Developing Interactive ICT Tools for the Teaching and Learning of Vectors at A-Level

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The aim of this qualitative study is to embed ICT in the teaching and learning of A-level vectors. Initially, students' learning difficulties and misconceptions in A-level vectors were diagnosed. Then, based on the difficulties identified, concepts in vectors together with interactive tools were developed and integrated in a webpage to enhance the teaching and learning of vectors at A-level. Finally, the tools developed were exposed to trainees for feedback and evaluated using the framework proposed by Pillay and Clarke (2008). The tools met the following criteria: learner focus, integrity, usability, and accessibility.

The world is experiencing a major shift in educational practices under the umbrella of Information Communication and Technology (ICT) enabled learning environment. Many studies have shown that the use of ICT in the teaching of Mathematics tends to improve learning, motivate and engage learners, promote collaboration, foster enquiry and exploration, and create a new learner centered learning culture (Glenn, 2004; Ng, 2005; Resta, 2002; Zhu, 2003).

Countries like United States, Australia, and the U.K. have developed high quality of online curriculum content for schools using dynamic and interactive software. This research project targets the creation of a dynamic online tools for A-level vectors for both teachers and students. There are hardly any online mathematical tools with connected pedagogical knowledge and technological tools suitable for the teaching and learning of A-level vectors for the Mauritian context. This study is an initiative to kick start research on the use of interactive ICT tools in the teaching and learning of mathematics at the secondary level in Mauritius.

During the last few years, the average pass rate in A-level Cambridge mathematics examination was close to 86 % with only around 54 % of the candidates managing to get at least a grade C. These figures show that students are encountering difficulties to perform well in A-Level mathematics. Some of the chapters in which students faced difficulties are integration, complex numbers, differential equations, and vectors. This study will focus on vectors.

Learning of vectors represents an important step in developing students' ability to solve problems. Knowledge of vectors is crucial both in solving problems from other chapters of mathematics and in other subjects like physics. However, students at A-level encounter difficulties in understanding and applying the concepts of vectors especially in three-dimensional space.

Both teachers and students have often commonly reported that the concepts of vectors are abstract and hard to apply. Every year, the general comment from the chief examiner of A-level Cambridge Mathematics examination includes the remark "Candidates have difficulties to work with the vector question and generally show poor mastery of its concepts". Examination reports also commonly include comment like the main difficulty

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 85-92). Melbourne: MERGA.

of the candidates is that they do not grasp the geometry of the situation and work with irrelevant pairs of direction vectors.

This problem is also specified by many teachers who believe that students are not able to understand and visualize the physical geometry of vector questions. Such students are unable to convert the semantic structure of the vector question into a good visualization of the physical problem of the question, especially when both lines and planes are involved.

This study targets the arousal of visualization sense in students so that they can convert verbal abstract vector questions into visually sound physical problem using ICT. It also helps to promote conceptual understanding and will help in improving performance in A-level, as displayed by ICT-based research studies (BERA Professional User Review, 2014).

Aims

The general aim of this study is to embed ICT in the teaching and learning process of the A-level Vectors to fulfil good pedagogical principles, that is, to make teaching and learning more engaging, motivating and interactive. Moreover, ICT-based teaching promotes conceptual understanding and retention ability. The following objectives are targeted in this research work:

- To diagnose students' learning difficulties in A-level vectors.
- To develop interactive tools to enhance learning of vectors in A-level.
- To implement and evaluate the teaching and learning tools developed.

Methodology

A mixed-method approach was used and the study was scheduled on different phases. The participants were pre-service trainee educators from the Mauritius Institute of Education (MIE) and A-level students.

During Phase 1, the research team reviewed the literature and Cambridge reports on the difficulties faced by teachers and students with respect to A-level Vectors. In addition, the existing ICT tools for the teaching of mathematics was documented. In parallel, a questionnaire, based on A-level vectors, was designed and administered to 45 MIE trainees. The sample contained 26 trainees enrolled on Teacher Diploma Secondary (TDS) and 19 Trainees enrolled on Bachelor in Education (BEd) programmes. The data collected gave an insight of the difficulties (in key concepts) faced by trainee teachers.

In Phase 2, based on the findings of Phase 1, the research team designed appropriate lessons (instruments) to develop interactive tools, which hold pedagogical promise on A-level vectors. The content area vector was divided into three subsections, three dimension vectors (including coordinates in 3D), lines and planes. The whole work was collaboratively vetted by the research team. The use of Tablets PC was also used by research team in the development of the teaching tools. The help of a graphic designer was needed for the setting up of the electronic page. The learning tools cater for different learning styles and allow the user to (i) review their previous knowledge, (ii) interact with the tools so that they can create or develop new ideas for better understanding, and (iii) solve problems on the concept to which they are exposed.

In Phase 3, once the learning resources developed, they were piloted with pre- and in-service educators and A-level students, and refined accordingly. The tools will also be accessible through Tablets PC. The webpage will soon be uploaded on the MIE website to provide access to everyone. The webpage has been designed so that it is user-friendly. A mathematics community will be developed to promote discussions and sharing of ideas between educators and students among others.

Evaluation of the Teaching Tools

The teaching tools were evaluated using the framework "evaluating ICT-based materials" proposed by Pillay and Clarke (2008). The framework consists of two phases. The first one deals with the pedagogical principles that guides the educational soundness of ICT-based tools. Under this phase, the framework considers the following criteria: learner focus, integrity, usability and accessibility of the tools. These criteria for ICT-based tools are expected to successfully promote learning. The second phase of the framework focused on the design, use and impact of the ICT-based tools. The design evaluation criteria relate to the quality of the educational soundness of the tools, the use of evaluation criteria relates to the factors influencing the utilization of the tools, and the impact of evaluation relates to the learning outcomes of the students. The framework is displayed next.

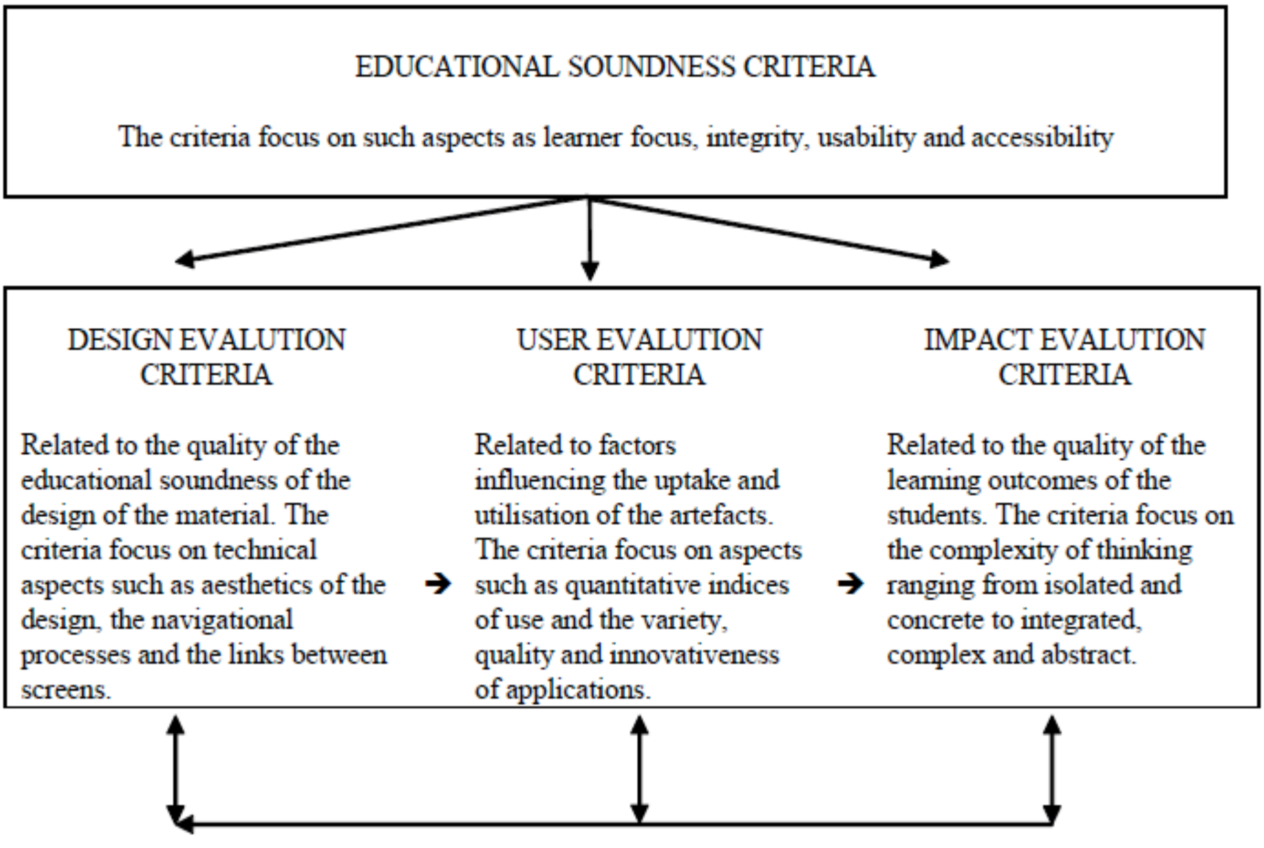

Figure 1. Framework for evaluating ICT-based learning materials (Pillay & Clarke, 2008).

Analysis

Learning Difficulties in A-Level Vectors

A questionnaire developed and administered to gauge the difficulties in A-level Vectors by MIE trainees (TDS and BEd). The findings from the questionnaires are surprising. Many questions were left unanswered while most of the solutions provided, for the questions attempted, were wrong. A major issue found in Cambridge reports is the lack of diagram in solutions to problems on vectors. Many of the difficulties faced by the trainees matched those found in the Cambridge reports. These observations demonstrate that the trainees still faced difficulties in vectors and this is quite surprising, in particular for TDS and BEd trainees who are aspiring mathematics educators. A sample for one question is next presented.

Relative to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 10 \\ 0 \\ 6 \end{pmatrix}.$$

(i) Find angle ABC.

The point D is such that $ABCD$ is a parallelogram.

(ii) Find the position vector of D.

Figure 2. Sample question (June 2011, p. 1, Question 8).

Extract of Examiner's Reports

This caused most candidates difficulty and there were only a small number of correct solutions. Most candidates did not realise that vector *OD* could be calculated directly from *OA* + *AD* and that *AD* = *BC* since *ABCD* is a parallelogram. Confusion often resulted from the fact that candidates were not prepared to sketch a diagram. Following the observations made from an examiner's report, a similar question was set to verify whether students are still facing difficulties with such problem. Extracts of trainees' work are shown below.

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}, \overrightarrow{OD} = \begin{pmatrix} 8 \\ 1 \\ 8 \end{pmatrix}.$$

The point C is such that ABCD is a parallelogram
Find the position vector C relative to the origin.

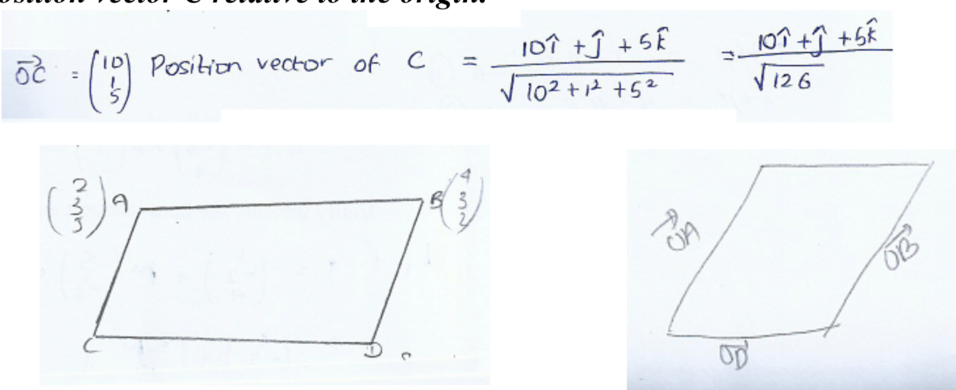

Figure 3. Extracts of trainees' work.

The above question (Number 3) tested trainees' knowledge of simple vector in 3D. These concepts have already been learned in Additional Mathematics at O level but still many respondents could not find the position of C given the vectors of A, B and D. Some were looking for the unit vector of C while some did not even label their diagram properly as can be seen in the above extracts. The findings from the survey confirmed those observed past examination reports.

It was disappointing to see that trainees did not know the basic concepts learned in vectors/coordinates, namely how to write coordinates in 3D. Base vectors or column vectors were used instead. Trainees demonstrate much difficulties on concepts such as scalar product, unit vectors, parallel and perpendicular vectors. Several wrong answers are due to incorrect calculations.

The trainees surveyed also faced difficulties in writing the general vector equations for lines and planes. Worst, many could not explain the notations used in the vector equations.

Trainees had difficulty to find the angle between two vectors. Some could not even recall the formula. Many of them did not realise that $\cos 90^{o}$ is equal to 0. In addition, they do not even know the meaning of scalar product of vectors.

Many trainees do not know how to verify whether two lines intersect or not. Many do not even know how to find the point of intersection in case it exists. Many could not even find the vector equation of a line. Incomplete answers were frequently observed.

Developing Interactive Tools to Enhance Learning of Vectors in A-Level

Following the identification of learning difficulties on vectors, lesson notes were written and interactive ICT tools were developed. Geogebra software was used for the development of the interactive tools, which are expected to enhance the teaching and learning of Vectors. The process of developing the webpage was time consuming. The tool will be available soon on the MIE website. Two snapshots of the webpage are shown below.

Figure 4. An example of the interactive tool.

6.5.1 Geogebra link: Point of intersection of line with plane

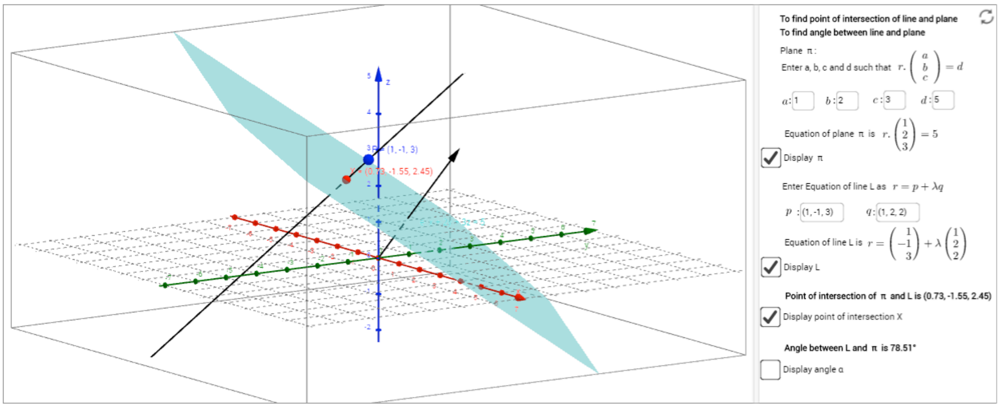

Figure 5. Second example of interactive tool.

Implementing and Evaluating the Teaching and Learning Tool Developed

Once the interactive tool was ready, it was presented to MIE trainees (enrolled in TDS and Post-Graduate Certificate in Education [PGCE]). Various requests were made by teachers for the availability of the tools, which they found amazing. Some issues were also highlighted and propositions made for the improvement of the tools. During the final stage of the study, the tools were evaluated using the framework "evaluating ICT-based materials" proposed by Pillay and Clarke (2008). The framework consists of two phases. The first one deals with the pedagogical principles that guides the educational soundness of ICT-based tools (that is, learner focus, integrity, usability and accessibility of the tools). The second one, the evaluation criteria, relates to the quality of the educational soundness of the tools (that is, Design, Use and Impact). These criteria for ICT-based tools are expected to successfully promote learning. A second questionnaire was developed (to evaluate the tools) and administered to PGCE FT, PGCE PT, and TDS.

Indices were created for each of the following: Learner Focus, Integrity, Usability, Accessibility, Design Evaluation, User Evaluation and Impact Evaluation. For example, the index (cumulated rate) for Learner Focus varies from 5 to 25 as it includes five items that are rated from 1 to 5. Similarly, the indices for the other variables are as follows: Integrity, 4 to 20; Usability, 5 to 25; Accessibility, 2 to 10; Design Evaluation, 3 to 15; User Evaluation, 3 to 15; and Impact Evaluation, 1 to 5. The frequency of the indices is displayed below. Based on the rate provided by the respondents, the tools under study were found to meet the following criteria:

Learner focus 84.4% of the respondents provided a cumulated rate of at least 20 out of 25.

Integrity 93.7% of the participants gave a cumulated rate of at least 15 out of 20.

Usability 78.1% of the respondents assigned a cumulated rate of at least 20 out of 25.

Accessibility 96.8% of the participants provided a cumulated rate of at least 7 out of 10

The second phase of the framework focuses on the design, use and impact of the ICT-based tools. The design evaluation criteria relate to the quality of the educational soundness of the tools, the use of evaluation criteria relates to the factors influencing the utilization of the tools, and the impact of evaluation relates to the learning outcomes of the students. Based on the cumulated rate provided by the trainees and educators, the following Evaluation criteria were met:

Design 83.9% of the respondents provided a cumulated rate of at least 10 out of 15.
User 92.3% of the participants gave a cumulated rate of at least 10 out of 15.
Impact 80.6% of the participants provided a cumulated rate of at least 4 out of 5.

The findings are encouraging with positive feedback. It was found that the tools developed met the following (pedagogical) criteria: learner focus, integrity, usability and accessibility of the tools. The tool developed also met the three evaluation criteria: design, use and impact.

The participants who have been exposed to the interactive tools by now are very satisfied with visual aspect and the interactivity of the tools. The latter will be uploaded on the MIE website to provide access to all teachers and students in the country. The interactive ICT tool will be presented to colleagues during an in-house session and at least one workshop will be conducted to expose the tools to educators from various secondary schools. More sessions may be organized to increase the awareness of the existence of the interactive webpage. This interactive webpage is a first of its kind and will be the baseline for many others.

Feedback. Written feedback were also gathered from a TDS FT cohort. The feedback was useful for the refinement of the tool. Most feedback was very encouraging. A sample of two extracts is presented next.

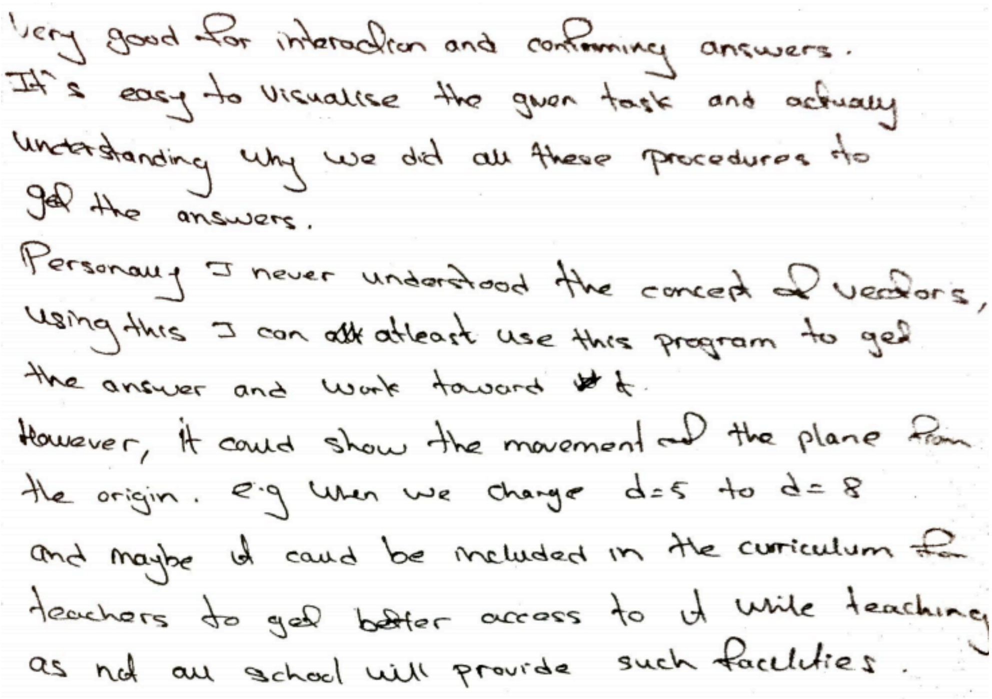
Very good for interaction and confirming answers.
It's easy to visualise the given task and actually understanding why we did all these procedures to get the answers.
Personally I never understood the concept of vectors, using this I can at least use this program to get the answer and work toward it.
However, it could show the movement of the plane from the origin. e.g when we change d=5 to d=8 and maybe it could be included in the curriculum for teachers to get better access to it while teaching as not all school will provide such facilities.

Figure 6. Sample extracts (T21).

Conclusion

This project was initiated following observations made about poor performance in mathematics at A-level and instrumental understanding of the topic of vectors by MIE trainees. The aim of the study is to embed ICT in the teaching and learning process of the A-level vectors that fulfil good pedagogical principles, hence making teaching and learning more engaging, motivating and interactive. Moreover, ICT-based teaching promotes conceptual understanding and retention ability.

The study was conducted in three phases and used mixed method approach. In the first phase, the aim was to diagnose students' learning difficulties and misconceptions in A-level vectors. As such, an analysis of Cambridge reports of A-level Mathematics (Syllabus 9709) was undertaken to find out the main difficulties faced by students in A-level Vectors. Various difficulties were noted but only a sample presented in this paper. Based on the difficulties identified, interactive tools were developed and integrated in a webpage to enhance the teaching and learning of vectors at A-level. Finally, the tools developed were evaluated using the framework proposed by Pillay and Clarke (2008). It was found that the tools developed met the following (pedagogical) criteria: learner focus, integrity, usability, and accessibility of the tools. The tool developed also met the three evaluation criteria: design, use and impact; and received positive written feedback from respondents.

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