Improvising in the Primary Mathematics Classroom

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If classrooms are dynamic, then mathematics teachers will need to improvise mid-lesson from time to time. Teachers' capacity to improvise is usually analysed via a cognitivist lens. This study contrasts a cognitivist analysis of a primary teacher of mathematics with an ecological analysis. The ecological approach was able to develop a more detailed characterisation of teacher improvisation where attention to, and manipulation of, environmental entities supported improvisation. This characterisation of improvisation is posited to have potential in developing novice teachers' capacity to 'think on their feet' in mathematics lessons.

Teachers' capacity to improvise mid-lesson has been posited to be a key element of teacher expertise which facilitates student learning in mathematics (Hatano & Inagaki, 1986). While there are theoretical frames which are argued to be able to model this capacity to improvise mid-lesson (Schoenfeld, 2011), a conceptualisation of teacher improvisation that can enable the structured development of this capacity is lacking (Grossman et al., 2009). This report compares two theoretical frameworks – Schoenfeld's (2011) model of teacher in-the-moment decision making and a framework developed from ecological psychology (Gibson, 1979) – which both seek to enable structured analysis of teachers' mid-lesson improvisation. Data collected as part of a larger study of mathematics teachers' in-the-moment practice is used to contrast each account of teacher improvisation. The data were collected using Head-mounted, Video-cued recall interviews (HMV interviews), a data collection method which was developed in firefighting (Omodei, McLennan, & Wearing, 2005), and deployed in a Foundation-level (first year of primary school) mathematics class in a primary school in Melbourne.

Literature Review

This report is the result of a lucky event that occurred during data collection. The larger study that this report draws from (Jazby, 2016) investigates how mathematics teachers adapt mid-lesson by combining both video and interview data collection techniques. It was envisaged that during the head-mounted camera data collection phase of the study, participants knew that a researcher was coming to their school to attach a camera to their head, so it would be unlikely that improvisation would occur. Luckily, given the researcher's interest in how teachers think on their feet, one of the study participants (Hannah) found that her meticulously planned lesson was not going to work, and she needed to improvise. Hannah had provided 5 and 6 year-old students with collections of 30-40 icy pole sticks. She had expected the children to group the sticks into bundles of ten to be able to count the collections of sticks effectively. Instead, the children were able to count the sticks by ones accurately and quickly, and Hannah felt that she had to improvise if she was going to be able to meet her goal for the lesson, which was to explore grouping strategies and place value with the students. The data captured from this lesson provides a rare opportunity to analyse an improvised sequence of teaching that occurred spontaneously on a day when cameras happened to be rolling. Given the rarity of such

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occurrences (Lipshitz, Klein, Orasanu, & Salas, 2001), this event provides a means to contrast and compare different theoretical models of teacher improvisation, so that the way in which each model frames the phenomena can be contrasted.

The research question which guides this study is: what characterisation of teacher improvisation in a mathematics lesson emerges when Hannah's episode of improvisation is analysed using different theoretical lenses? Two lenses are used to provide contrasting characterisations; a cognitivist lens, and an ecological lens. For the sake of brevity, one cognitivist model – Schoenfeld's (2011) theory of teacher in-the-moment decision making – is presented as being representative of cognitivist approaches. While Schoenfeld is not the only researcher to present a model of mathematics teachers' in-the-moment behaviour, his decision making model provides an example of an information processing account of teacher improvisation which is particular to mathematics education, and is well established in the field. Schoenfeld (2011) argued that teachers engage in a process of goal prioritisation which leads to the selection of particular courses of action mid-lesson. When decision points are reached mid-lesson, teacher actions can be modelled by assuming that the teacher engages in a calculation of subjective expected values relating to the options that are available to the teacher. In the situation faced by Hannah – realising that her lesson plan was not going to work several minutes into the lesson – Schoenfeld's (2011) model would characterise her behaviour as being driven by a change in goal prioritisation. A new goal, developing a new course of action for example, would be prioritised by Hannah, and she would start to engage resources such as her knowledge to think of alternative courses of action. As alternatives are arrived at, a calculation of subjective expected values can be used to select the best course of action. Schoenfeld (2011) acknowledges that this particular process (running through each option in your mind and working out the perceived costs and benefits of each course of action) cannot be performed at a conscious level in time-pressured decision making, but he claimed that this calculation enables accurate modelling of what decisions teachers are likely to make. Schoenfeld's model (2011) is characteristic of an information processing account of in-the-moment cognition (Jazby, 2016; Lipshitz et al., 2001). The main drivers of improvisation in this kind of account are internal mental processes and mental entities.

The alternate lens considered in this study is derived from ecological psychology (Gibson, 1979). Jazby (2016) developed an ecological account of mathematics teacher noticing which draws on ecological models of in-the-moment behaviour developed in the research area of Human Factors (Kirlik, 1995). In an ecological model, a person's cognition is viewed as occurring within an environment, and how a person interacts with their environment – largely through perceptual interaction – is considered as drivers of behaviour with less emphasis on internal mental processes and entities. While entities such as knowledge are not seen to be completely irrelevant to skilled task performance (Kirlik, 1995), less weight is accorded to mental entities and processes when behaviour is viewed through an ecological lens. Instead, a skilled performer is argued to have developed perceptual routines (ways of moving through a task environment and deploying their attention) which enable them to put themselves in the right place at the right time to perceive environmental entities that can provide guidance for behaviour (Gibson, 1979; Jazby, 2016). In the situation faced by Hannah, the realisation that her lesson plan would not work could result in her engaging a perceptual routine (a way of moving around the classroom and deploying her attention) which would enable her to find environmental structures that could help her know what to do next. As she perceived environmental structures (such as student movements, utterances and the mathematical representations

students were making) she would perceive these entities as being meaningful. The way a student piled counters, for example, could be seen by a teacher as indicating that student is engaging a particular type of mathematical thinking.

Kirlik (1998) pointed out that skilled performers not only are able to identify meaningful environmental structures mid-performance, they are also able to manipulate their environment in ways which create or increase the likelihood that such structures will exist. If you want to know what a student is thinking about a counting task, for example, you could direct a student to count objects and watch what they do. This would count as a manipulation of environmental structure according to Kirlik's ecological model. Unlike the characterisation produced by an information-processing model, teachers are not passively waiting for environmental structures to emerge mid-lesson in an ecological model; they actively manipulate the classroom so that particular environmental structures emerge. As a performer attends to meaningful environmental structures, they perceive an affordance structure (Kirlik, 1995) which provides a sense of what is possible within the current environment. Gibson (1979) argued that perception of an affordance is the perception of what can be done. Perception of a chair, for example, carries with it perception of 'sit-ability' with very little deliberative cognitive processing required (Gibson, 1979). Kirlik (1995) argued that as a skilled performer attends to multiple meaningful environmental structures simultaneously, they develop an awareness of an affordance structure rather than perceiving individual affordances and constraints. In Hannah's situation, Hannah is theorised to be attending to multiple meaningful environmental structures as she improvises. This will give her an awareness of what she can and cannot do mid-lesson, as her attention to her environment will enable her to 'pick up' information that can guide her behaviour more than relying on an internal cognitive process that relies on memory and information processing (Gibson, 1979).

Table 1
Primary Drivers of Teacher Improvisation in Cognitivist and Ecological Models

	Driver	Description	Relevant research
Cognitivist account	Teacher knowledge	Often described in terms of PCK, CK and PK – what teachers know about the content and teaching	Schoenfeld (2011)
	Teacher goals	What a teacher wants to do; these change as the lesson progresses	Schoenfeld (2011)
	Prioritisation of goals	A mental process which selects which goals a dominant at a particular time	Schoenfeld (2011)
	Calculation of subjective expected values	A mental calculation regarding the perceived benefit/cost of a particular course of action	Schoenfeld (2011)
Ecological account	Perceptual routines	The way in which a teacher moves and deploys attention mid lesson	Jazby (2016)
	Meaningful environmental structures	The specific structures within the classroom that a teacher sees as meaning something relevant to teaching	Jazby (2016), Kirlik (1995)
	Manipulation of environmental structures	Teacher actions which create new sources of environmental structure	Jazby (2016), Kirlik (1998)
	Perception of an affordance structure	What a teacher perceives as being possible within current classroom conditions	Jazby (2016)

When data relating to improvisation is analysed from a cognitivist or an ecological perspective, particular drivers of improvisation are posited to exist. Table 1 provides a summary of key drivers of teacher improvisation in Schoenfeld's (2011) model and contrasts them with the drivers posited by researchers who employ an ecological lens. Each lens posits a different ontology of entities which drive behaviour – a cognitivist lens directs researchers to look for mental processes and entities, while an ecological lens directs researchers to analyse perceptual behaviour and environmental structure. In order to ascertain how each theoretical lens characterises Hannah's improvisation, the same set of data are analysed, but different drivers of behaviour are used to guide coding of the data. Particularly in the post-lesson interview stage of data collection, a cognitivist lens has been used to code evidence of cognitive process or epistemic claims that can be used to develop a model of Hannah's knowledge and cognition during the period of improvisation. The ecological approach leads researchers to code perceptual interaction and the environmental structures Hannah attends to. By contrasting these two approaches to analysing the same data set, it is hoped that two characterisations of Hannah's improvisation can be developed and contrasted.

Method

Head-mounted, Video-cued recall interview method (HMV interview method) has developed from research concerned with investigating in-the-moment decision making in fields such as firefighting in the 1990s (Omodei et al., 2005). As part of a larger study (Jazby, 2016), three teachers were asked to wear a head mounted camera as they taught a mathematics lesson they had planned themselves to their regular class. Approximately 10 minutes after the lesson, teachers reviewed the head-mounted footage and provided a commentary on what they had been thinking, feeling, and attending to following a free-recall protocol developed by Omodei et al. (2005). The aim of the method is to use the head-mounted video footage to prompt 're-experiencing' of the activity of research interest, so that the detail of recall data collected is more detailed and less narrativised than data collected without use of a head-mounted camera (Omodei et al., 2005).

This study analyses data collected from Hannah's classroom. Hannah, unexpectedly, found that her lesson plan was not working mid-lesson and felt that she had to improvise. Hannah was in her 6th year of full-time employment as a primary teacher and her students were Foundation students (first year of school – approximately 5-6 years old). Hannah first noticed that her lesson plan was not working during a phase of between-desk instruction. This phase of the lesson occurred after an introduction phase of the lesson and prior to a summing up phase at the end of the lesson. In order to analyse Hannah's improvisation, data collected from the approximately 15-minute period of between-desk instruction was prepared.

During this period, three sources of data were collected: stimulated recall data collected post lesson, video data collected from the head-mounted camera, and video data collected from a fixed position camera. Data that could provide evidence of Hannah's cognitive processes – including the knowledge activated mid lesson, her goals, how she prioritises goals and her calculation of subjective expected values may be present in the recall data collected. The researcher asked Hannah to describe such processes as the free-recall interview began, and a few prompts such as, "what were you thinking?" and "what was your goal here?" were used if Hannah paused the video and singled out a segment of the lesson as being significant.

Coordination of head-mounted and fixed-position camera data enabled perceptual routines to be identified. The fixed position camera captured how Hannah moved around the classroom while the head-mounted camera captured what was in Hannah's field of view as she moved her head. Camera data could not be used to ascertain what she was attending to. Recall data provided evidence of this, as Hannah made statements such as, "I was looking at what these two were doing" while pointing to a pair of students when reviewing the head-mounted footage. When Hannah made statements that provided evidence of what she had been attending to, the head-mounted video data provided information relating to the way in which the environment was structured when Hannah was attending to it. Hence, the video data was not used to infer what Hannah was attending to, as the post-lesson interview provided Hannah's recall of her what drew her attention mid-lesson, but by coordinating the recall and video data the structures within the environment that Hannah recalled attending to could be identified. This enabled meaningful environmental structures to be identified, and as Hannah took action, changes in these structures could also be seen in the video data.

While no particular prompts were provided regarding perception of an affordance structure, Hannah made comments regarding 'seeing' what she could or could not do during the lesson. Description of meaningful environmental structures were frequently followed by comments relating to what could and could not be done in relation to those structures. For example, Hannah recalled, "I saw them counting by ones and then I looked over there and I saw them bundling and I thought, of course the ones are going to be quicker because you're just going 1, 2, 3, 4, 5, 6, 7 and the tens are going to be longer because they actually have to stop and bundle them … so it [the lesson plan] wasn't going to work; I'm going to have to change the lesson". When viewed in terms of an affordance structure, perception of the students' counting strategies via attention to the meaningful structure of the students moving icy pole sticks also carries information about what can or cannot be done if the lesson goals are to be accomplished. Hence, these particular data have been taken as evidence that Hannah perceived a constraint in relation in continuing with the lesson plan, and this perceived constraint, which guided her subsequent behaviour, was perceived via attention to particular elements of environmental structure with negligible recourse to deliberative cognitive processing. She also perceived affordances from attention to meaningful environmental structures. When asked why she stopped the class at a particular point in the lesson, she recalled that, "I saw that they'd counted by ones but then I could hear these two boys saying something about bundling … so I wanted to use them as an example and get them to show that they'd changed their mind". The improvised class discussion that took place after Hannah perceived this opportunity for action involved asking the boys questions about their strategy and why they'd changed their minds. This was also coded as evidence of an affordance structure.

Results

Direct Evidence of Cognitivist Drivers of Mathematics Teacher Improvisation

Hannah made 5 epistemic claims during this period of improvisation. Three of these claims related to knowledge of particular students, one claim was a general claim regarding how children learn, and the last claim related to mathematics. None of the epistemic claims made directly relate to how Hannah developed an unplanned task while concurrently engaged in teaching a lesson. Hence, there are not enough directly stated epistemic claims made which would enable an account of Hannah's improvisation to be constructed.

Hannah did not recall her teaching goals without prompting during the post-lesson interview. Her stated goal was "to teach the kids about grouping into 10s". When asked whether her goal changed during the lesson, she stated, "no". As she stated that she only had one goal during the lesson, there is also no evidence of goal prioritisation evident in the available data. There is also no evidence of calculation of subjective expected values in Hannah's recall data.

Direct Evidence of Ecological Drivers of Mathematics Teacher Improvisation

In terms of perceptual routines, at the beginning of the between-desk instruction phase of the lesson, Hannah stood at the front of the classroom and made small head movements left and right. She recalled attending to what the students were doing with icy pole sticks at this point. She claims that the realisation that the planned activity would not work occurred during this perceptual routine. She then began to walk between desks and switched her attention between student faces and the mathematical representations that students were constructing on their desks. She then directed students to move with her between desks, looking at the mathematical representations that had been created, before directing the students to stand on one side of a table while she led a discussion at three different tables centred around the representation that had been created. In this phase, she was able to keep most students' faces within her field of view. She then returned to her between-desk routine, before returning to the front of the class. When she returned to the front of the class, she split her attention between the area where the students were working, and a table which had unused manipulatives on it.

Primarily, Hannah attended to environmental structures which related to students' mathematical activity. In the recorded lesson, this meant icy pole sticks. Figure 1 shows two different ways in which students structured icy pole sticks while counting. When Hannah pointed to the image on the left during the recall interview, she recalled that this pair of students "got it" – referring to grouping by tens, which was the mathematical focus of the lesson. When reviewing the image on the right, Hannah recalled that she took this structure to mean that the students were counting by ones mid-lesson. Hence, these are examples of environmental structures which Hannah perceived as meaningful mid-lesson. Hannah's comments and gestures during post-lesson interview provides evidence of the environmental structures she was attending to mid-lesson. The images captured from the head-mounted camera, like those presented in Figure 1, provide evidence of how the environment was structured, while the language Hannah used to describe each structure provides evidence of the meaning perceived.

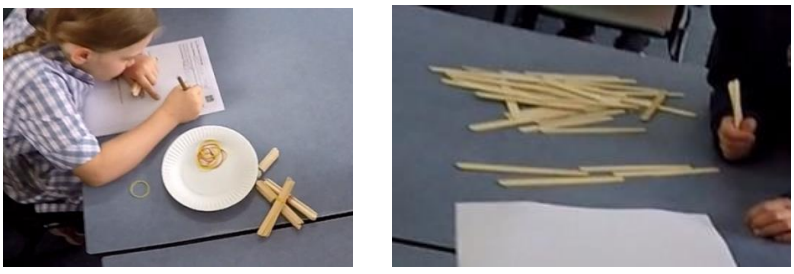

Figure 1. Student use of manipulatives which conveyed student thinking.

Hannah manipulated the task environment before the lesson began. As the previous literacy lesson ended, she added mathematics manipulatives (icy pole sticks) and worksheets to each table. As Hannah moved between desks, she gave students directions which changed the environmental structures which were present. At one point she asked all students to draw the strategy that they had used when counting sticks. When asked why she did this, she said, "well, I know this isn't going to, you know, help them. But I'm hoping that I'll get something from this". She then engages in a between-desk perceptual routine which gives her perceptual access to the drawings that students are creating. Some of these representations were perceived as meaningful and related to student thinking (e.g. "I could see how they were making uneven groups").

As Hannah recalled what she could see or what she noticed during the lesson during the post-lesson HMV interview, she frequently described how perception of a meaningful environmental structure carried a sense of what she could or could not do. During the 15 minute segment of the lesson analysed, 17 instances could be identified where Hannah recalled what she attended to, and this could be coupled with a statement which described what she perceived she could or could not do in the lesson.

Discussion

Each theoretical lens enabled a different approach to analyse of this single data set. Contrasting the two lenses, a cognitivist approach led to limited direct evidence of the drivers of improvisation in this episode. Hannah could not recall what she was thinking or the knowledge she accessed in-the-moment in the post-lesson interview in a high degree of detail. Of course, this does not mean that Hannah was not thinking or relying on knowledge, but that even when HMV interviews – a research method which is a well regard form of Cognitive Task Analysis (Omodei et al., 2005) – are used to gather data, limited evidence of these internal mental processes can be identified. König et al. (2014) found that mathematics teacher noticing required teacher knowledge that was organised into complex mental schema which were activated with automaticity. Perhaps teacher improvisation also requires the use of mental processes that are so automatic, little direct evidence of these process can be gathered by researchers.

In contrast, applying an ecological lens to Hannah's improvisation enabled the identification of drivers of improvisation to be directly identified in the HMV data in much more detail. The way she deployed attention mid-lesson via perceptual routines could be mapped from coordination of data sources. Her perceptual routines put her in the right place at the right time to see meaningful environmental structures. These structures were described by her in the interview data and were also captured in the camera footage. She also took action mid-lesson to create environmental structures, and attention to these structures gave her a sense of what she could and could not do (an affordance structure) mid-lesson and her recall of events provides some evidence regarding what she saw as being possible during the period of improvisation. This creates a characterisation of her improvisation as being driven by providing instructions and materials to children that would increase the likelihood that meaningful environmental structures would be created during the lesson. She employed perceptual routines which increase the likelihood that she would be able to see meaningful environmental structures if they arose. When she saw environmental structures which meant 'counting by ones' on most students' tables, this gave her a sense that she was constrained in following her lesson plan (a perception of the affordance structure of the class at that point). In order to ascertain a new course of action, rather than engaging in a mental process of searching her mind for an alternative, she

engaged in perceptual behaviour: she moved around the class, manipulating environmental structures, "hoping that I'll [Hannah] will get something from this". Though an ecological lens she got a sense of what she could and could not do in the environment, and her subsequent unplanned activity that was deployed on the fly, was arrived at by active perceptual behaviour which attended to what was happening in the moment more than by engaging in any calculated or cognitively demanding internal mental analysis.

Conclusion

This study demonstrates the potential utility of an ecological framework by contrasting it with the more commonly employed approach of analysing teacher improvisation using a cognitivist framework. Teachers' perceptual interaction with a classroom can be captured and analysed using data collection techniques such as HMV interviews. Teachers' internal cognitive processing – particularly processes which are likely to be automatic – are more difficult to capture and analyse. Because ecological entities such as perceptual routines and meaningful environmental structures can be identified directly in the presented data set, a more detailed characterisation of teacher improvisation can be developed. This characterisation identifies teacher behaviours that could be directly taught to novice teachers. These behaviours could enhance novices' capacity to know where to look for information to guide their teaching mid-lesson, how to create conditions which will increase the likelihood that there will be entities which are worth attending to in the classroom, and how to think on their feet when go awry. This would address the issues raised by Grossman et al. (2009) who argued that teacher education lacks structures which render these elements of classroom practice teachable to novices. The potential utility of an ecological framework provides justification for further ecological research of mathematics teachers' in-the-moment practice.

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