Arithmetical Strategies of a Student with Down syndrome

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Kayla was a 15 years old girl with Down syndrome attending a special education school in Indonesia. A modification of Wright et al.'s (2006) approach to assessment documented her number knowledge and arithmetical strategies. This paper discusses the assessment process and the results focusing on her ability to solve number problems. Results show that Kayla's stages in early arithmetical learning and base ten arithmetical strategies are the same as those of typical developing students of a much younger age. This supports the notion that a student with Down syndrome may be capable of learning arithmetic similar to that learned by typical developing children, but their speed of learning appears to be much slower.

Introduction

Many researchers have focused on the development of typical students' number knowledge (Bobis et al. 2005; Clarke, 2006; Gervasoni, 2007). However the mathematics education community gives scant attention to such research on students with special needs. Since traditionally, educators in many countries accept that every child has an equal right to receive high quality mathematics education, it seems timely for us to give more attention to the mathematics teaching and learning of children with special needs, including students with Down syndrome. Being able to use effective and efficient strategies to solve addition and subtraction problems is one of the important aspects of number learning. This paper describes and discusses the strategies which were used by a student with Down syndrome in solving arithmetic problems. Her strategies are compared to strategies used by typical children; and implications, limitations and further research are described.

Literature Review

A study by Brigstocke, Hulme and Nye (2008) shows that children with Down syndrome have difficulty mastering number skills, counting and simple arithmetic. This result is supported by Porter (2000) which shows that children with Down syndrome have difficulty in learning the number string. Compared to typically developing children, children with Down syndrome produced significantly fewer number words altogether, shorter standard number word sequences, and could not count larger sets (Nye, Fluck, & Buckley, 2001). Further, research shows that even though children with Down syndrome have a deficit in counting, appropriate teaching approaches during intervention was of benefit (Abdelhameed, 2007, 2009). Several methods and teaching materials have been suggested for more effective teaching (Haslam, 2007; Horner, 2007; McConnochie & Sneath, 2007; Wing & Tacon, 2007).

Despite this evidence, it was suggested that individual differences among children with Down syndrome should be studied to better understand their learning (Buckey, 2007). Children with Down syndrome can do and learn mathematics (Faragher, Brady, Clarke, & Gervasoni, 2008). Understanding the number knowledge possessed by them prior, during and after teaching is an important factor in order to advance their number knowledge. The Zone of Proximal Development proposes that students learn best if they are challenged

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within close proximity to their current level of development (Vygotsky, 1978). If a teacher accepts this principle, then it is very important to understand their students' current levels of development before they devise appropriate teaching activities. We are yet to determine what people with Down syndrome are able to achieve (Buckley, 2007). Determining and documenting what a child with Down syndrome is able to achieve is an important key to advancing their number skills through appropriate teaching strategies.

Method

At the time of interview Kayla, a 15 year old student with Down syndrome [DS], was attending second grade of a Special Junior High school for the intellectually disabled in Yogyakarta Special District, Indonesia. The school is a co-educational school catering to 126 students from kindergarten to high school with ages from five to 19 years. These students were assigned to a class or level according to their chronological age. There were 49 teachers and the ratio between teachers and students at this school was 1:2.6, which is considered ideal in Indonesia. Twelve students (seven girls and five boys) were diagnosed as individuals with DS based on physical observation and a Dermatogliphy test carried out by a medical specialist. Kayla was interviewed by the author.

The interview was modified and simplified from the assessment techniques and tools which are used in the Mathematics Recovery (MR) Program (Wright, Martland, & Stafford, 2006), which draws on research-based theories about how young children progress in arithmetical learning. These theories include Steffe and Cobb's (1988) theory about children's counting types and Cobb and Wheatley's (1988) theory about base-ten arithmetical strategies. The Stages of Early Arithmetical Learning (SEAL) are presented in Table 1 and Base Ten Arithmetical (BTS in Table 2. These models were also used to document the number knowledge of children in the first and second grade of an Indonesian School (Rumiati & Wright, 2010). The interviews consisted of three parts: Part 1 had the purpose of assessing her ability to identify numerals. Numerals are the written and read symbols for numbers, for example the numeral "3" is read as "three". Kayla was shown one-digit, two-digit and three-digit numerals, and not in numerical order. Part 2 was designed to determine her early arithmetical strategies. In this part the interviewer used counters, a screen and written number problems. Table 3 shows the interview procedure for determining Kayla's early arithmetical strategies.

Table 1
The model for stages of early arithmetical learning

Stage	Name	Characteristic
0	Emergent	Cannot count visible items
1	Perceptual	Can count visible items only.
2	Figurative	Can count invisible items, but starts from one.
3	Advanced-counting-by-ones	Can count invisible items, using a counting-on strategy to solve addition or missing addend tasks, and may use a counting-back strategy (counting back-from or counting-back-to) to solve missing subtrahend or removed items tasks.
4	Facile	Can use non-counting-by-one strategies, such as doubles, add through ten, compensation, etc.

Table 2
The model for the development of base- ten arithmetical strategies (Wright, et al., 2006)

Level		Characteristic
1	Initial concepts of ten	Not able to see ten as a unit composed of ten ones. The child solves tens and ones tasks using a counting-on or counting-back strategy.
2	Intermediate concepts of ten	Able to see ten as a unit composed of ten ones. The child uses incrementing and decrementing by tens, rather than counting on by ones to solve an uncovering board task. The child cannot solve addition and subtraction tasks involving tens and ones when presented as horizontal written number sentences.
3	Facile concepts of ten	Able to solve addition and subtraction tasks involve tens and ones when presented as horizontal written number sentences by adding and/or subtracting units of ten and ones.

Part 3 was designed to assess her strategy for solving two-digit number problems. Initially, ten-dot strips were presented one by one and the interviewer observed whether she incremented by tens or counted on. The interviewer showed a card with a two-digit number problem written on it and said, ‘please solve this problem.’ When she gave a correct answer, she was asked how she found the answer. The task was repeated several times with different two-digit number problems. The interview of 30 minutes was videotaped and constitutes the main data source. Secondary sources such as field notes and conversations with her teachers were also used. The videotapes were watched and transcribed in Indonesian and translated into English. They included what was heard and observed. Her stage and level were determined using the models in Tables 1 and 2.

Table 3
Interview procedure for determining Kayla’s arithmetical strategies

Type of tasks	Procedure
Unscreened tasks (addition)	The interviewer places a number of counters (yellow) on the table, and then asks, “How many are there?” Kayla’s first response noted. The interviewer then places the second set of counters (black) on the table and asks, “How many are there?” Kayla’s second response noted. The interviewer then asks. “How many are there altogether?” Kayla’s response (solution sequence) noted.
Screened tasks (addition)	Same as unscreened tasks, but after the second response, all the counters were screened so Kayla was not able to see these counters
Horizontal addition written problems	She was asked to solve one-digit addition problems in horizontal format written on a piece of paper. Her responses was observed and noted.
Removed items tasks	The interviewer places a number counters (yellow) under the screen; “I have ... counters here and then I take ...?” take several counters and asked. “How many counters are there now?” Her strategy was observed and noted
Horizontal subtraction written problems	Same as horizontal addition written problems

Results and Discussion

Kayla was able to identify all one and two-digit numerals which were shown to her. One-digit numerals were 7, 4, 5, 1, 0, 9, 8, 3, 6 and 10 and two-digit numerals were 59, 12, 48, 15, 21, 23, 19, 11, 13, 83, 77, 56, 77, 56, 25, 20, and 86. The presentation of these numerals which were not in order provides greater difficulty than if these numerals were presented in order. Thus Kayla was assessed as fluent in identifying one- and two-digit numerals. However, when she was shown three-digit numerals, she made mistakes. For "234" she said "twenty three thirty four", for 543 she said "fifty three forty three", for "710" she said "seventy one ten", for 110 she said "a hundred" and for "999" she said "ninety nine ninety nine". Compared with typical developing students, Kayla's ability in identifying numerals is similar to students at the first grade level. These students are around seven or eight years old. Typical developing students at the same age as Kayla have been able to identify numeral up to six-digits. This result accords with the findings of Porter (2000) and Nye, Fluck and Buckley (2001) that children with Down syndrome produced significantly fewer number words and shorter standard number word sequences than their counterparts. Even though, Kayla could not identify three-digit numerals, some of her incorrect responses indicate a common pattern, that is, she breaks the three-digit numeral into two two-digit numerals and reads it as two two-digit numerals. In conversation after the interview, her teacher said that she had not been taught to identify three-digit numerals.

Table 4 shows Kayla's response on unscreened tasks. The table shows that Kayla is able to recognise three counters without counting, while for more than four counters she needs to count one by one. She uses a count by one strategy to solve unscreened tasks.

Table 4
Kayla's response on unscreened tasks

Problem	Response One	Response Two	Solution Sequence
4 + 3	"four" counts counters quickly pointing one by one.	"three" without counting	"four" *Interviewer repeats question.* Counts the counters quickly by pointing one to one. 1, 2, 3, 4, 5, 6 7...7
5 + 8	"five" counts counters one to one quickly	"eight" counting one by one	Counts the counters quickly pointing one to one 1, 2, 3, 4, 5, 6, 7 ...13 so 13

Table 5 shows Kayla's response on screened tasks. Despite her failure to solve all screened tasks, it seems that her strategies for solving screened tasks were quite clear. Her strategy involved counting by ones from one and attempting to keep track with her fingers. For small numbers 3+2 this strategy worked correctly, and also for 5+6, since even though she ran out of fingers, she was still able to imagine that there was one more left before she reached the correct answer. For a larger number of counters, 9+9 and 7+8, she stoped at an incorrect number because she could not keep track of the counters in the second collection.

Table 5
Kayla's response on screened tasks

Problem	Response One	Response Two	Solution Sequence
3+2	"three" without counting	"two" without counting	She uses her fingers to count, 1,2,3, and then continue 4,5. "five
5+6	counting quickly by pointing the counters one by one "5"	counting quickly by pointing the counters one by one "6"	raise her fingers and start to count one by close the fingers one by one, 1,2,3,4,5, on one hand and continue to the right hand, 6,7,8,9,10...and 11
9+9	counting quickly by pointing the counters one by one "9"	counting quickly by pointing the counters one by one "9"	raise her fingers and starts to count by closing the fingers one by one, 1,2,3,4,5, on one hand and continues to the right hand, 6,7,8,9,10..and stops there are 10
7+8	counting quickly by pointing the counters one by one "7"	No response	raise her fingers and close the fingers one by one, 1,2,3,4,5,6,7 stop a moment and continue 8,9,10, then using the fingers she already used 11,12,13,14,15, 16 and stops at 16. There are 16.

She answered the two horizontal addition written numbers correctly. Her strategy still involved using her fingers but in different ways. Her strategy for solving 5+6 as a horizontal written number was the same as she would solve the same problem when presented as a screened task. However, for the task 8+7, she raised her 8 fingers to represent 8 and then counted by one, then when all fingers were closed she raised her 8 fingers to represent 7 and continued to count. This strategy seemed to be more effective that the strategy she used to solve the tasks involving screened collections, since she was able to find the correct answer for 8+7. Table 6 shows Kayla's solution for the horizontal written number problems 5+6 and 8+7.

Table 6
Kayla's solution on horizontal written number problems (addition)

Problem	Solution
5+6	Raise her fingers and start to count by closing her fingers one by one, 1,2,3,4,5, on her left hand and continue on her right hand, 6,7,8,9,10... 11.
8+7	Raise her 8 fingers and start to count from one by close the fingers one by one, 1,2,3,4,5,6,7,8 and stop. Then she raise 7 fingers and while closing one by one she continue to the right hand, ,9,10,11,12,13,14,15. It is 15.

Table 7 shows Kayla's solution on removed items tasks. The table shows her responses quickly for the first and second task. She seemed to be just guessing the answer. She was not able to explain her strategy. When the interviewer continued to probe she tried to explain a count on strategy. This response was interesting since she did not use this strategy for previous tasks.

Table 7
Kayla's solution on removed items tasks

Problem	Response	Solution
8−...=5	3 (quickly)	She was not able to explain her strategy
11−...=4	4 (quickly)	She said she does not know then she said 4...then 5, 6, 7, 8, 9, 10, 11. There are 11.

The interviewer decided to present horizontal subtraction written number problems. Table 8 shows Kayla's solution on these tasks. For the first task she used a counting back strategy and used her fingers to keep track. This strategy was successful since she had five fingers to keep in track. However for the second task, she failed because she did not have 12 fingers to solve this task correctly.

Table 8
Kayla's solution on horizontal written number problems (subtraction)

Problem	Solution Sequence
13−5=...	She opens her 5 fingers and close one by one while saying 12, 11, 10, 9, 8. The answer is 8
16−12=...	She said she doesn't know, then she opens 10 fingers and counts down 15, 14, 13,... 6 and said that the answer is 6.

Kayla was able to identify two-digit numerals. It was also important to know her strategies for solving two-digit horizontal addition written number problem tasks. Before assessing her ability in solving two-digit additions, a sequence of tasks involving incrementing by tens in which the interviewer progressively placed out ten-strips of dots was presented. The result showed that for each increment, she used a count by one strategy. Kayla did not regard ten as a unit composed of ten ones.

Table 9 shows her strategy in solving two-digit horizontal addition written number tasks. The table shows that Kaila used count on strategy in solving these problems. However, she still depended very much on her fingers. When the task involved more than 10, and she could not represent numbers more then 10 with her fingers, it led her to reach the incorrect answer.

Table 9
Kayla's solution on two-digit horizontal written number tasks (addition)

Problem	Solution Sequence
16+10	Looks at her 10 fingers and start counting 17, 12, 13,.., 26. The answer is 26
29+18	Looks at her 10 fingers and start counting 30, 31, 32, .., 39. The answer is 39

Analysing the overall results of the interview, Kayla was between stage 2 and stage 3 for early arithmetical learning and still in level 1 for base ten arithmetical strategies. It also appeared that in the beginning of the interview Kayla used a strategy that was less advanced than the strategy she used later in the interview. The stage was determined by her performance in solving tasks involving screened collections and tasks written in horizontal format. She was able to count screened items, but counted from one in doing so. However later in the interview, when she was asked to solve a two-digit addition problem, she used a count-on strategy. It seems that her count-on strategy was only used after she was not able to represent the first addend using her fingers. According to her teacher, she had been taught the count-on strategy by her teacher, but she was not fluent in the strategy. She preferred to use the count-on strategy which made sense for her when solving one-digit addition tasks. Furthermore, the level of based ten arithmetical strategies was determined by her inability to see ten as a unit composed of ten ones. She consistently counted by ones to add a ten, even when solving two-digit addition tasks.

Kayla's stage on early arithmetical learning and base ten arithmetical strategies appears less advanced compared to strategies which were used by participants in another study which involved first and second grade children in a regular school (Rumiati & Wright, 2010). In this study, participants were 7 to 8 years old, they were at least in the stage 3 or 4 of early arithmetical learning and most of them were at least on level 2 or 3 of base ten arithmetical strategy. This finding indicates that a child with Down syndrome may be capable of learning arithmetic similar to that learned by typical developing children; however the speed of her learning was much slower than that of typical developing children. It is not impossible that Kayla's number knowledge and strategies will be advanced by appropriate instruction. Nevertheless, the findings of this study accord with the study by Brigstocke, Hulme and Nye (2008) that number, as well as arithmetic could be a particular area of difficulty for children with Down syndrome.

Furthermore, the result of this study could be compared to the results of the study by Eriksson (2008) which suggested that "the arithmetic of the child" appeared to be active even for pupils registered as intellectually disabled. Eriksson noted that it was interesting as well as surprising that eight participants in her research developed the same behaviour patterns or mental structures for solving arithmetical problems as those emerging from investigations involving children in the compulsory school. This finding shall be taken into consideration in an effort to advance arithmetical skills and strategies of students with an intellectual disability as well as students with Down syndrome.

Conclusion, Limitation and Further Research

This paper describes arithmetical strategies of a student with Down syndrome. Kayla seems able to use counting on and counting back strategy in solving number problems. The results cannot be generalised to all students with Down syndrome. More research is needed to investigate whether students with Down syndrome developed counting in similar ways as typical developing students (Clarke & Faragher, 2013). This case study highlights that a student with Down syndrome can do, and learn, mathematics. The school system and teachers should neither underestimate nor overestimate their ability in mathematics. This is very important to remember when planning to teach them (Faragher, 2004). Further important question following this finding is, given what we already know about Kayla's arithmetical strategies, what teaching strategies we shall use to help her to advance her strategies.

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