Developing a Theoretical Framework to Assess Taiwanese Primary Students' Geometric Argumentation

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Geometric competences of students have sparked great concern in Taiwan since the release of last TIMMS assessment. Geometric argumentation is viewed as to play an important role to enhance the competences of geometry and reasoning. This study adopts Toulmin's (2003) model to develop such indicators, including naming, supporting ideas, and transformation reasoning. It is expected that further research will provide empirical evidence in these indicators to apply to topics in mathematics other than geometry.

Introduction

Taiwan has participated in several international assessments since 2000 and although Taiwanese grade 4 students are ranked top 4 in the Trends in International Mathematics and Science Study (TIMMS) in comparison with 62 other countries, results reveal some weaknesses of students in particular areas of mathematics. More specifically, TIMMS results suggest that grade 4 students are weaker in geometry and reasoning when compared with their performances in other mathematical topics (Mullis, Martin, Foy, & Arora, 2012). These learning problems may be caused by the curriculum and teaching instructions. National Council of Teachers of Mathematics (2000) indicates that in many countries the curriculum in geometry overemphasises the naming shapes and ignores the relationships between geometric properties. This may result in Taiwanese students being successful when it comes to memorise each shape's name and its specific geometric property correctly, yet less able to use geometric properties to identify or categorise shapes. Most eastern Asian countries (Taiwan, Hong Kong, Japan, Korea, and China) are ranked in the top 10 in TIMMS, but these countries have each specific teaching instruction (Li & Shimizu, 2009). Taiwanese primary teachers prefer to use the direct instruction (Chiang & Stacey, 2013) instead of cooperative learning in teaching mathematics (Mullis et al., 2012). In mathematics classes, students are used to repeating several tests and are trained to solve various types of problems in order to get higher scores in Taiwan. To score higher points, Taiwanese teachers may focus on students' cognitive skills in problem solving and ignore how students think mathematically. Some Taiwanese educators have noticed the consequences after participating in these international assessments and have put effort into reforming the educational system in Taiwan (Yang & Lin, 2015).

Although the educational reform is a controversial issue in Taiwan, research shows that educators have tried to improve students' weaknesses in learning mathematics (Yang & Lin, 2015). Since 2000, Argumentation in Taiwan has been shown to have several functions that could lead students to improve their competences in problem solving and communication in mathematics (Horng, 2004). The underlying nature of these competences is reasoning and the competence of reasoning is an essential one to support students to learn mathematics (Horng, 2004). Moreover, Lin and Cheng (2003) also state that the core competence of geometry is argumentation and learning geometry is tightly linked to learning argumentation in Taiwan. Therefore, we assume that reinforcing the use of argumentation in mathematics classroom may play an important role for improving students' mathematical competences in terms of reasoning and geometry.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 365–372. Sunshine Coast: MERGA.

Argumentation is one useful tool to help students improve their competences of reasoning and geometry for class teachers. Students' oral or written discourse is able to reflect on what they think and how they solve problems. Vanderhye and Zmijewski Demers (2007) also claim that class teachers are able to utilise students' mathematical conversation to understand their thinking. Although there are some tools that are used to assess students' argumentation (Healy & Hoyles, 1999; Lin & Cheng, 2003), they do not emphasise students' cognitive abilities. In this study, we will develop a theoretical framework to assess students' geometric argumentation from the cognitive perspective. In the following sections, we will identify the definition of geometric argumentation and develop the theoretical framework with indicators in greater detail.

The Definition of Geometric Argumentation

Mathematical argumentation is related to mathematical concepts and reasoning abilities in students' discourse. Durand-Guerrier, Boero, Douek, Epp, and Tanguay (2012) define mathematical argumentation as either a written or oral discourse and the discourse links between premises and a conclusion through reasoning. The processes of reasoning combines mathematical rules with plausible statements and the plausible statements are formed by knowledge that is valid (Durand-Guerrier et al., 2012). Discourse is one kind of communication and students should discuss with each other. According to this definition, Krummheuer (2000, 2007) admits that argumentation is one type of social interaction and students' understanding is constructed through social interaction. Social interaction has been recognised as improving students' learning since students' ideas will be justified and clarified with their classmates (Wood, 1999). From this perspective, although Durand-Guerrier et al. (2012) regard argumentation as a process, we hold a different perspective. In the next section, we will discuss the differences with several reasons.

Students' oral or written discourses can take various types of formats and *proof* is one specific format which has to use deductive reasoning in argumentation (Aberdein, 2005; Durand-Guerrier et al., 2012). Although there are some similarities between argumentation and proof for mathematics educators (Durand-Guerrier et al., 2012), we have to point out that both argumentation and proof can take different *shapes* when considering primary students. We can distinguish between argumentation and proof from three perspectives: types of reasoning; pedagogical meaning; and formats. From the first perspective, while students are able to use inductive, deductive, and abductive reasoning in argumentation (Douek, 1999), students are only allowed to use deductive reasoning in proof (Ayalon & Even, 2008). From a pedagogical perspective, the purpose of argumentation is to cultivate students' thinking and engage students' understanding (National Council of Teachers of Mathematics, 2000; Wood, 1999) while proof aims at, amongst others, providing means for mathematicians to discuss the validity of mathematics results and communicate with each other (Department of Elementary Education, 2008). Finally, while students use their daily language to explain their thoughts in argumentation (Wood, 1999), they are expected to use formal mathematical language to reason and explain their ideas logically (Ayalon & Even, 2008). For these reasons, we conclude that argumentation is different from proof and both argumentation and proof play two different but important roles at the primary level.

In summary, geometric argumentation in this study refers to the activity that occurs when students (in our case mainly primary students) employ geometric concepts or properties to form plausible statements in order to link premises and a conclusion through their daily language. There is a theoretical framework with three indicators that are essential to analyse students' geometric argumentation: naming, supporting ideas, and transformation reasoning. We will explain the theoretical framework and three indicators in greater detail.

Constructing a Theoretical Framework for Assessing Students' Geometric Argumentation

The theoretical framework is originally from the definition of geometric argumentation and there are three indicators that are developed from Toulmin's (2003) model in this framework. Toulmin's model is a structure of argumentation (Aberdein, 2005) and identifies some specific elements in argumentation (Toulmin, 2003). In the following sections, we will introduce the relationships between the definition of geometric argumentation and Toulmin's model, and describe three indicators.

The Relationship between the Definition of Geometric Argumentation and Toulmin's Model to Develop the Theoretical Framework

As stated previously, geometric argumentation refers to an oral or written discourse to link between premises and a conclusion with some geometric properties reasons. The theoretical framework in this study refers to a conceptual structure and the structure means that each element in the structure has some relationships with other elements. These elements are developed by Toulmin's model and their meanings come from the definition of geometric argumentation in this study.

The structure of the theoretical framework is originally from Toulmin's model, but we modify and simplify some elements in mathematics education settings (Krummheuer, 1995). Toulmin's model is a structure of argumentation and is used to argue others' claims (Hitchcock & Verheij, 2006). Since Toulmin's model does not only focus on mathematical communication, the definitions of elements in Toulmin's model are general statements. In Toulmin's model, the meaning of *data* refers to facts and information that students know; *warrants* represent evidence that is used to support their conclusion; *qualifiers* have no clear definition, but the function of qualifiers is to justify whether evidence is correct or not; *backing* is defined as theories that are to challenge evidence which people give; *claims* refer to drawing a conclusion from data, and *rebuttals* are other claims to criticise the conclusion (Toulmin, 2003).

Although Toulmin's original model encompasses six elements; namely, data, claim, warrant, backing, qualifier, and rebuttal (Toulmin, 2003), Krummheuer (1995) states that researchers tend to ignore the elements of rebuttals and qualifiers when they analyse students' argumentation, which may come from the definition of argumentation itself. The function of rebuttals is to justify and clarify students' thinking. Krummheuer (1995, 2000, 2007) regards students' argumentation as a product and *rebuttals* play a role to help students produce an appropriate response. Inglis, Mejia-Ramos, and Simpson (2007) also claim that although *qualifiers* have no psychological value, but can help students think logically. Both rebuttals and qualifiers are to improve students' argumentation, and have pedagogical meanings. Thus, both rebuttals and qualifiers seem to have pedagogical value when it comes to improving students' argumentation and we argue that these two elements should also be considered when modelling argumentation from a mathematics education perspective. Adopting Toulmin's work, Aberdein (2005) defines these four terms clearly: *Data* refers to the information given in a mathematical problem such as mathematical concepts and geometric properties. Students use this information to reach a conclusion. *Claim* means that students use the above-mentioned information to reach a conclusion.

Backing and *warrant* have different meanings when adopted in mathematics education. The meaning of backing refers to a mathematical theory and warrants represent evidence (Aberdein, 2005). Both of them are often called *reasoning* and the process of reasoning itself combines both backing with warrants (Prusak, Hershkowitz, & Schwarz, 2012). Supported by the above-mentioned research, we hypothesise that using argumentation enhances students' geometric concepts and reasoning, but Toulmin's model is limited in that although it introduces a theoretical framework to describe the role of argumentation, it just illustrates a structure of argumentation. For this reason, this study uses the structure to develop some indicators with the definition of geometric argumentation to assess students' geometric argumentation.

We adopt Toulmin's model and identify elements in the model in the definition of mathematical argumentation in this study: premises refer to what Toulmin calls data, mathematical rules refer to backing, plausible statements can be viewed as warrants, and conclusions refer to claims in mathematics. Therefore, within the mathematics education setting, geometric argumentation could be viewed as students' use of geometric properties or geometric concepts to link the relationships between premises and a conclusion through their oral or written explanations. Figure 1 shows the relationships between the definition of geometric argumentation and Toulmin's model.

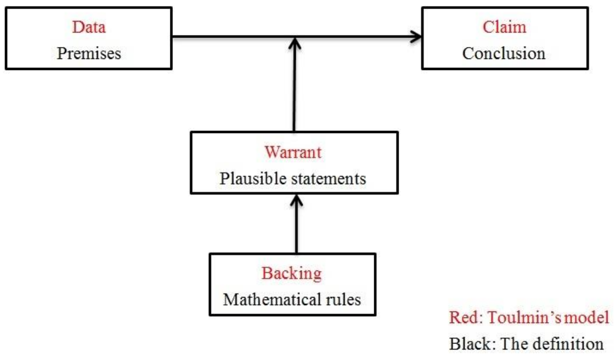

Figure 1. The relationships between the definition of geometric argumentation and Toulmin's (2003) model

Three Theoretical Indicators to Assess Students' Geometric Argumentation

As stated earlier, there are several existing frameworks designed to evaluate students' mathematical argumentation and each framework provides different information to mathematics educators. Healy and Hoyles (1999) investigated how secondary students construct a mathematical proof and analysed the proofs from two different perspectives: by analysing forms of arguments used and by attributing a score for correctness. There are four categories to distinguish students' proof which are the outcome of the two perspectives: "No basis for the construction of a correct proof, No deductions but relevant information presented, Partial proof, and Complete proof" (Healy & Hoyles, 1999, p.19). In a similar study, Lin and his colleagues adapted the categories from Healy and Hoyles to evaluate secondary students' arguments in Taiwan and developed four levels: "intuitive response, improper argument, incomplete argument and acceptable proof" (Lin & Cheng, 2003). In our view, these two frameworks do not seem suitable to be used as is, in a primary school setting. While both frameworks are useful to analyse students' argumentation, they may not reflect on students' learning problem in argumentation from the cognitive perspective, especially in the process of reasoning. Furthermore, both frameworks emphasise the concepts of *proof* for secondary students and may be not useful for primary students' *argumentation* (cf. our earlier discussion about the distinction between proof and argumentation). On another level, students' argumentation analysed by both frameworks is categorical data and categorical data is used to reflect on the types of students' argumentation. The aim of this framework is expected to understand students' geometric argumentation, especially in their cognitive competences, such as reasoning. For these reasons, we will develop three indicators to assess primary students' geometric argumentation in order to solve these problems in this study.

Reasoning is one essential component in argumentation (Mercier, 2011), and the indicators have to reflect students' competence in reasoning. There is one kind of writing style that is called *argumentative writing*, and a core competence of argumentative writing is reasoning (Reznitskaya, Kuo, Glina, & Anderson, 2009). Reasoning is the core competence in both geometric argumentation and argumentative writing. Argumentative writing has two scales to analyse students' works: the analytic and holistic scales (Reznitskaya et al., 2009). The former one lists several indicators and each indicator has different scales, and the later one is to rate students' writing holistically. Reznitskaya et al. (2009) develop five indicators, which are called the analytic scales, to evaluate students' argumentative writing, including *fluency, flexibility, alternative, focus, form*. Both fluency and flexibility are related to how students employ their ideas and alternative means whether students are able to give the opposite perspective to justify their ideas. Focus represents that students are able to utilise their ideas and form means that the structure in students' writing is complete. These five indicators are divided into two parts: content (fluency, flexibility, and alternative) and organisation (focus and form). However, mathematical argumentation is one kind of discourse and has no regular format. Thus, the dimension of organisation can be ignored.

On the other hand, the holistic scales have several points that combine with many criteria at one point. The scales are complex and students who get the same point may have different performances. The scales at each level are related to several factors, such as students' writing structure, supporting evidence, reasoning abilities, and giving opposite evidence. Each of these factors does not have the same criteria to assess. It may be difficult to reflect on the cognitive competences of geometric argumentation in this study. Hence, we adapt the analytic scales to develop three indicators: naming, supporting ideas, and transformation reasoning.

Naming. The indicator of naming relates to students' ability in identify the name of geometric shapes correctly. The indicator has two subscales: premises and conclusions. The subscale of premises means whether students are able to gather correct information from what they were taught and what a task is given. Premises have an important position in argumentation and they will influence other components of Toulmin's model. On the other hand, the subscale of conclusions means whether students are able to make a conclusion correctly. However, students have to do the first step correctly since the first step can be justified and needs to be correct in mathematical argumentation (Aberdein, 2005). For example, students have to choose correct shapes (premises) and name them correctly (conclusions) or they will do invalid reasoning in the sort task of geometric shapes.

Supporting ideas. The indicator of supporting ideas relates to students' ability of employing an appropriate geometric property in order to link premises and a conclusion. The scale in this indicator is affected by the indicator of naming. Even if students give complete and correct geometric properties, but the premises are incorrect, students cannot get any point in this indicator.

Transformation reasoning. As we previously discussed, reasoning plays a significant role in argumentation, thus the need to incorporate this indicator in the assessment of students' argumentation. The term of transformation reasoning has been developed by Simon (1996) as:

> The mental or physical enactment of an operation or set of operations on an object or set of objects that allows one to envision the transformations that these objects undergo and the set of results of these operations. Central to transformational reasoning is the ability to consider, not a static state, but a dynamic process by which a new state or a continuum of states are generated. (p. 201)

According to this definition, transformation reasoning is one kind of reasoning, which may be seen as in between inductive and deductive reasoning. Primary students learn geometric concepts through their visual cues and operations (Duval, 1998) and transformation reasoning is that students use visual cues and operations to reason. Based on this perspective, we define the indicator of transformation reasoning that students are able to provide what they operate, measure, and see and convert their actions into geometric concepts. Therefore, students have to provide their actions (evidence) and connect their actions to geometric concepts (theoretical reason). The indicators of transformation reasoning and supporting ideas influence each other: their actions determine how students employ what geometric property or using which geometric property decides how students confirm their ideas. Figure 2 shows the relationships among three indicators, the definition of geometric argumentation and Toulmin's model.

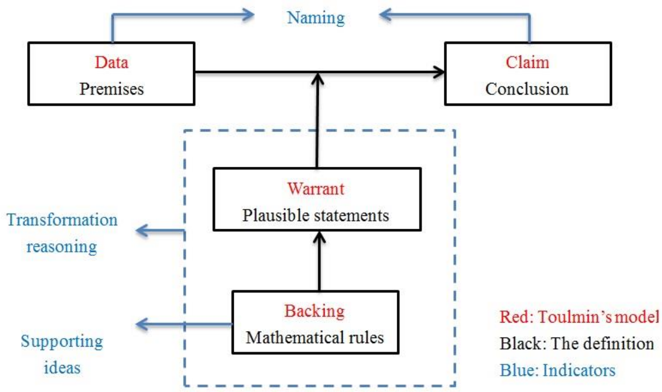

Figure 2 The relationships among the indicators, the definition of geometric argumentation and Toulmin's model

Discussion and Conclusion

Three indicators have been introduced in these sections, but how to use these indicators may be questioned by researchers. However, we do not develop the specific scales to assess students' geometric argumentation in each indicator and here are some reasons: first, there are several geometric activities that are related to geometric argumentation. Each activity has some pedagogical purposes and teachers or researchers should develop scales based on their purpose. Second, the scales are related to the curriculum design. The curriculum in different countries has different perspectives for educating students. While the curriculum in Eastern countries has a content orientation, Western countries adopt the creative approach. Therefore, the criteria of the scales should reflect their curriculum design. For both reasons, we encourage researchers to develop scales to reflect the students' learning problem in geometric argumentation.

The assessment tools have two points that should be of concern: one is that the tools are handy to understand students' cognitive competences in geometric argumentation for the practical purpose, and the other is that the tools have empirical evidence to support their use for the academic purpose. We cannot deny that there exist several scales, schemes, or rubrics to evaluate students' thinking and reasoning in argumentation. The theoretical framework in this study emphasises students' cognitive competences and points out three indicators to assess. Researchers are able to use this framework to understand students' learning problem and whether argumentation can improve students' weakness in mathematics in Taiwan. On the other hand, we still have to be concerned about the academic purpose. Although the indicators may be appropriate to assess students' cognitive competences in geometric argumentation, these indicators lack empirical evidence such as validity and reliability. Both validity and reliability support researchers and class teachers to use assessment tools confidentially. Hence, researchers should put effort into constructing validity and reliability in these indicators and these three indicators have theoretical evidence to put them into practice.

In summary, this study adapts different theoretical perspectives and these perspectives converge into one framework. It may be easy to use these three indicators for researchers, yet they still have empirical data to support. In the future, it still has a long way to go to construct the theoretical framework in geometric argumentation and we expect that the framework can be applied into other mathematical argumentation.

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