Entangled Modes: Social Interaction in Collaborative Problem Solving in Mathematics

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This paper reports a study conducted in a laboratory classroom with the capability to record classroom social interactions in great detail using advanced video technology. The social interactions of student groups during collaborative problem solving were analysed based on transcript data. This analysis suggests that meaning negotiation in mathematics classrooms can be usefully distinguished as social, sociomathematical, or mathematical. We suggest that all three modes coexist in an entangled form in the negotiative interactions documented in the mathematics classroom and we envisage all three as constitutive of learning.

Introduction

We propose that student socially performed negotiative activities constitute both an essential aspect of the learning process and a key learning product on which more sophisticated intellectual activity is dependent. In order to investigate this proposition, we have employed student collaborative problem solving as a suitable activity by which the negotiative aspects of mathematics learning can be made more visible. Intact classes of Year 7 students (13 years old) with their usual mathematics teacher were filmed in a laboratory classroom completing a sequence of mathematics tasks individually, in pairs and in small groups. Our goal in the research reported in this paper was the identification of negotiative patterns of social interaction within the context of collaborative mathematical problem solving. We see the identification of such patterns as an essential precursor to the modelling and ultimately the optimisation of student collaborative group work and associated learning in mathematics classrooms. This paper reports the first step towards this long-term goal.

Conceptualising this Study

In associating learning with participation in practice, Lave and Wenger (1991) assert that "participation is always based on situated negotiation and renegotiation of meaning in the world" (p. 52). As legitimate sites of situated mathematical practice, classrooms provide settings in which these negotiative processes can be documented. Clarke (2001b) suggested that the presumptions of meaning are community, purpose and situation, since "it is futile to discuss the meaning of a word or term in isolation from the discourse community of which the speaker claims membership, from the purpose of the speaker, or from the specific situation in which the word was spoken" (p. 36). Contemporary social theories of learning accord a central role to the situated construction of shared meaning, through such constructs as the didactic contract (Brousseau, 1986) and sociomathematical norms (Yackel & Cobb, 1996).

Yackel and Cobb (1996) advanced the notion of sociomathematical norms by investigating the particular regularities in the social interactions within mathematics classrooms. They suggested that individuals develop their personal understandings of the social interactions in the mathematics classroom as they participate in the negotiation of classroom norms, some of which are specific to mathematics. Examples of

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 133-140). Melbourne: MERGA.

sociomathematical norms include: what counts in that classroom as mathematically different, mathematically sophisticated, mathematically efficient, mathematically elegant, and what is considered to be an acceptable mathematical explanation and justification. Such norms can be distinguished from questions of mathematical correctness (mathematical norms), while also being distinct from social norms that govern other forms of social interaction of a non-didactical nature. We follow the European use of "didactical" here, referring to discipline-specific pedagogical concerns (Brousseau, Sarrazy, & Novotná, 2014).

While teachers play an important role in the classroom, peer interactions also appear to be particularly important for student learning. This observation can appear self-evident in many "Western" educational systems, where student-student interactions are an institutionalised aspect of classroom practice, but it is also consistent with research in classrooms in which such interactions are much less frequent. Results from the Learner's Perspective Study (Clarke, 2006; Clarke, Keitel, & Shimizu, 2006; Kaur, Anthony, Ohtani, & Clarke, 2013), for example, suggest that students across all cultural settings attach particular significance to explanations provided by their peers in all mathematics classrooms where this occurs. Educational reform prioritising collaborative group work is being undertaken in countries such as China and Korea that had previously made very limited instructional use of student-student interaction. In combination, both research and contemporary reform make the investigation of such interactions and their function in the learning process an international educational imperative.

In terms of research design, some studies of social interaction in settings characterised by collaborative problem solving have constrained the social complexity of the situation by using clinical designs focusing on the interactions of individual dyads, frequently triggered by digitally-delivered problems (e.g., Olive & Steffe, 1990; Steffe & Wiegel, 1994). Other attempts to seek structure in the extreme diversity of such social interactions have included comparative studies, in which aspects of instructional setting, culture and social interactive norms can provide the variation needed to reveal underlying structure or consistency of pattern (e.g., Clarke, 2001a, 2006; Clarke et al., 2006). The first approach (clinical designs) compromises validity in the interest of experimental control. The second (comparative studies) relinquishes control over key variables in an attempt to capture social interaction in naturally occurring settings. Design experiments (e.g., Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) represent one approach to resolving the tension between the need for control in an experimental environment and the freedom for the participants to interact and behave as they would in a naturalistic classroom setting. The balance between validity, experimental control and the accumulation of a substantial body of systematically generated, structured data continues to pose a challenge for research investigating student learning in social settings.

Research Design

The analysis reported in this paper addresses the research question: What are the foci of the students' social interactions during collaborative problem solving in this project? The research was conducted in a laboratory classroom situated within the Melbourne Graduate School of Education at the University of Melbourne, Australia. The classroom is equipped with 10 built-in video cameras and up to 32 audio channels. Intact Year 7 classes were recruited with their usual teacher in order to exploit existing student-student and teacher-student interactive norms. Two classes of Year 7 students (12 to 13 years old; 50 students) provide the focus for this report. Each class participated in a 60-minute session in the laboratory classroom involving three separate problem-solving tasks that required them to produce written solutions.

Problem-Solving Tasks

The problem-solving tasks used in the project were drawn from previous research (e.g., Clarke, 1996; Clarke & Sullivan, 1990, 1992; Sullivan & Clarke, 1988, 1991, 1992). All three tasks had multiple possible solutions, included symbolic or graphical elements, and afforded connection to contexts outside the classroom. Despite having these similar features, the content foci of the three tasks are disconnected to avoid carry-over effects between tasks. Task 1 provided students with a graph with no labels or descriptions (Figure 1) with the following instructions: "What might this be a graph of? Label your graph appropriately. What information is contained in your graph? Write a paragraph to describe your graph."

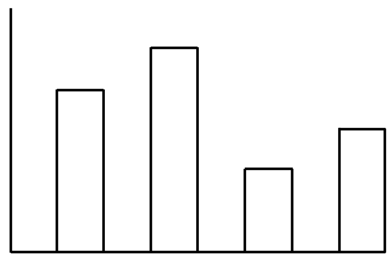

Figure 1. Task 1 stimuli.

Task 2 was specified as follows: "The average age of five people living in a house is 25. One of the five people is a Year 7 student. What are the ages of the other four people and how are the five people in the house related? Write a paragraph explaining your answer."

Task 3 stated that "Fred's apartment has five rooms. The total area is 60 square metres. Draw a plan of Fred's apartment. Label each room, and show the dimensions (length and width) of all rooms."

The students attempted the first task individually (10 minutes), the second task in pairs (15 minutes), and the third task in groups of four to six students (20 minutes). Figure 2 illustrates the grouping for the three tasks for a single group of students.

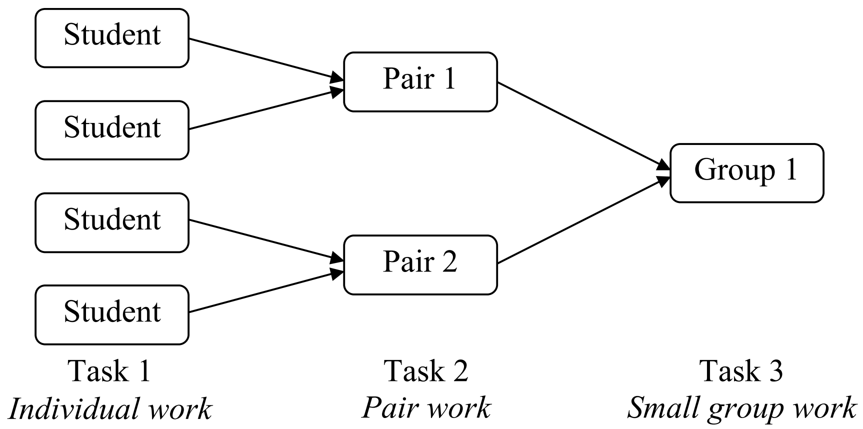

Figure 2. Student grouping for the three tasks.

Findings

As noted above, the analysis reported in this paper concerns the identification of regularities in the negotiative interactions of students engaged in collaborative problem solving. Analysis of the transcripts employed the negotiative event as the unit of analysis (Clarke, 2001b). In this analysis, a negotiative event is defined as “an utterance sequence constituting a social interaction with a single identifiable purpose.” The third task provides the focus of this paper due to the diversity of forms and foci of negotiation evident in the data arising from this group’s interaction during that task.

Analysis of the transcripts of student-student interactions identified three qualitatively distinct foci of interaction, which we have characterised as:

- Mathematical: a concern with mathematical correctness of fact and procedure
- Sociomathematical: a concern with the didactical norms of the classroom
- Social: a concern with social obligations and agency within the group, other than those included in the other two categories.

Below are some illustrative examples of each mode based on the group interaction during Task 3.

Negotiative Event 1 - Mathematical

Pandit: Okay, okay, okay. So, wait, this side’s 20.

Anna: Yeah.

Pandit: This is…

Anna: Wait. Let's just say that's - no, Pandit, it won't work.

Pandit: It does. It does.

Anna: It doesn't. We have to get a 30 there and then look, up to there is 30. Do you have a brain?

Pandit: (Laughs) I have a brain. No. Wait, isn’t that has to times?

Anna: Yeah.

Pandit: Twenty times thirty is like 600.

Anna: Six hundred.

Pandit: It has to be 60.

Anna: Yeah.

Pandit: You did it wrongly. That’s why.

Negotiative Event 2 - Sociomathematical

Anna: Guys, let's actually change the scale.

Pandit: We can't.

Anna: Why not?

Pandit: We're not allowed to change.

Anna: You are. Let's make two centimetre square equals one metre.

Negotiative Event 3 - Social

Pandit: Oh my god. The progress we put in is miserable.

Anna: I don't think they [John and Arman] like you as well.

Pandit: Yeah. I know, right.

Anna: Ha, ha. Both hating.

The first negotiative event involves Anna and Pandit realising that they need to multiply the length and width of the apartment to calculate the total area. Their apartment plan of 20m × 30m had a total area of 600sqm, which was much larger than the 60sqm specified in the task. The focus of the negotiation was on the correctness of the mathematics (Pandit: “You did it wrongly. That’s why.”).

The second negotiative event involves the students deciding on the scale of their apartment plan. Anna wanted to change the scale from 1 cm:1 m to 2 cm:1 m, but Pandit did not agree with her. The focus of the interaction was based on the students’ perceived expectations for task completion (Pandit: “You are not allowed”). The pair was negotiatively revisiting the “rules of engagement” for task completion – that is, the sociomathematical norms prescribing what is and is not a permissible approach to task solution.

The exchange between Pandit and Anna in the third negotiative event about John and Arman’s attitude towards Pandit illustrates an interaction focus that is neither about mathematical correctness nor didactical expectations. Their exchange was about the group dynamics in terms of social relations and connected these social relations to the group’s productivity.

The three negotiative events illustrate three distinct foci for student-student interaction: mathematical, sociomathematical, and social. Negotiation with respect to each of these appears to employ its own lexicon and can be considered as a distinct mode of interaction. Mathematical interactions and the negotiation of mathematical meaning not only invoke mathematics terminology, but employ distinctive logical connectives, invoking notions of truth (e.g., “is” and “wrong”) and also appeal to utility warrants such as “it works”. Sociomathematical negotiations are more likely to be phrased in conditional or relativist terms, such as “might” and involve reference to approval or permission. Negotiations of social matters can involve responsibility and expectation (like sociomathematical considerations) but the invoked authority is likely to appeal to moral obligation rather than didactical convention and also to make more frequent reference to affect. The utilisation of each lexicon within an interactional sequence can be thought of as analogous to the familiar phenomenon of “code-switching” documented in classes of bilingual learners. As students shift from one mode to another, they employ language drawn from the relevant lexicon to express themselves.

The excerpts provided give an impression of relatively clear distinction between the three modes of interaction, but, in practice, all three modes coexist in an entangled form in the negotiative interactions documented in the mathematics classroom. Below is an excerpt that illustrates such entanglement.

John: The area of the … of each room is the same or not same?

Arman: John. Fifteen times four is 60 and 15 times five is 75.

Anna: I don’t know.

Pandit: I don't care about that. No, wait. Wait.

John: We care about that - because maybe some rooms are bigger and some rooms are smaller.

Arman: Fifteen times five is 75 and 15 times four is 60.

John: Huh? What?

Arman: Fifteen times four is 60, but we need five rooms. Five, five.

Anna: Twenty, three. Twenty, no, no, no.

Pandit: You do like 10 times 6 or something like that.

Anna: Okay. Let's just do 10 times 6 then.

Pandit: Yay.

Anna: Yay.

Pandit: Oh my God.

Anna: Oh my God.

Pandit: Yeah.

Anna: Yeah, yeah, yeah…

Each person in the dialogue had a different focus, where John was concerned about the task expectation (sociomathematical) in terms of whether the rooms can be of the same size or not, Arman was concerned about John's calculation of the size of each room of 15 sqm which produced an overall size of 75 sqm instead of 60 sqm as specified in the task (mathematical). Neither of the issues raised by Arman nor John was of concern to Anna and Pandit who were focused on working out the overall size of the apartment (mathematical). Anna's parroting of Pandit's utterances towards the end of the excerpt after they reached agreement could either be expressing collegiality or sarcasm towards Pandit (social).

In relation to the unit of analysis, the excerpt can be interpreted as three overlapping negotiative events: one consisting of the interchange between John and Arman focusing on the size of each room; one between Pandit and Anna focusing on the overall size of the apartment; and one concluding exchange between Pandit and Anna that is basically social in focus. Entanglement exists in both the actuality of the overlapping speech (negotiative events) and also in the shifts in focus between one utterance and another. The term "entanglement" also recognises that each pair (John-Arman and Pandit-Anna) is aware of the actions/statements of the other pair, as indicated by Anna's statement: "I don't care." The potential exists for the convergence of the separate negotiations into a further negotiative event involving all four students, but this does not occur within the brief transcribed sequence.

At this point, it is also appropriate to ask, "In what way does access to the video record (as distinct from the transcripts) provide additional insight into the student-student interactions?" From the excerpts above, there is evidence of discord between Anna and Pandit, while John was trying to become involved in the girl's discussion and Arman was shifting between being involved in the task and carrying on an independent conversation. However, it was striking when viewing the video recording, that the interactions between the students appear far less confrontational than might appear from reading only the transcribed verbal interactions. There was frequent laughter between the students and the interactions between Anna and Pandit, and other interactions within the group appear to have a good-humoured character, when one takes into consideration the students' body language, facial expression, and tone of voice. The video record also provides important information concerning whether or not interacting pairs of students were aware of each other's actions/statements and whether a silent student was a "silent participant" in a particular negotiation (Remedios, Clarke, & Hawthorne, 2008) or disconnected from that exchange. In addition, the connection between student utterances and physical actions, not only gestures, but also writing and the drawing of diagrams, can be made with much greater confidence with the support of the video record, with obvious benefits for transcript interpretation.

Discussion and Conclusions

As an entry point to the investigation of the nature of the social interactions evident in the data generated in the Social Unit of Learning project, this paper addressed the research question: What are the foci of the students' social interactions during collaborative problem solving in this project?

The transcript analysis suggests that meaning negotiation in mathematics classrooms can be usefully distinguished as a focus on either social, sociomathematical, or mathematical concerns. As was noted earlier, negotiation with respect to each of these foci appears to employ its own lexicon and can be considered as a distinct mode of interaction. However, to interpret the three modes as a stratification of social process (e.g., in terms of scale (grain size) or a logical or socially-normed interactive sequence) suggests a separation and a hierarchy that is not evident in practice. All three modes coexist in an entangled form in the negotiative interactions documented in the mathematics classroom. Future analysis will involve delineating each of the three lexicons and investigating the nature and consequences of their interaction.

The capability to document classroom interactions continuously and simultaneously in fine-grained detail using multiple cameras and microphones represents a technological advance in classroom research. Such an advance offers possibilities for the structured, rigorous, fine-grained investigation of the complex processes involved in learning in various social settings. This includes the simultaneous documentation of student production of written solutions to problems undertaken collaboratively and the recording of the social negotiative exchanges of which those written solutions are one outcome. A consequence of the type of tasks employed in this study is that student written solutions, supplemented by transcript and video data, are amenable to quite detailed classification indicative of mathematical decisions and associated reasoning in which the students engaged. Such an analysis was not the purpose of this paper, but the connection between multiple data types is a major affordance of the particular research facility.

The distinction between the different negotiative foci (mathematical, sociomathematical, and social) makes visible the dynamics of collaborative problem solving. The entanglement of these three foci within the negotiative dialogue between students is highly significant if we are to understand the dynamics of collaborative problem solving and associated learning. This project envisages all three interactive modes as constitutive of learning and as providing distinct entry points for teacher instructional intervention (or scaffolding). We suggest that each negotiative focus must be accommodated in a social theory of learning and each represents one avenue to improved learning outcomes in our mathematics classrooms. All three must be studied in situ and in relation to each other as they occur in authentic classroom activity. Such postulated interconnectedness poses challenges for theory and for research. This paper offers a starting point for further theorisation and investigation.

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