Hypothesis of Developmental Dyscalculia and Down Syndrome: Implications for Mathematics Education

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In this paper, the hypothesis that Developmental Dyscalculia (DD) is a characteristic of Down syndrome (DS) is proposed. Implications for the hypothesis are addressed: If it were to be confirmed that DS implies DD, what would be the consequences for the mathematics education of learners with DS? The use of prosthetic devices to overcome the impaired calculation capabilities of the brain is essential. Fortunately, electronic calculating devices are readily available. Their routine use opens the possibility of studying areas of mathematics that were once inaccessible.

Introduction

Despite improved teaching, family circumstances and higher expectations, learners with Down syndrome (DS) continue to experience severe challenges learning arithmetic (Faragher & Clarke, 2014). These learning difficulties are disproportionate to accomplishments in other areas of the learner's life and persist despite being functionally necessary, of interest to the learner, and with considerable opportunities for regular practice – just the environmental factors usually advocated for effective learning. In addition, these difficulties have been observed around the world and across decades, reducing the likelihood that teaching methods or some other environmental factor is the cause.

Studies of the neurological basis of the development of early number and arithmetical skills (such as, Butterworth, Varma, & Laurillard, 2011; Dehaene, 2011; Dinkel, Willmes, Krinzinger, Konrad, & Koten, 2013; Shalev & Gross-Tsur, 2001) have led to an understanding of processes in typical brain function, and a growing awareness of atypical development. Discovery and description of a specific learning disability named Developmental Dyscalculia (DD) is one outcome of this research effort.

In this paper, the hypothesis that DD is a characteristic of DS is proposed. In raising this hypothesis, current understandings of DD are presented and the symptoms of DD, are described. The research on the development of related areas of mathematics by learners with DS is then summarised before the evidence for the hypothesis of a co-morbidity of DD with DS is discussed. Implications for the hypothesis that DS implies DD follow, focussing on the consequences for the mathematics education of learners with DS.

Background

Before considering disabilities with learning and using mathematics, and how that might affect people with Down syndrome, typical mathematics cognition as scientists have so far revealed is discussed.

Human Perception of Number

Humans have a remarkable capacity to make sense of the world through the use numbers. How our brains do this has been the subject of a great deal of recent research. For

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a review of the field and to note the development in the first decade of this century, the reader is referred to the seminal work of Dehaene (1999, 2011).

> Animal research on number pointed to an age-old competence for processing approximate quantities. This "number sense", which is also present in infants, gave humans the intuition of number. Cultural inventions, such as the abacus or Arabic numerals, then transformed it into our fully-fledged capacity for symbolic mathematics. (Dehaene, 2011, p. x)

Although details are still being refined, research undertaken by psychologists and neurobiologists have revealed a model of human cognition with two systems that are now widely accepted. In Figure 1, a diagrammatic representation has been developed as a synthesis of a number of research descriptions (especially Dehaene, 2011; Lanfranchi, Berteletti, Torrisi, Vianello, & Zorzi, 2015).

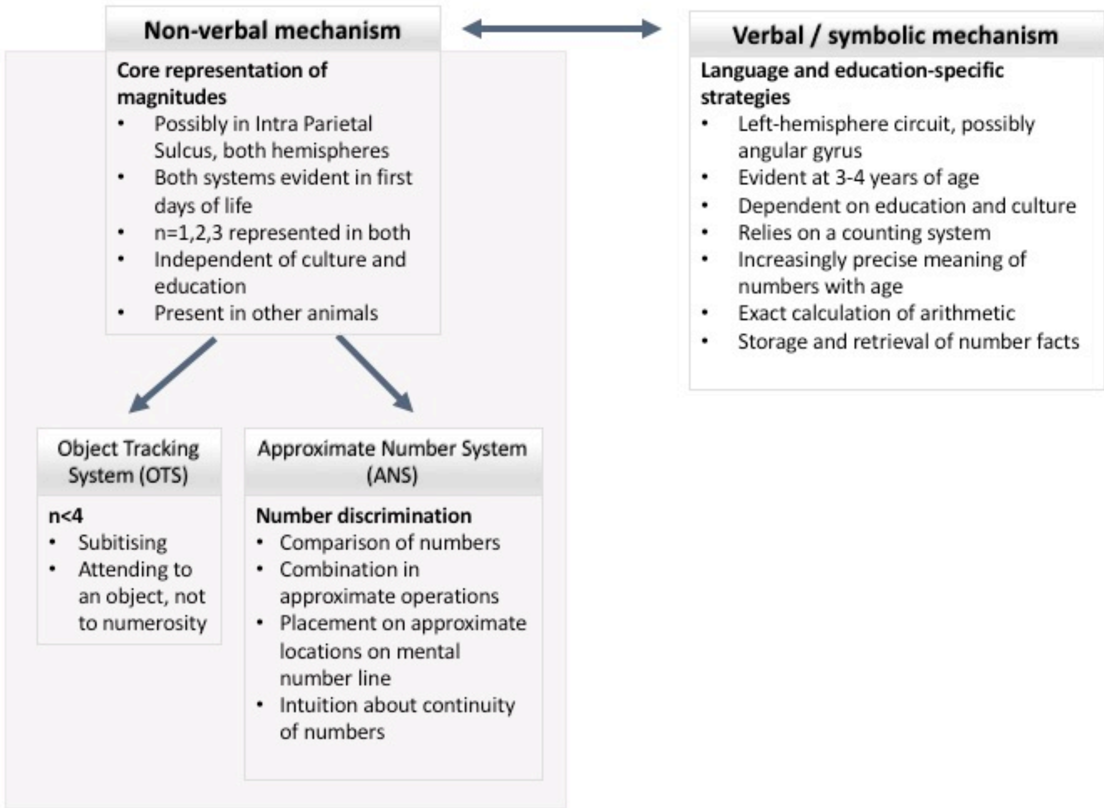

Figure 1. Model of the human mathematics processing systems.

There are two mechanisms – non-verbal and verbal. The non-verbal mechanism has commonly been called 'pre-verbal' however, it continues to function after the development of the verbal mechanism and throughout life, therefore 'non-verbal' is a more apt description. Within the non-verbal mechanism, there are now considered to be two systems – an object tracking system (OTS) and an approximate number system (ANS). These two systems are important in the discussion of mathematics cognition for learners with DS, as it would appear that these may not be completely intact. The OTS is a system that allows tracking of up to four objects in space, without attaching ideas of quantity to them (Xu, Spelke, & Goddard, 2005). The ANS allows approximate discrimination of quantities, including location on a mental number line, with a sense of continuity of numbers.

Comparison and combination of quantities can be undertaken, though not by exact calculation.

Humans share both the OTS and ANS with many other animals. It is the verbal mechanism that humans alone exhibit. At three to four years of age, humans link their quantitative knowledge to language – number words or symbols, building an increasingly precise number meaning, leading on to exact arithmetic and further mathematics (Dehaene, 1999).

Butterworth, Varma, and Laurillard (2011) summarise current evidence from brain imaging research concluding "almost all arithmetical and numerical processes implicate the parietal lobes, especially the IPS [intraparietal sulcus], suggesting that these are at the core of mathematical capacities" (p. 1050). Therefore, the area of the brain implicated in the non-verbal mechanism is the IPS, in both hemispheres. For the verbal mechanism, a major area of the brain is likely to be the pre-frontal cortex or the angular gyrus. These are parts of the brain that are used by the working memory and are engaged when non-routine, automatic thought processes are in action (Dehaene, 2011).

In this section, the model that expresses the current understanding of human number cognition has been presented. This serves as background to consider the system when it does not operate as expected – when calculation is extraordinarily difficult.

Definition and Presentation of Developmental Dyscalculia

It is not a new phenomenon for there to be learners in a primary class that experience significant challenges in learning arithmetic, while having no problems learning other areas of the curriculum. Developmental dyscalculia was coined as a term to describe this condition and was defined several decades ago by Kosc (1974):

> Developmental dyscalculia is a structural disorder of mathematical abilities which has its origin in a genetic or congenital disorder of those parts of the brain that are the direct anatomico-physiological substrate of the maturation of the mathematical abilities adequate to age, without a simultaneous disorder of general mental functions. (p. 47)

More recently, Kucian and von Aster (2015) defined DD as "a specific learning disability affecting the development of arithmetic skills" (p. 2) and "a heterogeneous disorder resulting from individual deficits in numerical or arithmetic functioning at behavioural, cognitive/neuropsychological and neuronal levels" (p. 4). DD is described in the Diagnostic and Statistical Manual of Mental Disorders V (American Psychiatric Association, 2013) as a specific learning disorder that cannot be explained by inadequate learning opportunities, and is "characterized by problems processing numerical information, learning arithmetic facts, and performing accurate or fluent calculations" (p. 67). There is a high incidence of co-morbidities with DD such as ADHD. Some researchers (such as Kucian & von Aster, 2015) suggest this could be due to the broad functional and structural differences across the brain observed in individuals with DD. Down syndrome may be another disability paired with DD.

Consensus is emerging from neuroimaging and other behavioural evidence that DD has a basis in neurological impairment (Price & Ansari, 2013). Early evidence for highly specialised areas of the brain performing various aspects of quantity, calculation and other mathematics came from observations of people who had experienced brain injuries (Kosc, 1974).

Diagnosing DD

Growing evidence of the neurobiological basis of DD would suggest diagnosis could be made using neuroimaging techniques. Dinkel and colleagues (2013) describe using functional magnetic resonance imaging (fMRI) to diagnose DD however, research using this technique is in its infancy. Some fMRI studies of people with DS have been undertaken (for a review, see Key & Thornton-Wells, 2012) however, it would appear no studies of calculation have occurred to this point.

In the absence of reliable imaging techniques, diagnosis of DD has been made on the basis of clinical assessments of arithmetic skills (Shalev & Gross-Tsur, 2001). Timed tests are commonly used because the answer alone will not provide the evidence of DD; correct answers may eventually be found but by very laborious or inefficient strategies. For example, in determining the bigger of two sets of objects, people with DD would count both sets of objects to compare rather than being able to know at a glance (subitise). Diagnosis of DD is considered when the "performance in arithmetic is significantly lower than expected for the child's aptitude" (Shalev & Gross-Tsur, 2001, p. 340).

Arithmetical Development of Learners with DS

The mathematical development of learners with DS remains an emerging field of research. Most studies in the area have investigated basic arithmetic, the area of interest in a discussion of DD, and these studies indicate considerable difficulties (see Faragher & Clarke, 2014, for a review of the literature on attainment). Of significance, the areas of difficulty in arithmetic almost completely match the areas of impairment in DD where the evidence exists.

Perhaps as a result of the pervasive view of mathematics as hierarchical, with attainment of basic arithmetic considered a pre-requisite for any further study, research on the mathematical development of learners with DS into areas beyond calculation is rare. Some research has emerged, reporting success in areas such as algebra and coordinate geometry (see, for example, Faragher, 2014; Monari Martinez & Benedetti, 2011; Monari Martinez & Pellegrini, 2010).

Research literature on the mathematics attainment of learners with DS indicates universal difficulties with basic arithmetic. Studies report a range of achievement. However, in none is the achievement on par with matched participants without DS (Faragher & Clarke, 2014). In addition, studies (e.g., Lanfranchi et al., 2015) indicate that arithmetic is a specific difficulty, over and above other difficulties, such as language.

Hypothesis

The hypothesis proposed here is that people with DS experience DD. While occurring in 3-6% of the general population (Rotzer et al., 2009), in the subset of DS, the hypothesis is that DD is comorbid and likely to affect the majority.

Evidence

DS is a chromosomal pattern marked by triplication of some or all of chromosome 21. While it is known that DS affects the brain in a number of areas such as decreased brain volume, including in frontal and occipital lobes and different brain activation patterns (Key & Thornton-Wells, 2012), this remains an area of research. With the invention of less invasive analysis techniques, such as fMRI, there is potential for advances in understanding the neurobiology of people with DS.

At this time, there is no direct evidence from brain imaging studies of DD in DS. Indirect evidence, however, abounds. In recent years, research studies from the field of psychology have begun to explore mathematical cognition in Down syndrome. These fine-grained studies are beginning to shed light on the sub-skills involved in representing quantity and give clear indication of aspects of the non-verbal mechanism that are not operating as they should – that is, there is evidence of DD.

Belacchi and colleagues (2014) studied approximate additions and working memory. The model of number cognition would suggest approximate additions would be part of the ANS and therefore undertaken by the IPS. Working memory is known to be part of the verbal mechanism and to use the frontal cortex of the brain. Their study observed impairment of the ANS with significant impairment of numerosity estimation involving one set. When participants were able to make use of working memory resources (the verbal system), they were successful with estimating the numerosity of additions.

Numerical estimation has been studied by another research team. Lanfranchi and colleagues (2015) researched the ability of people with DS and two groups matched either by mental or chronological age to estimate the location of numbers on number lines. This would be a feature of the ANS. Their results suggest that this aspect is within findings expected for developmental stage of the participants. The paper also reported findings from measures of numerical intelligence and arithmetic knowledge, including statistically significant poorer performance on non-verbal calculations, in which participants had to add or subtract one or more dots from a given set. The operands were in the single-digit range. If four or less, this would involve the OTS. Greater than four would imply the ANS was activated. Their results indicate impairment of the non-verbal mechanism, a marker of DD.

The research team had previously reported findings from related data (Sella, Lanfranchi, & Zorzi, 2013), studying enumeration skills. They note evidence for a specific deficit in the OTS for individuals with DS, which would again suggest DD. Number acuity (the ability to distinguish the larger of two numbers) and the understanding of cardinality was in keeping with mental age.

The work of this research team noted that the groups matched on mental age were very much younger (DS mean age was 14 years; MA matched mean age was five years). The superior lexical performance of individuals with DS was suggested to be due to their longer experience with number words and symbols. This finding would appear to be an indication of the importance and value of education. Dehaene (2011) emphasises the critical role of teaching the cultural tools necessary for moving from the non-verbal to the verbal mechanism which are essential for exact arithmetic. A number of studies have indicated that children with DS can make use of the verbal system (Lanfranchi et al., 2015; Nye, Fluck, & Buckley, 2001; Sella et al., 2013), which employs other parts of the brain than the IPS.

The challenge in this hypothesis is that scientific understandings of DD are still emerging and there continues to be definitional confusion about the condition, its causes, symptoms and impact on learners (Rubinsten & Henik, 2009). A further challenge relates to diagnosis of DD in learners with DS. Timed arithmetic tests are problematic as the results can be confounded by difficulties with completing the tests – understanding what is being asked, recording responses etc. – all of which take time. Observational tests or task-based interviews (Clarke & Faragher, 2014), may provide evidence of the use of inefficient strategies, such as counting small sets rather than subitising. Key here would be the discrepancy between arithmetical achievement and the learner's general achievement profile.

Discussion

Much can be done without linguistic labels (number count words, for example). The major concepts of arithmetic can be accomplished: quantity; comparison; approximate operations. These are the within the non-verbal mechanism. The linguistic labels help, and indeed are necessary, to move beyond the non-verbal system.

Implications of the Hypothesis for Education

Researchers in DD have considered implications of the diagnosis for learners and some propose cognitive training to attempt rewiring the brain. Many also assume that calculation and number sense are essential pre-requisites for further mathematics study. These positions are problematic for learners with DS (and perhaps other learners as well).

Some evidence for the efficacy of cognitive training in ameliorating the symptoms of DD is emerging (Butterworth et al., 2011; Kucian & von Aster, 2015), though intense effort is needed to achieve this improvement. If DD is a person's only cognitive impairment, the learner may well be sufficiently motivated and the potential gains significant enough to justify the effort. For learners with DS who have many other challenges to contend with, devotion to brain training may not be a feasible intervention.

The second implication proposed by others is that calculation is considered to be a precursor to further study in mathematics and therefore, learners with DD will have limitations of further study of mathematics. Even though this has been the accepted and rarely questioned view in the field of mathematics education until recent times (Forgasz & Cheeseman, 2015), there is growing evidence to suggest this is not the case: it is possible for learners with DD to learn other mathematics. Indeed, evidence is available in the work of researchers in DD (Butterworth et al., 2011; Dehaene, 2011).

As noted earlier, some learners with DS have been able to accomplish mathematics from a number of areas, including algebra, trigonometry, and percentages. Each of these students, it must be noted, could not calculate without the support of an electronic calculator.

For learners with DS, the following are alternative implications:

- Learners should use an electronic calculator as a prosthetic device, that is, a device that replaces the function of a part of the body. In this case, a calculator is used to overcome the impaired calculation functions of the brain.
- Explicit and lifelong attention to supporting conceptions of number must be made, making use of a variety of visual supports such as number lines. This encourages and reinforces alternative neural activity.
- Previously so called "functional mathematics" programs for learners with DS need to be fundamentally changed to focus on the use of prosthetic devices such as calculators and electronic banking methods in areas of finance and measurement.
- Learners should not be required to demonstrate accomplishment on basic number work before they are taught mathematics from across the discipline. The content should be from the curriculum of their schooling level and adjusted as needed.

Testing the Hypothesis

The indirect evidence for the hypothesis of Developmental Dyscalculia being a feature of Down syndrome lies in the pervasive nature of calculation difficulties for this population across the world and over decades of research in the field. Direct evidence to confirm the hypothesis is needed and may come from fMRI studies, particularly to examine if the IPS is affected. Alternatively, clinical assessment tools, such as task-based interviews need to be developed that are designed for learners with DS and explicitly probe areas of number development observing strategy use.

Conclusion

Should the hypothesis of DD as a characteristic of Down syndrome be confirmed, the implications for mathematics education are profound. Until now, a great deal of effort has been expended in school mathematics programs trying to teach arithmetic to learners with DS with limited success. This has led to students being taught the same material over and over again, often for all their years of formal schooling. An understanding of the source of the difficulties as being an impairment in calculation, frees those involved in the development of learning programs to include the routine use of electronic calculating devices as a support to enable the teaching of other areas of mathematics. As we await further research from the fields of neurobiology and psychology, mathematics programs can be planned that continue to reinforce understanding of number concepts while teaching year level mathematics content with adjustments.

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