# The Interplay Between Pre-service Teachers' Intentions and Enacted Mathematical Content Knowledge in the Classroom

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Pre-service teachers (PSTs), like practising teachers, enact their mathematical content knowledge (MCK) in pursuit of instructional goals during lessons. In this study, I explored the relationship between six secondary mathematics pre-service teachers' goals and the MCK that they chose to enact in 10 lower secondary algebra lessons. The findings indicate that PSTs enact stronger aspects of their MCK when they pursue goals that pertain to making mathematical connections rather than procedural mastery. Also, live classroom interactions with confused students can positively impact the instructional goals that pre-service teachers form and the quality of MCK that they enact.

Mathematical content knowledge (MCK) is recognised as a crucial type of teacher knowledge held by effective mathematics teachers. Pre-service teachers (PSTs) begin to develop their MCK specifically for the work of teaching while undertaking their university studies but improvements are needed in the way that this development occurs. A study of over 20 mathematics teacher education programs (including Australia) by Tatto, Lerman, and Novotna (2010) indicated that insufficient attention is paid to equipping PSTs with the MCK needed to provide high quality mathematics instruction. Mathematics education stakeholders in Australia, such as Professor Ian Chubb, past Chief Scientist of Australia (Chubb, Findlay, Du, Burmester, & Kusa, 2012), echo the concerns raised by Tatto et al. (2010). The creation of evidence-based mathematics teacher education programs, which Beswick and Goos (2012) argue are needed in Australia, require teacher educators to possess accurate gauges of PSTs' MCK and, just as importantly, their willingness to enact certain aspects of that knowledge in front of a classroom of students. To contribute to this area of research, this study investigated the link between the MCK that six secondary PSTs shared with their students during an algebra lesson and the goals behind those actions.

## Background to the Study

Specialised subject knowledge was highlighted three decades ago by Shulman (1987) as one of seven knowledge types necessary for teaching. Research undertaken more recently in the field of mathematics education reinforces MCK as one of the foundational knowledge types needed for teaching. In a German study of secondary practising teachers (Baumert et al., 2010), pedagogical content knowledge (PCK) was found to be the better predictor of student results. However, the researchers noted that PCK cannot be developed adequately without strong MCK (Baumert et al., 2010), highlighting the need for studies such as this one to focus on PST MCK. This paper focuses more specifically on the MCK of algebra that secondary PSTs hold and choose to enact in a live lesson.

Scholars have further developed Shulman's notion of content knowledge needed specifically for mathematics teaching. For example, the regularly cited Mathematical Knowledge for Teaching (MKfT) framework developed by Ball and colleagues (Ball & Bass, 2009; Ball, Thames, & Phelps, 2008) emphasises the need for mathematics teachers to not simply know more mathematics content than students but to hold that knowledge in a qualitatively unique form that is well connected and unpacked. Within the topic of

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algebra, studies either have explored multiple knowledge types for teaching algebra (McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012) or have focused on a particular type of algebraic knowledge that teachers should enact (Driscoll, 1999).

Guided by the literature, this study explored three types of PST MCK of algebra, namely conceptual knowledge, procedural knowledge, and algebraic ways of thinking. Conceptual knowledge and procedural knowledge are two vital knowledge types popularised by Skemp (1976) and a decade later, Hiebert and Le Fevre (1986). Researchers such as Star (2005) have continued to develop these two knowledge types and it is widely accepted that teachers should ideally be enacting both knowledge types regularly in their teaching. Algebraic ways of thinking (AWOTS) are specific mathematical habits that form the third knowledge type investigated in this study. Here, three AWOTS noted by Driscoll (1999) and Harel (2008) that are of particular relevance to lower secondary algebra are addressed. The first is "manipulating with purpose", which highlights how one manipulates symbols in algebra to achieve a particular mathematical purpose (Harel, 2008). Harel (2008) also identifies "algebraic invariance" as an important AWOT which recognises that in algebra, a purposeful manipulation can include changing one aspect of a mathematical object (such as the form of an expression) while holding a second aspect unchanged (such as the value of the expression). The third AWOT, "doing/undoing", reflects one's ability to understand a process so well in algebra that it can be reversed (Driscoll, 1999).

In this study, putting one's MCK into practice is referred to as enacting that MCK. A review of mathematics education literature revealed that measures of PST MCK have usually occurred in contexts where the participants enacted their MCK outside the classroom, in the form of written tests and interviews (Ball, 1990; Even, 1993). These measures give a strong indication of the MCK that PSTs may take into the classroom but they do not capture which aspects of that MCK PSTs may heavily emphasise in their teaching or perhaps which aspects they may avoid enacting. Lave and Wenger (1991) argue that knowledge is dynamic and located within a particular community of practice. In the mathematics classroom, teachers need to be able to use their MCK "in the ebb and flow of practice, responding quickly and accurately to student thinking" (McCrory et al, 2012, p. 601). There have been limited studies that have investigated PSTs' MCK in a live classroom setting (e.g., Borko & Livingston, 1989; Thwaites, Jared, & Rowland, 2011) but as these studies did not focus solely on MCK enactment, the MCK was not systematically reported as it is in this study.

There are limitations in studying only the visible aspects of enacted MCK within a live lesson because the thoughts behind particular actions cannot be observed. Beswick and Goos (2012) argue that research is needed that provides detailed examinations of both the mathematical knowledge and thoughts that underpin teaching actions of PSTs. To capture a more detailed slice of PSTs' MCK in action in this study, the goals that lay behind PSTs' MCK related actions were also chosen for investigation. Teachers perform actions during their lessons in pursuit of a particular goal or goals, according to researchers of mathematics teacher decision making (e.g., Schoenfeld, 2010; Simon, 1995). The PSTs' choice of goal, a critical aspect of the thinking that precedes MCK related actions, can provide additional insights about the MCK that they subsequently enact. Therefore, this study contributes to the limited research undertaken of enacted MCK of algebra and related thinking with the following research questions:

1. What MCK do secondary mathematics PSTs enact in lower secondary algebra lessons?
2. How do PSTs' instructional goals impact the MCK they decide to enact?

## Methodology

The results reported form part of a doctoral study (Daniel, 2015) that investigated six secondary mathematics PSTs' MCK related teaching actions and instructional goals. The PSTs comprised two females and four males aged in their early twenties, in their third or fourth year of study in a Bachelor of Education degree at a regional Queensland University. Ten secondary mathematics PSTs were originally invited by the researcher, who was also their lecturer for a mathematics methods course, to participate in the study. For ethical reasons, the researcher was not made aware of which PSTs had agreed to participate until the completion of the course. All the invited participants agreed to participate in the study but the findings presented in this paper, using aliases to protect the PSTs' identities, pertain only to 6 of the 10 participants who provided data relating to lower secondary algebra lessons during a practicum phase. The lessons focused on manipulating algebraic expressions and solving simple linear equations (six Year 8 lessons) and solving sets of simultaneous linear equations (four Year 10 lessons).

Qualitative data collection techniques were used to capture insights about the PSTs' MCK related thoughts and teaching actions in a live classroom setting. To investigate the MCK that manifested in PSTs' live teaching actions, lesson data were collected from the PSTs' practicum lessons via lesson observations, digital video footage, and lesson artefacts, such as lesson plans. To gather data related to the PSTs' goals, each PST participated in a stimulated recall interview within 48 hours of each lesson observation. Lesson excerpts from the video footage that featured sets of MCK related actions were chosen by the researcher prior to the interview. The excerpts were retained for three reasons: (a) they typified the MCK that the PST enacted throughout the lesson, (b) they featured surprising MCK related actions in the opinion of the researcher, an experienced secondary mathematics teacher, and/or (c) they featured teaching actions where the researcher sensed a PST may provide a particularly rich or insightful reflection. During the interviews, the reduced lesson footage was played to each of the participants who were encouraged to pause the footage and reflect on their thinking at the time. The researcher also paused the footage at particular times and prompted the participants with questions such as, "What were you thinking here?" during the interview. The interviews were videotaped with the video camera focused on a laptop screen containing the lesson excerpts so that the on-screen footage and the participants' retrospective thoughts could be recorded simultaneously.

After the lesson and interview data were collected, the lesson data were reduced further by keeping only those excerpts of lesson footage for which the participants provided commentary during the interview. If a PST did not comment on a set of teaching actions, which happened quite rarely, those actions were discarded for further analysis because it was not possible to discern any connections between the participants' MCK related thoughts and actions. The remaining lesson and interview data were then analysed for evidence of enacted MCK and for any goals identified by the PSTs.

The goals shared by the participants during their interviews were used to partition the lesson data into units of analysis called "episodes". In this study, an episode is defined as a set of one or more MCK related teaching actions that a PST undertakes in pursuit of one or more instructional goals. The researcher noted that during the stimulated recall interviews, the participants either explicitly stated or strongly implied one or more instructional goals as they reflected on their actions in the footage. Across the ten algebra lessons, the researcher discerned 174 goals from the participants' commentary. Those 174 goals,

referred to as episode goals, were used to demarcate the footage upon which the participants commented into 137 episodes, underpinned by either one or two episode goals.

The episode data were analysed for the type of MCK enacted by the participants. Literature pertaining to the MCK needed for teaching algebra (e.g., Ball & Bass, 2009; Ball et al., 2008; Harel, 2008; Hiebert & Le Fevre, 1986; McCrory et al., 2012; Skemp, 1976; Star, 2005) was used to develop an initial coding framework to analyse the MCK manifesting in the episodes but the researcher reconfigured several categories and subcategories as patterns began to emerge in the data. The interview data associated with each episode were inductively coded for the instructional goal(s) that the PSTs were pursuing when they enacted certain MCK and then categorised using pattern seeking techniques. It was also noted whether the PSTs' goals appeared to have been formed prior to the lesson or if they were the result of live classroom interactions prompted by an event such as a student question. Finally, cross-variable analyses were undertaken to determine trends in the presence of particular goal types and different types of enacted MCK.

## Results

### *Procedural Knowledge Overshadows Other Types of Enacted MCK*

The participants explicitly shared three types of MCK of algebra when teaching, namely algebraic ways of thinking, conceptual knowledge, and procedural knowledge. However, the types of MCK were not enacted in similar proportions because procedural knowledge heavily dominated the teacher talk and written work in the lessons. Figure 1 provides an overview of the MCK that the participants enacted in isolation or in combination in the 137 episodes examined.

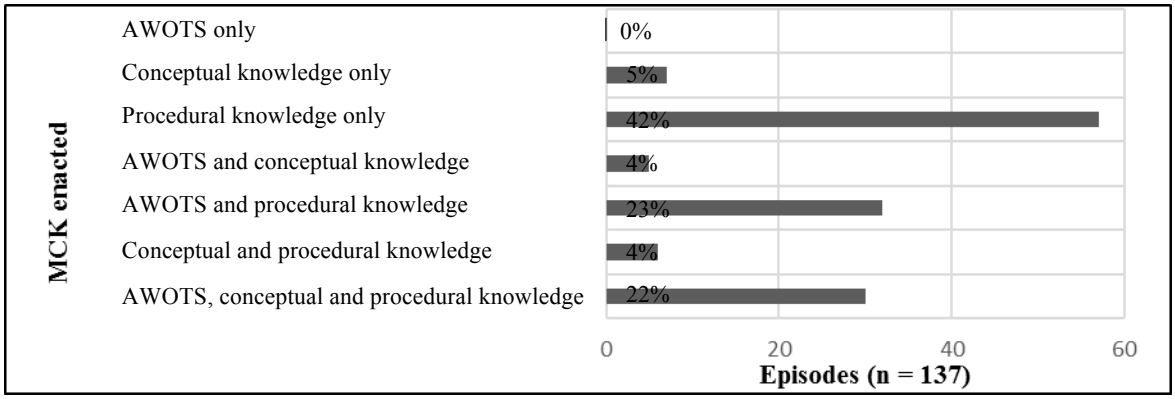


*Figure 1*. Presence of MCK types in teaching episodes.

The heavy emphasis by the participants on teaching procedures is reflected in Figure 1, which shows that 91% of all episodes ($n$ = 125) evidenced procedural knowledge. The most common manifestations involved the PSTs explaining or questioning students about performing procedural steps. Given the high proportion of episodes with only procedural knowledge (42% of all episodes) and the view that substantial conceptual knowledge and AWOTS enhance the teaching of procedures (e.g., Harel, 2008; Skemp, 1976), the PSTs did not enact conceptual knowledge or AWOTS often enough in their algebra lessons.

When PSTs chose to enact AWOTS, they only did so in combination with another type of MCK which was usually procedural knowledge. The addition of AWOTS in the MCK

that the participants enacted tended to support their explanation of procedural steps. One participant, Ben, for example, repeatedly enacted the "manipulating with purpose" way of thinking, which the PSTs enacted more often than any other way of thinking (52% of the 67 episodes featuring AWOTS). In one episode, Ben commented to his students, "What do we want to do to the $x$? Isolate it. So, you wanna get it on its own." Overall, the AWOTS appeared to be explicitly taught by the participants as a supportive mechanism to assist students to develop procedural mastery, extending the earlier finding that not only were AWOTS not enacted often enough but additionally, not deeply enough by the PSTs.

Conceptual knowledge was the type of MCK that the PSTs enacted the least, featuring in only 35% of all episodes. Unfortunately, across the 10 lessons, no participant spent a significant amount of time specifically addressing a mathematical concept. Rarely enacted in isolation, conceptual knowledge tended to be presented alongside either AWOTS, procedural knowledge, or both knowledge types. The conceptual knowledge that the PSTs explicitly taught consisted of their knowledge of arithmetic operations and of algebraic objects. The participants spoke of the different representations of arithmetic operations (e.g., words and symbols) and conceptual features of algebraic objects. One participant, William, highlighted the concept of equivalence when he spoke explicitly about the meaning of the equals symbol in an equation. He explained to his students as he gestured to the expressions on each side of the equation, a + 6 = 13, written on the board, "It means it's like the same on both sides. So, we're saying that this half is the same as this half."

Although the presence of conceptual knowledge in episodes strengthened the mathematical quality of those episodes, all too often conceptual knowledge featured in brief supporting statements that were interspersed amongst longer procedural explanations in the same way that AWOTS were enacted. This resulted in few instances where strong, explicit connections were made between algebraic procedures and related concepts and ways of thinking. An analysis of the participants' instructional goals that lay behind these stronger and weaker MCK episodes revealed important connections between certain goal types and the subsequent MCK enacted. These connections are now described.

### *Different Goal Types Lead PSTs to Enact Stronger and Weaker Aspects of Their MCK*

The PSTs referred to 174 episode goals when they commented on the MCK that they chose to enact in the episodes. Those goals were inductively sorted into nine episode goal types, shown in Table 1, then categorised into two major goal types, namely content focused goals and student focused goals. Content focused goals, comprising the first five of the nine goal types in Table 1, were goals where the participants were intent on teaching particular mathematical content, regardless of their students' understandings, such as a goal to explicitly teach a certain algebraic procedure during a lesson. Student focused goals housed the final four goal types in Table 1 and reflected the participants' desire to present content that aligned with their students' mathematical knowledge, such as a goal to address a particular point of confusion for a student. Table 1 also shows the frequency with which the participants enacted each type of MCK when they pursued each type of goal in an episode. Due to the pervasive nature of procedural knowledge enacted in the majority of episodes, goals referring to procedural knowledge episodes within the table pertain only to those episodes where procedural knowledge was enacted in isolation within an episode so that trends between PST goal types and enacted procedural knowledge can be revealed.

The table reveals that PSTs tend to enact only procedural knowledge in pursuit of the content focused goal types "Develop students' knowledge of procedures" (58% of episodes

with this goal type) and "Associate procedure with certain types of solutions" (60% of episodes with this goal type).

Table 1
*Type and Relative Frequency of Episode Goals for Episodes with AWOTS, Conceptual Knowledge and Only Procedural Knowledge*

| Type of episode goal | Total goals for type | Goals for AWOTS (% of total goals by type) | Goals for conceptual knowledge (% of total goals by type) | Goals for procedural knowledge only (% of total goals by type) |
|---|---|---|---|---|
| Develop students' knowledge of procedures | 60 | 22 (37%) | 8 (13%) | 35 (58%) |
| Teach students appropriate use of mathematical language | 16 | 10 (63%) | 13 (81%) | 1 (6%) |
| Connect procedure with a concept | 12 | 8 (67%) | 8 (67%) | 2 (17%) |
| Associate procedure with certain types of solutions | 10 | 2 (20%) | 3 (30%) | 6 (60%) |
| Connect procedure with mathematical purpose | 9 | 9 (100%) | 5 (56%) | 0 (0%) |
| Address student confusion | 32 | 23 (72%) | 18 (56%) | 7 (22%) |
| Value and/or encourage student contribution | 12 | 4 (33%) | 3 (25%) | 8 (50%) |
| Gauge student knowledge | 12 | 4 (33%) | 3 (25%) | 6 (50%) |
| Avoid student confusion | 11 | 6 (55%) | 5 (45%) | 4 (36%) |
| Total | 174 | 88 (51%) | 66 (38%) | 69 (40%) |

Additionally, when the PSTs aimed to align their teaching with their students' understandings by aiming to "Value and/or encourage student contribution" or "Gauge student knowledge", they again enacted mainly procedural knowledge. These findings suggest that when PSTs focus on teaching their students algebraic procedures in particular or attempt to ascertain their students' knowledge regarding algebraic procedures, their attention is not necessarily drawn concurrently to the concepts and ways of thinking that support those procedures and their teaching remains mathematically superficial as a result.

More encouraging findings revealed in Table 1 pertain to the goals that led the PSTs to enact stronger aspects of their MCK. In this study, stronger MCK refers to the PSTs sharing conceptual knowledge and/or AWOTS in addition to procedural knowledge. Table 1 reveals that two content focused goals that focus on making mathematical connections explicit appear to be constructive ones for PSTs to form. PSTs enacted conceptual knowledge and/or AWOTS in the majority of episodes underpinned by the goals, "Connect procedure with a concept" and "Connect procedure with mathematical purpose". The desire to make a mathematical connection explicit appears to be one that PSTs should be

encouraged to prioritise, given the relatively high proportion of episodes with conceptual knowledge and AWOTS that followed. This finding has implications for teacher educators.

A particularly interesting result was the PSTs' enactment of stronger MCK episodes when they attempted to address student misconceptions. Table 1 shows that in the majority of episodes with the goal, "Address student confusion", PSTs enacted AWOTS (72% of episodes with this goal) and/or conceptual knowledge (56% of episodes with this goal). It was noted during the data analysis phase that the majority of the "Address student confusion" episodes occurred spontaneously, prompted by a student error or question that manifested during the lesson. It is therefore quite surprising that some of the PSTs' strongest MCK episodes occurred when they taught unplanned episodes with only moments to form goals. A possible reason for this phenomenon may lie in the PSTs' perspectives of what MCK they believe they need to share with their students. During unplanned episodes, the PSTs based their MCK related decisions, in part, on their interactions with their students. Their own MCK related teaching actions in one episode prompted their students to provide verbal and written contributions in response to those actions. The student contributions, in turn, informed the PSTs' choice of MCK in subsequent episodes, reflecting the cyclic nature of instructional decisions, teaching actions, and live classroom interactions as espoused by Schoenfeld (2010) and Simon (1995) and as illustrated with the following example.

One participant, Sam, presented a careful explanation about how to solve an equation using the backtracking method after having used the balance method in a previous lesson. Sam's explanation made no explicit mention of any AWOTS and, of particular note was his failure to share the purpose of performing the procedure (the "manipulating with purpose" AWOT). After Sam completed his explanation, he was confronted with one student's question, "Is that how we get the answer, Sir?" Sam then offered an explanation, rich in AWOTS, about choosing different methods to solve equations as he confirmed his student's hunch. At this point, the "manipulating with purpose" AWOT was no longer hidden but was highlighted by Sam as a key feature of the procedure. When reflecting on the episode in his interview, Sam laughed at the footage, commenting, "Well I thought, 'Why, why, why was I doing it all this time if not to get to an answer!'". Sam's surprise at having to spell out the point of the procedure was echoed by three other participants during their interviews, suggesting that live exchanges with students can direct PSTs to share aspects of their MCK that might otherwise remain hidden from their students.

## Discussion and Conclusion

Existing research suggests that consistently stronger MCK than what the PSTs enacted is needed for more effective teaching. Hence, the findings reinforce calls in Australia for PSTs' MCK to be further developed during their tertiary studies; however, development of only MCK is not enough. The findings suggest that secondary PSTs' MCK related goals do not always lead them to enact the strongest aspects of their MCK of lower secondary algebra. Teacher educators must pay attention to instructional goals that lead PSTs to share weaker and stronger aspects of their MCK in mathematics methods courses and practicums. Those goals can be used by teacher educators as a valuable lens for reflection on the quality of MCK that PSTs enact. In this way, the development of PSTs' MCK related decisions and actions might be developed in a more intentional manner.

This study was limited to one contextual setting, one mathematics topic, and only a small number of participants. The findings are also limited by the loss of data that occurred when participants' enacted MCK that could not be connected to an instructional goal was

discarded. The findings do offer, however, a rich slice of both the visible and less visible aspects of secondary PSTs' live teaching practice. The study highlights the methodological benefits of studying the MCK that PSTs enact in live teaching actions alongside the goals that lie behind their MCK-related actions.

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