Forty Years of Teaching Problem Solving

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This paper presents a reflection on problem solving, stimulated by re-reading my paper on teaching problem solving, after 40 years. It describes how seeing problem solving as the ultimate goal of mathematics education reached its zenith in the early 1990s, and how subsequently this has been largely replaced by a less ambitious agenda where working on interesting problems is seen mostly as a teaching methodology to serve other curriculum goals. Equipping students to use whatever mathematics they know to solve problems that arise within and outside mathematics is an elusive goal, but it is the most important.

At the first MERGA conference, I presented a paper entitled "Teaching Problem Solving" (Stacey, 1977). I was in my third year of working in teacher education and I had a special responsibility for the mathematics discipline studies for pre-service primary and secondary teachers. During my doctoral studies in pure mathematics, I had regularly participated in research conferences, but MERGA 1 provided my first opportunity to engage with research in mathematics education. It was a very exciting occasion.

Too much has happened in the intervening 40 years to give a fair and comprehensive summary of progress in this field. There are multiple reasons. First, solving problems is the central activity of mathematics, and to help students do it well is for me the central goal of mathematics teaching. Consequently, this field encompasses all mathematical topics and all year levels, along with applications to other subjects and to life beyond school. As theoretical analysis and empirical studies have clearly demonstrated across the years, most aspects of the classroom environment and student learning have an impact on achieving the problem solving goal: conceptual understanding, procedural fluency, heuristic strategies, understanding the problem solving process, cognition and metacognition, student attitudes and emotions, productive teaching practices, equity, socio-cultural aspects and more.

A summary is also difficult because mathematics educators working in this field are engaged in three mutually supportive but different activities:

- advocacy (influencing teachers and curriculum authorities to address problem solving goals more thoroughly);
- curriculum development and evaluation (e.g., what to teach about problem solving, how to teach for it and assess it, how to design good problems for teaching); and
- systematic research into task and student variables, teacher behaviours, success of professional development, curriculum design, and many other questions.

(Niss, Blum, & Galbraith, 2007 link these with successive time periods.) As a consequence, this short article is a personal account touching only on a few aspects.

Work in problem solving has always been plagued by communicating definitions, especially for advocacy to teachers. In 1977, I saw problem solving as the most important goal of mathematics education (I still do), and I used the phrase to include all problems where mathematical thinking or knowledge contributes to a solution. However, the focus was to assist students to tackle non-routine problems, rather than those that good students "should" be able to solve routinely (e.g., easily recognising that proportional reasoning applies to a certain situation and being able to use it correctly, or solving an equation).

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The 1977 paper and beyond

The paper described an experimental course in problem solving that Susie Groves and I designed at what was then Burwood State College, now Deakin University. We were team-teaching it in 1977 for the second time. The 60-hour problem solving course was nearly half of the first year optional mathematics major (or minor). The paper discussed issues arising in teaching the course, illustrated by examples. The sample problems included designing a car park (geometry of turning circles involved), predicting patterns that can be produced by the Spirograph drawing toy, problems involving mathematics in sport, and finding the best size for a 500 mL drink can (It is not the minimum surface area).

The paper reported a small evaluation of the 1976 course: Half of the students liked it more than their traditional mathematics subjects, one quarter liked it less, and one quarter were neutral. I remember that one "neutral" girl explained that she appreciated how much effort we had put into the course, but whatever we did it was still just work to her. In subsequent years, I have often found a similar 50:25:25 ratio when evaluating innovation.

It is clear from the set of problems described that the label 'problem solving' included both intra-mathematical and real world problem solving. Indeed, what is now called mathematical modelling was the central component. The problems were intended to be non-routine (i.e., not standard applications of learned mathematics) with many of them requiring substantial investigation and a solid written report for assessment. The problems were always open; some open at the start, some in the middle, and some at the end. Solving them successfully required some independence of thought, originality, and common sense, directed by strategic thinking and metacognitive control, and supported by deep mathematical knowledge and a productive disposition. Both Susie and I were strongly motivated to share the joy of mathematical discovery with our students, and to show how mathematics gives insight into real world situations. Our central goal was "teaching for problem solving" to help our students become better problem solvers, giving them confidence and strategies to use whatever mathematics they knew to understand the world.

The Context

In 1977, problem solving was still a fringe activity in teaching mathematics but it was attracting growing interest. For example, Georg Polya had written his famous books about heuristics and made an influential film (Polya, 1966) demonstrating how he worked on a challenging problem with a class. Henry Pollack (1968) was conducting an experimental course at Teachers' College (Columbia) that used mathematical modelling of accessible problems to motivate and illustrate mathematics.

At that time, interest in problem solving was especially strong in university teacher education. Prospective teachers have less need for specific content in tertiary mathematics, but especially need a broad view (Stacey 2008). Sadly, the opportunity to design tertiary mathematics programs specifically for prospective teachers is now rare in Australia.

At a similar time, to support the developing research into learning mathematics, work in psychology on human problem solving was maturing, developing information processing theories and using research methods such as protocol analysis for looking at cognitive processes. This research opened up beyond the cognitive in later decades.

Beyond 1977

Susie and I ran the course for several years, developing it in many ways. We built a wonderful collection of rich problems from many sources – far too many to use them all.

We learned that problem solving was better taught by looking at a fewer problems in depth, encouraging students to generalise and extend solutions, thereby achieving reflection on the solution (Polya's "looking back" phase) by "looking forward". Many teachers beginning to teach problem solving feel they do not have enough good problems, but in fact they "waste" good problems with narrow interpretations of the task until they come to appreciate the gems hiding below the surface, and see multiple solutions.

We also learned to engineer problems for classroom use: for example, to present them so that everyone can start, and to ensure that there was something interesting to find out (ideally a surprise). Many of the problems that we developed, and what I learned from our students' solutions, were included in *Thinking Mathematically* (Mason, Burton, & Stacey, 2010), written mostly in 1980 and now frequently described as "a classic".

Soon, we began to take our problem solving pre-service students to teach problem solving in primary schools. They (and we!) were able to observe children's thinking first hand, and to appreciate how a problem can be solved in multiple ways and at multiple levels. For example, upper primary students could often find a general solution to Arithmogon puzzles when the problem was presented as finding an unknown number of beans hidden in matchboxes arranged around a triangle or square. Students working cooperatively could often create convincing proofs of their method, expressed in concrete terms. At the same time, a general solution using algebra involves the independence/dependence of systems of linear equations that the pre-service teachers were learning in Linear Algebra. These experiences led us to work with teachers in schools to develop problem solving lessons for early secondary school students, and thence into mathematics education research. This work culminated in "Strategies for Problem Solving" (Stacey & Groves, 2006) first published in 1985. The word "Strategies" in the title refers to both teaching strategies and mathematical heuristic strategies.

The Zenith of Problem Solving as the Central Goal of School Education

Australia's attention to problem solving was greatly boosted when the NCTM's Agenda for Action (1980) declared that "Problem solving should be the focus of mathematics education in 1980s" (p. 1). Advocacy exploded and research and development blossomed. Within a few years, every educational jurisdiction in Australia proclaimed problem solving as central to all levels of the school curriculum. In MERGA's 1988 four-yearly review, we published an annotated bibliography (Groves & Stacey, 1988) of 238 recent Australian articles on problem solving. Although classification is somewhat arbitrary, 56 articles described innovative practices, 59 mainly discussed the importance of problem solving in the mathematics curriculum, 65 addressed aspects of teaching problem solving, 51 looked at cognitive processes and seven were concerned with assessment. Lester (1994), a speaker at MERGA1, provides a detailed summary of mainly American work from the 1970s until 1994, when a socio-cultural perspective was added, and Schoenfeld (2008) extends the timeframe.

Just a few years later, assessment of problem solving became a highly controversial issue. Ambitious curriculum and assessment change was implemented in Victoria to cement a place for problem solving and mathematical modelling in the mathematics subjects of the Victorian Certificate of Education. Students worked on substantial 20 hour mathematics projects (investigations) and solved challenging problems, writing up solutions for assessment that contributed directly to tertiary entrance scores. These changes to Year 12 assessment quickly rippled down throughout the secondary school years. Implementation problems caused a backlash, and the initiative has not yet been regained.

Where is Problem Solving now?

Until the 1990s, the strongest thrust in advocacy, research, and development was directed to teaching for and about problem solving, where the main goal is to make students better problem solvers. However, from about 1990, problem solving was split so that reasoning, communication, and connections became separate proficiencies (e.g., in the NCTM Standards of 1989), and the main goal became "teaching through problem solving" in order to teach the specified curriculum better. Whilst this has always been the main inclination of most teachers, it was also boosted by changing national accountability and assessment regimes for schools, as well as national and international thinking. The Australian Curriculum: Mathematics v8.3 (ACARA, n.d.) illustrates this very clearly. The proficiency strands, of which problem solving is one, are said to "describe how content is explored and developed [and] provide a meaningful basis for the development of concepts […]". Making students better problem solvers in any broad sense is not a prominent goal.

At best, the themes that originated in the early problem solving movement are now harnessed in a cluster of related new styles of teaching. Labels include inquiry teaching, sense-making, reform teaching, and standards-based. The themes include the importance of purpose developing autonomy as a learner and as a thinker, understanding the purpose of what you are learning, developing productive habits of mind including persistence, appreciating rigorous arguments, making conjectures. The evaluation of the performance of students who have been consistently taught in this way (mainly US studies comparing 'reform' and traditional curriculum programs) generally shows that 'reform' students perform about the same on skills, and better on concepts and applying their knowledge.

Problem solving in the 1977 sense is *the* reason for teaching mathematics. It cannot just be thought of as a teaching method or one of a number of goals. We need to reinvigorate efforts to value and work towards this most elusive, but most fundamental, benefit of learning mathematics.

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