Developing Young Students' Meta-Representational Competence through Integrated Mathematics and Science Investigations

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This paper describes students' developing meta-representational competence, drawn from the second phase of a longitudinal study, *Transforming Children's Mathematical and Scientific Development*. A group of 21 highly able Grade 1 students was engaged in mathematics/science investigations as part of a data modelling program. A pedagogical approach focused on students' interpretation of categorical and continuous data was implemented through researcher-directed weekly sessions over a 2-year period. Fine-grained analysis of the developmental features and explanations of their graphs showed that explicit pedagogical attention to conceptual differences between categorical and continuous data was critical to development of inferential reasoning.

The development of informal statistical reasoning has received increasing attention in mathematics and statistics education research and related curriculum development (ACARA, 2012; English, 2012; Makar, Bakker, & Ben-Zvi, 2011). Engaging children in fundamental aspects of data modelling includes structuring and representing data, identifying variation in data, making predictions and drawing informal inferences. Here we see synergies with science education where student-led investigations involve the process of investigation and scientific process, including the representation of changes in phenomena over time (Prain & Tytler, 2012).

Background to the Study

Recent studies on young children's mathematical development have highlighted the importance of representations and tools to promote structural awareness and generalisation, albeit emergent, from an early age (Mulligan & Mitchelmore, 2009). Our studies have shown that structural awareness is an underlying feature of mathematical development that we identified as Awareness of Mathematical Pattern and Structure (AMPS), a construct which can be reliably measured. A longitudinal study of 5-6 year olds, *Reconceptualising Early Mathematics Learning,* evaluated a structural approach to early mathematics learning (Mulligan, English, Mitchelmore, & Crevensten, 2013). This study showed that the scaffolding of structured tasks over time can significantly advance the development of such mathematical processes as patterning and unitising, spatial structuring, multiplicative and pre-algebraic reasoning. The question remained about whether AMPS may play a significant role in development of meta-representational competence in data modelling.

A new study, *Transforming Children's Mathematical and Scientific Development,* integrated the idea of developing AMPS through student-led investigations in data modelling (see English, 2012; Mulligan, Hodge, Mitchelmore, & English, 2013). This study built on an aligned longitudinal study of Grade 1 students (English, 2012), which indicated that children as young as 6 years old can successfully collect, represent, interpret, and argue about the structure of data. In our new study, developmental features of how students represent data through the integration of mathematical and scientific investigations

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(Prain & Tytler, 2012) were analysed. This paper describes examples of fine-grained analysis of core features of meta-representational competence, i.e., how students conceptualised and represented data through scaffolded learning experiences. In particular, we focus on the conceptual changes that were necessary for students to interpret and represent continuous data.

Theoretical Perspectives

Data Modelling

Data modelling involves a number of inter-related components that enable the development of both conceptual and meta-representational competence. English (2013) proposed an adapted model of data modelling (Lehrer & Schauble, 2004) that originates with students' initial questions and investigations of real-life situations. This leads to students realizing the need to generate and measure particular attributes of the data collected, and to organize, structure and represent that data in meaningful ways. Developing concepts of informal inference and variation can emerge from these investigations (Lehrer & Schauble, 2005; Makar, Bakker, & Ben-Zvi, 2011; Watson & Fitzallen, 2010). The model highlights conceptual and meta-representational competence as inter-related but individual components that play a dynamic role in the modeling process (English, 2013). What we need to consider in the analysis of young students' representations of data is the symbiotic nature of that interrelationship. English describes this as "tightly interactive… rather than rigidly sequential" (p. 69).

Structural Development

The analysis of students' graphical representations has been described as increasingly sophisticated stages of development (Prestructural, Unistructural, Multistructural, and Relational) ranging from simple showing attributes, methods of displaying data, to understanding relationships and variation (Watson & Fitzallen, 2010). Similarly, a structural approach based on the construct of Awareness of Mathematical Pattern and Structure (AMPS) can be applied to the analysis of students' development of data-structuring skills (see Mulligan & Mitchelmore, 2009). Students' AMPS can be described by identifying common features of underlying structures in their representations. For example, our data has revealed the importance of representing intervals and constructing scale.

In our earlier studies (Mulligan, English, Mitchelmore, & Crevensten, 2013), the notion of equal-sized units was found to be critical to the construction of scales necessary for representation on number lines, measures of length, area and coordinates. The vertical-horizontal structure of graphs is identical to the structure of a rectangular grid/array and includes ideas of congruence and co-linearity. Students with a good understanding of the number lines and the rectangular grid structure may therefore be able to acquire graphing skills more quickly than others. Allowing young students to create their own pictographs initially, without scale, was a basis for developing concepts of attribute, frequency and variation, to which they could later add scale.

Method

Participants were 21 students in an academically selective Grade 1 class, all male, of an independent school in an Australian capital city. These students were followed for 22 months from the beginning of Grade 1 through to the end of Grade 2. At the beginning of Grade 1 (February), the students ranged in age from 6 years and 1 month to 7 years and 10 months (mean 6 years and 6 months), following their participation in PASMAP during Kindergarten. Students came from high socio-economic backgrounds and a range of cultural/ethnic groups.

As measures of students' high ability, the researchers administered the Peabody Picture Vocabulary Test, 4th edition (PPVT4) (Dunn & Dunn, 2007) and the Raven Coloured Progressive Matrices (RCPM) (Raven, 2004) in October of their Kindergarten year. The median score on each test was at the 95th percentile. Students were also administered two forms of the PASA interview at the beginning of Kindergarten and at the end of Grade 1 (Mulligan & Mitchelmore, in press). Broadly, the majority of students were classified as operating at the structural or advanced structural level at the Grade 1 interview.

Data Collection and Analysis

Students were placed in two learning groups: advanced (10 students) and less advanced (11 students). Each group was withdrawn from the regular class mathematics lesson for one hour per fortnight for four consecutive school terms (16 sessions) in Grade 1 and for two consecutive terms in Grade 2. The lead researcher and an assistant led the data modelling investigations based on the questions posed by the students. The classroom teacher was consulted about the planning and implementation of the program and was debriefed following each session. The students did not receive explicit instruction in their regular classroom program on data modelling or advanced graphical representations. Data collected were the scanned completed work samples, including students' written accounts of their activities, and the researcher's observation and evaluation notes taken during and after the learning sessions. (Collection of video data was not permitted).

Student data were collated in a student profile. Work samples were analysed for features of AMPS and subsequently coded for level of structural development based on the features revealed. Analysis utilized iterative refinement cycles, comparing prior learning with new structural features (Lesh & Lehrer, 2000). Each work sample response type was coded by the first author, and checked by the senior research assistant; consensus was reached on all coding.

Data Modelling Learning Sessions

The learning program adopted a design-based research approach that enabled the development of particular forms of graphing to build effective data representation skills (Kelly, Lesh, & Baek, 2008). The learning sessions aimed to develop children's interest in data modelling and build their skills in structuring data through representations based on the investigations that were part of their classroom program or those that spontaneously arose as a result of the children's questions about everyday events. Such topics included Pets in our class, Birthdays, Holiday destinations, Daily temperature, Melting ice, Growth of chickens, and Growth of onions. Because these students had already highly developed literacy skills, they were encouraged to explain their representations in writing and to produce reports that emphasised the process of representing data as a 'story'. Another strategy was to allow students to pose questions about their graph that others could answer. The sessions began with opportunities to focus on data that may have already been collected and to represent and describe it as clearly as possible. Considerable time was devoted to developing skills of visualising and sketching data freehand, where students were given minimal instructions. Careful attention was paid to the shape of data sets and determining which types of displays best showed the variation in a data set. Various ways of representing the data were encouraged. An important learning feature was students' ability to determine the most effective graph to displays these data. Students' iterative refinement of their representations was particularly reinforced. Students were always presented with their previous representations and others' attempts; some scaffolding enabled them to improve the clarity and mathematical detail and accuracy of their graphs. Students were required to justify why they had represented their data in particular ways.

Students had already experienced the construction of two-way tables and picture graphs, with attention to coordination of vertical and horizontal grid lines to structure the data. Students were also familiar with using 2cm grid paper to assist in recording data as a vertical pictogram, a task that most students found easy (following each session students were required to reproduce their graphs from memory).

Analysis of Students' Graphical Representations

An analysis of students' graphical representations for three investigations is provided: *Melting Ice*, *Growth of Chickens* and *Growth of Onions*. In the first investigation, *Melting Ice*, students predicted how long it would take for an ice cube to melt when considering variation in temperatures. They posed further questions about what might happen if the melted ice was re-frozen. The students decided on the intervals of time that they would observe of the melting ice. They attempted to construct horizontal and vertical axes showing intervals of time and measures of temperature. They explained the process in writing. Figure 1 shows Franz' interpretations that the ice would melt in a period of 5 hours indicating the rise and later the fall in the temperature. He poses new questions.

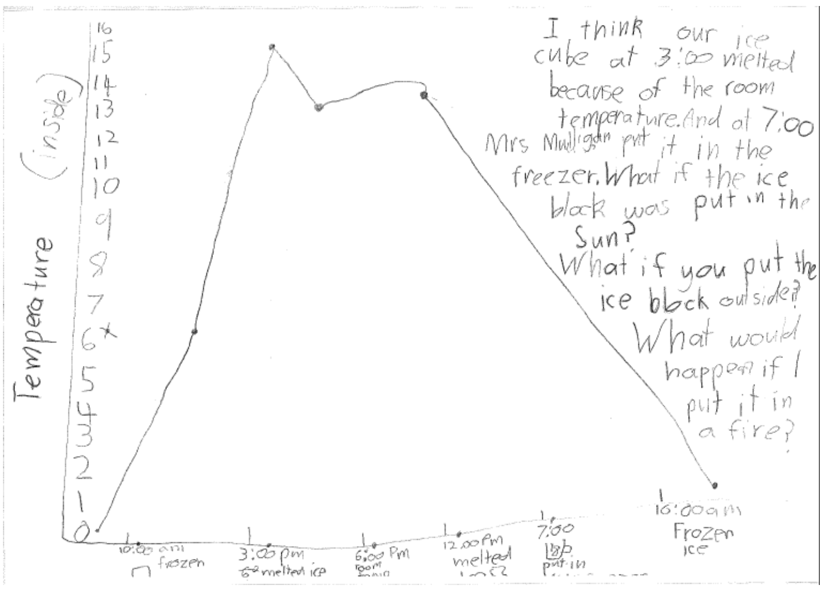


Figure 1: Franz' representations of ice melting over time.

Figure 2 shows Edward’s representation and report of his observations of the development of eggs to 10-day-old chickens. Edward uses pictorial icons on the horizontal scale to represent the stage of chicken growth. On the vertical scale he uses equally spaced points to indicate number of days (10), although in his report he understands that the total number of days for the chicken to hatch and grow is 21.

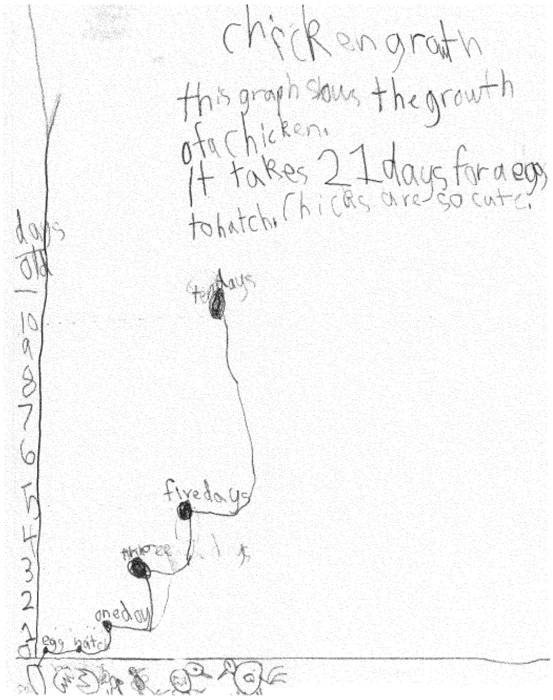
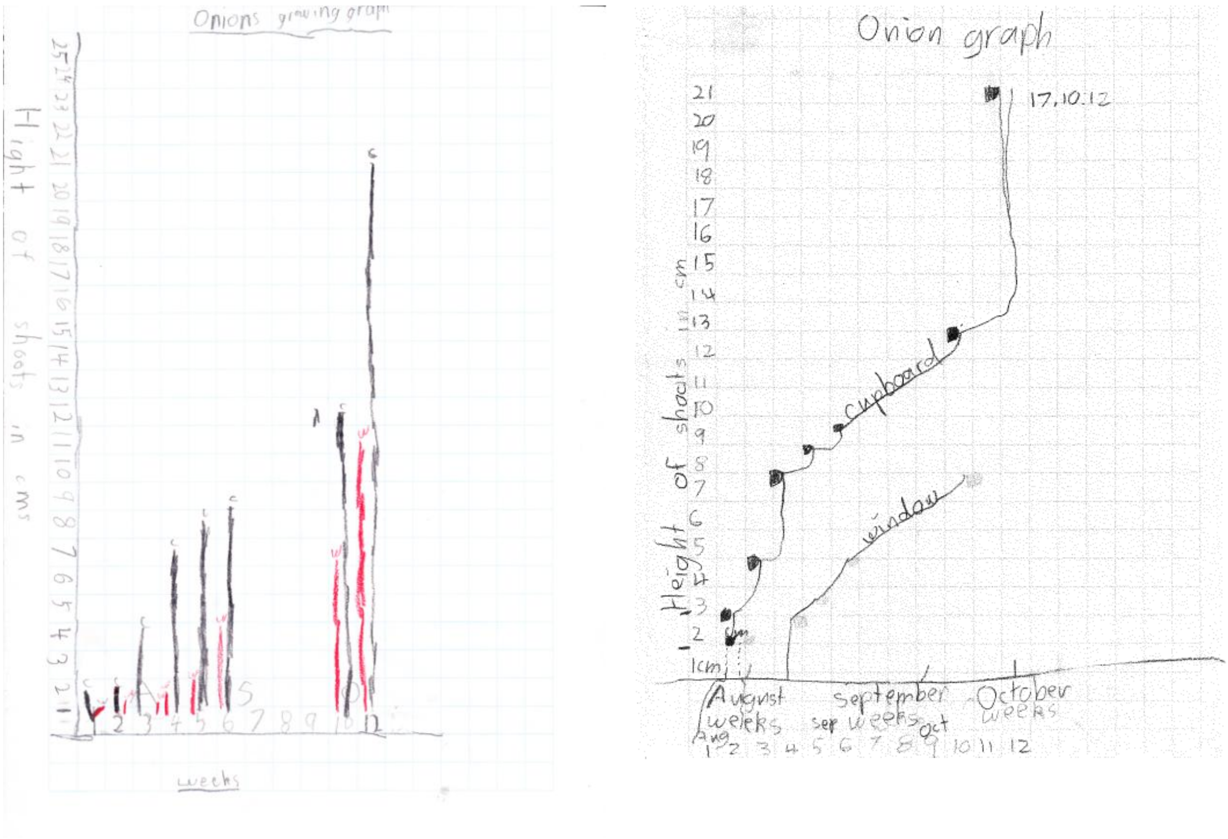


Figure 2: Edward’s representations of chicken growth over time

Edward’s icons depict increasing growth but he is not able to represent the interval of time showing growth within the egg (i.e., 21 days). While his graphical representation shows features of coordinated vertical and horizontal axes, the representation is incomplete. Thus we would categorise this as partial-structural level. An important inference drawn from this example is that Edward focuses on representing data that he could observe first hand (the actual growth of the chicken) but is not able to visualize the 21-day period prior. He combines his personal observations (“Chicks are so cute”) with mathematical features. In terms of his understanding of the shape of a line graph, he is yet to understand the difference between the curved and straight line between each point.

In Figures 3a and 3b, contrasting graphs show the growth of onions under two different conditions— in a cupboard and on a windowsill. No data were collected for three weeks (7-9) while the students were on holidays. Contrary to the boys’ predictions, the onion shoots grew longer in the cupboard. James uses grid paper to accurately depict the vertical scale (height of onion shoots in centimeters) and intervals of time on the horizontal scale (number of weeks). He draws pairs of column to show the two different conditions, labeling and colour-coding each column as red ‘c’ (cupboard) and black ‘w’. James has not yet developed a representational strategy for showing growth over time, and he uses strategies built up from his experience of drawing column and bar graphs. James is at a structural stage for categorical data, but his understanding is still at an emergent level for representing continuous data.

In Figure 3b, we see a more sophisticated attempt to graph the growth over time using two line graphs. Franz draws the horizontal axis first to show intervals of time over 12 weeks. Although this is accurate, he starts the window condition data at week 3, ignoring that the two conditions began simultaneously. The vertical axis represents the growth in centimeters, for which a flexible tape was used when a ruler was found to be ineffective. Franz' line graph shows no understanding of the difference between a straight and a curved line. Franz is in the partial-structural stage for representing continuous data, and it would be expected that both examples might have inaccuracies without the support of the grid paper.



Figures 3a and 3b: James' and Franz' graphs of onion growth over time

Discussion

The first phase of this present study focused on the collection and representation of categorical data, e.g., drawing picture graphs of categories of Pets in Our Class. Students' attempts to use one-to-one matching as a baseline were readily surpassed when they were able to conceptualise the need for a common scale. In the second phase, continuous data were collected and represented leading students to coordinate the horizontal and vertical axes. One of the important features was the students' ability to conceptualise the meaning of the data, e.g., temperature changing over time. This required more integrated understanding of variation and coordination of more than one element of structure at a time. In representations of continuous data, spatial structuring is necessary in visualising and organising equal spacing, coordination of axes, and scale. A common scale must be used for each graph, conveniently drawn on a vertical axis on the left hand side. This scale defines points, not intervals. Not all values on the vertical axis need to be labelled (numbered), and the points should be evenly spaced horizontally.

More importantly, the development of conceptual understanding of the meaning of the data and the inferences that can be drawn from these data are critical.

A crucial observation in terms of developing meta-representational structure was the students' ability to notice and then coordinate all of the elements required for constructing each type of graph. In the case of Franz, we observe high-level AMPS, because he is able to integrate many aspects of structure simultaneously. His conceptual understanding of the meaning of the graph is also high. We regard this as an example of the interrelatedness of the model described earlier (English, 2013). The idea of drawing a line to display change over time was built over time from experiencing real-life investigations in different contexts. From this representational view, the student can explain the shape and meaning of the data. In this process students may begin to see patterns and relationships in the data, a move towards informal inference.

Our data support the findings of English's (2012) analysis of representations in data modelling contexts. We found a diverse range of icons, including pictorial imagery, and structural features such as grid lines and symbols that reflected student's individual forms of representation. Further, these students' ability to explain their graphs and refine (re-represent) their representations was impressive. This may have been masked if the graphs and the structural features had been provided for them. Challenging students to explore structural features and to discuss these with others of similar ability provided opportunities to co-construct meaning in what often became an increasingly motivating context for engagement in mathematics learning.

Conclusions and Implications

This second phase of the analysis of student-led investigations provided new evidence of students' ability to develop statistical concepts alongside, or interrelated with meta-representational skills. The development and coordination of meta-representational features promoted effective discussion in the group about what they were trying to convey in their graphs. The requirement to "explain and raise further questions about your data" supported a scaffolded pedagogical approach that centred on developing coherent ways of representing the meaning of the data. At times students reverted to, or integrated pictorial and idiosyncratic 'fragments' of their images of the data. We concluded, tentatively, that young, able students can develop critical developmental features of data modelling. This may have been difficult to observe and describe systematically with less able students. While this sample indicated a risk of underestimating young children's capacity for meta-representational competence, our findings do not permit generalisation. Nevertheless, we have been able to describe critical features that can be shaped into a pedagogical framework for data modelling. Implementation of curriculum priorities in statistical reasoning and associated teacher pedagogical learning may then be more easily achieved.

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