Aligning Online Mathematical Problem Solving with the Australian Curriculum

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The current Australian Curriculum mandates that technology be utilised to support students to "investigate, create and communicate mathematical ideas and concepts" (ACARA, 2018). However, research reports suggest that use of digital technologies in Australian primary mathematics often focusses on the lower-order drilling of algorithms and basic facts. In this study, an online environment provided the medium for Year 5 students to engage in collaborative mathematical problem solving. The students' online dialogue, along with uploaded diagrams and spreadsheets, provide evidence of their development of the Australian Curriculum: Mathematics' proficiencies of Problem Solving and Reasoning.

Introduction

Since the proliferation of the Personal Computer, much has been made of the potential for technology to transform teaching and learning within primary and secondary education. The Australian Curriculum mandates that technology be utilised to support students to "investigate, create and communicate mathematical ideas and concepts" (ACARA, 2018). For this technology to assist learning, teachers need to think carefully about when and how it is used. Niess (2005), for example, argued that:

> for technology to become an integral component or tool for learning, science and mathematics preservice teachers must also develop an overarching conception of their subject matter with respect to technology and what it means to teach with technology—a technology PCK (TPCK) Pedagogical Content Knowledge, Technological Pedagogical Content Knowledge)] (p. 510).

Currently, it is rare to observe digital technologies-based mathematics teaching occurring in the symbiotic manner to which Niess (2005) refers. Instead, as discussed below, we see a focus on drill and practice-based activities. This paper reports an approach that may offer opportunities for teachers to make more effective use of technology integration within their primary mathematics teaching. The research question addressed is:

How does student engagement with online mathematical problem solving align with the Australian Curriculum?

A brief review provides background related to technology use in mathematics education. This is followed by details of the study, data analysis, then results, discussion and implications.

Digital Technology use in Mathematics Education

Zbiek, Heid, Blume, and Dick (2007) draw attention to research driven, historical, alternative approaches to technology integration in the mathematics classroom. The most common approach, they note, develops technical (or skill and procedure focused) proficiencies while the other promotes conceptual development (finding patterns, conjecturing, generalizing, connecting representations, predicting). However, they suggest that while technology may 'free' students from laborious computation, in many cases, students must have conceptual understanding of the mathematics to successfully operate the technology to execute the required operation. Much of the work of Zbeik et al. (2007) refers to the use of sophisticated computer algebra systems and hence allows us only limited

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understanding within the primary school mathematics setting. As Day (2013), Kuiper and de Pater-Sneep (2014), Turvey (2006) have reported, a large proportion of time spent on technology integration within the primary mathematics classroom is often assigned to drill-and-practice mathematics software that is conveniently and freely available on the Internet. Day's (2013) study of 118 primary schools in Western Australia shows that teachers, school administrators and pre-service teachers all believe that ICT integration has the potential to lead to conceptual knowledge development. However, the evidence presented suggests that the resources most commonly used do not target this goal. She found that 80% to 90% of ICT integration for mathematics referred to 'the Internet' and typically involved students playing games that encourage practice of routine procedures. Day's (2013) findings support the claims of Herrington and Kervin (2007) who stated that, technology was often employed for all the wrong reasons, for example: pressure from school administrators and the belief that students need to be entertained. While Sinclair and Yerushalmy (2016) note reports of primary teachers sharing mathematical thinking and pedagogy online but this did not extend to primary students. Changes to mathematical discourse, whether linguistic or non-linguistic, may correspond to changes in students' mathematical thinking and access to mathematical software potentially enriches the possibilities for mixed discourse.

The conclusions of this earlier research suggest that achieving the Australian Curriculum goals will entail pedagogy promoting discourse and deeper conceptual understanding of mathematics. This paper informs such an approach to the use of ICT in primary mathematics.

Background and Method

Context

The study was conducted over a nine-week period in a Melbourne state primary school with an Index of Community Socio-Educational Advantage closely matching the state average. Participants were Year 5 students: 26 boys and 28 girls (10 to 12 years old). They were allocated to 10 mixed ability groups of 3 to 6 students within an online space. Groups were created based on prior judgments, by their teacher, classifying students as below, at or above level in mathematics. The context of the online work was Edmodo, a freely available online computer supported collaborative learning environment. Throughout the intervention students were supported by the first author with a weekly classroom session that typically took the format of a short review of online interaction from the previous week, followed, in weeks one to seven, by discussion of a new problem to be solved, this included technical information that might help students use available software (Edmodo, MS Excel, MS Word etc). Early on, time was spent outlining appropriate online behaviours and promoting discussion that goes beyond superficial chat. The last two weeks of the intervention were different. No discussion preceded online mathematical problem solving; students immediately started their work in the online space. Work in weeks 8 and 9 was therefore less influenced by the researcher (facilitator).

Coding and Analysis

The mathematical proficiency strand of Problem Solving as prescribed by the Australian Curriculum: Mathematics (ACARA, 2018) is summarised in Figure 1 as progressive steps towards problem solutions. Figure 2 summarises the description of the Reasoning proficiency of the Australian Curriculum: Mathematics (2018). This also sets out sequential strategies although each could be evident with different levels of sophistication. These formed frameworks used for the analysis of students' online work. The purpose of using the Australian Curriculum descriptors as a source for coding and analysis of the data was to explicitly look for evidence of if and how student engagement with online mathematical problem solving aligned with the Australian Curriculum. This research considers all data shared via Edmodo: both the text of students' online discussion and artefacts (graphs, diagrams, tables, pictures)

Analysis of data was supported using qualitative analysis software, NVIVO (International, 2015). Data consisted of individual students' online work. Classroom discussion was not recorded. The themes from Figures 2 and 3 were designated as nodes. Each file (discussion or added artefact) was separately multi-coded for problem solving abilities, and then again for the analysis of reasoning. This indicated whether students were likely to engage in skill or conceptual development during online discussions or while constructing supporting artefacts. Coding was undertaken by two researchers independently of one another. Over 85% inter-rater reliability was achieved, and the remainder was agreed upon through discussion.

Mathematics	Problem Solving: Students Developing ability to:	Make Choices
		Interpret (understand the problem)
		Formulate (Use Procedure)
		Model and Investigate Problem Situations

Figure 1. Australian Curriculum Framework for Analysis of Problem Solving.

Mathematics	Reasoning: Students Developing ability to engage in:	Analysing
		Evaluating
		Explaining
		Generalising (use of explicit equation formula)
		Inferring (placing understanding in new context)
		Justifying
		Proving

Figure 2. Australian Curriculum Framework for Analysis of Reasoning.

Extracts of two examples of set problems, with layout compressed, are shown in Figure 3.

Week 3: Wallpaper patterns are an example of symmetry in our everyday lives.
Record your groups' understanding of the different types of symmetry in this space …
You can find a very good explanation of these in the folder attached to week 3.
Now your group will design a wallpaper 'block'. You will need to decide how to incorporate symmetry into your block. Your group will create (and upload to this space) one sheet of wallpaper using Microsoft Word. …Most importantly… Your group will write a summary of which types of symmetry you have used and how you have incorporated these into your wallpaper designs. These will be posted within this forum.

Week 4: What is the biggest breed of dog?
Research a variety of dogs using your netbook. Decide what 'biggest' means. Provide a definition. Your group will have to decide whether they think 'biggest' means heaviest, tallest, longest etc How do breeders measure this? Create a graph in Excel representing the data you have found.
Horizontal axis (x axis) should be breed of dog and vertical axis (y axis) should be height/ weight/ length etc.
Upload the graph that you have made to this message board.
Which dog, according to your definition, is the 'biggest'? Discuss any other facts that you can 'read' from the graph that your group has created?
Think about another measurement you can use to define 'biggest' e.g. If you defined 'biggest' as height of the dog last time, you might like to use weight this time. Create a new graph.

Figure 3. Extract of two examples of problems set for students (Symons, 2017)

Results and Discussion

Problem Solving

Coding and analysis of students' online discussion data and uploaded artefacts shows that individual students made use of the online environment in subtly different ways. Table 1 shows that when students used Excel, the files that they created and uploaded almost all involved formulation (the use of procedures), interpretation (evidence that the students either fully or partly understood the problem), making choices and modelling and/ or investigation of some aspect of the problem. Of the 97 Excel files 95 displayed evidence of the development of these themes.

Table 1
Frequency of examples of Problem Solving Categories evident in uploaded artefacts

Category	Excel Artefacts	Paint Artefacts	Word Artefacts
Formulate (Use Procedure)	96	3	59
Interpret (Understand the Problem)	95	3	59
Make Choices	96	3	59
Model and Investigate Problem Situations	96	3	60

Of the 65 Word Files uploaded, 60 showed evidence of at least one of the mathematical problem-solving categories. In the five files where these themes were not detected students were providing general (non-mathematical) background information about the problem. The three Paint artefacts showed evidence of all four problem solving categories.

Table 2 provides an indication of differences between student use of the various software for different mathematics. While across the nine weeks the overall focus was on problem solving, as indicated, each week the problem drew on one Australian Curriculum mathematics content area: Measurement and Geometry (M&G), Statistics and Probability (S&P) or Number and Algebra (N&A). Students were encouraged to use any software, depending on which they believed would best support their thinking and communication of ideas. Uploading supporting artefacts was introduced to students in week 2, hence. the absence of artefacts in the first week.

Table 2
Comparison of Student use of Software in Uploaded Artefacts Across Problems

	Excel Artefact	Paint Artefact	Word Artefact
Week 1 - Toilet Roll (M&G)	0	0	0
Week 2 - 10 Hour Day (N A)	0	0	1
Week 3 – Symmetry (M&G)	0	0	30
Week 4 - Biggest Dog (S&P)	26	0	8
Week 5 - Animal Ages (N&A)	30	0	5
Week 6 – Shapes (M&G)	0	3	15
Week 7 - Pet Names (S&P)	10	0	0
Week 8 - Mr M's iPhone (N&A)	27	0	0

Week 9 – Geese (N&A)	3	0	6

It is apparent that when students developed and communicated their thinking in a problem where M&G was a focus, while we might have expected them to use Paint, students preferred to use Microsoft Word. This was the package with which they were most familiar. In week 3, students were asked to create a panel of wallpaper, representing their understanding of symmetry. Of the 30 artefacts uploaded all utilized Microsoft Word for this problem. When students were asked to investigate four sided shapes in week 6, again most chose to use Microsoft Word. In this problem, while the auto-shapes function within Word was heavily employed students also organized their thinking by using tables in Word

Table 3 (below), represents how students engaged in the four categories of MPS in online discussion compared to their level of engagement with these categories when constructing artefacts. It indicates that students more commonly showed evidence of engaging with these important categories when representing their mathematical thinking through creating a representation within their uploaded artefacts. It is interesting that in the 'Interpret' category the distribution is more evenly shared between artefacts and online discussion. This indicates that students unpacked and discussed their ideas with each other in order to ensure they fully understood the problem.

Table 3
Comparison of Development of Problem Solving Concepts in Artefacts Vs Online Discussion

Category	Examples within all Uploaded Artefacts	Examples within Online Discussion	% of Developing Concepts in Uploaded Artefacts	% of Developing Concepts in Online Discussion (Excluding Artefacts)
Formulate	158	54	75	25
Interpret	157	109	59	41
Make Choices	158	103	61	39
Model and Investigate Problem	159	67	70	30

Reasoning

Analysis of data suggests that the types of reasoning engaged in over the period of the intervention changed from week to week. It appears that students were able to engage in analysis, evaluation and explanation throughout, however it is not until week four that consistent evidence of students engaging in generalizing, inferring and proving occurs. The evidence here suggests that students' development of reasoning skills progressed over the course of the intervention. A comparison of week 2, when students devised timetables for 10-hour days with 100 minutes per hour and week 8, when students investigated an iPhone that progressively halved its battery life, suggests improvement in reasoning skills. By week 8 students were giving evidence of their thinking not just results. In weeks eight and nine students did not have the prompt of classroom discussion prior to their online work. Therefore, the fact that students showed evidence of all areas of reasoning in these weeks was encouraging.

Table 5 (below) shows the degree to which students who were assessed (by their teacher) as below level, at level and above level, in mathematics, engaged in reasoning throughout the intervention. Across the three groups there was a fairly consistent strong level of analysis, evaluation and explaining occurring. This is interesting because it might be assumed that the students described as *below level* (by their teachers) would show less ability to engage in all areas of reasoning. Additionally, the lower ability students' engagement with the remaining skills (except 'generalizing') was of a similar level to that of their peers. All students demonstrated fewer instances of generalizing, inferring and proving. This is not unexpected given that these aspects of reasoning are considered to involve more sophisticated processes.

Table 4

Reasoning used Across Nine Weeks Evidenced by Online Discussion and Artefacts

	Analysing	Evaluating	Explaining	Generalising	Inferring	Justifying	Proving
Week 1	1	3	22	0	0	13	0
Week 2	3	4	6	0	0	1	0
Week 3	1	4	35	0	0	2	0
Week 4	32	43	64	0	2	6	0
Week 5	23	26	37	3	7	8	6
Week 6	12	15	24	0	1	1	0
Week 7	12	20	30	1	0	3	1
Week 8	35	38	50	3	5	17	5
Week 9	11	18	21	5	2	11	3

Table 5

Reasoning Categories and Teacher Allocated 'mathematical Ability'

	Below Level	At Level	Above Level
Analysing	36	51	43
Evaluating	48	72	51
Explaining	80	116	93
Generalising	0	4	8
Inferring	3	7	7
Justifying	18	26	18
Proving	4	3	8

Table 6 (below) splits the "ability levels" into girls and boys. There is evidence that the ability of boys to reason increased according to the ability group they had been assigned. For example, the *Below Level Boys* exhibited 8 instances of *Analysing*, the *At Level Boys* exhibited 13 instances of *Analysing* and the *Above Level Boys* exhibited 32 examples of *Analysing*. It is worth noting here that students had been evenly distributed across the three ability classifications. Thus, the tendency of this pattern to be replicated across the various reasoning categories is important. The reasoning of girls did not follow the pattern of increasing according to teacher assigned ability group. *At Level* and *Below level* girls showed evidence of a greater volume and variety of approaches to reasoning. This may indicate that the procedural tests conducted for the purpose of allocating ability groups may not provide teachers with adequate information about their students' ability to engage in mathematical reasoning. This is an issue for further research.

Table 6
Comparison of reasoning between Genders

	Below Level Boys	Below level Girls	At Level Boys	At Level Girls	Above level boys	Above Level Girls
Analysing	8	28	13	38	32	11
Evaluating	9	39	20	52	39	12
Explaining	16	64	29	87	67	26
Generalising	0	0	1	3	7	1
Inferring	0	3	4	3	3	4
Justifying	3	15	11	15	13	5
Proving	1	3	2	1	7	1

Conclusion and Implications

Across the world, curricula are now requiring teachers to teach and assess problem solving and reasoning. It is no longer enough to require students to ‘do mathematics’ rather, there is an increasing expectation that students should be able to demonstrate an ability think, behave and communicate mathematically (Boaler, 2008). In the USA, The Common Core Standards for mathematics, released in 2010 (NGA Center, 2010) for the first time included *Standards for Mathematical Practice*. These detail the level to which students should be able make sense of, persevere with, reason, argue and critique, model and choose the appropriate tools and strategies when engaging in mathematical activity. In Australia, our national curriculum placed a new emphasis on problem solving, reasoning and communicating mathematics.

Traditional modes of assessment that privilege summative above formative approaches and focus on a student’s ability to perform procedures, for example, through the use of regular mathematical content driven pre- and post-tests, provide limited information about the student’s ability to engage mathematically. These traditional methods of assessment may provide the teacher with a skewed view of the range of abilities and levels of understandings of their students. Students who may correctly execute basic computation and procedures, sometimes beyond the level expected of them, may struggle to apply these skills to problem-based contexts. Conversely, some students who may not perform as well on the narrowly targeted tests may demonstrate ability to engage in the mathematical communication, investigation and reasoning associated with collaborative problem solving.

The online approach taken to problem solving in this study can circumvent some of these issues. The online platform provided a detailed record of interactions and supported the production of meaning making artefacts so necessary for teachers to make judgments about students’ problem solving and reasoning.

The results of this study have shown that when engaged in MPS in the online environment students are more likely to use software that they are familiar with, even when other software might be more suited to a task. We have shown that when making mathematical meaning online student thinking associated with MPS is evident in the artefacts that they create, while their interpretation both of their own work and each other’s work is seen in their online discussion. When considering student mathematical reasoning we have found that only after a number of weeks engaged in the scaffolded online MPS process were students starting to consistently display higher-order reasoning skills.

At best, a teacher within a classroom will have the opportunity to observe each group of students for a few moments within a session. From the online environment, a teacher can review all interaction and discussion. In a traditional classroom where discussion is encouraged, it can be very difficult for the teacher to know that the discussion ace in each small group is productive and related to the mathematical problem being investigated. The approach offered in this study allows teachers easy access to data related to the amount of time each group remained 'on task'.

Whilst there have been intentions for the integration of digital technologies within mathematics instruction to allow for the communication, representation and investigation of mathematical ideas and concepts over many curricula, for many years, as Day (2013) has reported this is rarely achieved in Australian primary mathematics classrooms.

The data from this study supports the value of online mathematical problem solving with upper primary students as a strategy for achieving the goals of the Australian Curriculum Mathematics (ACARA, 2014) in the proficiencies of problem solving and reasoning.

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