Beginning teachers learning to teach mathematics through problem-solving

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New Zealand curriculum documents have referred to mathematics as a problem-solving endeavour for more than 25 years. Although an intended curriculum focus, problem-solving seems to be an aspect of mathematics that many beginning teachers are not familiar with. This research follows three beginning teachers in their first year of teaching as they incorporate problem-solving within their mathematics programmes. Data shows that familiarity with a structure for mathematics lessons that fosters problem solving and reasoning (Sullivan, Walker, Borcek, & Rennie, 2015) supported beginning teachers' subsequent successful efforts to teach a problem-solving lesson.

The New Zealand primary classroom is a multi-faceted, complex context in which teachers are required to educate children in mathematics. In recent decades there has been a focus on problem-solving in mathematics in New Zealand curriculum documents. For example, Mathematics in the New Zealand curriculum (Ministry of Education, 1992) advocated for the use of a problem-solving approach, and the current New Zealand Curriculum (Ministry of Education, 2007) also explicitly refers to mathematics as problem-solving, an approach expected for all levels of schooling. However, although problem-solving has been an intended curriculum focus for more than twenty years, it has often been overlooked in many mathematics classrooms (Holton, 2009).

Research with pre-service teachers indicates that experiencing a problem-solving approach within their teacher education programme enables the beginning of envisaging how such mathematics pedagogies could be enacted in future practice (Bailey & Taylor, 2015). In this current research study, a group of beginning teachers who experienced problem solving within their teacher education programme have been followed into the classroom to explore what enables and constrains them in adopting a problem-solving approach for the teaching and learning of mathematics.

Literature review

Mathematics is a social, constructive and creative human endeavour (Mason, 2008; Solomon, 2009), with problem-solving an integral part of this discipline (Schoenfeld, 2007; Liljedahl, Santos-Trigo, Malaspina & Bruder, 2016). Mathematics is neither separate to one's self, nor is it finite, but a creation that is "never finished, never completed" (Barton, 2008, p. 144). Moreover, mathematics is created by communication between people (Barton, 2008), and involves experimentation, observation, abstraction and construction.

For more than 25 years New Zealand curriculum documents have encouraged the teaching of mathematics as a social, constructive endeavour with a focus on problem-solving. In the most recent document (Ministry of Education, 2007) mathematics and statistics are presented as an active endeavour, with learners expected to be creating, exploring, investigating, justifying, explaining, communicating and making sense

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(Ministry of Education, 2007). The lists of mathematics achievement objectives also highlight problem-solving, prefacing the objectives for every year of schooling (year 0-13) with the statement, "In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will *solve problems* (emphasis mine) and model situations. ..." (Ministry of Education, 2007).

Mathematical problem-solving is a term that is interpreted in different ways, by teachers and texts alike. In this research mathematical problem-solving refers to "the solution of problems, the method of which is not immediately obvious to the potential solver" (Holton, Anderson, & Thomas, 1997, p. 3). It differs considerably from traditional "triple-x" mathematics lessons of teacher explanation followed by examples and exercises (Foster, 2013). In a problem-solving approach, students develop a conceptual mathematical understanding in the process of creating their own strategies to solve problems. In such a classroom mathematics is regarded as creative, imaginative and includes an emphasis on the communication of emerging ideas and concepts (Boaler, 2016; Ministry of Education, 1992, 2007) rather than being a solitary experience with the teacher being the dispenser of mathematical knowledge.

Despite problem solving being at the heart of mathematics and a problem-solving emphasis in curriculum documents this vision of mathematics, and mathematics teaching and learning has not been consistently adopted (Holton, 2009). For many beginning teachers their previous experiences of learning mathematics are more likely to be aligned with lessons such as those described as "triple-x" (Foster, 2013) lessons. As pre-service teachers however, they may begin to encounter a different conception of mathematics with a likely focus on core practices of ambitious teaching. Problem solving is central to ambitious teaching and refers to supporting all learners to develop conceptual understanding, procedural fluency, strategic competence and adaptive reasoning to solve authentic problems (Lampert & Graziani, 2009; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010). Research has shown that *experiencing* problem solving firstly situated as a learner is an important first step towards pre-service teachers learning about mathematics as a problem-solving endeavour (Bailey & Taylor, 2015). Cavanagh and McMaster (2017) suggest such experiences need to be further supported by having opportunities to observe experienced teachers teaching problem solving lessons, followed by engaging in co-teaching a problem-solving lesson.

In the late 1990s a research project specifically investigating teachers' learning about teaching mathematics by problem-solving concluded that with more widespread use of problem-solving there will be challenges for teachers and a need for more professional development (Holton, Anderson, & Thomas, 1997). Cavanagh and McMaster (2017) echo this sentiment, writing, "a problem-solving approach to teaching mathematics presents a major challenge for many PSTs [pre-service teachers] in primary education" (p. 48). Sullivan, Walker, Borcek and Rennie (2015) comment that even though there is agreement about the importance of incorporating problem-solving and reasoning into mathematics teaching there is limited specific advice about how to do this. They propose a defined structure for lessons that support children's learning when using tasks intended to prompt problem solving and reasoning.

A Structure for Lessons that Fosters Problem Solving and Reasoning

The lesson structure suggested by Sullivan, Walker, Borcek and Rennie (2015) comprises four phases described as the 'Launch', an 'Explore' phase, a 'Summary' phase followed by a 'Consolidation' phase where additional experiences are posed to consolidate the learning activated by the initial task.

Posing the task is a critical aspect of structuring a problem-solving lesson. Two key tasks are establishing a common language, so the task is interpreted appropriately, and deliberately maintaining the cognitive demands of the task (Sullivan, Walker, Borcek, & Rennie, 2015). During the 'Explore' phase children work individually or in small groups, with the teacher thoughtfully walking around the desks. A key aspect of this phase is the teacher having already anticipated ways that different students might respond to the challenge by pre-planning questions/tasks that differentiate the experience. It is suggested this is done by the provision of enabling prompts and extending prompts. Enabling prompts involve "reducing the number of steps, simplifying the complexity of the numbers, and varying the forms of representation for those students who cannot proceed with the task" (Sullivan, Walker, Borcek, & Rennie, 2015, p. 44). It is important to note that this is done with the explicit intention that the children subsequently return to work on the initial task. Extending prompts are offered to students who "complete the original task quickly which ideally elicit abstraction and generalisation of the solutions" (Sullivan, Walker, Borcek, & Rennie, 2015, p. 44).

In the 'Summary' phase the way student activity on a problem/task is reviewed, including solutions and strategies, needs to be carefully managed. Sullivan, Walker, Borcek and Rennie (2015) in referring to the work of Smith and Stein (2011) describe the key elements as:

> Selecting responses for presentation to the class and giving those students some notice that they will be asked to explain what they have done;
>
> Sequencing those responses so that the reporting is cumulative; and
>
> Connecting the various strategies together (p. 45).

The last phase, consolidating the learning, involves posing a task similar in structure and complexity to the original challenging task. Some elements of the original task remain the same while other aspects change to help the learner avoid over generalisation from solutions to one example.

Building on Sullivan, Walker, Borcek and Rennie's (2015) research with teachers who "were a mix of age and experience, although skewed toward being more experienced" (p. 46) this article investigates the use of the lesson structure by beginning teachers. This is one aspect of an ongoing research project following a small group of beginning teachers through their first two years of teaching, with a focus on their learning to incorporate problem-solving within their mathematics programmes.

The Context, Data Collection and Analysis

In this qualitative action research project, data were gathered from three beginning teachers who responded to an invitation to be involved in the project at the end of their one-year graduate diploma in primary teacher education. The beginning teachers (hereafter called teachers) were teaching a diverse range of year levels at three different schools. Julia (pseudonyms have been used for all names) was teaching year 0-2 children at a small rural school; Charlotte, year 5-6 children at an urban city school; and Reine, year 7-8 children, at another small rural school.

Action research has an emphasis on the participation and collaboration of all involved in the research. My role has been that of a facilitator of this small group contributing my experience as a mathematics educator and researcher. A facilitator acts as a co-participant within an inclusive network, aiming to improve practice, challenge and reorient thinking, and transform contexts for learning (Locke, Alcorn, & O'Neill, 2013). Congruent with the principles of action research activities and procedures were negotiated throughout the year in a co-constructed, responsive and emerging way as the research evolved.

The first step in the process was a one-and-a-half-hour focus group discussion at the beginning of the year before teaching had begun. The aim of the first discussion was primarily determined by me as facilitator and included inviting the teachers to reflect on problem-solving aspects of the mathematics education paper (completed as part of their teacher education year); brainstorm how they might introduce this approach in their first year of teaching; and identify and plan what actions might be needed to support their trialling of this approach in their first year of teaching. It was agreed during this first focus group discussion that some workshops would be useful to support their learning and reflections. Three of these were subsequently held, each for three hours, at the end of terms one, two and four (the beginning teachers opted for these to be held during school holidays). It was also decided that I would visit and observe the teachers teach a problem-solving lesson in term three. All focus group discussions, workshops, lesson observations were audio-taped and transcribed.

Data from the workshops was analysed using an emergent analytical approach (Strauss & Corbin, 1994; Borko, Liston, & Whitcomb, 2007) as the year of research unfolded. Transcripts were read and re-read with notes taken as particular issues and themes emerged. These notes not only constituted data analysis but also provided direction for subsequent workshops. During the analysis of the first workshop the teachers all expressed concern about catering for the diverse range of learners within their classes. They pondered that it would be useful to have 'extensions' ready to give those children who solved problems more quickly. It also became apparent that they needed to know the problems they were using more thoroughly and understand in more depth how a problem could be planned to cater for a range of children. In response to this finding I searched for a resource that might be helpful to share during the second workshop, and in the process located the article by Sullivan, Walker, Borcek and Rennie (2015). The specifics around sharing ideas from this article, the teachers' responses (during workshops two and three – workshop three was primarily a feedback session about the year, and planning for the next year) and brief reference to observations of all three teaching a problem-solving lesson, constitute the bulk of the data presented below.

The Second Workshop

The professional learning in the second workshop began with a discussion based on the teachers' problem-solving experiences in term two. During the first workshop we had co-constructed a format for a problem-solving session (like that proposed by Sullivan, Walker, Borcek, & Rennie, 2015). During the second workshop I asked the teachers what had worked well, what they found easy and where they encountered challenges. We then discussed the structure for a problem-solving lesson as suggested by Sullivan, Walker, Borcek and Rennie (2015) (referring to a one-page handout constructed by the author based on the four phases described by Sullivan, Walker, Borcek and Rennie (2015)). The teachers then engaged in solving a "river crossing" problem (an algebra problem based on the linear relationship $y=4x+1$) as the author deliberately modelled the sequence of phases suggested by Sullivan et al. (2015). During this experience teachers varied their role between 'being teachers' thinking about the suggested structure in relation to their previous teaching experiences and being in the role of mathematical problem-solvers. After the "river crossing" problem was solved (with two of the three having begun a consolidation task) time was spent discussing their experience making specific links to each of the four phases. Another one-page handout was provided, giving examples that linked the "river crossing" problem to each of the four phases. At the end of each workshop there was an opportunity for written reflection about what had been learned, and what might be tried during the following term.

Results and Discussion

An analysis of the discussion from workshops two and three yielded two key findings pertaining to the use of the lesson structure (Sullivan, Walker, Borcek, & Rennie, 2015): the first being that the teachers found the structure helpful for their learning about how they might teach mathematics through problem-solving. The structure, appeared to enable two of the three teachers, Charlotte and Reine, to engage more fully in teaching mathematics through problem solving. Julia had already gained some experience through teaching problem-solving lessons one day a week for most weeks in terms 1 and 2. The second finding was that examining the structure alongside the first-hand experience of solving a problem appeared to engender personal reflection for improving their future practice. All teachers were able to identify what they needed to change to more successfully conduct problem-solving lessons in the future.

Problem-Solving Lesson Structure Helpful

During the first workshop one of the tasks was co-constructing a 'list' of useful tips for teaching a problem-solving lesson: it was envisaged this could become a 'guide' for teaching mathematics through problem-solving. As the facilitator I envisaged the teachers would contribute ideas based on their problem-solving experiences from term one. Julia had been trialling problem solving lessons approximately once a week. Charlotte and Reine had attempted to use a couple of problems at the beginning of term one and not since then. The compiled 'list' read as follows (NB. The list was co-constructed with ideas contributed by all participants hence some aspects of the list have a range of ideas):

Begin lesson with teacher reading through the problem with whole class.

Give individual thinking time. Maybe answer, clarify some questions from children about the problem.

Organise class to work on the problem individually or in pairs or small groups up to 3. If children, choose to work independently it's OK for them to also have 'talking-time'. Maybe have a table for those who choose to work independently.

Think carefully about how to visually present the problem. Maybe not too many problems. Maybe have the piece of paper folded in half with the beginning of the problem on the first side, and extension questions on the second.

Offer equipment but don't be specific, so children are doing thinking from the start. Allow choice.

Say to children: If you know what to do, go and begin. And then maybe work with those who still have questions (but make sure you don't start solving the problem for them!).

Group children according to teacher choice OR mixed ability OR might try letting children group themselves.

Need to teach group behaviour. E.g. Set up expectation that teacher can ask anyone. Shift the focus from the answer.

Have the same problem for all children and know how to extend the problem.
Once children are working, teacher observes the groups, and asks questions.
Monitor groups. Choose when to extend. Check they all really understand the answer. Avoid the temptation to rush on. E.g. have they solved the problem in different ways?
Use scrapbooks and groups record how they solve it.
Whole class feedback. Think, pair, share for older children. Maybe share between two groups. Pick random or pick specific groups/individuals.

It was during the second workshop the structure for a problem-solving lesson proposed by Sullivan, Walker, Borcek and Rennie (2015) was shared, experienced and analysed. Consistent with the finding in Sullivan et al.'s (2015) study that teachers found the lesson structure useful and achievable, these *beginning* teachers experienced likewise. From the beginning of the discussion all were able to make connections to each of the four phases, and even offered some suggestions building on those offered in the article by Sullivan et al. (2015). For example, Reine suggested having a two-stage launch process. He said, "I wonder if it's like two parts – like you just initially clarify words you're not sure of… and then you go a bit deeper after the start ...". Reine was referring to a possible need for further clarification once the children had had an opportunity to work with materials and begin solving the problem. At the end of the second workshop after the teachers had engaged in solving the "River Crossing" problem and analysed their experience against the structure, they were asked what the main thing they had learned was. All of them referred to the structure specifically identifying the enabling and extending prompts as useful.

> Reine: That you can give the entire class a problem, you've just got to have a plan, [plus] your enabling and extension prompts.
>
> Charlotte: Yeah, I think the main thing is that using the same one – it's important and it's useful for your whole class to be working on the same thing. And kind of [how] easy it is to have enablers and extenders to make sure that everyone feels successful. And, that approaching it, and how to present it to your class. Yeah.
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> Julia: … I think around that planning. Yeah ensuring that I plan, and trialling the problem, planning it well so that we've got enabling and extending prompts. But the other thing I'm going to try and make sure I do is how they're recording what they're learning.

During the next term all three teachers were observed teaching a problem-solving lesson. All three lessons delighted the respective teacher, with them noting the prolonged engagement of the children, the resulting learning and being able to cater for all learners with the one problem. In Charlotte's words, "it really worked".

During the final workshop of the year feedback was sought about which aspects of the research throughout the year had been most useful. All three teachers identified the lesson structure proposed by Sullivan, Walker, Borcek & Rennie (2015). Julia said "I think for me, it was talking about what a problem is. And the framework [referring to the lesson structure] of how to present a lesson that way". Reine commented,

> I like the framework. So, from start to finish, how you go through that whole lesson. So how you set it up. And then you go through the phases. Then off that I like the, the prompts that we went through…. knowing where, where you could go, if they're like, 'What do I do?' And then like if they get it too easy, then where can you go? So, you've got all these little avenues.

Charlotte said,

> Definitely that the – like the problem that you've experienced the problem – that whole like, setting it out. Yeah. And at the start I didn't – I guess I didn't use that very well, or that much. And then that time that you came and observed I planned it that way. And it really helped, and really worked. So, I found that really useful. And I think it was, it was really useful for me and my class. 'Cause they really understood. And just – I think also making sure that you know, yeah, like all the ins and outs of a problem. So where could they go? What do they... What do you need to know? What do they need to know? And all that stuff.

It is interesting to note that during term two (after workshop one where we compiled the 'list') Julia had once again been trialling problem-solving lessons approximately once a week, whereas Charlotte and Reine had not made any further significant attempts. In contrast following the second workshop all three teachers were keen to be observed and to trial the lesson structure they had experienced and analysed. Despite the similarities between the compiled 'list' which all the teachers had contributed to, and the lesson structure, it is possible the defined and focused nature of the four phases in Sullivan, Walker, Borcek & Rennie's lesson structure (2015) was more effective. It is also likely that aligning the lesson structure against the first-hand experience of solving the "River Crossing" problem facilitated more understanding of the lesson structure. During the first workshop we had aligned the generated 'list' against a problem for new entrant children (in an effort to respond to a previous question from Julia who was teaching new entrants). Maybe because the mathematical content did not constitute a genuine 'problem' for the teachers, connections were not made at that initial stage.

Identifying Needed Changes in Problem-Solving Teaching Practices

Examining the lesson structure alongside a first-hand experience of solving a problem also enabled all three teachers to reflect on and identify what aspects of their practice they needed to change to more successfully conduct problem-solving lessons. Charlotte explained that she had previously not known how to extend the problems, explaining,

> I think that's what I'm kind of missing out is the... Like these kind of extensions, write it down as a general equation, write it as an equation. You know? Like I think I'm skipping that, 'cause they go, 'Oh yeah, but it's this many trips.' But then the generalising and the recording of it....

Julia realised that she had probably been under-planning. She said, "I feel like at times I've been underprepared with the problems I've used. Like I haven't spent enough time thinking about – or tried it, or spent any time thinking about what the kids might do with it". Reine agreed commenting that, "yeah sometimes I just print it all out the morning before". This is an important learning for these teachers, given Sullivan, Walker, Borcek and Rennie's (2015) similar finding that teachers who are more successful in terms of improvement in students' responses are those who did the task/problems before the lessons. It seems likely that being able to utilise the lesson structure to reflect on their current practice has potential for improving the teachers' future practice.

Summary

Given this is a small study care needs to be taken with making claims and generalising. However, it appears that for at least this small group of beginning teachers, providing and analysing the lesson structure (Sullivan, Walker, Borcek, & Rennie, 2015) in conjunction with a genuine problem-solving experience facilitated their learning. This finding has implications for teacher educators in that beginning teachers need to not only have first-hand experience of solving problems, but also opportunities to analyse their experience against the lesson structure. The lesson structure also appears to offer a reflective framework against which the teachers could identify what needs to change and improve in their current problem-solving teaching practice.

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