Forty Years on: Mathematical Modelling in and for Education

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The topic from MERGA1 (structural modelling) was abandoned as a topic of study within months. This paper reflects on mathematical modelling, also a research interest since that time. Mathematical modelling has grown enormously in currency, and this is a mixed blessing, given the propensity for private interpretations to muddy its meaning and purpose. In this presentation, live issues from the field are discussed, including conceptions of modelling, modelling cycles, competencies, and developments in metacognitive understanding. Opportunities presented by recent international initiatives are indicated.

Background

The topic I spoke to at MERGA1 was abandoned within months as a research interest, and consequently, this present contribution is not a continuation of that theme. Instead it reflects on developments, opportunities, and challenges that attend the incorporation of mathematical modelling into educational theory and practice. This is a field with which I have been continually engaged over the same period of time.

It is perhaps significant that the first memorable contributions to the field of mathematical modelling in education were not put forward by professional educators, nor did they emerge from conventional educational settings. One of the earliest papers noting the lack of attention to real world problem solving in mathematics teaching (Pollak, 1969) made no mention of mathematical modelling as such. Simply titled "How can we teach applications of mathematics?", it focused on the unrealistic nature of word problems posing as "application problems" in school textbooks. In later contributions, he included descriptions of mentoring learners to become modellers – where the setting was not a classroom, but the research laboratory of the Bell Telephone Company. Similarly, the origins of the International Community for the Teaching of Mathematical Modelling and Applications (ICTMA) featured staff in British Polytechnics who had previously been engaged in applying mathematics to solve problems in industrial settings. They were concerned that their tertiary students did not know how to go about solving problems in real-world contexts, and sought to promote this ability. Around the same time, Treilibs (1979) demonstrated that secondary school students had little idea how to apply existing mathematical knowledge to solve problems located in the world outside the classroom. His identification of the formulation stage of modelling as a key gatekeeper to success has been reinforced and extended in many later studies, situated both at school and university level.

To engage with the subject of mathematical modelling in mathematics education we should first clarify the goal of the latter. Here is an attempt at a succinct statement

> Mathematics education aims to enable/enhance the teaching/learning of more mathematics, to more people, more effectively.

Secondly, we should agree on how to determine success. In their respective fields, chemists and engineers (for example) choose and apply mathematics in ways appropriate to their professional needs. Our field of application for mathematics is education, so in our case the interest is in promoting its effective teaching/learning, for which a key component is the scaffolding provided to facilitate this goal. Sadler (2008) argues that for some practitioners scaffolding becomes so elaborate, and the level of assistance so detailed, that

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the learner cannot help but ‘succeed’. They then fail when it is removed. Yet scaffolding is supposed to be a temporary arrangement that supports the building process, following which the building needs to stand on its own merits. So, a test for the effectiveness of learning/teaching in mathematical modelling would be:

> When scaffolding is withdrawn, what are students able to do independently in solving real-world problems that they could not do before?

Models of Modelling

Statements indicating the importance of students enabled to apply mathematics to real-world situations, including everyday life, society, and the workplace, are found increasingly in national curricula (e.g., ACARA, 2016; CCSSI, 2012).

Rarely do these seem to progress beyond lip service and, from the literature, readers new to the field could be forgiven for inferring that there are many modelling genres. The contention here is that there are but two – depending on where the ultimate authority is located (Julie & Mudaly, 2007). Either “modelling” is used to give a context for lesson material aimed at developing conceptual mathematics – the various approaches being controlled ultimately by curriculum priorities and perceived constraints on teaching. Alternatively, the purpose is to provide experiences requiring students to call on mathematical knowledge to address problems in the real world beyond the text book and the classroom. Authority here is vested in the requirements of the problem being addressed, as providing the evaluation standard for a modeller’s activity. Such modelling cannot live entirely in the classroom, as a captive of the education industry. This is also the approach necessary for participation in the recently inaugurated international Mathematical Modelling Challenge (IM2C, 2017). That word problem culture continues to be an issue for the treatment of real life problems in mathematics is illustrated specifically in recent studies on text-book content (e.g., Gatabi, Stacey, & Gooya, 2012), and more generally (e.g., Verschaffel & Greer, 2010).

Modelling Cycles

Arguments still engage around the structure and characteristics of the modelling process. A typical essential modelling cycle has the following components:

Understanding the task → formulating mathematical model → solving the mathematics → interpreting outcomes → validating/evaluating the outcomes in terms of the real context → documenting the solution process and outcomes

The components are necessary in the sense that every thoroughly conducted modelling project contains them; and analytical in the sense that they describe an objective problem solving process. In providing support consistent with the activities of professional modellers, they do not describe the activities that take place inside a modeller’s head. The pathway of these mental activities is anything but cyclic, as to and fro movement between stages occurs in response to how a solution is progressing. The essential cycle acts as a guide in terms of checking and progressing solution attempts. Novices typically refer to it constantly as an external referent, experienced modellers internalise it as a mental construct as noted in the *Stepping Stone* article (Galbraith & Clatworthy, 1990). The term *mathematising* is sometimes used to describe activities across the first three components of the cycle. This is legitimate, and an important emphasis, but creates problems when presented as if this is all that modelling entails.

The essential modelling cycle may be augmented for purposes of pedagogy or research. The cycle devised by Blum and Leiß (2007) contains additional structure of "real model" and "situation model", introduced to scaffold the formulation phase of the modelling process. These are not necessary components, as a variety of other successful modelling implementations do not employ them, but they have been used successfully in teaching modelling, especially in Germany. The approach using modelling eliciting activities also inserts pedagogical criteria into its carefully designed framework (Lesh & Doerr, 2003).

Stillman (2011) provides an example of a research augmented modelling cycle. The essential cycle is included, but is augmented by structure addressing mental activity, that enables examination and release of blockages that impede modelling performance. The additional structure is included for research purposes, and is not necessary for scaffolding the modelling process. Confusion in the field would be lessened by recognition that not all representations have the same purpose, nor contribute equally to enhancing performance.

Modelling Competencies

Sadler (2008) exposes the inadequacy of fragmented criteria to assess performance on holistic tasks, arguing that overall competence involves more than ticking off success on a set of individual sub-competencies. It requires also a demonstrated ability to integrate them is a complete solution. In terms of modelling, the following statement, deriving from ICMI Study 14, addresses competency as a synthesis of sub-competencies in the sense of Sadler:

> mathematical modelling competency means the ability to identify relevant questions, variables, relations or assumptions in a given real world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation. (Niss, Blum, & Galbraith, 2007, p. 12)

Contained in this statement are sub-competencies that contribute to the whole, but whose sufficiency can only be evaluated in terms of performance on the whole. In an extensive study Maaß (2006) addressed the matter of sub-competencies for modelling, identifying in excess of 20 components such as the following: to carry out the single steps of the modelling process; to solve mathematical questions within a mathematical model; to interpret mathematical results in a real situation; to validate solutions; to structure real world problems; and to work with a sense of direction for a solution.

The question remains as to whether modelling sub-competencies, when developed and tested separately, can be identified as contributing definitively to overall modelling competence. The alternative of enhancing sub-competencies through reflection and analysis within the context of complete modelling problems also has substantial support, and the arguable advantage that sub-competencies and overall modelling competence are then developed and assessed in a single setting.

Anticipatory Metacognition

The importance of reflection on actions undertaken in addressing real-world problems, whether checking mathematical accuracy, evaluating a solution against contextual implications, or examining interim decisions is well known, and such classical metacognition remains important. However, studies of metacognition are moving in additional directions. Anticipatory metacognition refers to modellers' metacognitive processes, as they attempt to anticipate necessary cognitive actions, and identify opportunities, not yet undertaken, but essential for the success of their modelling endeavour. Work to date suggests there are three distinct dimensions: (1) meta-

metacognition (2) implemented anticipation, and (3) modelling oriented noticing (Stillman et al., 2016).

Concluding Thoughts

Mathematical modelling as real world problem solving has two complementary goals: to enable the solution of specific real-world problems, but over time to nurture and enable modelling abilities for students to apply in their own life environments. What learning endures when scaffolding support is removed is an issue. Other more restricted purposes for modelling exist, and it is important to keep the different purposes conceptually distinct. In this short paper, some issues of continuing importance to the former purpose have been discussed. Of immediate significance is the initiation of the International Mathematical Modelling Challenge (IM2C, 2017). Australian participation is managed by a reference group containing mathematicians, mathematics educators, teachers, AAMT representation, and representatives from industry. A visit to the website will access the extraordinary modelling quality of which students are capable when properly guided.

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