# Year 3/4 Children's Forms of Justification

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Engaging children in justifying, forming conjectures and generalising is critical to develop their mathematical reasoning. Previous studies have revealed limited opportunities for primary school children to justify their thinking, form conjectures and generalise in mathematics lessons. Forms of justification of Year 3/4 children from three schools in Victoria, Australia will be examined. Evidence from children's written explanations and their verbal explanations captured in video recordings revealed that some children employed sophisticated mathematical ideas in their justifications. The value of making children's reasoning explicit through written explanations and verbal communications is highlighted.

Developing students' capacity to form and test conjectures is seen as central in mathematical reasoning (Carpenter, Franke, & Levi, 2003). Engaging students in the process of testing conjectures and justifying reasons are critical in forming generalisations. However, recent studies in an Australian context have revealed that primary school children lacked experience to generate or test conjectures, justify their reasons and generalise mathematical relationships (Clarke, Clarke, & Sullivan, 2012; Stacey, 2010).

Studies involving primary school children revealed common use of analogical reasoning in mathematical problem solving (Carpenter et al., 2003; Ellis, 2007). Carpenter, Franke, and Levi (2003) classify students' levels of justification in three broad categories: "appeal to authority, justification by example, and generalisable arguments" (p. 87). They contend that the most common justification used by primary school children is justification by example. They argue that while 'young children' do not commonly use generalisable arguments, they display more general forms of arguments which can be classified as "restating the conjecture" (p. 88) , "provide concrete examples that are more than examples" (p. 89) and "building on already justified conjectures" (p. 90). Furthermore they call for providing experiences for primary school children to form and test conjectures as the experience will lay "the groundwork for developing ideas of justification in more depths in later years" (p. 102).

This paper presents findings from the analysis of Year 3/4 children's reasoning emerging during the second demonstration lesson in the *Mathematical Reasoning Professional Learning Research Program* [MRPLRP].

## The Study

This study is part of a larger research project MRPLRP, which employed demonstration lessons, teacher observation, and collaborative discussion of these lessons followed by trialling in teachers' own classroom to support teachers to develop understanding and enactment of mathematical reasoning. Data in this paper were taken from three classes from three schools in Victoria, Australia. The focus of qualitative analysis was to analyse children's written explanations to the mathematical reasoning tasks and children's reasoning and justifications captured on video-recordings of the demonstration lesson. Salient examples that illuminate various forms of justification evident in both written and

2014. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). Curriculum in focus: Research guided practice *(Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia)* pp. 694–697. Sydney: MERGA.

verbal explanations will be analysed for the three classes of justifications (Carpenter, et al., 2003).

### *The Magic Vs problem*

"The Magic V" is an open-ended problem (http://nrich.maths.org/6274) that offers opportunities for children to develop, test conjectures and form generalisations. The MRPLRP project extended the "Magic V" problem by drawing students' attention to a conjecture to encourage children to form generalisations related to the properties of odd and even numbers. The lesson was planned for 60 minutes with 4 main components: 1) children working in pairs to find and record different Magic Vs (Figure 1); 2) whole-class discussions to justify "How do you know this is a Magic V; 3) form and test conjectures about Magic Vs, including 'Sam's conjecture'; 4) reflect on explanation justifications. It should be noted that throughout the lesson, students were encouraged to notice similarities and differences in various Magic Vs and to share their thinking during the whole class discussion. This noticing of similarities and differences offers opportunities for students to form their conjectures before they are asked to test a given conjecture.

This paper will focus children's forms of justifications evident in their written responses to Sam's conjecture. After forming conjectures concerning numbers in the vertex through whole class discussion of various Magic Vs, students were asked to work with a partner to respond to a conjecture "Sam said 'It is impossible to make a Magic V with an even number at the bottom with a set of numbers from 1 to 5.' Is Sam right? Explain why or why not." Children were asked not only to discuss and test Sam's conjecture with their partners but also to record their reasoning on a sheet. During the final whole class discussion, children were encouraged to articulate their justifications and to form generalisations based on their reasons to 'prove' Sam's conjecture.

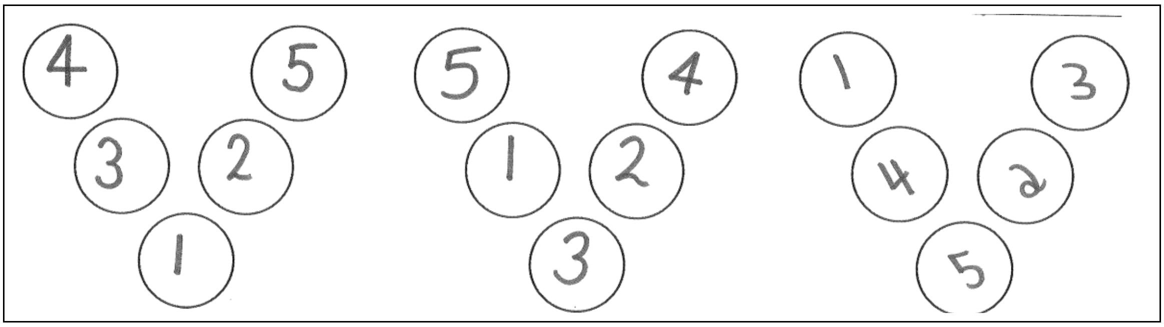


*Figure 1*. Samples of students' Magic Vs.

## Children's forms of justifications

*Re-stating the conjecture or no justification*

This study does not find evidence to indicate justification by appealing to the authority on their written or verbal exchanges during whole class discussion. Some students struggled to provide justifications and did not record a response on their worksheet. The following responses do not show evidence of justification because students only re-state the given conjecture or conclude that it is impossible "because you can't".

Sam is right that is impossible to make an even magic V at the bottom. [L & I, School C]

It's impossible to have an even number at the bottom because you can't. [J & S, School C]

It is impossible becass [because] you cannot put a even number in the magic vie (sic) [L & J, School A]

*Justification by example*

The following written explanation by students T and J illustrates a justification by examples in which students tested Sam’s conjectures by re-arranging the numbers to show some examples (see Figure 2a) that supported Sam’s conjecture.

We think that Sam is right because we tried the two even numbers and none of them worked and we mixed the numbers around and only the odd numbers worked. [T & J, School D]

Similarly, students C and A said “I think it is impossible” and provided examples with some calculations to show that the sum of the two arms were not equal in their justifications (see Figure 2b). These children needed to be encouraged to progress to more general forms of arguments beyond examples.

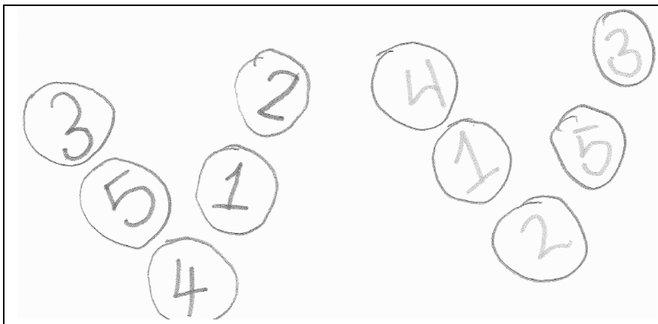

*Figure 2a.* [T & J’s examples , School D]

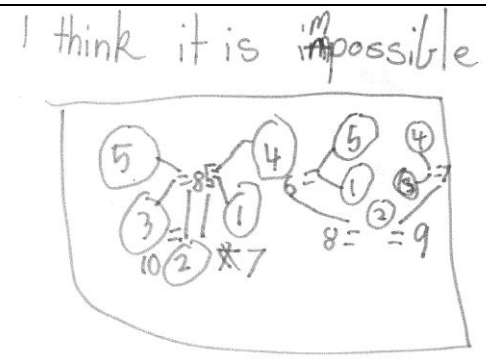


*Figure 2b.* [C & A’s examples , School C]

*Figure 2.* Justification by examples

*Justification with a conjecture*

The following responses indicate justification using some form of generalisable argument.

It is impossible because the odds need to go there to make an even number but if you put a 2 or 4, it will not be even or right. It might be close but it is impossible. [L & J, School A].

There is more odd than even. It would work if you put another number to 6. [B & L – School D]

In the first example, L and J state a conjecture that the vertex has to be odd based on what they notice in their Magic Vs. However they decided to test the conjecture further with more examples. During the whole class discussion, J expanded on her thinking

… because we were really close… we were really close in getting to the same number but it turned out that one was 10 and that one was 9 so we're kind of like unsure because we never know if that could happen with other even numbers not just like 1,2,3,4,5. [J’s verbal explanation – School A]

*Justification by generalisable arguments*

Analysis of our findings shows that Year 3/4 children are able to provide justification by generalisable arguments. The written explanation by L and B from School A suggests that they test Sam’s conjecture with both even numbers and refer to the general relationships about the sum of even and odd numbers.

> I agree with Sam we have tried every number, you need two even numbers to make the even number and you need two odds and one even to get the even number. [L & B, School A]

Another justification by generalisable arguments by S and O clearly indicate that they start by noticing that the sum of three odd numbers (1+3+5) is bigger than the sum of two even numbers (2+4). During paired work, O and S tested Sam's conjecture by placing the 4 at the vertex and noticed that 5+1=6 and 2+4=6 which they called "a perfect pair" because "they equal the same amount". S and O use this example as a case of generality. They notice that 4 cannot be at the vertex because "if you put 3 in then it would be an odd number and you can't divide it [the total] into two equal groups".

> It is impossible to make a magic V with an even number because you need an odd number down the bottom because it has one more number than the even number or more and it won't work up the top the odd number will be more than the other side [A & S, School A]

## Concluding remarks

Our study found that Year 3/4 children are capable of justifying, forming conjectures and generalising in various forms. Our findings concur with Carpenter et al.'s (2003) observation that justification by examples is the most common form of justification among primary school children. It is evident that in forming conjectures by generalised arguments, students started by exploring examples and then thinking of a specific example in a general way. It is encouraging to find no evidence of justification by appealing to the authority. This reflects the design of the lesson in which students are encouraged to justify their thinking to their peers and invites students to expand each other's ideas. In this lesson, students take ownership of the lesson as they are actively involved in the process of exploring, noticing, justifying, forming and testing conjectures. The teacher plays a critical role in challenging students' thinking by prompting them to explain why and recording students' thinking on the board so students can attend to the key mathematical ideas. This observation resonates with Lobato, Hohensee, and Rhodehamel's (2013) point on teachers' critical role in setting up expectations for students to offer satisfactory justification and in directing students' attention toward central mathematical ideas. Similarly, Swan (n.d) calls for making reasoning "the object of attention" in the mathematics lesson and the importance of making reasoning "visible and audible" through written explanations and verbal explanations.

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