

Learning from our Neighbours: The Value of Knowing Their Number History

Kay Owens
Charles Sturt University
<kowens@csu.edu.au>

Recent research has supported and extended earlier research on how and for how long Indigenous people of Australasia have been counting. This history values the long history of Indigenous knowledge and re-writes the limited and sometimes false history that many Australian teachers accept and teach about number systems. The current views on the spread and innovation of number systems are critiqued in terms of how oral cultures used and represented large numbers.

Forty years ago, ethnomathematics was beginning to be recognised more widely (Bishop, 1979; Lancy, 1978; Van der Waerden & Flegg, 1975; Wilder, 1974; Wolfers, 1971; Zaslavsky, 1973). Many linguists were recording the counting systems of the people whose language(s) they were learning (Panoff, 1970; Pumuye, 1975); Wurm and his colleagues (Wurm, Laycock, Voorhoeve, & Dutton, 1975) were documenting the languages in New Guinea, and others were considering Oceania (Lynch, 1977; Ross, 1988). Anthropologists were incorporating counting into the various complex activities and beliefs of people (Strathern, 1977) while cognitive, developmental psychologists (e.g., Saxe, 1979) were considering how cultural context affected concepts. Papua New Guinea University of Technology had a Mathematics Education Centre and the University of Papua New Guinea had an Education Research Unit, both of which supported research that took account of cultural difference in mathematics learning. Some of this work was focused on difference using Piagetian studies and some on cognitive development for mastery of concepts. It would be another decade before ethnomathematics was widely discussed (Ascher, 1994; Bishop, 1988; D'Ambrosio, 1990).

Counting System Diversity and How They Developed

With written records of number, there are symbols used in the various languages for numbers. These reflect the ways that the people combined numeral words to make new number names. For example, the Romans, at least in their later history, said four was IV, that is, one before five. In one sense, place or position was important. We are also cognisant that the Arabs used and modified the original Indian ways of recording numbers to give us our current Hindu-Arabic system of base-10 with place value and a zero. But what happened before this? In the second half of the 18th century, it was considered that numerals were a hallmark of civilisation (Crawford, 1863), and then there was a debate about whether different civilisations invented their counting and numerals or whether there was a diffusion from either Egypt around 4,000 BCE (G. E. Smith, 1933) or the Sumerians (Raglan, 1939). The diffusionist ideas prevailed and Seidenberg (1960) suggested that counting systems diffused from the Middle East civilisations starting with two-cycle systems having one and two as frame words around 3,500 BCE “spread out over the whole earth; later, other methods of counting arose and spread over almost all, but not quite all, of the world” (p. 218). His view was based on the anthropological evidence that suggested these systems only occur on the vestiges of the world in southern Africa, southern America, and Australia. Some systems with numerals for three and perhaps four, but

without combinations for higher numbers he claimed as systemless and an aberration of the two-cycle system. Seidenberg then suggested that paired systems developed (He called them neo-2 systems) in which $6 = 2 \times 3$ and $7 = 2 \times 3 + 1$ and so on. Then, the base-10 system developed in conjunction with these paired systems spreading around the world.

Seidenberg went on to suggest that the North American Inuits that have a (5, 20) cycle system came after the 10-cycle and there was a fusion, slurring (his term), or modification made with the (2, 10) cycle system resulting in (10, 20), or 20-cycle systems. He suggested that when the 10-cycle system developed in different places it was diffused, unlike the two-cycle system that began in the Middle East. From African systems, he suggested that the (2, 10) and (5, 20) systems came together to make the second pentad (6 to 9) as five plus one to four or another word plus one to four. Finally, Seidenberg believed that the body-part tallying was a combining of the two-cycle system with the (5, 20) system as they linked the idea of body part tallying to represent numbers when *tabu* prevented them from using counting words.

Glendon Lean began collecting the counting systems of his students from across Papua New Guinea (PNG) and Oceania in the late 1960s and located first contact records in linguistic and government records and in research journals. Through careful collation and analysis, he was able to publish these data in 1988 for each province of PNG. These data formed part of his doctoral thesis (Lean, 1992) together with counting system data for Oceania (West Papua – the name refugees use – then called Irian Jaya, Indonesia; Island Melanesia; Micronesia; and Polynesia). G. Smith (1984) had also studied the systems in Morobe Province. Both used Salzman's (1950) method of ascertaining the frame words with which all other numerals were named, deciding the operative patterns by which the frame words were combined to make the other numerals and then deciding on the primary and superordinate cycles. For example, if a system has frame words of 1, 2, 5, and 20 by which all other numerals were named then the system was a (2, 5, 20) cycle system. In an earlier paper (Owens, 2001), much of Lean's work was summarised.

Countering the Diffusionist Theory

Seidenberg claimed the two-cycle system was once widespread, but Lean found that there were virtually no systems that were pure two-cycle systems in PNG, and they have not been found en route to South America (e.g., in North America). In PNG, as elsewhere, most two-cycle systems, in fact, went on to have a (2, 5) or (2, 5, 20) cycle system and these existed well before the 10-cycle systems introduced mainly by Austronesian (AN) Oceanic speakers. Furthermore, the paired system was a variation of the AN Oceanic speakers on the southern coast of PNG and generally not found in the non-Austronesian (NAN) systems although there is evidence of different group counting. These Oceanic languages spread from their homeland near New Britain (Owens, Lean, Paraide, & Muke, 2017).

Seidenberg thought 10 cycles preceded (5, 20) and body-tally systems, but Lean countered this by recognising that finger gestures while counting are so widespread among Indigenous people that digit-tally systems would have developed across the world and not after base-10 systems were in place. It is not commonly considered that the body tally systems were older than (5, 20) systems as they are more complex and in many cases, there is no evidence that this practice was linked to sharing body parts in rituals, as Seidenberg claimed. Furthermore, Seidenberg has not explained how the systems with 1, 2, 3 and/or 4 evolved into a system with 10. Finally, having a system that is efficient such as the base 10 system may not be the only reason for a group to take up a counting system from a

neighbour. A few AN 10-cycle systems in PNG changed to a two-cycle system when the people moved inland along a major river valley, the Markham, and were building relationships with people using two-cycle systems, including variants like (2, 5) cycle systems (Holzknecht, 1989; Owens, Lean, Paraide, & Muke, 2017). There are many other reasons: friendliness, trade, length and intensity of contact, desire to be separate or similar, or for what is valued such as multilingual skills, care, confounding others, and for extravagance or prestige or just confusion (Jett, 1971).

Extending Our Understanding of How Counting Systems Develop

Not only does Lean's (1992) evidence contradict Seidenberg on a number of major issues but his analysis also found several other factors of interest in discussing the spread of counting systems. First, it was found that in West Papua, PNG, and the Solomon Islands that NAN and older languages had borrowed some words, mostly in the second pentad from Austronesian (AN) languages. Similarly, some AN languages borrowed words from NAN languages. Table 1 indicates these phenomena. However, these borrowings are not particularly common and more significant is where a whole system is adopted as mentioned above in the Markham Valley (Owens, Lean, Paraide, & Muke, 2017).

Table 1

Summary of Five-Cycle Systems in Austronesian and Non-Austronesian PNG Languages

Phylum or Cluster	(5)/(5, 20)	(5, 10)	(5, 10, 20)
Non-Austronesian	79	21	13
Austronesian	63	113	46

The majority of (5)/(5,20) cycle systems are found in the NAN Sepik-Ramu and Trans New Guinea (TNG) Phyla; (5, 10) in East Papuan Phylum; and (5, 10, 20) in West Papuan Phylum. Among the AN languages, the majority are in North New Guinea, Papuan Tip and Vanuatu; Vanuatu; and New Caledonia, respectively.

Evidence of Large Numbers and Concepts of Infinity

Non-Base-10 Systems

In order to illustrate how these Indigenous systems used large numbers, three language groups are selected: Yu Woi (Mid-Wahgi) from the Jiwaka Province of PNG that displays a digit tally system with vestiges of a four-cycle and from some informants, a body-tally system; Iqwaye from the Eastern Highlands Province bordering Morobe that is a digit-tally system with (2, 5, 20) cycles; and six-cycle systems from a large island off the southern coast of West Papua and languages around the border of West Papua and Western Province, PNG.

Yu-Woi. Muke (2000) obtained data from a number of people from his four tribal groups that have different dialects. The system is a typical (2, 5, 20) cycle system, e.g.

27 is *angek yem yemsi simb yem yemsi, angek yemsi, tak*
 both hands both legs half hand two
 40 is *hi ende simb angek*
 2nd man's legs hands

However, they also count items by twos and in hundreds by grouping them in tens and then tally each ten using their fingers and toes to reach 200. Thus, as one participant said, “for six hundred pigs, they would say that they will kill pigs equal to the hands and legs of three man” (Muke, 2000, p. 134). For the larger thousands, people used the fingers and toes for groups of ten and when they had ten groups of ten, they referred to a specific body part, starting from the head towards the legs, aiding memory of giving for reciprocity (Table 2).

Table 2
Yu Wooi (Mid-Wahgi) Counting System for Large Numbers

Numeral	Yu Wooi and Explanation		
100	<i>elsi or angek yem yemsi peng ngond or peng</i> 10th ten or head 1st 100		
200	<i>komuk</i>	ear – 2nd 100	600 <i>angek daro</i> left hand – 6th 100
300	<i>gnumb</i>	nose – 3rd 100	700 <i>buk</i> back – 7th 100
400	<i>gupe</i>	mouth – 4th 100	800 <i>kumbuk</i> belly – 8th 100
500	<i>angek woiro</i>	right hand – 5th 100	900 <i>simb woiro</i> right leg – 9th 100
1,000	<i>simb daro or hi ende simb angek poro bekenj</i> left leg – 10th 100 or whole body parts of one person		
2,000	<i>hi tak</i> two persons		3,000 <i>hi takendeka</i> three persons

Source: Owens, Lean, and Muke (2017)

One variation of this was the use of the names of fingers for adding almost as a vestige of a body-part tally system. Furthermore, when it came to counting and having the opportunity to use hands, people often counted in twos. They would fold down two fingers at a time saying *eraksi* meaning “take two” each time: two then two on one hand followed by two and two on the other and bringing the four folded fingers of each hand together being *mam erak* followed by the two thumbs with the words *angek yem yem* “together hands” (some also folded two, two, then thumb *eraksi eraksi el* and then two and two and thumb repeated before bringing together and saying *angek yem yem*).

When deciding the number of pigs to be given by each person in a compensation claim, the leader asked people to take the number that they would give from a bunch of small banana fruit. When everyone had offered as they wished, the banana fruit were put together and tallied in groups of 10, each group matched with a digit tally part starting with the fingers.

Iqwaye. Building on a binary system of relationships in which a pair is one or where another number is linked to one (the whole), it seems that PNG and Oceania cultures have a richer understanding of number. There is order in the counting, but temporality related to this order may not be of the Western kind but pulsating back and forth (Mimica, 1988). One such group, *Iqwaye*, has a digit-tally system starting with the thumb of either open hand, then the index finger, etc., moving to the other hand, then counting the toes. (Most language groups seem to count by bending the fingers down starting with the little finger.) Twenty is two hands and two legs or a person. *Iqwaye* refer to the link between the creator and each of the five children represented by a finger as one child or one to five children so one can be one or five can be one.

Using this digit-tally counting system, by which each digit represents a counting word in order, the man standing up becomes one denoted by the thumb again. So, each digit then

represents a multiple of 20. Three fingers could represent three persons or 3×20 . The next iteration is for the digits to represent 20^2 counted off by the crouching man with fingers touching the toes. Thus, two groups of 20 men ($20 \times 20 = 400$) is represented by the thumb and index finger so 1,000 is represented by two fingers and then 10 fingers (each finger representing 20).

400 is Aa' `mnye, aa'mnye, toqwozni tepu hyelaqa kokoloule hyule hwolye hyelaqapu

Person person this-me this that-all their leg hand that-all

'[as] this many persons [as] me this [one] person [speaker] all their legs and hands'

1,000 is 'two persons [as] me this [one] person all their legs and hands and to another person's two hands (= ten persons) all their legs and hands' (Mimica, 1988, pp. 35-36)

Thus, the notion of infinity is generated (Mimica, 1988). This self-generating system of numbers is reminiscent of some modern Western mathematical and binary systems. He suggested that a study of the system shows an intuitive non-Western origin of number, capable of developing into a system and purpose for counting (Brouwer, 1975).

Six-Cycle Systems Near the Border of West Papua and PNG. Donohue's (2008) study of the languages of Kolopom Island has shown that languages like Kanum have developed an interesting variety of counting systems to manage their base-six system for large numbers. There are in fact three systems for small numbers, moderate numbers, and a complex system for one to large numbers, 6^5 . Some number words occur in more than one system, but the complex system is well established except that Indonesian currency note 1,000 has brought confusion:

Some younger speakers are reinterpreting nta_{ma}no '1296' as '1000', ... nta_{ma}no tampa is effectively ambiguous between '5000' (1000×5 ; new reading) or '6480' (1296×5 ; old reading), although only the latter is prescriptively correct. (Donahue, 2008, p. 427)

Evans (2009) has shown that languages of the Morehead-Maros language group in Western Province, PNG and further west also have large numbers for six-cycle languages. For example, Nen count to 6^5 or 6^6 . The counting in these languages seems to relate to counting yams (three and three) six times, since Williams (1936) recounts that in Keraakie with two counters had a yam representing 36 yam in a daisy pattern. Interestingly, different groups represented six by different parts or gestures of the hand.

Base-10 Systems of the Region

The AN languages, and in some places neighbouring NAN languages such as Nasioi and Uisai on Bougainville, used numeral classifiers for large numbers. In most cases, the classifying prefix or suffix were for counting specific groups of objects such as single bananas, a hand of bananas, long thin objects, food items and so on (Chapter 8 of Owens, Lean, Paraide, & Muke, 2017). Fisher (2010) noted, for example, the prefix *po-* for 100,000. Bender and Beller (2006) also suggest that:

The Samoan expression refers to just 2 coconuts whereas the corresponding article in Tongan (2) multiplies this amount by 10-score (200), thus yielding 400 coconuts. It is only when numeral and classifier change their position (as *infua-lua*) in Samoan that a numerical change occurs (from 2 to 20). (p. 396)

Rennelles, a Polynesian Outlier, in the Solomon Islands also shows multiplication for large numbers so the practice was widespread.

Evidence for the Longevity of Counting Systems in Our Region

Lean (1992) drew heavily on linguistic data and archaeological linguistics, and his thesis is supported by more recent evidence. For example, the dating of the spread of Oceanic languages is based on Proto-Oceanic (POC) community and the cultural complex associated with the Lapita-style ceramic tradition found throughout Island Melanesia and western Polynesia (Allen, 1996; Pawley & Green, 1985; Spriggs, 2011).

The deep conceptual structure of counting systems, namely their cyclic nature, rather than just loanwords suggests a long-standing existence of these counting systems. Table 3 indicates the diversity. TNG Phylum accounts for most of the mainland of New Guinea outside of the coastal AN areas has a diversity of types of systems. Eighty percent of body-part tally systems are found in one of the sub-phylum. There is also a cluster of four-cycle systems but they are quite diverse in themselves and so seem to be localised innovations. The majority are variants of two-cycle systems and mainly (5, 20) cycle systems.

Table 3

Distribution of Counting System and Tally Types Among the NAN Phyla

Types	West Papuan	East Papuan	Torricelli	Sepik- Ramu	Trans New Guinea	Minor Phyla	Total
(2)	0	0	0	3	39	0	42
(2, 5)	0	1	16	5	86	1	109
(2', 5)	0	1	3	5	17	1	27
(2'', 5)	0	0	5	3	31	1	40
(5, 20)	0	1	2	17	52	7	79
(4)	0	0	0	1	6	2	9
(6)	0	0	0	0	5	0	5
Body-Parts	0	0	0	8	58	4?	70?
(5, 10)	2	12	0	3	4	0	22
(5, 10, 20)	5	0	0	0	4	3	13
(10)	1	8	0	1	2	0	13
(10, 20)	2	0	0	0	1	0	3

Note. These are numbers from Lean's (1992) collected data, which are most languages but not all languages in PNG and Oceania. They exclude 11 West Papuan languages in North Halmahera.

The Sepik-Ramu Phylum tend to have body-part tallies as well as (2, 5, 20) systems with some variants due to the nearby AN languages. There are also body-tally systems in south-east Australia and Torres Strait Islands. The southernmost languages of the Sepik Hill Stock, especially Hewa, have been influenced by East New Guinea Highlands Stock and other TNG Stock (Wurm, 1982). Body-part tallies may have been introduced into the Sepik-Ramu Phylum languages by such influence and not been an original feature (Lean, 1992). The two branches (Kanum and Moro) of Morehead-Wasur languages have six-cycle systems with large numbers suggesting the Proto-Morehead-Wasur language contained a six-cycle system, thousands of years ago (Evans, 2009). The East Papuan and Bougainville languages tend to have classifiers and there appears to be some influence with AN.

POC contained at least terms for 100 and 1,000 with some northern areas losing these for various reasons, perhaps due to NAN influence with other languages inventing further

systems. Higher powers are common in Polynesian languages sometimes as 2 x powers of ten and probably due to tributes to chiefs. According to Harrison and Jackson (1984), higher powers, at least for Micronesia, may have a later history and several innovations.

The genealogy suggested for the age of the systems is given in Figure 1.

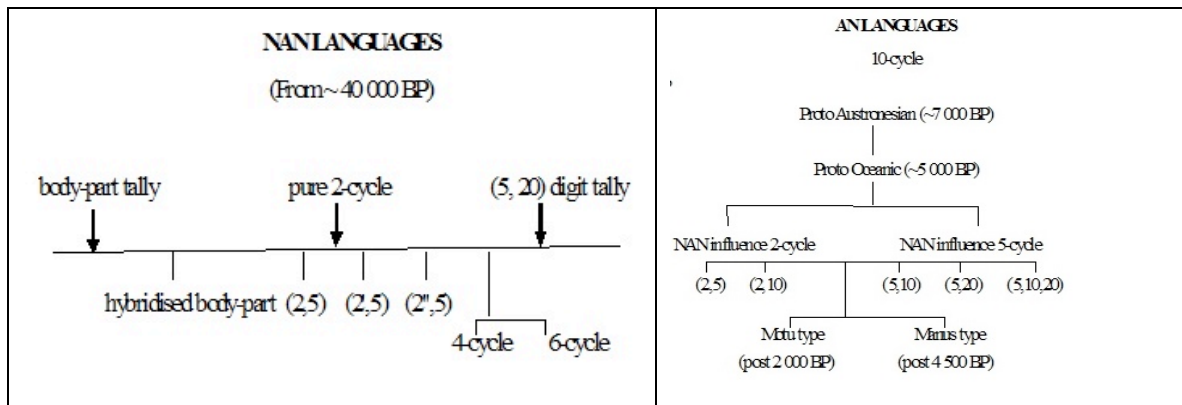


Figure 1. Genealogy of Languages of PNG and Oceania (Source: Owens, Lean, Paraide, & Muke, 2017).

Implications for Changing our Teaching on the History of Number

One of the most significant aspects of this research is that a complex connection between archaeological linguistics, an understanding of counting systems and their cycles, and an acceptance of oral understandings has resulted in new insights into how counting began and was used. Learning about these systems extends students' understanding of counting systems, respects Indigenous cultures, and provides a global mathematics perspective. It counters limited conceptions of history including the development of number and mathematics. It values archaeological linguistics of which Australasia is rich. The importance of groups in early arithmetic and higher levels of mathematics and relationships between numbers can also be enriched. For some students, the link to culture will be a critical way of engendering an interest and understanding in mathematics.

References

- Allen, J. (1996). The pre-Austronesian settlement of Island Melanesia: Implications for Lapita archaeology. *Transactions of the American Philosophical Society*, 86(5), 11-27. doi:10.2307/1006618
- Ascher, M. (1994). *Ethnomathematics: A multicultural view of mathematical ideas*. New York, NY: Chapman & Hall.
- Bishop, A. (1979). Visualising and mathematics in a pre-technological culture. *Educational Studies in Mathematics*, 10(2), 135-146.
- Bishop, A. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht, The Netherlands: Kluwer.
- Brouwer, L. (1975). Guidelines of intuitionistic mathematics. In A. Heyting (Ed.), *Collected works, Vol. 1: Philosophy and foundations of mathematics*. Amsterdam, The Netherlands: North Holland.
- Crawford, J. (1863). On the numerals as evidence of the progress of civilization. *Transactions of the Ethnological Society of London*, 2, 84-111.
- D'Ambrosio, U. (1990). The history of mathematics and ethnomathematics: How a native culture intervenes in the process of learning science. *Impact of Science on Society*, 40(4), 369-378.
- Donohue, M. (2008). Complexities with restricted numeral systems. *Linguistic Typology*, 12(3), 423-429. doi:10.1515/LITY.2008.043
- Evans, N. (2009). Two plus one makes thirteen: Senary numerals in the Morehead-Maró region. *Linguistic Typology*, 13, 321-335.

- Fisher, J. (2010). Enriching students' learning through ethnomathematics in Kuruti elementary schools in Papua New Guinea. Lae, Papua New Guinea: ResearchGate.
- Harrison, S., & Jackson, F. (1984). Higher numerals in several Micronesian languages. In B. Bender (Ed.), *Studies in Micronesian linguistics* (pp. 61-79). Canberra: Pacific Linguistics.
- Holzknicht, S. (1989). The Markham languages of Papua New Guinea. *Pacific Linguistics*, C-115.
- Jett, S. C. (1971). Diffusion versus independent development: The bases of controversy. In C. Riley, J. Kelley, C. Pennington, & R. Rands (Eds.), *Man across the sea: Problems of pre-Columbian contacts* (pp. 5-53). Austin & London: University of Texas Press.
- Lancy, D. (1978). Indigenous mathematics systems: An introduction. *Papua New Guinea Journal of Education*, 14, 6-15.
- Lean, G. (1992). *Counting systems of Papua New Guinea and Oceania*. (Unpublished doctoral dissertations), Papua New Guinea University of Technology, Lae, Papua New Guinea.
- Lynch, J. (1977). On the history of the Tanna numerals and number markers. *Te Reo: Journal of the Linguistic Society of New Zealand*, 20, 3-28.
- Mimica, J. (1988). *Intimations of infinity: The mythopoeia (cultural meanings) of the Iqwaye counting and number systems*. Oxford, England: Berg.
- Muke, C. (2000). *Ethnomathematics: Mid-Wahgi counting practices in Papua New Guinea* (Unpublished master's thesis), University of Waikato, Waikato, NZ.
- Owens, K. (2001). The work of Glendon Lean on the counting systems of Papua New Guinea and Oceania. *Mathematics Education Research Journal*, 13(1), 47-71.
- Owens, K., Lean, G., & Muke, C. (2017). Chapter 3: 2-cycle systems including some digit-tally systems. In K. Owens, G. Lean, P. Paraide, & C. Muke (Eds.), *History of number: Evidence from Papua New Guinea and Oceania*. New York, NY: Springer.
- Owens, K., Lean, G., Paraide, P., & Muke, C. (2017). *The history of number: Perspective from Papua New Guinea and Oceania*. New York, NY: Springer.
- Panoff, M. (1970). Father arithmetic: Numeration and counting in New Britain. *Ethnology*, 9(4), 358-365.
- Pawley, A., & Green, R. (1985). The Proto-Oceanic language community. In R. Kirk & E. Szathmary (Eds.), *Out of Asia: Peopling the Americas and the Pacific* (pp. 161-184).
- Pumuye, H. (1975). The counting system of the Pekai-Alue tribe of the Topopol village in Ialibu sub-district in the Southern Highlands District, Papua New Guinea. *Science in New Guinea*, 3(1), 19-25.
- Raglan, B. (1939). *How came civilization?* London, England: Methuen and Company.
- Ross, M. (1988). Proto Oceanic and the Austronesian languages of western Melanesia. *Pacific Linguistics*, C-98.
- Salzmann, Z. (1950). A method for analyzing numerical systems. *Word*, 6, 78-83.
- Saxe, G. (1979). A comparative analysis of the acquisition of numeration: Studies from Papua New Guinea. *The Quarterly Newsletter of the Laboratory for Comparative Human Cognition*, 1(3), 37-43.
- Seidenberg, A. (1960). The diffusion of counting practices. *University of California Publications in Mathematics*, 3.
- Smith, G. (1984). *Morobe counting systems: An investigation into the numerals of the Morobe Province, Papua New Guinea* (Unpublished master's thesis). Papua New Guinea University of Technology, Lae, Papua New Guinea.
- Smith, G. E. (1933). *The diffusion of culture*. London, England: Watts and Company.
- Spriggs, M. (2011). Archaeology and the Austronesian expansion: Where are we now? *Antiquity*, 85(328), 510-528.
- Strathern, A. (1977). Mathematics in the moka. *Papua New Guinea Journal of Education*, 13(1), 16-20.
- Van der Waerden, B., & Flegg, G. (1975). *Counting I: Primitive and more developed counting systems*. Milton Keynes: Open University.
- Wilder, R. (1974). *Evolution of mathematical concepts*. London, England: Transworld.
- Williams, F. (1936). *Papuans of the Trans-Fly*. Oxford, England: Clarendon Press.
- Wolfers, E. (1971). The original counting systems of Papua and New Guinea. *Arithmetic Teacher*, 18(1), 77-83.
- Wurm, S. (1982). Papuan languages. In R. May & H. Nelson (Eds.), *Melanesia: Beyond diversity* (pp. 225-240). Canberra: Research School of Pacific Studies, Australian National University.
- Wurm, S., Laycock, D., Voorhoeve, C., & Dutton, T. (1975) Papuan linguistic prehistory, and past language migrations in the New Guinea area. *New Guinea area languages and language study. Vol. 1: Papuan languages and the New Guinea linguistic scene, Pacific Linguistics (C-38)* (pp. 935-960). Canberra.
- Zaslavsky, C. (1973). *Africa counts: Number and pattern in African culture*. Boston, MA: Prindle, Weber, and Schmidt.