Does (Problem-Based) Practice Always Make Proficient?

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The current study compared the rate at which problem-based practice increased the use of retrieval-based strategies for students identified as displaying accurate min-counting with students identified as displaying almost proficient performance. The findings supported the prediction that the rate at which problem-based practice promoted retrieval use was lower for students in the accurate min-counting group; in fact, it had no effect on their retrieval development. Implications for teaching practice are discussed, in particular, the notion that such students may require exposure to different problem representations (e.g., visual imagery) to move them away from conceptualising addition as counting.

Education standards indicate that children will solve most simple (single-digit) addition problems using retrieval-based strategies by second or third grade. Retrieval-based strategies encompass direct retrieval as well as decomposition strategies, where a number is partitioned to make use of a retrieved fact (e.g., 3 + 4 = 3 + 3 + 1 = 6 + 1).

It has slowly come to light that many children are not solving simple addition problems in ways that match curriculum expectations. Children's use of retrieval is considerably lower than expected in second and third grade (Cowan et al., 2011; Geary, Hoard, Byrd-Craven, & deSoto, 2004), and their use of min-counting is much higher than expected (Cumming & Elkin, 1999). This is a concern for educators given that the frequency with which retrieval-based strategies are used to solve simple addition problems has been found to be strongly associated with (i) maths achievement (Geary, 2011a), (ii) understanding key principles such as the commutativity of addition and the complementary relationship between addition and subtraction (Canobi, 2009), and (iii) the use of flexible mental strategies for adding and subtracting multi-digit numbers (Carr & Alexeev, 2011).

Hopkins and Bayliss (2017) found that in seventh grade, around 35% of students were still reliant on min-counting to accurately solve simple addition problems. This group of students was referred to as displaying an accurate min-counting pattern of performance and was distinguished from students who displayed (i) an inaccurate min-counting counting pattern, (ii) almost proficient performance (i.e., the predominate use retrieval-based strategies), and (iii) proficient performance (the exclusive use of retrieval-based strategies). Students in the accurate min-counting group showed lower achievement in maths compared to students who predominately or exclusively used retrieval-based strategies. Hopkins and Bayliss (2017) suggested that students in the accurate min-counting group had not developed problem-answer associations in memory that allowed them to directly retrieve answers or use retrieved facts as part of a decomposition strategy because they had a high confidence threshold for using retrieval. However, it was not clear if min-counting was the most efficient strategy these students had access to in their strategy repertoire or if they simply had chosen to min-count on the one occasion they were assessed. The aims of the current pilot study were to test the assertion that such students do not use retrieval-based strategies because of a high confidence threshold for using retrieval.

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Background

Children generally learn retrieval-based strategies for solving single-digit addition problems as a result of problem-based practice; that is, practice solving single-digit addition problems using strategies of choice. Children may initially solve simple addition problems using counting-all strategies but with continued practice, children will abandon the use of less efficient strategies and replace them with the min-counting strategy (where the smaller addend is counted on), direct retrieval and decomposition strategies (Canobi, 2009; Hopkins & Lawson, 2002; Siegler & Jenkins, 1989).

The transformative role practice can have on strategy use is sometimes overlooked in educational literature. For example, in a report by the U.S. National Research Council (2001), it was stated that, “The role of practice in mathematics, as in sports or music, it to be able to execute procedures automatically without conscious thought. That is, a procedure is practiced over and over until so called automaticity is attained” (p. 351). While practice can automatize the execution of a strategy, this is not the only role of practice. Specifically, the benefits of problem-based practice extend beyond automaticity to encompass evolution in strategy use (Hopkins & Lawson, 2002).

Problem-based practice is arguably the most important teaching approach for strategy development as it leads to the use of multiple strategies and promotes adaptive strategy use (Verschaffel, Luwel, Torbeyns, & Van Dooreen, 2009). Other approaches include strategy-based practice, where students are directly taught a strategy like min-counting and are required to practice using the particular strategy (e.g., Fuchs et al., 2010), and fact-based practice, where students rehearse each problem with the correct answer (e.g., Powell, Fuchs, Fuchs, Cirino, & Fletcher, 2009).

However, not all children develop accurate retrieval-based strategies when exposed to problem-based practice. Two explanations are commonly put forward elucidating why this is so. Firstly, it is suggested that children make too many errors during problem-based practice as a result of losing track of the count (e.g., Geary, Bow-Thomas, & Yao, 1992). Frequent errors form competing (incorrect) problem-answer associations in memory leading to retrieval errors or the continued use of backup strategies (Shrager & Siegler, 1998). Secondly, it is contended that some children have a learning disability, characterised by particularly low mathematics achievement and retrieval deficits (e.g., Fuchs et al., 2010; Geary, Hoard, & Bailey, 2012). These deficits appear to be related to difficulties inhibiting irrelevant information during the retrieval process (Geary, 2011b). However, neither of these explanations can explain an accurate min-counting pattern of performance. Children in this group rarely make mistakes and, although low achieving as a cohort, are generally not among the lowest achieving students (Hopkins & Bayliss, 2017).

There is a third explanation, however, which can be used to explain an accurate min-counting performance pattern; confidence. Shrager and Siegler (1998) argued that although practice using a back-up strategy builds the associative strength of an answer in memory, the answer will only be stated if its associative strength exceeds an individual’s response threshold for determining confidence in its correctness. Students with a high confidence threshold for retrieving answers will need more practice before becoming sufficiently confident to use retrieval, compared to students with a less restrictive confidence threshold. Bailey, Littlefield, and Geary (2012) found that boys and girls displayed different developmental paths towards using retrieval for simple addition and suggested that girls have a higher confidence threshold for retrieval than boys. Similarly, Hopkins, and Bayliss (2017) found that students in the accurate min-counting group were more likely to be girls.

In the current study, we compared the rate at which problem-based practice increased the use of retrieval-based strategies for students identified as displaying accurate min-counting with students identified as displaying almost proficient performance. We predicted that this rate would be less for students in the accurate min counting group because students have a higher confidence threshold.

Method

Participants attended one primary (elementary) school located in Melbourne, Australia. Initially 94 students were assessed on how they solved single-digit addition problems. Students were in Grade 2 (n = 14), Grade 3 (n = 20), Grade 4 (n = 24), Grade 5 (n = 15), and Grade 6 (n = 21). Students were individually withdrawn from class and assessed on how they solved 36 single-digit problems on a trial-by-trial basis, using a combined method of observation and self-report. Immediately after each problem was solved, the child explained to a research assistant (RA) who had observed them how they had solved the problem. This method for assessing strategy use has been shown to be a reliable and valid approach (Reed, Stevenson, Broens-Paffen, Krischner, & Jolles, 2015; Siegler, 1987) and used extensively in studies of single-digit addition skill (e.g., Canobi, 2009; Geary et al., 2004). Reaction times (RTs) for each trial were recorded, representing the time the problem was displayed to the time when the child gave their answer. Mean RTs separated by strategy use are reported for correct trials in Table 1. These data corroborated children's self-reports with RTs to correct retrieval trials generally under three seconds. The problem set encompassed 36 single-digit problems written in the form $a + b =$; where $a \leq b$ and $1 < a, b \leq 9$.

Table 1
Strategies Reported During the Initial Assessment with Corresponding RTs for Correct Trials

Strategy	Example	No. correct trials (%)	Mean RTs (*SD*)
counting-all	3+4=1, 2, 3, 4, 5, 6, 7	52 (1.7)	13.1 (7.2)
counting-on-right	3+6=3: 4, 5, 6, 7, 8, 9	90 (2.9)	10.8 (7.2)
min-counting	3+6=6: 7, 8, 9	1,074 (34.5)	6.4 (9.0)
decomposition	4+5=4+4	106 (3.4)	5.4 (4.3)
other	2+4=2+2+2; 3+3=2x3; 3+5=4+4	55 (1.8)	6.5 (5.8)
retrieval	7 (*just knew it*)	1,735 (55.8)	2.3 (1.4)

Note. No. of correct trials was 3,112, representing 92% of trials.

Results of this initial assessment were used to cluster students into groups based on performance characteristics. Students who always used retrieval-based strategies were first identified (proficient group), followed by students who frequently used a counting-all strategy (inefficient-counting group). A k-means clustering technique was used to group the remaining students into three clusters, similar to Hopkins and Bayliss (2017). These clusters included (i) students who exhibited typical retrieval development (almost-proficient group), (ii) students who frequently used min-counting and were accurate (accurate-min-counting group), and (iii) students who frequently used min-counting and were inaccurate (inaccurate-min-counting group). Performance characteristics for each group are summarized in Table 2.

Students in the accurate min-counting group were distributed across grade levels: Grade 2 (*n* = 6), Grade 3 (*n* = 12), Grade 4 (*n* = 9), Grade 5 (*n* = 6), and Grade 6 (*n* = 6). This performance pattern was the focus of investigation. Students in the almost-proficient group exhibited more typical retrieval development consistent with curriculum expectations, and used for comparative purposes. Students in this group were distributed across grades as follows: Grade 2 (*n* = 1), Grade 3 (*n* = 7), Grade 4 (*n* = 8), Grade 5 (*n* = 7), and Grade 6 (*n* = 12).

Table 2

Percentage for Trials Indicating Strategy Frequency and Accuracy (in Parentheses) by Group

Group	No. students	Retrieval	Min-counting	Decomp. + other
Proficient	7	91.7 (98.8)	0	8.3
Almost-proficient	35	67.7 (98.2)	25.7 (96.7)	4.5 (85.0)
Inaccurate-min-counting	8	58.3 (93.6)	27.4 (58.9)	1.0
Accurate-min-counting	39	35.5 (96.5)	54.8 (90.1)	6.5 (84.0)
Inefficient-counting	5	19.4 (92.2)	11.1 (77.4)	4.4

Note. Accuracy not shown if based on fewer than five students.

Participants in the study included six students: three students who were randomly selected from the accurate-min-counting group and three students who were randomly selected from the almost-proficient group. Participants were monitored as they engaged in problem-based practice for 15 consecutive school days (where possible). Each day, students were individually withdrawn from class and strategy use was determined using the same combined method of observation and self-report, and the same 36-problem set, described above. To estimate the rate at which problem-based practice leads to retrieval development, a regression line was fitted to each students' data representing the number of problems correctly retrieved each practice session. It was not clear if students' errors should be corrected during the study as corrective feedback could influence retrieval development differently for students with different profiles. For this reason, the influence of corrective feedback was controlled for using a multiple-baseline design. During the uncorrected condition, no feedback was given. During the corrected condition, the RA would respond with "let's check that" if an incorrect response was given. She then proceeded to model the correct use of the same strategy that the child had reported using.

Results

Participants were monitored each day for 15 consecutive days. Area graphs in Figure 1 illustrate the range of strategies correctly applied and the number of errors made at each time interval (i.e., practice session) by each child, as well as the influence of corrective feedback. Numbers along the y-axis indicate the number of problems. Numbers along the x-axis indicate consecutive school days. A dotted line separates the no feedback condition from the corrective feedback condition. Data are missing for Kyle and Karen due a computer fault.

Visual inspection of graphs in Figure 1 suggest that participants selected from the accurate min-counting group did not benefit from problem-based practice in terms of improved correct retrieval; whereas, two of the three participants selected from the almost proficient group did show marked improvement in correct retrieval. The third child in the accurate min-counting group (Karen) did not exhibit performance consistent with her group: she made frequent errors, particularly min-counting errors (21.2%) rather than retrieval errors (0.8%). In addition, the third child in the almost proficient group (Casey) did not benefit from problem-based practice, appearing to use retrieval less frequently during the corrective feedback condition.

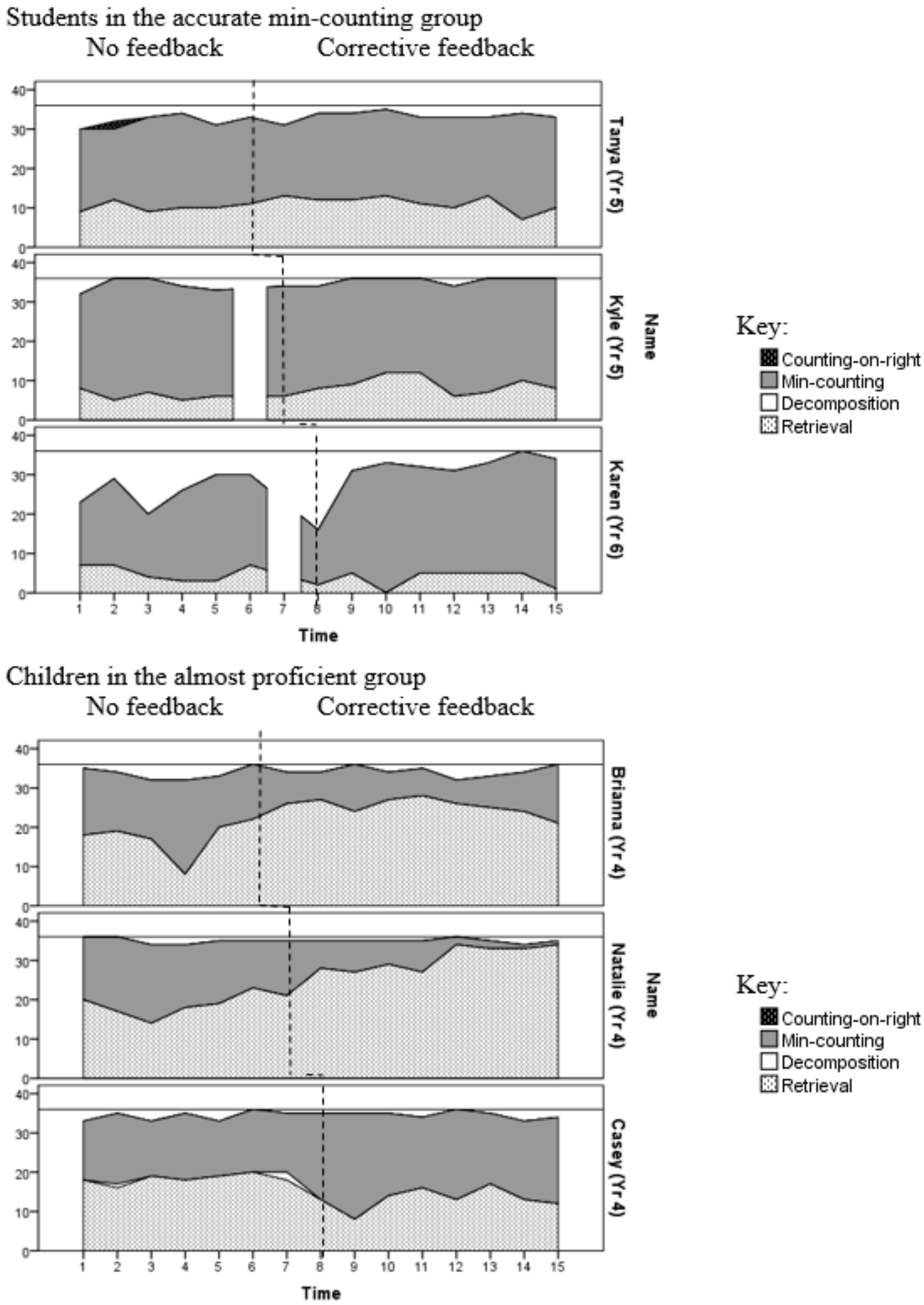

Figure 1. Area graphs displaying the strategy mix and errors across time by participant.

Table 3
Changes in the Frequency of Correct Retrieval as a Result of Problem-Based Practice

	Parameter	Estimate*	SE	t	p
Tanya	Intercept	10.74	.98	10.92	.000**
	Slope	.011	.12	.10	.922
Kyle	Intercept	5.9	1.18	5.00	.000**
	Slope	.23	.18	1.80	.10
Karen	Intercept	5.6	1.17	4.79	.000**
	Slope	-.17	-.37	-1.36	.199
Brianna	Intercept	16.56	2.38	6.95	.000**
	Slope	.70	.26	2.66	.020***
Natalie	Intercept	13.9	1.35	10.24	.000**
	Slope	1.41	.15	9.49	.000**
Casey	Intercept	19.49	1.59	12.27	.000**
	Slope	-.46	.18	-2.64	.020***

*Unstandardized coefficients **$p < .001$ ***$p < .05$

Results of the regression analyses confirmed these patterns (see Table 3). Specifically, it was revealed that two of the three students whose performance was consistent with an accurate min-counting pattern (Tanya and Kyle) did not benefit from problem-based practice and their frequency of correct retrieval remained constant across sessions. By contrast, two of three students whose performance was consistent with an almost proficient pattern (Brianna and Natalie) benefited from problem-based practice with an estimated rate of increase of 0.7 and 1.4 problems per practice session respectively (as indicated by slope estimates in Table 3).

Discussion

A considerable proportion of students are not using retrieval-based strategies for solving single-digit addition problems when they are expected to do so (Hopkins & Bayliss, 2017) and these students are likely to show lower achievement than their peers who do use retrieval-based strategies (Geary, 2011a; Hopkins & Bayliss, 2017). This pilot study investigated if students who were identified as predominately using an accurate min-counting strategy for simple addition benefitted as much from problem-based practice as peers who showed higher retrieval use. The findings supported the prediction that the rate at which problem-based practice promoted the retrieval-based strategies was lower for students in the accurate min-counting group; in fact, this type of practice had no effect on retrieval development for these students. This is an important finding as problem-based practice is the principal type of practice used to promote retrieval, and retrieval is thought to be "dependent on sufficient and appropriate practice" (U.S. Department of Education, 2008, p. xix).

While it is recognised that students who make frequent errors with backup strategies are less likely to benefit from problem-based practice (Shrager & Siegler, 1998), the findings of this study indicate that confidence may also be a factor that hinders retrieval development. The notion of a high or restrictive confidence criterion has been referred to many times in the literature (e.g., Bailey et al., 2012; Geary et al., 2004; Shrager & Siegler, 1998) but this idea has remained largely unexplored empirically. Findings from this study suggest that further research examining this factor may prove to be important for research and teaching practice. Further research is needed to test the assumption that confidence is restricting retrieval use for these students, rather than another factor such as *Einstellung*; that is, a disposition towards applying a familiar strategy (Bilalić, McLeod, & Gobet, 2010). This is the focus of a current research project involving the authors.

The findings of this study also indicated that the methods adopted for categorizing and selecting students from groups based on what strategies they use and how accurate they are need to be refined. At least one participant did not exhibit performance characteristic of her group. For the purpose of testing predictions, particularly using a single case study design, it would be more efficacious to purposefully select students from each group (rather than randomly select them) so that they represent typical performance for that group. There was also some evidence that corrective feedback can inhibit retrieval use and so larger-scale studies will need to consider feedback in their design.

Despite these limitations, findings from the cluster analysis of data collected using the initial assessment were consistent with that found by Hopkins and Bayliss (2017) using a separate sample of students. This analytical approach shows much potential. A similar cluster method has been used to identify cognitive subtypes of mathematics learning difficulties (Bartelet, Ansari, Vaesson, & Blomert, 2014) and profiles for core numerical competencies (Reeve, Reynolds, Humberstone, & Butterworth, 2012).

Importantly, the current study goes one step further than identifying different groups of students and elucidating their performance profile. Specifically, it presents some initial evidence indicating that students with different profiles might in fact respond differently to a particular type of teaching practice. This has definite implications for classroom teachers. For example, students identified as almost proficient in simple addition are likely to be able to continue to improve their capacity to directly retrieve answers through conventional problem-based practice. We are not suggesting that this group can be ignored by teachers, but rather, that the main role of teachers would appear to provide sufficient opportunities for them to engage in problem-based practice. By contrast, it is likely that students with an accurate min-counting performance profile require exposure to a variety of different problem representations in an attempt to move them away from conceptualising addition equations primarily as "counting problems". It is worth noting that a subitising-based intervention has shown promise in improving retrieval, and reducing reliance on min-counting (Hopkins & de Villiers, 2016). It may be that primary school teachers could consider exposing accurate min-counters to additional games and activities involving subitising and other number visualisation tasks. These students might then be able to apply such imagery to aid retrieval when confronted with addition problems, and in time, as retrieval improved, perhaps start to simultaneously benefit more from engaging in problem-based practice. These ideas provide new directions for both classroom-based practice and future research.

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