

## Identifying Core Elements of Argument-Based Inquiry in Primary Mathematics Learning

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Having students address mathematical inquiry problems that are ill-structured and ambiguous offers potential for them to develop a focus on mathematical evidence and reasoning. However, students may not necessarily focus on these aspects when responding to such problems. Argument-Based Inquiry is one way to guide students in this direction. This paper draws on an analysis of multiple primary classes to describe core elements in Argument-Based Inquiry in mathematics.

The inclusion of inquiry-based pedagogies into classroom mathematics teaching has the potential to engage students in mathematics in authentic ways (Fielding-Wells & Makar, 2008). Students engage with inquiry as they are supported to work with ambiguous and ill-structured problems (Makar, 2012); ill-structured problems being considered those which have no *correct* solution, may have multiple solutions, or have unclear solution processes (Eraut, 1994). An advantage of working with such questions is that:

their inherent ambiguity allows for multiple interpretations of a question, a range of pathways, and numerous solutions with varying degrees of efficiency, applicability and elegance. This requires students to focus on decision-making, analysis and justification. Rather than a 'correct' answer or strategy, there is a claim which requires evidence, explanation and defense – in short, an argument (Fielding-Wells & Makar, 2012, p. 149).

Blair (2012) describes a view of argumentation that essentially sees it as a form of inquiry in which argumentation is utilised to explore a problem and to arrive at a solution through examination of the evidence and grounds that can be employed towards solving the problem. By implementing such a model of argument, students may be explicitly focussed on obtaining evidence, using evidence to make a claim, and articulating how the evidence leads to the claim through reasoning. Thus, argumentation offers potential to purposefully direct students engaged in inquiry to focus on the discipline content, and the ways in which the discipline content can be used, to respond to a problem or dilemma.

Argumentation is not new in mathematics, there is a great deal that mathematicians do that incorporates reasoning and argument. For example, mathematical proof must stand up to rigorous, critical, dialectical argument by other mathematicians and be open to argument as attempts are made to examine, generalise, extend, and simplify the proof. Another area of argumentation research in mathematics has been as it applies to procedure (see, for example, Goos, 2004; Yackel & Cobb, 1996). Here it is “the strategies used for figuring out, rather than the answers, that are the site of the mathematical argument” (Lampert, 1990, p. 40).

There is a third type of argumentation, one that would appear to have been the focus of less research and that is the use of argumentation to address authentic, ill-defined mathematical problems in which *neither* the procedural pathways *nor* the solutions are limited in terms of ‘correctness’; that is, *inquiry* (Anderson, 2002). This is the focus of the research described in this paper and which differs from the existing body of literature somewhat in that both the solution process and the answers are considered the site of the

argument. Hence, the term *Argument-Based Inquiry* (ABI) has been adopted to describe this view of argumentation.

## Argumentation

Toulmin, Rieke and Janik's (1984) seminal work on argument structure enables an argument to be identified by components of claim, grounds, backing, warrants, and so forth. However, such a structure focuses on the components of an argument rather than providing a focus on evaluating the logic or strength of their claim. A simpler model than that proposed by Toulmin et al. would appear to be indicated for children, such as the Claim-Evidence-Reasoning model devised by McNeill and associates (McNeill & Martin, 2011; Zembal-Saul, McNeill, & Hershberger, 2013). This enables a more general focus on the core components of classroom argument. The claim and evidence components align with Toulmin et al.'s claim and grounds: claim being the conclusion that addresses the original question and evidence being the scientific data that supports the claim. In explaining their model, Zembal-Saul et al. (2013) maintain that the data needs to be both appropriate and sufficient to support the claim. The third component, reasoning, encompasses the warrants and backing; that is, the logic that enables the grounds to be used to establish the claim (McNeill & Krajcik, 2012).

### *The Nature of Argumentation*

Various theories of argumentation can be found in the literature with Toulmin et al.'s (1984) classical work on argumentation structure forming a basis for most. For instance, van Eemeren and Grootendorst (2004) extended Toulmin et al.'s structural (product) approach to pragma-dialectical argumentation that incorporated the process of argument also. Lumer (2010) and Siegel and Biro (1997) proposed a model of Epistemic Argumentation, which distinguished itself through a focus on the strength and validity of an argument, based on epistemic criteria (Nettel & Roque, 2012) rather than structure and use of emotive devices. It is this theory of argumentation that was adopted throughout the research described here: the rationale being that science (and mathematics) value accuracy, logic and verifiability over persuasive devices seen in other forms of argument. Furthermore, the ability to challenge the argument is offered on an epistemic level, giving potential rise to challenge about what is acceptable evidence and reasoning within a discipline (Simon & Richardson, 2009).

Traditional ways of teaching do not provide a classroom culture that is necessarily conducive to the introduction of ABI practices and thus there are many practical considerations to developing such approaches. In order to facilitate the research undertaken, argumentation was introduced into primary classrooms that were already fluent in the use of inquiry-based learning (IBL) in mathematics. What signature elements of Inquiry-Based Argument can serve to guide children's mathematical argumentation?

## Methodology

The larger research study from which this report stems was conducted using Design-Based Research; selected because this methodology entails engineering forms of learning and then systematically studying the learning within its context, which was ideal for the research purpose (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003).

### *Participants and Data Collection*

In total, five classes of students were engaged in Inquiry-Based Argumentation units with some carrying out one unit and others as many as three across the course of a year. A total of nine units were recorded in full from classes (at the Prep, Year 1, Year 3, Year 5 and Year 7 levels) at a metropolitan government primary school in Queensland. This school is a relatively large primary school with approximately four drafts of each year level. At the commencement of the research, the school site had been part of an IBL research project for seven years and involved a number of teachers at the school. The teachers were all experienced with teaching using IBL; however, due to changing grouping over years some students were quite familiar with learning through inquiry whereas others had little or no experience. Teachers were provided with ongoing guidance and support; however, beyond a need for a Question-Evidence-Conclusion focus, the teachers were largely responsible for implementing their approach to the inquiry question as they chose.

### *Data and Data Analysis*

A selection of videotaped units (approximately 5-10 lessons each) was analysed using a process adapted from Powell, Francisco and Maher (2003). In line with their approach, the lesson videos were viewed and logged, lesson-by-lesson, along with time stamps, excerpts of students' work, and still shots of teaching materials to capture the essence of the lessons. Critical events, such as those that demonstrate a particular struggle or advancement in the inquiry were noted and transcribed in more detail. Logs were coded using adapted grounded theory (Corbin & Strauss, 2008) and this enabled cross comparison between the units for particular events and patterns in the development of the inquiry. In particular, commonalities and differences were highlighted in order to develop an overarching narrative of the ABI process. Four units that were felt to demonstrate deep engagement with ABI pedagogy were transcribed in full. For consistency of the story, all classroom illustrations provided are drawn from one unit: *Biased Bingo* (Year 3): a teaching unit designed around the game of addition bingo, which addressed the question 'What is the best card for addition bingo?' In the game, all possible combinations of the sum of two numbers (1 to 10) were each written on a slip of paper and placed in a box. Children had a card consisting of a 5 x 5 array of self-selected numbers (their predictions of what will be called). In order to address the problem, they needed to decide on the best numbers they could place on their card.

The purpose of the grounded coding was to enable the development of substantial codes to describe, name, or classify aspects of the study (Flick, 2009). The codes assigned were grouped into common themes and codes that were essentially duplications were amalgamated. These codes were clustered where appropriate into code categories and substantive categories and used to map themes and relationships.

## Results and Discussion

The analysis undertaken illustrated four key components or threads at the most basic level of ABI; that is, that were consistent across all classes and ages. While more advanced components were also able to be identified in older classes engaging in more than one ABI unit, the essential and consistent elements noted are the focus of this paper. At the very simplest level, mathematical argumentation was characterised by students addressing of a purposeful inquiry question, the advancing of evidence which was used to form a claim, the justifying of the evidenced claim through epistemically acceptable reasoning, and

acknowledgement of context. While the elements are presented here sequentially, in practice the teacher drew attention to different components and the relationships between different components, as required. Each of these elements will be addressed in turn.

### *Addresses a Purposeful Inquiry Question*

In order to present an argument, the students first required a question they could address. Questions were variously provided by the teacher, by the students or, most often, in a vague and unrefined way by the teacher and then refined by the students with teacher guidance to a topic that was mathematically researchable. The excerpt below illustrates a teacher working to help the students unpack the question being posed.

Mrs T: Can you create a bingo card with the BEST chance of winning? What does 'best' mean?

Jess: The best chance of winning doesn't mean like every number that gets pulled out that one person will always get that number. It means that like most of the time when you pull out a number that that person will have that number. If they have a like a good bingo card they have worked out like how many of each number they need to have to have a really good chance of winning.

[unidentified student] The best chance of winning is the most likely chance that it is going to get called out.

The inquiry question in this instance was posed by the teacher but in such an ill-structured way that the students needed to engage with it determine the meaning. The question need not be posed by the teacher. In another unit, a student's question was adopted after it was posed spontaneously in class. Students are capable of formulating their own inquiry questions even from a young age, although research indicates there is a need to teach students how to pose their own questions with a focus on what makes a *good* question (Allmond & Makar, 2010). While this may be time consuming it does more closely match authentic practices and teaches students an important skill – how to mathematise a problem so that it can be addressed.

The word purposeful has been added to the element *addresses a purposeful inquiry question*. In this instance, a purposeful question is deemed one that seeks to address a genuine problem. By purposeful, it is meant that the question has a genuine reason for being asked. Often when students are provided a question, the teacher already has a known answer. Because of this, even if the question is open-ended, students may not engage purposefully as they have no real need to persuade their audience (the teacher) of the answer or a method (Sandoval & Millwood, 2007).

### *Advances Evidence to Enable the Forming of a Claim*

In scientific/mathematic argument, evidence or data is sought and then attempts are made to make sense of it and to make a claim based on all the evidence, both supportive and contradictory (Sampson & Clarke, 2006). This is distinct from the role that evidence may have in advocative argument, where a claim is made and then evidence is presented in order to support or add weight to the claim. In ABI, the teacher needs to focus students on the obtaining of evidence to make a claim.

Mrs T: ... I wanted to just come back to our question, because our question was 'What Bingo card would give you the best chance of winning?' ...? Who can remember what you were doing yesterday and what you were hoping to achieve, or what were you trying to find out?

- Gen: If other numbers other than 12 would be pulled out mostly.
- Mrs T: Yes. Some people said, 'Wow, another 12 another 12' and so everyone decided 'OK 12 comes out the most' but we weren't really sure of that, so you guys had to find?
- Students: Evidence.
- Mrs T: So you went off to find some evidence for that [writes evidence on whiteboard]. To prove that. So, while you were finding evidence, what did you find? What did you discover along that track?
- Byron: That 18 was second most popular ...
- Gen: That um if you did 12, there was eleven of them. And when we did 11 there was ten. And you keep taking 1 from each one and then it makes how many ...
- Mrs T: I am hearing people saying, oh well actually, 10 is the most common. And I heard someone say, 'No, 11 is' And Bethany saying, '12' ...
- So now that you look at your book, can you tell me, from the evidence that you have got there, which number, definitely, and I mean definitely. Can you prove to me, which number is the most common? Or numbers. You can Jasmine, from your evidence there could you show me, and could you prove it to the rest of the class?

In this instance, the teacher is focussing on the students need for evidence to support their claims: one commonality throughout all the units observed was the repeated and consistent focus of each teacher to bring students' attention back to the need for evidence in order to lead them to a claim, but also the need to represent the evidence in ways that assisted students to see patterns in their evidence that would lead to a claim.

It was evident throughout the units analysed that students needed to envisage the evidence they could use to address the problem, plan to obtain that evidence, organise or represent the evidence, and then interpret and analyse it in order to make and support a claim.

### *Justifies the Claim through Epistemically Acceptable Reasoning*

Students need to use reasoning that is based on evidence to justify the making of a claim. There is potential for the connection between evidence and claim to be omitted, largely because the connection is either thought to be implicitly understood, or is left unaddressed unless challenged. However, this does not meet the purposes of IBA in mathematics, as the reasoning is the site of the actual mathematical understandings, connections, proofs, or concepts. In one class, the students engaged in three units over the year, and, by the end of that period, were explicitly stating their reasoning in terms of the mathematical underpinnings. However, this was not a stage typically reached by classes engaging in only one unit. Thus, a more *typical* response is shown from the Year 3 class:

Why do you think so?  
Because [1] said 12 are the most popular number  
because 11 has 10 chances/10  
of winning / 7  
100, 8 has 7 chances  
100, 12 has 9 chances 9  
100,

Because [ I ? ] said 12 are the most popular number because 11 has 10 chances 10/100, 8 has 7 chances of winning 7/100, 12 has 9 chances 9/100
--

While the suggestion here is that the signature components for argumentation should include claim-evidence-reasoning (McNeill & Martin, 2011; Zembal-Saul et al., 2013), it

is only essential that the teacher be able to recognise these components, particularly in younger students, and that these components may be elicited, for example verbally, pictorially, diagrammatically, or concretely. However, to have the students accustomed to providing evidence for and justifying their responses even at an earlier age would likely position the students for more formal learning and reasoning at a later time.

### *Reflects the Context*

The final element is the necessity of the claim, evidence, and reasoning to reflect the question context. In a unit contextualised outside of mathematics, there should be a reflection of what the student's response means in the context. While the claim would reflect the context and the reasoning would require a mathematical basis, the evidence may be constrained or guided by the context and this could potentially influence the evidence at several stages: envisaging (How many trials of the Bingo should we make?) and interpreting (What does the evidence mean in light of the context? How can anomalies be interpreted in light of the context?).

Justine: I keep losing on a 10.

Mrs T: This is an interesting comment. Laura says 'I keep losing on a 10'. How many times have you lost on 10?

Justine: Two.

Mrs T: So if this was happening as you predicted and as you expected, do you think Laura could have been winning?

Students: Yes.

Mrs T: Because she's been waiting for a 10 and it hasn't happened although I would have predicted, or I would have expected that we would have had more 10's. So would her choosing two 10's have been a reasonable sort of assumption to make? Do you think that would have been a good idea when she was making her card?

Students: Yes.

In this instance, these students have determined that ten is one of the highest frequency outcomes. However, in playing the game, ten has not been drawn as often as expected. The students recognise that is brought about through chance and accept that Laura has still designed a card that has a good chance of winning. Context plays an important role in the interpretation of mathematical evidence. In this instance, we see that students are able to take the numbers as drawn (experimental data) and explain why it doesn't behave as they predicted. According to Borasi (1992):

Mathematical applications require not only good technical knowledge but also the ability to take into account the context in which one is operating, the purpose of the activity, the possibility of alternative solutions, and also personal values and opinions that can affect one's decisions. Unfortunately, none of these elements is usually recognised as relevant to mathematical activity by people who have gone through traditional schooling. (p. 160)

## Conclusion

The purpose of this research was to begin to identify some key components of Argument-Based Inquiry as it might take place in primary mathematics classes. Four components that appear essential are suggested: the addressing of a purposeful inquiry question; the advancing of mathematical evidence to enable a claim to be made (in the illustrated unit the students' bingo cards formed the basis of their claim); the justification

of a claim through epistemically acceptable reasoning; and, the acknowledgement of context. It is suggested that these components are likely present, or a requirement, of all ABI in mathematics. However, at the level of the youngest children, there may not be an explicit acknowledgement of *claim*, *evidence*, *reasoning*, and *context* by the children. However, it is essential that the teacher can identify these components and guide students towards their development.

Argumentation structures and practices offer the means to focus students on the need for quality evidence and thus encourage students to focus deeply on mathematical content. Much of the work with argumentation that has already occurred in mathematics is associated with justification of procedural choices to arrive at a correct answer, or on the defence of the answer itself. By contrast, mathematical ABI offers the opportunity for students to engage in ill-structured, ambiguous problems that have neither a defined solution path nor a single correct answer. Thus, while this is only a small beginning, there appears to be potential for argumentation to be effective in deepening student focus on developing mathematical evidence and reasoning in inquiry-based learning environments.

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