Interpreting Association from Graphical Displays

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Research that has explored students' interpretations of graphical representations has not extended to include how students apply understanding of particular statistical concepts related to one graphical representation to interpret different representations. This paper reports on the way in which students' understanding of covariation, evidenced through their interpretation of scatterplots, was applied to the interpretation of split stacked dot plots. The outcomes of the study suggest that incomplete understanding of the characteristics of a graph and the data displayed can lead to students applying knowledge of statistical concepts relevant to one graph type to misinterpret a different graph type.

In recent times, the research agenda in statistics education research has focused on the nature and development of statistical literacy, reasoning, and thinking (Garfield & Ben-Zvi, 2004) with particular attention given to the key statistical concept of informal inference (e.g., Rubin, Hammerman & Konold, 2006; Watson & Donne, 2008; Watson & Wright, 2008). Much of the research has been conducted within the context of the learning environments afforded by new technologies (e.g., Paparistodemou & Meletiou-Mavrotheris, 2008) but little attention has been given to how students translate their knowledge and understanding of one particular graphical representation to another. For the most part, that research has focused on one particular graphical representation to explore the development of particular key statistical concepts. For example, Watson and Donne explored students' use of hat plots when making informal inferences, and Rubin et al. explored the sorting of data into bins and the display of visual and numerical information simultaneously when comparing groups. A number of studies, however, have given students the freedom to construct multiple representations to make sense of the data (e.g., Rubin et al.). Although favourable outcomes have been reported, Rubin and her colleagues and Bakker (2002) warn that providing open access to the full suite of features available in some graphing software programs may be overwhelming and distracting for some students. Given the ability that these new technologies have to produce non-traditional graphical representations in conjunction with traditional graphical representations (Watson & Fitzallen, 2016), it is worthwhile investigating the way in which students use their understanding of statistical concepts developed using one particular graphical representation to the interpretation of other graphical representations.

Interpreting Graphical Representations

Graphical representations have many characteristics that can be used when interpreting the data displayed. The characteristics, such as the mode, scale of an axis, or the variation in the spread of the data can be extracted directly from the graph (Roth, Pozzer-Ardenghi, & Han, 2005) or from calculations performed by graphing software (Watson & Fitzallen, 2016), such as *TinkerPlots Dynamic Data Exploration* (Konold & Miller, 2011). An understanding of the context and the nature of the variables of interest may be gained from personal experiences of the context, from information about the data, or from the details embedded in the scales and frameworks of the graphs (Roth et al., 2005; Watson & Fitzallen, 2016). Collectively, the elements of graphs are resources that provide a link

2016. In White, B., Chinnappan, M. & Trenholm, S. (Eds.). Opening up mathematics education research (Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia), pp. 222–229. Adelaide: MERGA.

between the visual two-dimensional representations and the real world measurement situations; relevant to this study are scatterplots and split stacked dot plots.

Scatterplots and Split Stacked Dot Plots

Developed in the 1800s, a scatterplot is a graphical technique used to display paired measurements of two quantitative variables and used to explore the relationship between two numerical attributes. They are characterised by data points that correspond to the measures of two variables designated at the same time on a Cartesian graph (Moritz, 2004). Each data point on a scatterplot corresponds to one unit of analysis between the two variables and the values of the two variables may be said to involve some form of relationship, association, function, dependency, or correspondence (Cobb, McClain, & Gravemeijer, 2003; Moritz; Zieffler & Garfield, 2009). Scatterplots have numerical attributes on both axes of a graph and are used to display covariation (Moritz, 2004).

Split stacked dot plots are distinctly different to scatterplots. They are non-traditional graphs that are easy to construct using interactive graphing software, such as *TinkerPlots*. These graphs display a numerical attribute on one axis and a categorical attribute on the other axis, which facilitates a comparison of categories or multiple data sets for the one numerical attribute. Like scatterplots, split stacked dot plots can be used to display association. They are, however, used primarily to make comparisons between groups and support the analysis of data that go beyond direct comparisons (Watson & Wright, 2008). It is, however, the direct comparisons of the visual characteristics of the plots that students are able to use to determine if there is an association between the two attributes displayed.

Two examples of split stacked dot plots are provided in Figure 1. Added to the graphs are hat plots that divide the distribution of the data into three sections. The hat plot resembles a hat and is made up of two main components. The crown of the hat is a rectangle that shows the middle 50% of the data and the brim of the hat is a line that extends across the full range of the data set. The lower 25% of the data is represented by the line to the left of the crown and the upper 25% of the data is represented by the line to the right of the crown. A hat plot can only be applied when one or more of the axes of a graph have a continuous scale. The graph on the left shows there is an association, thereby a relationship, between the gender and height of adults. This is determined by the distinct differences in height of the two groups—the ranges of the crowns of the hat plots and the means (denoted by the △ symbol) are different for the genders. The graph on the right, shows there is no association, therefore no relationship, between the gender and height of children. This is determined by the similarity of the height of the two groups—the ranges of the hat plots and the means are essentially the same for both genders.

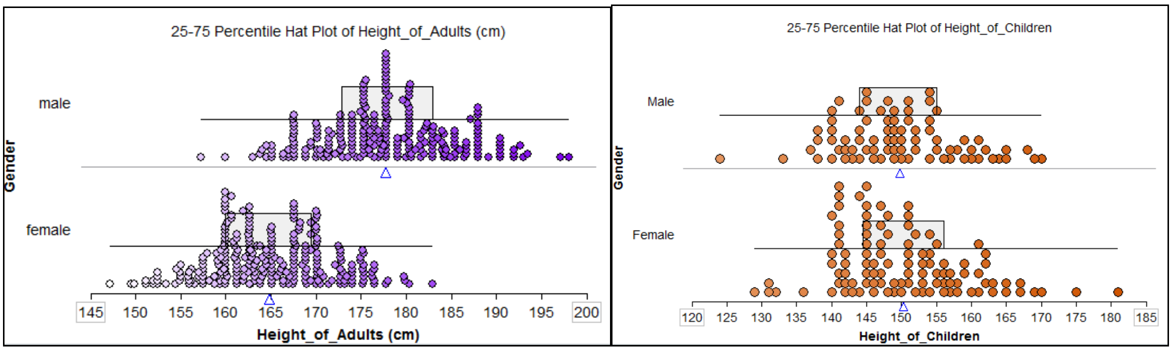

Figure 1. Stacked dot plots.

Distinguishing Between Covariation and Association

Although related, covariation of numerical attributes is a distinctly different form of association from that seen in Figure 1. However, the definition of covariation is often included in the definition for association (Batanero, Estepa, Godino, & Green, 1996; Moritz, 2004; Zieffler & Garfield, 2009). Batanero and her colleagues describe covariation as a form of association. They note that association is "the analysis of contingency tables, the determination of correlation between quantitative variables, and the comparison of a numerical variable in two or more samples" (p. 151). Covariation is "[r]easoning about *association* (or *relationship*) between two variables, also referred to as *covariational reasoning*, or reasoning about *bivariate data*, involves knowing how to judge and interpret a relationship between two variables (Zieffler & Garfield, p. 7). Similarly, Moritz suggests, "[c]ovariation concerns association of variables; that is, correspondence of variation. Reasoning about covariation commonly involves translation processes among raw numerical data, graphical representations, and verbal statements about statistical covariation and causal association" (p. 227). Moritz also adds, "The more general term *statistical association* may also refer to associations between two categorical variables, commonly represented in two-way frequency tables, and between one categorical and one interval variable, often formulated as the comparison of groups" (p. 228).

The Study

The research reported in this paper is part of a study that explored students' use of *TinkerPlots Dynamic Data Exploration* (Konold & Miller, 2011) to construct and interpret graphical representations. It involved 12 Year 5/6 students (11-12 years old) students worked in pairs with the teacher/researcher (45 minutes, twice a week for 6 weeks) through a sequence of learning experiences designed to provide them with the opportunity to develop an understanding of various statistical concepts and graphical representations using *TinkerPlots*. The learning sequence included activities related to distribution, variation, and increasing sample size as well as the construction of dot plots, bar graphs, value bar graphs, scatterplots, and stacked dot plots. At the end of the learning sequence the students were interviewed individually as they used *TinkerPlots* to create various graphical representations of their choice to show the relationship between two attributes from the data set provided. The data set included both categorical and numerical data. The activities were set up in *TinkerPlots* as an interview protocol and were designed to provide the opportunity for the students to demonstrate what they had learned during the sequence of learning experiences. On screen capture video was used to record the students' actions as they used *TinkerPlots*. The video also captured audio recordings of the students' explanations of actions taken and responses to questions posed by the teacher/researcher. Prior to the analysis of the data for this paper the video data were analysed to determine their level of understanding of covariation. The results of that analysis provide reference points from which the analysis of the data for this paper is discussed.

Students' Level of Understanding of Covariation

Paparistodemou and Meletiou-Mavrotheris (2008) contend that *TinkerPlots* enhanced young students' opportunities to find relationships between two variables in the data and to draw conclusions from the data. In this study, the students' statements, descriptions, and justifications about the relationships seen in the graphs were analysed to evidence the complexity of the responses and the level of understanding attained.

To determine the students' level of understanding of covariation, the data were analysed according to the level demonstrated according to the SOLO taxonomy (Biggs & Collis, 1982). Of the 12 students interviewed, six students' responses were uni-structural, three students' responses were multi-structural, and three students' responses were relational (Table 1). At the uni-structural level the students made statements that often involved a declaration that there was or was not a trend evident in a scatterplot but little or no justification or reasoning was offered to explain how they made the judgement. At the multi-structural level, the students used multiple characteristics of scatterplots to explain and justify their thinking as they described the covariation identified. Students who demonstrated understanding at the relational level also used multiple characteristics of graphs to make their decisions. They also went further to identify the variation in the graph that did not meet their expectations for a relationship to exist (Fitzallen, 2012).

Table 1.
Students' Achievement for Covariation According to the Levels of the SOLO Framework

Uni-structural	Multi-structural	Relational
Jake, Natasha, Rory, Johnty, Natalie, Kimberley	Shaun, Blaire, Jessica	James, William, Mitchell

Data Analysis

The audio data from the video recordings were transcribed verbatim and descriptions of the students' actions observed on the videos were added to the transcribed data. Content analysis of the transcripts (Miles & Huberman, 2003) involved analysing the transcripts line-by-line to code the responses and actions according to the four dimensions of the Graphing in EDA Software Environments framework—Generic [ICT] knowledge, Being creative with data, Understanding data, Thinking about data (Fitzallen, 2012). After this initial categorisation, analysis of the grouped data was dominated by, but not restricted to, the data coded for the dimensions *Understanding data* and *Thinking about data*. That analysis focused initially on determining the students' level of understanding of covariation (Fitzallen, 2012). A second round of data analysis was undertaken to determine the way in which the students constructed and interpreted split stacked dot plots, particularly in relation to their selection for showing the relationship between two attributes. The data from the second data analysis iteration is reported in this paper and discussed in relation to the results of the first iteration of data analysis.

Results

Students' Interpretation of Split Stacked Dot Plots

As the students worked through the activities and questions in the interview protocol, they constructed a variety of graphs, including scatterplots and split stacked dot plots. An example of the graphs created is provided in Figure 2. The final task in the interview protocol set up in *TinkerPlots* required the students to look at all the graphs created during the session and determine which graph showed the strongest relationship between two attributes. It was anticipated that the students would select scatterplots that displayed covariation. Unexpectedly, of the 10 students that completed this task, seven indicated that split stacked dot plots were the ones that displayed the strongest relationship between two attributes (see Table 2). Jessica and Kimberley did not make a contribution to the following results as they ran out of time and did not complete the final question on the interview protocol.

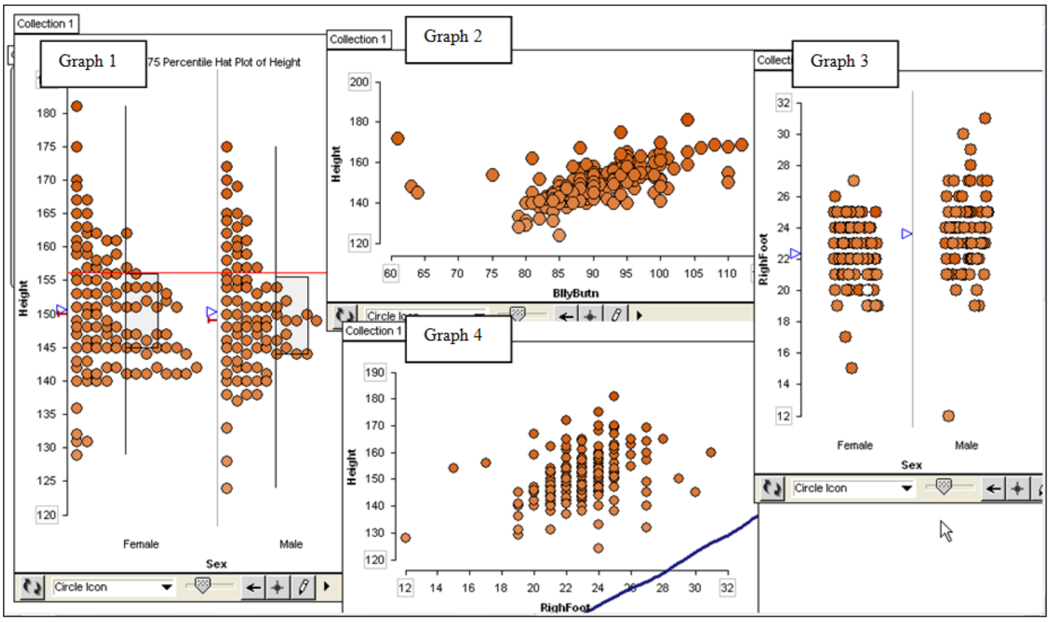

Figure 2. Graphs constructed by Mitchell using the interview protocol.

Table 2.
Students' Selection of Graphical Representations to Display the relationship Between Two Attributes

Split stacked dot plots	Scatterplots
Blaire, Jake, Johnty, Natalie, Mitchell, Rory, Shaun	Natasha, James, William

All of the students had examples of scatterplots and split stacked dot plots similar to those in Figure 2 from which to choose. The seven students who chose split stacked dot plots selected graphs that displayed the association between one numerical attribute and one categorical attribute. All of the split stacked dot plots created by the group of students were similar but varied somewhat. An example is provided in Graph 1 in Figure 2. The students individualised their graphs by accessing different features of *TinkerPlots*, such as the mean, hat plots or reference lines. Regardless of the attributes chosen, the features added and the scale of the axes selected for the split stacked dot plots, each of the students indicated that it was the similarity or closeness of the information from the visual features in each section of the graphs that influenced their decisions. As can be seen in the split stacked plot in Graph 1 in Figure 2, the mean for the males and females is the same and the range of the crown of the hat is also very similar for both genders. When making the decision about the relationship between two attributes from this type of graph, the students gave little attention to the distribution of the data across the outer reaches of the hat brim. On occasions, the overall range of the data for each group was considered but students only mentioned the range of the data when there was a large difference in the ranges.

Johnty, Rory, and Blaire chose split stacked dot plots that displayed the association between gender and height to be the one that showed the strongest relationship between the two attributes chosen. Blaire used the mean to make her decision. Blaire said she was confident it was the "best" graph because the mean height was the same for both genders. Johnty also used the mean to draw his conclusion but justified his decision further by comparing the crowns of the hat plots. As Rory made his choice he said, "[t]he hat plots [are] exactly the same. So is the average."

Jake and Natalie also used the crowns of the hat plots to justify their choices of a split stacked dot plot that displayed gender and foot length. Adding to Natalie's confidence was the distribution of the data. She said, "It is not as spread out [pointing to a split stacked dot plot]. This one is not as neat and you don't know how many dots there are [pointing to a scatterplot]." Shaun also selected a graph with gender and belly button height and said, "They've got the same amount of people like under the [hat] roughly about 50% and roughly about the same … outside the 50%."

The four graphs in Figure 2 were constructed by Mitchell. They are typical of the graphical representations the students constructed during the student interview. From the selection of graphs, Mitchell asserted that Graph 1 showed the strongest relationship between height and gender. Mitchell determined that the height of the males and females were the same by comparing the two groups using the mean, the position of the crown of the hat, and the spread of the data. He equated the closeness of these characteristics of the data to infer there was a strong relationship evident in the graph. By convention, interpretation of this graph would show there was no relationship or association between gender and height.

William, Natasha, and James selected scatterplots that displayed the relationship between belly button height and height to be the ones that showed the strongest relationship between two attributes. In all three cases, the decision was based on the trend evident in the data. For example, when William pointed to the scatterplot with height and belly button height displayed, he said: "Umm … probably this one, because umm, it shows us that the strongest, was umm, it shows you that the height, depending on where it is, chances are that that's where the bigger belly button height is, probably." Like the other seven students, these three students had split stacked dot plots showing no association in their collection of graphs but chose to focus their decision making on the scatterplots.

Discussion

It is customary to use scatterplots to determine if there is a relationship between two numerical attributes (Moritz, 2004). It was within the context of scatterplots that students demonstrated their understanding of covariation, which involves reasoning about the relationship displayed in the data. In doing so, they identified the trend and, in some cases, were able to describe the way in which one of the attributes increased in much the same way and in conjunction with the other attribute (Fitzallen, 2012). Although split stacked dot plots may display an association, thereby the relationship, between two attributes, different interpretation strategies than those used to interpret scatterplots are needed. Analysis of the results presented in this paper suggests the students transferred their naive understanding of covariation to interpret split stacked dot plots. Their interpretations of the split stacked dot plots were based on the way in which the data were the same for each gender. With these types of graphs, the "sameness" of the data is an indication that there is no relationship between the two attributes. With scatterplots, the idea that the attributes are behaving in the same way is an indication that there is a relationship between the two attributes.

The decision by seven of the students to use split stacked dot plots that displayed the association between two attributes to show the relationship between one numerical attribute and one categorical attribute suggests that they did not appreciate the limitations of the data or the graphical representation chosen. The association they identified due to the similarity of the visual representations suggests that some of the students had transferred their understanding established within the context of scatterplots to the context of split stacked dot plots. This revealed that the students had not established fully an understanding of the purpose of the two different graphical representations and the meaning they embodied.

Of the seven students, five demonstrated the lowest level of understanding of covariation, that is, the uni-structural level. Only Natasha, demonstrated a higher level of understanding of covariation (relational) than demonstrated previously when she chose a scatterplot as the one that showed the strongest relationship between two attributes. Conversely, Mitchell performed at the highest level of understanding for covariation (relational) previously and then transferred this understanding to the interpretation of a split stacked dot plot. It appears Natasha's and Mitchell's levels of understanding of covariation were not stable. Had the question about making a choice from a selection of self-generated graphs not be included in the interview protocol, it may not have become evident that some students had not established fully their understanding of the utility of scatterplots and split stacked dot plots. These results add support to the warning offered by Rubin et al. (2006) and Bakker (2002) that students may be overwhelmed when given access to software packages that give them access to various graphical representations. The students in this study did not appear to be overwhelmed by the features of *TinkerPlots* but when given the opportunity construct multiple graphical representations for the same data. The issue is that some of the students did not always choose graphs appropriate for answering the question asked.

Conclusion

The results reported in this paper are limited as the study only involved 12 students. They do, however, offer insights into the way in which students make sense of and interpret various exploratory data analysis representations. They also draw attention to the need to extend research to encompass student interpretation of non-traditional graphs made possible through innovative graphing software. A search of the literature did not reveal any research that explored specifically how students transfer understanding of statistical concepts to various graphical representations. To date, research has treated the exploration of student use of graph types discreetly (e.g. Watson & Donne, 2008) rather than focusing on the interconnectedness or not of various graph types. Further exploration of the development of key concepts such as data, distribution, centre, variability, outliers, sampling, and comparing groups (Biehler et al., 2013) within the context of multiple graph types, is required. Research has a role to play in developing models of learning that support teachers to counter the problems faced by students when transferring knowledge developed in one graphing context to other graphing contexts. An appreciation of the different thinking needed and used by students to interpret various graphical representations has the potential of empowering teachers to guide student learning towards an understanding of the particular learning outcomes targeted. Teachers will then be in a better position to choose the most appropriate data sets and graphical representations to explore statistical questions and promote understanding of statistical concepts.

Acknowledgements

This research is funded by an APA(I) Scholarship associated with an ARC Linkage Project LP0560543 and the industry partner, Department of Education, Tasmania. Thanks to Jane Watson for her support and collegial conversations about the paper.

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