvement.

In Australia in 2008, the National Assessment Program-Literacy and Numeracy (NAPLAN) assessment was introduced for all school children in Years 3, 5, 7 and 9 (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2011). The aggregated national data are reported annually, providing information on children's achievement against a variety of factors including location, gender, Indigeneity and parental education. The NAPLAN assessments are intended, at least in part, to facilitate systemic improvement over the long term:

> It is important that there be consistent and well understood measures of student achievement around the country, and that the outcomes of these assessments be used to inform future policy development, resource allocation, curriculum planning and, where necessary, intervention programs. (ACARA, 2011)

As well as providing feedback about the education system as a whole, NAPLAN is also intended to provide diagnostic feedback at the local school level to drive local improvements over a shorter time-scale. These results can be used to identify individuals who require intervention by noting low scores, or progress that was slower than might be expected from previous assessments. The Ministerial Council on Education, Employment, Training and Youth Affairs (MCEETYA) (2007) highlighted that NAPLAN's diagnostic

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capability was an important part of its purpose by choosing May as the month in which to hold the assessment: "It is early enough in the year for the test results to be used as a diagnostic tool" (p. 1). To facilitate this, detailed results for each child are reported to the school. Because the NAPLAN assessment measures the cumulative effect of children's mathematical education to date, this process is less useful for providing a focus for reflection on the individual teacher practices at that grade level.

There is much useful information in the NAPLAN data; however, using the data constructively poses a challenge (Leder, 2012). Scores provide a general picture of numerical achievement, while individual items provide an opportunity to gain insight on children's thinking, and therefore reflect on what additional experiences would be helpful for children. This paper focuses on the results of a single item to examine what might be learned from them.

Although the following observation by Black and Wiliam (2006) was made in the context of children learning in a classroom, it was framed as a general statement that might be applied in a variety of contexts:

> When anyone is trying to learn, feedback about the effort has three elements: recognition of the desired goal, evidence about present position, and some understanding of a way to close the gap between the two. All three must be understood to some degree by anyone before he or she can take action to improve learning. (p. 6)

These three elements are discussed here from the perspective of a teacher wishing to learn from LSA results. The first element, the desired goal, is to develop children's numeracy skills to a greater extent. The assessment items themselves convey information about at least some aspects of the desired goal, an aspect that will be discussed later in this paper. The second element, evidence about the present position of children's attainment, is provided by the LSA results. However, LSA's themselves provide little information regarding the third element, the understanding of a way to close the gap. While this understanding may reside in the accumulated knowledge of teachers and education researchers, in terms of the formal processes involved in LSA's, the application of this knowledge remains ad hoc, and left to individual schools and teachers.

An ad hoc approach is unlikely to realise the goal of an education system where all children attain a rigorous mathematical education as this goal is yet to be achieved anywhere. The international and long-standing difficulty of achieving high levels of numeracy for any population suggests that the goal is difficult and systemic. This paper makes the assumption that substantial progress will require deep rather than shallow teaching. It is likely to be necessary to address issues that have already been recognised as difficult by teachers and in the research literature.

This paper sets out to draw on the existing research literature and provide an illustration of how Black and Wiliam's (2006) three elements of learning can be applied to NAPLAN results, using a single NAPLAN item on the topic of fraction addition as an illustrative example. The general area is recognised as being difficult and important:

> The most important foundational skill not presently developed appears to be proficiency with fractions (including decimals, percent, and negative fractions). The teaching of fractions must be acknowledged as critically important and improved before an increase in student achievement in algebra can be expected. (National Mathematics Advisory Panel, 2008, p. 18)

This section has described NAPLAN as providing the evidence of the present position of children's numeracy attainment, as part of a larger process of systemically improving the outcomes of mathematics education. This paper argues that NAPLAN items draw attention to issues of such a long-standing nature that they have become part of the background scenery. The NAPLAN data provide an opportunity to consider children's attainment at an aggregate level, and to draw on existing research to provide understanding on how to improve outcomes. The argument is illustrated by use of an item that assessed addition of fractions, which is a topic known to be difficult. The interpretation of the results of such an item relies on knowledge of children's understanding of fractions. The following section outlines the existing literature relating to this topic.

Literature Review

The history of research on children's errors involving items dealing with fractions is interesting. Brueckner (1928) reported on a large, detailed study of children's errors with fractions undertaken with the aim of comprehensively cataloguing the types of errors that children make. Hundreds of children from classes from Grades 4 to 8 in six Minnesota elementary schools were given a wide variety of fraction items to complete and their errors were recorded and classified. The study was extensive; on the topic of addition of fractions alone, 25,000 examples of written work were collected and analysed. Brueckner's study was concerned with children's ability to apply processes rather than children's understanding. Despite this, Brueckner's observation that children were not presented with a wide enough variety of drill materials is echoed in modern calls for children to experience a wider variety of tasks.

It is interesting to note that the error in which the numerators were added together and the denominators were added together was classified as a "Lack of comprehension of process involved" (Brueckner, 1928, p. 762), whereas more recent researchers might be more likely to identify this error as a misapplication of additive processes (e.g., Brown & Quinn, 2006). This type of error may arise from a mis-generalisation of an understanding of fractions as parts of a set of objects. Smith (2002) pointed out that older children who continue to make this error justify their reasoning along the following lines: 3 represents three parts out of five, and 2 represents two parts out of five. Adding the fractions together is then construed as five parts out of a possible eight parts.

This error suggests inappropriate generalisation from experiences with combining collections of objects. Learning algorithms prior to having understood underlying principles is known to be counter-productive (Clarke, 2005). Hart (1980) remarked that we "have ample proof that they [algorithms] are not remembered or sometimes remembered in a form that was never taught, e.g. to add two fractions, add the tops and add the bottoms" (p. 212). While this faulty algorithm may not be taught explicitly, it is quite possible that children are mis-generalising from marked tests that they see frequently in the classroom. For many mathematics tests and assignments, the number of marks attained for each question is represented as the numerator of a fraction, with the denominator being the total possible marks for that question. The score for an entire test is found by adding all of the numerators, to give the score attained. This is often expressed as a fraction of the possible score, which is obtained by from the sum of the maximum possible marks for each individual item, previously expressed as the denominators for the individual item scores. In other words, although teachers are not explicitly teaching the faulty algorithm, it is not a surprise that children construct it for themselves.

The fractional representation of the mark for each question is not, in itself, problematic. The problem lies in the apparent addition of fractions that do not represent the same quantity. It would seem that many children lack a sound understanding of the nature of a fractional number. At the most basic level, fractions represent a quantity that can be represented on a number line (Kilpatrick, Swafford, & Findell, 2001; Siegler, Thompson,

& Schneider, 2011). However, many students, and some teachers, lack a well-grounded sense of the magnitude of fractions (Clarke & Roche, 2009; Siegler, Fazio, Bailey, & Zhou, 2013).

Method

This paper sets out to illustrate that NAPLAN assessments have the capability of focusing attention on difficulties children experience with numeracy and thus lead to constructive implications for teachers. The secondary analysis presented in this paper presents an in-depth analysis of anonymised children's responses to a single item, Item 25, from the 2008 Year 7 NAPLAN Numeracy non-calculator assessment. This item was selected because it possessed the following features:

- The topic, addition of fractions, has been observed in the literature to be difficult.
- A higher proportion of children than had chosen a particular incorrect response than had chosen the correct response.
- The popular incorrect answer was implausible by applying a common-sense measure of magnitude.

This unusual combination of features is interesting, and is suggestive that an analysis may provide insights into difficulties children may have either with this topic, or perhaps more general difficulties.

The interpretation of the inherently limited information available from the NAPLAN data was informed by drawing on the research literature on this topic. NAPLAN data are used and reproduced with permission of the Victorian Curriculum and Assessment Authority (VCAA). Analysis and findings using that data are not connected with or endorsed by the VCAA.

Analysis of Children's Responses to Fractions Item

This section presents an analysis of Year 7 children's responses to Item 25 on the Victorian NAPLAN Numeracy non-calculator assessment. Item 25 was a multiple choice item that was answered correctly by 35% of children:

> A garden centre sells a potting mix made up of soil, compost and sand. Soil makes up $\frac{2}{3}$ of the mix and compost makes up $\frac{1}{4}$ of the mix. What fraction of the potting mix is sand?

The options were: $\frac{1}{12}$ (correct), $\frac{3}{7}$, $\frac{5}{12}$ and $\frac{4}{7}$ The option $\frac{3}{7}$, which corresponds to adding the numerators and the denominators to arrive at an answer, was chosen by 37% of children. The remaining two options were chosen by 12% and 13% of children respectively. This item is interesting because of the high proportion of children who added the numerators and the denominators of the two fractions without regard to the question, suggesting that some children are generalising from their instrumental understanding of adding whole numbers to apply the same process to adding fractions (Brown & Quinn, 2006).

The potting mix item involved both problem-solving skills and multiplicative thinking in the use of fractions. The apparent difficulty of this item is dependent on whether it is approached as an exercise with fraction operations or as a problem-solving exercise.

Multiple choice items ask questions of the form: *Given the answer is one of the options shown, which one of these options answers the given question*? (Morley, 2013). This section discusses a possible problem-solving approach in which the question might be restated as: Which of the following options: $\frac{1}{12}$, $\frac{3}{7}$, $\frac{5}{12}$ and $\frac{4}{7}$ could be added with $\frac{2}{3}$ and $\frac{1}{4}$ to make 1?

There are two aspects of fraction understanding required to answer this question using a problem-solving approach. The first is that the sum of the fractions given, including the unknown fraction of sand, is equal to one. The second is a sense of the size of the fractions. Since the amount of soil in the mix is $\frac{2}{3}$, the remainder of the mix totals $\frac{1}{3}$. The addition of compost allows the inference that the only plausible option is $\frac{1}{12}$, since this is the only option that is less than $\frac{1}{3}$. It is only necessary to consider the first term, and not at all necessary to perform the calculation $\frac{2}{3}+\frac{1}{4}$. From a diagnostic perspective, it would appear that most children who answered this item incorrectly children are not approaching this item by considering what a plausible answer might be in terms of magnitude. Either children have not yet acquired a sense of the size of such fractions, as suggested by previous research (e.g., Clarke & Roche, 2009), or they are not approaching the item by considering what a plausible answer might be prior to performing any calculations.

An alternative approach to the item that may have been taken, as an exercise in operating with fractions, is discussed here. In this case, obtaining the correct answer requires finding the sum of $\frac{2}{3}$ and $\frac{1}{4}$ which is $\frac{11}{12}$. Since the required amount, when added to $\frac{11}{12}$ would equal one, subtracting $\frac{11}{12}$ from one yields the correct answer, $\frac{1}{12}$. Finding the sum of $\frac{2}{3}$ and $\frac{1}{4}$ might present a high cognitive load for children who are under time pressure unless they are fluent with fractions and familiar with this type of problem to the point where it is routine.

The fact that 37% of children chose the option $\frac{3}{7}$, corresponding to adding the numerators and the denominators, suggests that children did not consider the magnitudes of each of the fractions stated in the item question as numbers with individual magnitudes. This finding supports Siegler et al.'s (2013) contention that:

> Learning to accurately represent and arithmetically combine the magnitudes of all types of real numbers – whole numbers and fractions; positives and negatives; common fractions, decimals, and percentages – is thus inherently central to numerical development

This analysis has provided findings from NAPLAN data that support the need for robust teaching practices regarding the numbers, particularly the introduction of fractional numbers. The next section discusses the implications for teachers of the above analysis and then discusses the limitations of individual teachers to effect the desired educational changes.

Implications of Analysis for Teachers

This section discusses the implications for teachers that arise from the analysis of the addition of fractions item. Haladyna, Downing and Rodriguez (2002) reviewed multiple choice item-writing guidelines and found that among those universally endorsed by textbooks were that distractors should be plausible and reflect common errors by students. The item analysed earlier clearly reflects common errors and therefore may reasonably be considered plausible to the children involved. Yet, this distractor would not be considered plausible if the magnitude of the fractions were considered. Whether this distractor could be considered plausible or not is therefore dependent on the children's expertise with the magnitude of fractions.

The data are consistent with Siegler et al.'s (2013) position that "interventions that improve fraction magnitude representations also improve other mathematical capabilities" (p. 17). Clarke (2005) suggests that the early introduction of written algorithms that operate on a number from the digit representing the smallest magnitude to the digit representing the largest magnitude interferes with children's natural thinking of magnitude:

> They tend to lead to blind acceptance of results and over-zealous applications. Given the focus on procedures that require little thinking, children often use an algorithm when it is not at all necessary. (p. 94).

Blind acceptance of results is antithetical to the goal of numeracy. Numeracy, at least in part, involves being able to detect whether a given result is plausible or not. The desirability of this skill is not limited by age or proficiency with mathematics. Even for people who are proficient with complex calculation, such as engineers, it is important to evaluate the plausibility of a result, whether the calculation is done by hand or by computer. It would appear that a consideration of what results might be accepted as plausible prior to performing calculations is a neglected area in Victorian mathematics education and therefore addressing this issue offers scope for the general improvement of numeracy outcomes.

Implications for the National Development of Numeracy in Australia.

This section discusses of the implications of this analysis on the broader question of the use of NAPLAN data to effect system wide improvements. Large-scale assessments such as NAPLAN are intended to provide direct feedback of children's mathematical achievement to teachers and thereby drive improvements to teacher practices. While assessments may exemplify aspects of the curriculum to an extent, generating improvements through feedback to teachers of the large-scale assessment results remains ad-hoc in approach, rather than systematic, because beneficial changes that might be made as a result of individual teachers or schools are not systematically identified and propagated back through the system.

For example, assume, for the sake of argument, the hypothetical ideal situation:

- feedback to teachers was clear and instantaneous;
- the current teacher was the only teacher whose practices were involved;
- the action required to attain desired outcomes was clear;
- a particular teacher managed to incorporate the feedback perfectly; and
- the subsequent assessment results reflected the improved teaching.

Even with these ideal conditions, the improvements that would result would remain local to the school unless successful classroom innovations were identified so that they might propagate through the school system, as a teachers' classroom practice does not extend far beyond their own class. While this occurs to some extent through the professional literature, conferences and other professional activities, it is not incorporated directly through the NAPLAN system. Further, competition between schools fostered by the posting of schools' NAPLAN attainment on the My School website may inhibit sharing. Large-scale assessments provide not only the opportunity to identify schools requiring additional support, but there is also the potential opportunity to identify and share practices that are found to be exemplary, as yet largely untapped. The required changes are system-wide, not local, and the feedback from assessments have implications for teaching practices beyond the current teachers of the assessed cohort.

Conclusion

The goal of improving educational outcomes for all is a daunting one. This paper addressed how the data from large-scale assessments can be used to attain educational improvement, illustrated by an analysis of an item that assessed proficiency with fractions.

Drawing on the research literature, the analysis of a fractions item from a Year 7 Numeracy assessment suggested that interventions that strengthen children's understanding of the magnitudes of fractional numbers may be beneficial (e.g. Siegler et al., 2013). This finding suggests that difficulties that may be evident in Year 7 are an issue in the short term for the current teacher of the child, but for sustained systemic improvement they are also an issue for teachers in earlier years. The implication for the education community is that findings from NAPLAN results have relevance for a wider section of the education community than the current teacher.

Generalising the findings from the analysis of the fractional item it is possible that interventions that encourage children to consider what answers may or may not be plausible for any given question may help develop children's ability to connect the results that they obtain from calculations with their broader understanding of numerical concepts. Further research in this area may be beneficial.

Children's difficulty with addition of fractions has been noted in the literature for many decades. The persistence of children's difficulty in adding fractions is one example that provides evidence that the attainment of such understanding is non-trivial. Drawing on Black and Wiliam's (2006) three elements required for learning, large-scale assessments such as NAPLAN may clarify the first element, specific attainment goals, and directly address the second element of providing evidence of children's current position. However, LSA's do little to address the third element: attaining an understanding of how to close the gap between the goal and the current position. This paper, therefore, recommends further research on the processes involved with large-scale assessments to encourage a more systemic approach to improving education rather than relying on ad hoc efforts duplicated throughout the nation.

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