Grade 10 Students' Mathematical Understanding and Retention in a Problem–Based Learning (PBL) Classroom

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In this study, we examined students' mathematical understanding and retention in a problem-based learning (PBL) classroom. The participants were 48 Grade 10 students, and the data were collected in December of 2016. After the end of the PBL lessons, the Mathematical Understanding of Function Test (MUFT) and the Retention Test (RT) were administered. The findings showed that most students demonstrated mathematical understanding of functions in all components and more than 50 percent of the students could pass both tests by the overall mean scores. Moreover, the overall mean difference between the MUFT and the RT was low, which means that the students had retention.

For several decades, mathematics teaching and learning have undergone a worldwide reform (Young-Loveridge & Bicknell, 2016). Moreover, studying mathematics with understanding is emphasized by several researchers (Stylianides & Stylianides, 2007). Fennema and Romberg (1999) argue that the development of understanding should be the outcome of teaching and learning mathematics. Furthermore, a greater amount of teaching and learning should utilize tasks that provide problem situations in order to promote learning for mathematical understanding. For these reasons, mathematics is best learned when students learn through problem-solving tasks. Students develop understanding when they engage in classroom activities to solve problems.

Mathematical understanding is accepted by some researchers as a procedural process that consists of an ability to carry out action sequences. Other researchers define it as a part of network of connections (Kinney & Kinney, 2002). Specifically, Hiebert and Carpenter (1992) define mathematical understanding as making decisions involving knowledge. However, mathematical understanding can also be defined as a network of ideas or representations about mathematics (Barmby, Harries, Higgins, & Suggate 2007).

One of the fundamental concepts in mathematics for secondary school is that of functions. Nevertheless, many students still have misconceptions about this topic. The Programme for International Student Assessment 2012 showed that the weakest ability of Thai students was in the change and relations topic in mathematics content. This topic is directly related to functions (The Institute for the Promotion of Teaching Science and Technology, 2014). Functions are important for students' learning of advanced mathematical topics such as calculus and advanced concepts of functions. Therefore, students' understanding of functions should be emphasized (Bardini, Pierce, Vincent, & King, 2014). Hollar and Norwood (1999) proposed mathematical understanding of functions with four components as follows: (1) Modelling: a real-word situation using a function, (2) Interpreting: a function in terms of a realistic situation, (3) Translating: translation among different representation of function, and (4) Reifying: the process of concept development that involves transformation from the operational to the structure phase. In this study, we apply Hollar and Norwood's study (1999) as a tool for assessing students' mathematical understanding of functions.

In addition, Kwon, Rasmussen, and Allen (2005) found that students' retention could be considered as a result of understanding mathematical concepts. Retention means students' ability to recall and organize information about mathematics from their memory.

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 349-356). Melbourne: MERGA.

Therefore, retention can be defined as having a retentive mind (Kundu & Tutoo, 2002). Several researchers use retention intervals in their studies that last between one week and four weeks (Driskell, Willis, & Copper, 1992). The retention interval is the time period from the latest students' learning to a retention test (Rohrer & Taylor, 2006).

One of the 21st century teaching approaches that support both students' understanding and retention is Problem–Based Learning (PBL). Albanese and Mitchell (1993) suggested that PBL is a teaching approach that promotes understanding while students are challenged and engaged with problem situations. Hung, Jonassen, and Liu (2008) noted that retention of content and problem-solving skills could be considered as a result of PBL classroom. PBL is carried out in classrooms through examining real-world problem situations, conducting research, discussing and working in groups, and giving presentations (Othman, Salleh, & Abdullah, 2013). PBL activities are not only provocative but also beneficial for students in an active learning context. The PBL processes employed in this study are adapted from Othman et al.'s study (2013) with five steps: (1) an introduction to the problem, (2) self-directed learning, (3) group meeting, (4) presentation and discussion, and (5) exercises. In this study, we were interested in exploring Grade 10 students' mathematical understanding of functions and retention in a PBL classroom.

Methods

Participants

We employed a mixed-method design to investigate students' mathematical understanding of functions. The investigation was comprised of 48 Grade 10 students (19 boys and 29 girls) from a high school in Chiang Mai Province, Thailand. Nine students with mixed mathematical abilities were selected in order to gain in-depth information from interviewing them about their mathematical understanding of functions.

Instrumentation

The data were gathered using the following instruments: (1) eight PBL lesson plans (100 minutes per lesson, two lessons per week), (2) the Mathematical Understanding of Function Test (MUFT) and the Retention Test (RT; the RT was parallel to MUFT), adapted from Hollar and Norwood (1999), (3) students' reflections, (4) the teacher's notes, (5) classroom observation forms, and (6) students' interview forms. The data were analyzed in both qualitative (descriptive analysis) and quantitative (descriptive statistics) ways.

Procedure

During the PBL lessons, the data were collected for four weeks in December 2016. One of the researchers was the teacher in the PBL classroom. A mentor teacher observed all of the PBL lessons. Moreover, students' reflections, classroom observation forms, and the teacher's notes were used to reflect on students' understanding and teaching. At the end of the PBL lessons, the MUFT was administered in order to examine students' understanding of functions. The MUFT consists of two problem situations. The New Year party problem is an example problem situation from the MUFT. This problem situation was adapted from Mathematics Assessment Resource Service (MARS, 2003) (see Table 1). Nine students were selected according to their mathematical abilities (three high, three average, and three low) to be interviewed about their mathematical understanding of functions. Four weeks after the end of the PBL lessons, the RT was used in order to examine students' retention. We also used video recording and voice recording to provide supporting data for each teaching period and interview.

Table 1
Problem Situation Example (The New Year Party Problem)

The end of the year is near, which means a new year is coming soon. The New Year party is an opportunity to celebrate with family and/or friends. Your mother has a plan for a New Year party. She assigns you to prepare tables and seats in the party. The pattern setting is that all tables are put together in a line; then, a chair is put on the top and the bottom of each table. Moreover, at the ends of the lines of tables, you will put two seats as shown in the example in Figure 1:

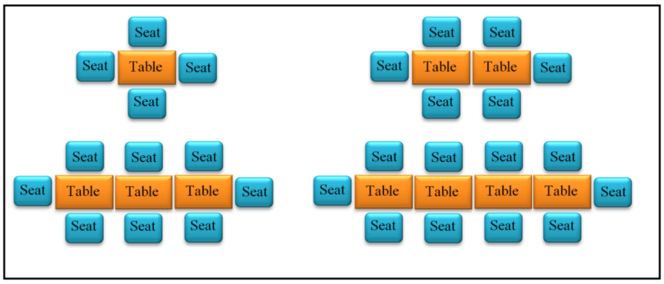


Figure 1. An example problem situation from the MUFT.

Results

Below, the findings are first reported with the students' scores on the MUFT and the RT. Then, information about students' mathematical understanding of functions is aligned with the five steps of the PBL classroom. Finally, details about the MUFT and interviews about students' mathematical understanding of functions in all components are described.

Part 1: The Mean Scores of the MUFT and the RT

The scores from the MUFT and the RT were computed in percentages for reporting the modelling, interpreting, translating, and reifying components and overall mathematical understanding of functions. The differences between the mean scores of the MUFT and the RT in each component ranged from 2 to 10 percent. The students' scores on the reifying component showed the highest mean difference, at 10 percent. The lowest mean difference was the interpreting component, at 2 percent. Moreover, the mean difference in overall understanding was at only 7 percent. This means that the students had high retention. In addition, the mean scores of the MUFT and the RT in overall understanding were 65 and 58 percent, respectively. More than 50 percent of the students could pass both tests by the overall mean scores as shown in Table 2.

Table 2
The Mean, Standard Deviation, and Mean Difference from the MUFT and the RT

Mathematical Understanding	MUFT (%)		RT (%)		Mean Difference
of Functions (Components)	Mean	SD	Mean	SD	(%)
1) Modelling	68	23	62	24	6

2) Interpreting	68	25	66	21	2
3) Translation	66	26	57	22	9
4) Reifying	56	33	46	33	10
Overall	65	23	58	20	7

Part 2: The Students' Mathematical Understanding of Functions in the PBL Classroom

Below, we describe the findings regarding the students' mathematical understanding of functions aligned with the five steps of the PBL process based on students' reflections, teacher's notes, and classroom observations.

1) Introduction to the Problem. In the first step, the students paid attention to the teacher while he introduced the real-world problem situation. Students' reflections showed that they preferred learning through the problem situations because it was interesting and helped them to get a better understanding of the lessons. However, some students expressed that they could not catch all main points; they just got some points of the problem situations. Thus, the teacher sometimes needed to restate the given problem situations to facilitate students' understanding about the problem situations.

2) Individual Work. In the second step, the students analysed the problem situations. Then, they created the problem situation's representations (modelling). Students' reflections showed that the familiarity of the problem situations to their daily lives helped them to be able to create the problem situations' representations. From the classroom observations, most students attempted and were engaged to create the problem situations' representations in their individual work. At the beginning of the PBL lessons, we found that only half of the students could correctly create representations of the problems. However, after the PBL lessons, most students could understand the problem situations quickly and they could correctly provide the problem situations' representations.

3*) Group Meeting.* In the third step, the students continued to analyze and solve the problem situations through small group activities. The students discussed their own ideas with each other about the mathematical understanding of functions in all components. The students' reflections showed that the students liked to meet in groups because it made them feel comfortable sharing their ideas with friends. From the classroom observations, we found that some students did not participate as they should in the group meetings. Therefore, the teacher often tried to motivate the students by asking for their ideas and connecting those ideas with other group members. This helped to increase students' participation in group work. After working together, most students were able to illustrate their understanding in group meetings with less of the teacher's facilitation. For example, the students could make a conclusion about the problem situations' representations from each group member's ideas (modelling), were able to connect the problem situations' representations to the description of a function (interpreting), were able to translate and create the multiple problem situations' representations (translating), and were able to correctly connect the solutions of the problem solutions to new conditions of the problem situations (reifying).

4) Presentation and Discussion. In the fourth step, the students presented their group work to the classroom. In the presentations, the students asked questions when they did not understand the presentations. In discussion, the students could offer their own mathematical understanding of functions by sharing ideas and discussing in the classroom. Finally, the students concluded with the mathematical understanding of functions from their discussions. The students' reflections showed that group presentations and classroom discussion helped them to get a better understanding of functions. From the classroom observation, the researchers found that the classroom discussion was a significant process that enhanced students' mathematical understanding of functions. Especially, most students interestingly discussed about connecting the operational process between the problem solutions and new conditions and concepts of the problem situations (reifying).

5) Exercises. In the last step, students individually worked on the exercises about mathematical understanding of functions. The students' reflections showed that the students could do the exercises and showed their mathematical understanding of functions in all four components. For example, students were able to create exponential equations from the problem situation about bacteria growth (modelling). They were also able to describe number of bacteria at various times (interpreting). Moreover, they were able to translate from exponential equation of bacteria growth to exponential graph (translating). Finally, they were able to understand the exponential shape when bacteria growth rate was changing (reifying).

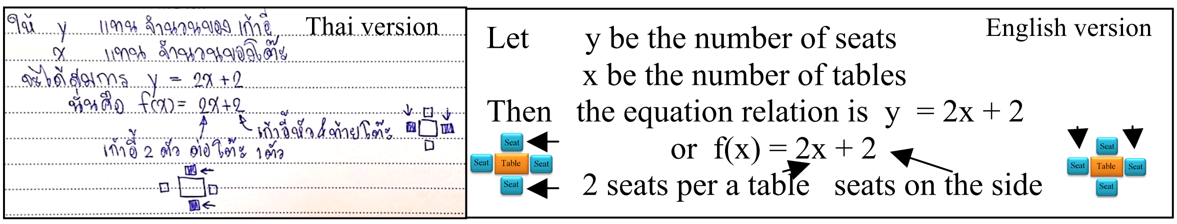
Part 3: The Students' Mathematical Understanding of Functions after all PBL Lessons

At the end of all PBL lessons, the MUFT was administered in order to examine students' understanding of functions. In the following sections, we provide examples that demonstrate the students' mathematical understanding of functions in all four components.

1) Modelling. The MUFT showed that most students could create meaningful models of the problem situations. The students could show complete modelling. For instance, some students' answers illustrated some of the equations for the situations by: (1) defining variables, (2) describing relationships between the variables, and (3) modelling the problem situations. Moreover, some students described modelling by using pictures (see Figure 2).

Figure 2. Example of modelling.

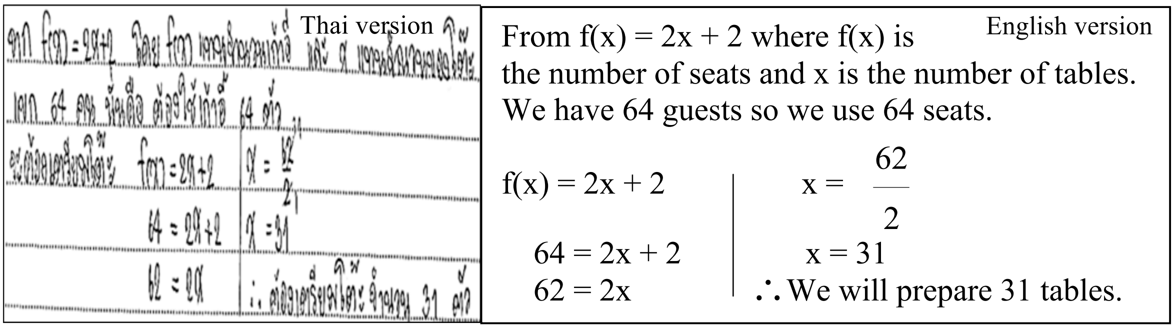
The students' interviews showed that students with high levels of achievement could give examples to clarify their explanations. In addition, they could make interesting references to the problem situations. However, students with average and low levels of achievement could correctly transform the problem situations to modelling, but they could not correctly refer to the problem situations.



2) Interpreting. The MUFT showed that most students could analyze the problem situations and correctly answer the questions about the problem situations. The students were able to find the value of input variables from the given situations (see Figure 3). However, most students couldn't completely describe or interpret the problem situation representations. For example, the students answered that the values of input variables were increased when the values of output variables were increased. Likewise, they did not correctly consider the rates of change and the constants in the problem situations.

Figure 3. Example of interpreting.

The students' interviews revealed that students with high and average achievement correctly described and interpreted situations. The students could specify what they have to do, what they need for interpreting situations, and what they already have. However, we found from the student interviews that students with low achievement were confused between output variables and input variables. Moreover, they incorrectly solved the equations.



3) Translation. The MUFT showed that most students could present multiple function representations (see Figure 4). In addition, the students had various ways to translate the representations. For instance, they could draw graphs from linear equations, ordered pairs, or by finding the x-intercepts and y-intercepts. Nevertheless, some aspects of the students' representations were still incomplete. For example, some students set inconsistent scales on the x and y axes, some students did not define the x and y axes on the graph or the name of columns in the table, and some students ignored the graph symbols.

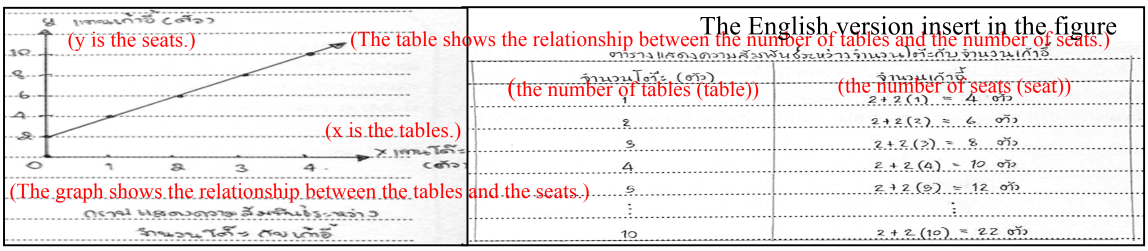
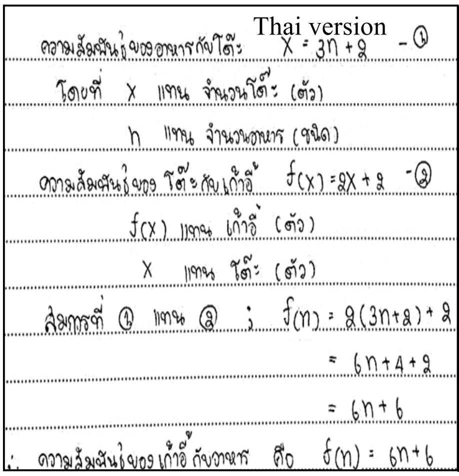


Figure 4. Example of translating.

The students' interviews showed that students with high, average, and low levels of achievement were able to explain how to translate one representation to another representation. However, the responses in the MUFT from low-achieving students were poor and incomplete. The students with high and average levels of achievement still presented their translation of the representations that linked to the problem situations and could explain the pros and cons of each representation.

4) Reifying. The MUFT showed that more than half of the students were able to connect the problem solutions with the new conditions of the problem situations in order to create new concepts. For example, the students could find a relationship between the number of tables (x) and the number of seats (f(x)) from the New Year party problem as the relevant equation $f(x) = 2x + 2$. In addition, the new condition of the New Year party problem is a relationship between a number of tables (x) and a number of food types (n) in the equation $x = 3n + 2$. The problem situation needs the relationship between the number of guests, which equals the number of seats (f(x)) and the number of food types (n). The students replaced $x = 3n + 2$ with $f(x) = 6x + 6$ as the relationship is $f(n) = 2(3n + 2) + 2$ or $f(n) = 6n + 6$ (see Figure 5). Nevertheless, some students' responses showed incomplete solutions, such as incorrectly replacing variables or solving the equations.

Figure 5. Example of reifying.



English version

The relation between the number of food types and the number of table is $x = 3n + 2$ …(1)

When x is the number of table (table)

n is the number of food types (type)

The relation between the number of tables and the number of seats is $f(x) = 2x + 2$ …(2)

When f(x) is the number of tables (table)

n is the number of seats (seat)

Replace (1) to (2); $f(n) = 2(3n+2) + 2$

$= 6n + 4 + 2 = 6n + 6$

$\therefore$ The relation between the number of seats and the number of food types is $f(x) = 6n + 6$.

The students' interviews revealed that students with high achievement could explain connections between the problem solutions and the new conditions of the problem situations. They were concerned with more references to the problem situations. In addition, they had several ideas for solving the problems. The students with average achievement were able to identify connections between the problem solutions and the new conditions. However, they could not explain the situations completely. On the other hand, students with low achievement could not identify the connection between the problem solutions and the new conditions.

Conclusion

In this study, we examined Grade 10 students' mathematical understanding of functions and retention in a PBL classroom. The findings showed that the PBL learning context gave students the opportunity to better gain mathematical understanding of functions in all four components: modelling, interpreting, translating, and reifying. Particularly, in the second step of PBL (individual work), the students were able to show modelling components. In the third, fourth, and fifth steps of PBL, the students were able to show mathematical understanding of functions in all four components.

In addition, the overall mean score of the MUFT was 65 percent. Meanwhile, the overall mean score of the RT was 58 percent. Both the MUFT and the RT showed that more than 50 percent of the students could pass the test, according to their overall mean scores. However, the mean scores of both the MUFT and the RT showed that the interpreting component had the highest mean scores and the reifying component had the lowest mean scores. Interestingly, the mean overall difference between The MUFT and the RT was only 7 percent. This showed that the students retained their mathematical understanding of functions.

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