Young Children's Number Line Placements and Place-Value Understanding

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In this paper we report on two assessment tasks extracted from a larger study. The tasks involved number-line placements on two different number lines (0-to-10 and 0-to-20) and place-value understanding. Participants were 119 children from four different classes (Years 1-3). Children's placements were more accurate on the 0-to-20 than the 0-to-10 number line but many found midpoint placements difficult. Children with good place-value understanding were better than their peers at making accurate number-line placements. The findings have implications for practitioners in making more explicit the connections between number and space.

Background

The representation of numerical quantity is a complex multi-dimensional domain. Dehaene, Piazza, Pinel, and Cohen (2003) propose three systems that contribute to the processing of number, each involving activation of different parts of the brain. The verbal system represents numbers as words and focuses particularly on the memorisation and recall of number facts. The other two systems are nonverbal, including a "visual system" that encodes numbers in terms of a mental number line running from left to right, and a "quantity system" that represents the size and distance relations between numbers. One type of magnitude estimation comprises translation from one non-numerical magnitude into another form of non-numerical magnitude, such as estimating a quantity by indicating it as a position on a number line. The other type of magnitude estimation is numerical, such as assigning line lengths to numbers.

The nature of the cognitive systems associated with magnitude estimation are strongly debated in the literature (Moeller, Pixner, Kaufmann, & Nuerk, 2009; Núñez, 2011; Núñez, Cooperrider, & Wassmann, 2012). Number seems to be initially coded *logarithmically* where the distances between adjacent numbers on the mental number line decrease as their magnitudes increase. It has been argued that formal schooling and other cultural practices lead to changes in coding from logarithmic to linear (Booth & Siegler, 2008; Siegler & Booth, 2004; Dehaene, Izard, Spelke, & Pica, 2008; Núñez, et al., 2012), and this is correlated positively with mathematics achievement. For example, learning to integrate tens and ones in the place-value system could help to explain the apparent transition from logarithmic to linear representations with age (Moeller et al., 2009).

Older children are better at magnitude estimation than younger children, and smaller numbers are represented more accurately than larger numbers (e.g., Barth & Paladino, 2011; Praet & Desoete, 2014; Rouder & Geary, 2014). Children's ability to place numbers on a number line is strongly related to their understanding of proportional reasoning and overall mathematical achievement (Rouder & Geary, 2014). Anchor points at the beginning and end of the line are used to help place numbers by children as young as six years old. Older children (7- to 10-year-olds) are able to make use of a third anchor point (the midpoint) to place numbers more accurately (Slusser, Santiago, & Barth, 2013). Understanding geometric ideas such as the axis of symmetry also helps children to use the

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midpoint in making placements on a number line (Mulligan & Mitchelmore, 2013; Spence & Krizel, 1994).

The research on numerical magnitude and number-line representation links to the distinction made by Yackel (2001) between counting-based and collections-based approaches to working with numbers. Both approaches are important for developing a deep and connected understanding of number. There is an 'inherent contradiction' in the way that Western children are initially encouraged to count by ones (unitary *counting-based* concepts), but then are expected to reorganise these into *collections-based* concepts involving units consisting of tens and ones when place-value instruction begins (Yang & Cobb, 1995).

Research on children's awareness of mathematical pattern and structure (AMPS) shows the importance of students developing an awareness of structural relationships in mathematics (e.g., Mulligan, 2011). Low levels of AMPS seem to be associated with having poor visual and working memory. Mulligan found that students with low AMPS tended to "rely on superficial unitary counting by ones" (p. 36), and did not develop efficient and flexible strategies for solving problems. AMPS also appears to impact on the development of measurement concepts and proportional reasoning. Mulligan's work on promoting awareness of pattern and structure is consistent with other research on the importance of helping children develop knowledge of place-value structure (Cobb, 2000; Fuson, Smith, & Cicero, 1997; Thomas, Mulligan, & Goldin, 2002).

The development of place-value understanding requires children to be familiar with the concept of unit, and appreciate the difference between units of ten and units of one. Children need to be part-whole thinkers in order to partition numbers into tens and ones (Fuson, Smith, & Cicero, 1997; Ross, 1989). A key feature of place-value development is the shift from a unitary (by ones) way of thinking about numbers to a multi-unit conception (e.g., tens & ones). Place-value knowledge has four major properties: positional, base-ten, multiplicative, and additive (Ross, 1989). Because place-value understanding is inherently multiplicative, it is more complex than additive thinking (Clark & Kamii, 1996; Vergnaud; 1994). Multi-digit arithmetic requires not only an understanding of the place-value system for the Arabic number system but also an understanding of the magnitude of numbers (Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011).

The theoretical perspective taken in this paper was informed primarily by the extensive work of Mulligan and colleagues on the importance of pattern and structure for mathematical thinking (e.g., Mulligan, 2010, 2011; Mulligan, Mitchelmore, English, & Crevensten, 2013) and the literature in the multiplicative conceptual field (e.g., Clark & Kamii, 1996; Vergnaud, 1994). These two fields of research led to the research question focussing on the relationship between young children's number line placements and place-value understanding. We explored how accurately young children mapped one- and two-digit numbers on number lines, their understanding of two-digit numbers, and the relationship between these constructs. This research was part of a larger study that focused on developing children's part-whole thinking through the use of multiplication and division problem-solving contexts. Selected baseline data from the study was analysed to answer the research question.

The Study

This exploratory study was set in an urban school (medium socioeconomic status) in New Zealand. The participants were 119 five- to seven-year-olds (59 girls and 60 boys) from Years 1 to 3 (average age at each year level: 5.5, 6.5, 7.3 years). There were 42 Year

1, 34 Year 2, and 43 Year 3 children from four different classes. The children were from a diverse range of ethnic backgrounds, with approximately one third Māori (the indigenous people of New Zealand), one third European, one fifth Asian, and the remainder from other ethnicities including African and Pasifika (Pacific Islands people). Approximately one quarter of the children were English Language Learners. The children were assessed using an individual diagnostic task-based interview designed to explore number knowledge and problem-solving strategies. The assessment tasks included: subitising, addition, subtraction, multiplication, division, basic facts, incrementing in tens, counting sequences, number-line placement, and place value. The two tasks reported in this paper focused on the latter two categories.

In the first task (number-line placements), children were shown a number line with 0 and 10 marked on it (see Figure 1). The interviewer said: "This number line goes from zero to ten. Where does five belong on this number line?" The children then indicated the estimated position on the number line, which was recorded by the interviewer. This was followed by questions about the placements of two and one. Children were then shown another number line with 0 and 20 marked on it (See Figure 1). The same process was used for the placement of 19, 10, and one. Later, the researcher measured and recorded the distance in millimetres between zero and the child's placement of the target numbers on the number lines. The number line placements were coded from 0 to 3 based on the accuracy of the position. Placements within 10 per cent of the target position were coded 3, 11 to 20 per cent were coded 2, 21 to 50% were coded 1, and the others were coded 0.

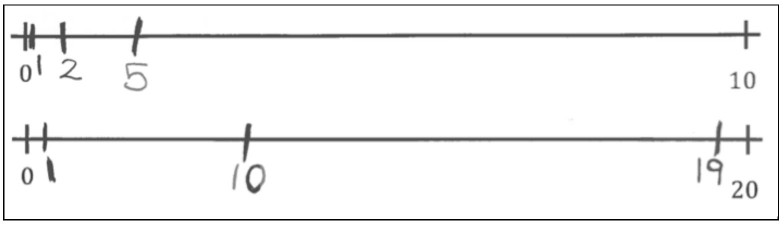

Figure 1. Record of one child's responses to number-line placements

In the second task (place value), children were shown a picture of two ten-sticks (each ten stick represented by a row of five grey boxes joined to a row of five white boxes) and four singleton boxes. The children were asked to find the total number of boxes (their strategy was recorded) and then to write this number above the picture. The interviewer circled the digit "4" in "24" and asked: "which boxes might the four mean?" The collection indicated by the child was circled and a line drawn connecting the boxes with the digit "4." The interviewer circled the digit "2" in "24" and asked: "Which boxes does the '2' in '24' mean?" This was recorded in the same way as the "ones" digit. Finally, the interviewer asked: "So, what is the '2' in '24' telling you?" The interviewer recorded how the children determined the total number of boxes, and whether they linked the "4" to four boxes, and "2" to 20 boxes. Figure 2 shows a record of one child's correct responses to the task.

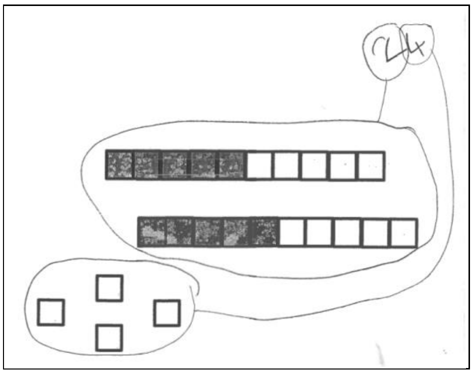

Figure 2. Record of one child's responses to questions about the meaning of 4 and 2 in 24

Results

The first task required children to estimate the placement of numbers on the two different number lines, 0-to-10 and 0-to-20. Table 1 shows the median number-line placements for 5, 2, and 1 on the 0-to-10 number line, and 19, 10, and 1 on the 0-to-20 number line (measured in millimetres) by year group. The table also shows the discrepancy between the median and correct position in brackets, and the minimum and maximum values (range).

Table 1
Median Number-Line Placement in mm, (Discrepancy), and Range for Each Year Level

	Correct Place	Y1 (n=42)	Y2 (n=34)	Y3 (n=43)
0-to-10 Line				
Place "five"	80	40 (40)	38 (42)	67 (13)
Range		1 to 169	15 to 100	21 to 85
Place "two"	32	12 (20)	12 (20)	14 (18)
Range		3 to 157	4 to 25	6 to 31
Place "one"	16	5 (11)	3 (13)	5 (11)
Range		1 to 170	1 to 12	1 to 19
0-to-20 Line				
Place "nineteen"	152	142 (10)	147 (5)	148 (4)
Range		2 to 169	71 to 155	79 to 156
Place "ten"	80	84 (-4)	74 (6)	78 (2)
Range		-2 to 162	29 to 132	33 to 130
Place "one"	8	7 (1)	5 (3)	6 (2)
Range		-2 to 145	1 to 11	1 to 15

Year 3 children were, on average, far more accurate with their placements than the other year groups. These children were also most accurate in placing 10 and 1 on the 0-to-20 number line, with a median discrepancy of only 2 mm short of the correct position. Their accuracy was greater on the 0-to-20 number line than on the 0-to-10 number line, and they were least accurate in placing 2 on the 0-to-10 number line, with a median discrepancy of 18 mm short of the correct position.

Year 1 and Year 2 children were most accurate in placing 1 on the 0-to-20 number line, with the median placements being less than 5 mm short of the correct position. The next most accurate placement for these children was 10 on the 0-to-20 number line, with the median position 4 mm beyond the actual position for Year 1 (shown as a negative value in Table 3), and for Year 2, the median placement was 6 mm to the left of the position (a positive value).

The second task (place-value) was given to all children who could successfully complete several ten-structured tasks such as subitising a ten-frame and finding half of 20. This reduced the sample size for the place-value tasks to 12 Year 2 and 35 Year 3 children (n = 47). Table 2 shows the strategies used by the 43 out of 47 children who correctly determined that there were 24 boxes in total (see Figure 2). These strategies included counting by ones, fives, and tens. Approximately half of the children counted by tens to determine the number of boxes. Almost one-quarter counted by ones (n = 12), while seven children counted by fives.

Table 2
Strategies Used to Count 24 Boxes and Make the Links between Digits and Quantity

Year	By ones	By fives	By tens	Links “4” to 4 Boxes	Links “2” to 20 boxes
2	6	1	4	8	3
3	6	6	20	32	20

When asked to link digits with quantities, most (85%) of these 47 children were able to link the “4” in “24” to four single boxes (see Table 2). Twenty-three children (49%) made the correct place-value link (the “2” in 24 to two tens or to 20). The children who could count 24 boxes by tens were not necessarily the same children who could link the “2” in 24 to 20 boxes. Individual profile data showed that one Year 2 and 17 Year 3 children counted by tens and made this place-value link. The 23 children who were able to connect the “2” in “24” to two tens or 20 were selected for further analysis to explore the relationship between place-value understanding and number-line knowledge. The accuracy of their number-line placements is shown in Table 3.

Table 3
Accuracy of Number-line Placements for the 23 Children who Correctly Linked 2 in 24 with Two Tens or 20

	0-to-10 number line			0-to-20 number line		
Number	"five"	"two"	"one"	"nineteen"	"ten"	"one"
Actual length	80 mm	32 mm	16 mm	152 mm	80 mm	8 mm
≤ 10%	10	2	1	22	13	8
11-20%	6	1	4	1	4	2
21-50%	3	7	5	0	5	9
> 50%	4	13	13	0	1	4

Placing 5 on the 0-to-10 number line (the midpoint) was easier for these children than placing 2 or 1 (43% vs. 9% and 4%, respectively). All but one child accurately placed 19 on the 0-to-20 number line (96%). They were not so accurate in placing either 10 or 1 on the 0-to-20 number line (57% and 35% within 10% of the target, respectively). The placement of 10 (the midpoint) was more accurate than the placement of 1, showing children did not notice that 19 and 1 are equidistant from the two endpoints. These children were more accurate in placing numbers on the 0-to-20 number line than the 0-to-10 number line. When the 23 children who made accurate place-value links were compared to the 43 Year 3 children (the oldest year group), the 23 children were slightly better at making number-line placements, with a greater proportion making the most accurate placements (within 10% of the target) for 5 and 10 (43% vs. 35%; 57% vs. 49%).

Discussion

The assessment tasks reported in this paper were designed to explore young children's number-line knowledge reflected in the placement of one- and two-digit numbers on number lines, and their understanding of two-digit numbers as represented by ten sticks (composed of two groups of five) and singleton boxes. The 119 children were more accurate in placing numbers on the 0-to-20 number line than 0-to-10. This could be explained by the fact that the 0-to-10 line (two anchor points) was presented first to help the children to become familiar with the task. The first placement question for the 0-to-10 line was 5, but most children did not recognise 5 (the midpoint) as a third anchor point that could help in making a more accurate placement (Rouder & Geary, 2014). This recognition of the midpoint relates to understanding about an axis of symmetry and proportional reasoning, which was evident in responses from older children in other studies (e.g., Mulligan & Mitchelmore, 2013; Spence & Krizel, 1994).

Overall, children were more accurate in placing 19 on the 0-to-20 line than 1, failing to recognise that both these placements were equidistant from the anchor points at each end. These young children had not yet established number-to-space connections that could have supported their number-line placements (Núñez, et al., 2012). A few children made negative placements (i.e., to the left of zero), lacking awareness that all whole numbers are to the right of zero. Other children placed 10 and 19 to the right of 20, suggesting that they had some weaknesses in their knowledge of number sequences.

Many of the children in this study did not appear to have developed a sense of the midpoint as the third anchor point because of little or no experience with number-line placement. This finding is consistent with researchers who have found that the use of the midpoint to make number line placements appears about the third year of school (e.g., Barth & Paladino, 2011; Slusser et al, 2013). This could be explained by the focus in many New Zealand schools on teaching number in isolation from the other domains within the mathematics curriculum. This practice does not help children to build the connections highlighted by the research on spatial structuring and number. For example, experiences with the axis of symmetry in the context of geometry, as well as halving quantities and shapes could help children build a deeper more connected understanding of the relationships among numbers (Mulligan & Mitchelmore, 2013).

In the place-value task, children used a range of strategies to determine that there were 24 boxes in the picture. Half of the children recognised that two groups of ten (as represented by ten-sticks) made 20 in total, and quickly determined that there were 24 boxes altogether. This is consistent with the work of Fuson and colleagues (1997) on the developmental trajectory from unitary to ten-structured thinking, and then progression to multi-unit conceptions of number. A few children (n = 7) took advantage of the quinary structure of the ten-sticks and counted by fives to 20 (Mulligan, 2010). One quarter of the children counted the ten-sticks by ones (unitary counting), and this included both Year 2 and Year 3 children. This could be explained by the continued emphasis in the early years of school on counting by ones, as reflected in curriculum documents such as the Mathematics Standards (Ministry of Education, 2009) which expect children after one year at school to add by counting all, and after two years at school, to add by counting on.

Only 18 children used groups of ten to determine that there were 24 boxes, and correctly linked the “2” in “24” to 20 boxes. Despite having the beginnings of place-value understanding (as reflected in linking digits to quantities), five of the 23 children had used counting by ones to determine the total, ignoring the groups of five and ten clearly evident in the picture. These results reflect the complexity of part-whole understanding and ten-structured thinking (Fuson et al, 1997; Ross, 1989).

The 23 children, mostly from Year 3, who successfully made place-value links between digits and quantities, were also reasonably competent with the number-line placement task. This provides evidence that their recognition of the linear aspect of number lines is consistent with research showing that older children perform better on magnitude estimation (Praet & Desoete, 2014; Rouder & Geary, 2014). However, these children were more accurate in placing 19 and 10 than 1 on the 0-to-20 number line, and placing 5 than 2 or 1 on the 0-to-10 number line. Perhaps it was easier for them to place the 5 and 10 because they used the midpoint as a third anchor point to make the placement. They also may have used their awareness of the axis of symmetry (Mulligan & Mitchelmore, 2013).

The findings reported here could be useful for classroom teachers in emphasising the importance of helping children make connections between different representations of two-digit numbers. Multiple representations for two-digit numbers, including using materials such as Unifix cubes, ten-frames, Slavonic abacus, numeral cards, and number lines, could help children strengthen connections between visual and non-visual systems. Links between measurement concepts, proportional reasoning, and numerical magnitude could be made as children learn to divide a distance on a number line in order to estimate more accurately the placement of numbers (e.g., by folding a number line in half). By varying the anchor point at the right-hand end of the number line (e.g., 0-10, 0-20, 0-100), an appreciation of proportionality could be further developed (e.g., half of 10 is 5, half of 100 is 50). This could strengthen understanding of relationships among numbers and enable children to make more accurate number-line placements.

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