Factors that Influenced Year 11 Students' Choices to Use CAS for Selected Algebra Problems

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This paper reports the factors that three Year 11 mathematics students identified as influencing their choice to use a Computer Algebra System (CAS) for solving selected problems, some of which were expected to be outside their pen-and-paper (p&p) range. In individual interviews, the students were presented with eleven algebra problems and were asked to indicate the items that they would choose CAS for, and to provide a rationale for their choices. Factors that influenced students' choices included their perception of the amount of p&p working required, their p&p facility, the complexity of the problem and the presence of words in the problem statements which were CAS commands (for example, solve).

Background

In Victoria, Australia, students and teachers in Year 11 and 12 mathematics subjects are expected to use technology for teaching, learning and assessment (VCAA, 2015). Although curriculum documents do not specify a specific technology to be used, it is anticipated that CAS is the predominant technology used in mathematics since CAS calculators contain all functionalities of technology outlined in the curriculum and are approved for use in the Year 12 mathematics examinations where technology is expected. Curriculum documents detail that a key skill when working with technology is to "select an appropriate functionality of technology in a variety of mathematical contexts and provide a rationale for these selections" (VCAA, 2015, p. 36). Thus, there is an imperative for teachers and students to make informed choices about appropriate use. Given CAS use is expected in Year 12 examinations, there is a motivation for Year 11 students to develop the required knowledge and technical expertise for doing and learning mathematics with CAS.

The findings presented here are situated within the context of a larger study investigating thirteen Year 11 students' use of, and attitudes towards, CAS for algebra while completing a subject called Mathematical Methods (MM). The research question investigated in this paper is "What factors do students perceive to influence their choice of CAS for eleven selected algebra problems?"

Literature Review

In classrooms where teachers provide advice about how CAS or p&p can be used for doing and learning mathematics, students' choices about CAS use may be influenced by their teacher's advice. Ball (2014), in a study of three Year 12 teachers, found that teachers wanted students to become good CAS users and to make informed choices about CAS or p&p. Thus, it is evident that some MM teachers encourage students to actively consider whether to use CAS or pen-and-paper to solve problems. In a study of two teachers, Kendal and Stacey (2001) found that, in the class where a teacher allowed free use of CAS, the students made greater use of CAS compared to students in a class where CAS use was restricted by the teacher. In this study, the teachers' use of CAS was found to influence students' CAS use. The teachers in Ball's (2014) study focussed on the need for students to make choices based on the number of marks assigned to a problem, the difficulty of the p&p working required and students' mathematical and technological facilities. However, the

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study did not report whether students considered these aspects when choosing between CAS and pen-and-paper. Thus, it could be expected that students might identify the teacher, the number of marks allocated to a problem, p&p facility or CAS facility as factors that influence their use of CAS.

A range of studies have investigated students' use of CAS and their reasons for using CAS. Students' perceptions of the speed of CAS and their p&p facility have been found to influence choices. Ball and Stacey (2005) reported that some students chose to use CAS to do problems quickly or to supplement their p&p skills; in this case, choice was related to perceived facility with p&p. It is not surprising that students perceive CAS to be quicker than p&p as CAS can condense (Flynn & Asp, 2002) intermediate steps of p&p working. In a study of six Year 11 students, three students used CAS to solve a given a calculus problem and three used p&p; all six students indicated that speed was the reason for their choice (Cameron & Ball, 2015). Geiger (2008) also reported that students used CAS to solve problems outside their p&p range, so perceived facility with p&p may have influenced their choices. A study of 327 Year 11 and 12 students suggested that greater technology confidence (i.e., perceived ability to work and learn with technology) resulted in greater CAS use (Orellana, 2016). It could be argued that students who make greater use of CAS will have greater confidence with technology, so it could be bi-directional rather than only having technology confidence influencing CAS use. Fuglestad (2005) reported that even students with limited technological facility can make choices about ICT tool use, so technological facility may not be a determining factor in making choices about CAS for solving problems.

This paper provides insight into three students' reasons for using or not using CAS to solve specific problems, which builds on studies which explore students' general intention to use CAS.

Methodology

In the larger study, students participated in four individual interviews during a school year. In this paper, we report on a task completed in the first and third interview, conducted in May and October. The reason for using this task twice was to collect data on whether students' choices about the use of CAS for a common set of algebra problems would change over time.

The task, which consisted of eleven common[1] items (see Table 1) related to three algebraic procedures (solving, expanding and factorising), required students to indicate whether they would use CAS to complete each problem, but they were not required to solve the problems. Once students had indicated where they would use CAS, the interviewer asked students to explain why this was the case for each problem where they would use CAS. This provided data on the rationale behind choices for CAS use. In May, the focus was on reasons for CAS use and the interviewer did not ask for reasons why CAS would not be chosen for the items that were not ticked. For the October interview the prompt was modified to ask students to explain why they would or would not choose CAS for each item, as the researchers noted that the reasons for not using CAS might provide additional insight about factors related to use of CAS, through discussion of why p&p was chosen.

[1] In October, the task included an additional five items (not reported here).

Table 1
Students' responses to the interview items

Item	Problem	Tom		Sara		Ben	
		May	Oct	May	Oct	May	Oct
a)	Solve $x^2+3x+2=0$	C	N	C	C	C	N
b)	Expand $(x–3)(x+1)$	C	N	C	N	C	N
c)	Find the values of x where $x^2+2x+1=0$	C	C	C	C	N	N
d)	Factorise $x^2–4$	C	C	C	C	C	C
e)	Expand $(2x+1)(x–2)(x+6)$	C	C	C	C	C	C
f)	Solve $x^4–2x^3+3x–4=0$	C	C	C	C	C	C
g)	Write x^2+5x+6 as a product of two linear factors	C	C	C	N	N	N
h)	Find the exact values of the x intercepts for $y=x^2+4x+4$	C	N	C	C	C	N
i)	Solve $^1/_5\,x^2+2x–^3/_7=0$	C	C	C	C	C	C
j)	Write the following expression in expanded form $(x+4)(x–2)$	C	N	C	N	C	N
k)	Factorise $x^3–x$	C	C	N	C	C	C

Key: C – CAS, N – No CAS, No change (i.e. C/C or N/N), Change (i.e. C/N or N/C)

The eleven problems were designed to be similar to, or an extension of, problems that had been completed in class. They were procedural problems, rather than problem solving problems, as the focus was on students' choice of CAS for a given algebraic procedure, rather than whether the students could use CAS as part of their problem-solving routines.

Items a, c and h were parallel items related to solving a quadratic equation but were presented in different contexts (solving an equation, finding x-intercepts, finding x-values to make an equation true). If problem context influenced students' choices, it was expected that choice of CAS may be different for these items. Parallel items (a, f, i) required 'solving' but differed in complexity (item a, a quadratic equation, within the expected p&p skills of students; item f, a quartic equation outside the p&p skills of students; item i, a quadratic equation with fractions as coefficients, which was more difficult than item a, but expected to be within the p&p skills of students). If the complexity of a problem influenced CAS use, it would be expected that students might choose CAS for items f and i.

Factors impacting choice of CAS were determined by noting key phrases in the interview transcripts that provided a rationale for each student's choice of CAS for each item. For example, Tom's statement in May that he would use CAS for item a, "*it* [i.e. CAS] *would make it a lot easier, like you don't have to write as much working out*" suggests that his perception about the amount of p&p working required to complete the problem influenced his choice.

The three students reported here were selected as their responses had the greatest change between May and October. These students responded differently to at least four of the 11 items, whereas other students only had one or two changes in intention to use CAS.

Results and Discussion

Table 1 provides a summary of Tom, Sara and Ben's (pseudonyms) indications of where they would use (C) or not use (N) CAS for each item. There were twenty-one instances

where their choices about CAS were the same in both interviews (i.e. CC or NN). While in many cases the students' choices about whether they would use CAS did not change, their reasons for choosing to use CAS did. Of the 11 items, there were four items (d, e, f, i) where all students indicated they would choose CAS in both interviews. There were two items (b, j) where all students indicated that they would choose CAS in the May interview, but not in October (i.e. C/N in table 1), and three items (a, g, h, k) where some students made different choices about CAS in October and May. All students indicated that they would choose CAS fewer times in October than in May. There was one instance where a student indicated a change from N to C (Sara, item k). Following is an analysis of the rationale that students gave for their choices.

Tom

In May, Tom chose CAS for all items. The amount of p&p working needed to complete problems influenced his choice for nine items (a, b, c, e, f, g, h, i, j), so one factor in his choice was the anticipated extent of p&p work. For item a, Tom stated "*it* [i.e. CAS] *would make it a lot easier, like you don't have to write as much working out"*. Ball (2003) found that CAS solutions were generally shorter than p&p solutions, so given this we might expect Tom to identify the amount of p&p working as influencing his choice for each problem. The results suggested that this was not the case. For item d (factorise x^2–4), which we expect to be within the p&p range of students at this level, Tom noted he would choose CAS to avoid errors, stating, "*I hate these ones. They look pretty easy, but I get stuck on them all the time. It's just to make sure that I don't get the easy ones wrong*". In this case, Tom's reflection on his pen-and-paper facility influenced his choice to use CAS.

The need to answer problems quickly in an examination and the extent of p&p working required for a problem worth only a few marks was another factor that influenced Tom's choices. Tom stated *"*[There's] *a lot of writing in that one* [i.e. item e], *like a calculator might be easier, especially on an exam, I don't want to waste time on writing out things that aren't worth a lot* [of marks]". This focus on marks allocated to an examination problem and the time required to solve using p&p was also noted to be a consideration of teachers in a study by Ball (2014), where teachers wanted their students to consider the number of marks allocated to a problem when choosing CAS or p&p.

For items c, d, e, f, g and i, Tom's choices to use CAS and the factors that influenced his choices were unchanged in the two interviews. Where Tom chose to use CAS in May, but not October (items a, b, h, j), his choice was influenced by his perception of the difficulty of a problem. In October items a and b were described as "*pretty easy*" and Tom chose not to use CAS. This contrasts with his choice to use CAS in May where he suggested there would be too much p&p work. Over the year these procedures may have become routine, thus requiring less p&p work, resulting in a change from C to N. This example demonstrates that there is the potential for factors that influence a students' choices to change over time.

Parallel items a and c had similar complexity (i.e., both items require solving quadratic equations with integer coefficients), so we would expect Tom to make the same choice (i.e., CAS or p&p) for both items. Given that Tom indicated he would choose to use p&p for item a, it might be expected that he would choose p&p for item c, but this was not the case. As Tom chose differently for these two items complexity may not be the determining factor for his choice of CAS.

Tom chose CAS fewer times in October than in May, which may be due to improvement in his p&p facility over the year. Tom identified some problems as easy in both interviews, but the identification of a problem as *easy* did not always result in him choosing p&p. For item b, in May, Tom stated "*I could probably do that* [item b] *easily without doing it on the*

calculator" but chose CAS. In October, Tom did not choose CAS even though his description of the item (i.e., it was "*pretty easy*") was similar to that provided in May. This suggested that he might not always make choices based on the perceived difficulty of a problem. A change in choice of CAS was also evident for items h and j which were identified as being "pretty easy" in October, despite Tom choosing CAS in May "*just to make sure it's right*". For these items, Tom's confidence in his p&p facility seemed to increase, which may have influenced his choices.

In conclusion, Tom made choices about CAS based on his p&p facility, a desire to avoid errors, and the amount of time that it may take to complete a problem with p&p.

Sara

Sara chose CAS for all items in May, except item k. For items where she chose CAS, her perception was that CAS would be quicker than p&p. For item d, Sara stated *"it would just take a lot longer* [i.e., with p&p]*, so it's quicker* [to use CAS]*"*. For item k (factorise $x^3 - x$), Sara's initial response was that she would choose CAS, but she changed her mind, stating *"I realised it would just be quicker to use my head instead of CAS"*. For this item, Sara considered the speed of mental calculation to guide her choice. In October, Sara chose CAS for item k to factorise a cubic, as *"I can't really think what numbers would go together"*. If Sara cannot easily identify the factors of the cubic, then CAS would be a good choice as it would enable her to obtain a correct answer.

There were six items (a, c, d, e, f, h) where Sara said she would choose CAS for speed in both interviews, so her perception that CAS would be quicker for these items remained unchanged. The efficiency of CAS was noted as important in making choices *"instead of using p&p to go through all the steps to work it out, I can just put it on the calculator and it's done"*. Efficiency appears to be related to the extent of p&p work required, rather than considering the time for syntax entry or the need to interpret a CAS output in the context of a solution.

There were seven items (a, c, d, e, f, h, i) where Sara chose CAS in both interviews. For six of these, Sara provided the same reason for her choice in both interviews. Item i was the only item where Sara made the same choice but for different reasons. Her rationale moved from a focus on speed of CAS in May, to compensating for her lack of p&p skills with fractions in October. Sara's concern was that she could not perform the fraction calculations correctly and this influenced her choice, noting, *"they're* [i.e., fractions] *a bit more challenging, I'm not really good at them"*. This example indicates that while a student's choice may not change, the factors that influence their choice can.

There were three items (b, g, j) where Sara identified CAS would be quicker in May but not in October. For item b, Sara noted *"it would just be faster to do it in my head and use p&p to get it done"* in October. The shift to p&p may be due to improvement in p&p facility or mental calculation, which is not surprising after an additional 5 months of study. For both Sara and Tom changes in their choices for some items were related to perceived p&p or mental ability.

Ben

In May, Ben said he would choose CAS for nine items (a, b, d, e, f, h, i, j, k). Of these nine items, there were seven (a, b, d, e, f, i, k) which contained words in the problem statement which were CAS commands (e.g., solve). In May, Ben identified that the wording impacted his choice for six of these items (all except f). So, while Ben's choices were influenced by the wording of the problem statement, item f demonstrated that this was not always the case. Ben identified the complexity of the problem as the factor that caused him

to choose CAS for item f as *"it's got the cubed and the four, x to the power of four, so I'd use a CAS"*. He noted that the quartic required several steps of working out (*"It makes it* [i.e., the problem] *more difficult to solve, because you have to do long division and all that and it takes way too long"*) so chose CAS. For other items, the appearance of a word that is a CAS command influenced his choice. Item e included the word 'expand' (a CAS command) and Ben stated, *"it just says expand out the front* [i.e. in the problem statement] *so I would always link that with the calculator"*. Further evidence that words that were CAS commands impacted his choices is provided by a comparison of items a and c. These parallel items both required a quadratic equation of similar procedural complexity to be solved, but in item a, the word 'solve' was used. For item a, Ben said he would choose CAS because *"it says solve"*, making a direct link between the wording of the item and a CAS command (i.e., solve). For item c, which does not contain the word 'solve' nor any word associated with a CAS command, Ben indicated that he would not choose CAS. This example suggests that inclusion of a word that is a CAS command in an item may be a factor that impacts Ben's choices; however, item f shows that this is not always the case.

Multiple factors can influence a student's choice to use CAS for solving a problem. Item i illustrated this, as Ben initially identified that the fractions in the equation influenced his choice of CAS noting *"it's got a half out the front, ah no, one fifth and then three sevenths"*. He confirmed his choice by noting a second factor, namely that the problem included a CAS command, *"it's just confusing* [i.e., the problem] *and it says solve out the front as well"*. This example demonstrates that the initial factor provided by a student may not be the only factor that influences their choice of CAS or p&p. This was also illustrated by Tom who described multiple factors that he identified as influencing his choices.

There were five items (d, e, f, i, k) where Ben chose CAS in both interviews. For items d, f, and i, Ben noted the same reasons for choosing CAS in both May and October. There were four items (a, b, h, j) where he chose CAS in May, but not in October.

For items e and k, Ben indicated he would choose CAS in both interviews, but he provided different reasons for his choices. In May, CAS was chosen due to presence of the words which were CAS commands in the item statement. In October, the anticipated extent of p&p work influenced his choice for item e and the ability to solve correctly using p&p influenced his choice for item k.

In October, Ben made several choices about CAS based on his p&p facility, giving different reasons for why this impacted his choices. For item k, Ben stated that the problem *"really confuses me"*. Ben's confusion could not be related to context, as the problem was not given in a context, therefore it appears that the perceived difficulty in the p&p work required to solve the problem was the factor that influenced his choice. For items a, c, and g, Ben explicitly identified a weakness in his p&p skills, but contrary to what might be expected (i.e., use of CAS to compensate), he did not choose CAS as he wanted to practise p&p skills: *"I'd use p&p* [for item a] *because I'm not very good at solving them* [i.e., problems of this type] *with p&p, I'd like to practise that"*. In this case, he did not wish to use CAS to supplement his p&p skills.

Some of Ben's choices appeared to be based on a different rationale to the other students, as he did not always choose CAS when he predicted that he might have difficulty solving a problem using p&p. However, there were problems identified as complex where he elected to use CAS, so although he wanted to practice p&p skills he also indicated that for some complex problems he would choose CAS. Ben made choices based on words in the problem statement in May, however in October, Ben seemed cognisant of the importance of considering CAS use, as well as the important role of p&p in mathematics.

Conclusion

Factors identified in this analysis included the extent of p&p working required to complete a problem, the complexity of the problem, the amount of time required to complete a problem, a student's p&p facility, and the appearance of words in the problem statement that are CAS commands. The findings of this paper build upon previous literature by identifying the presence of words in the problem statement that are CAS commands as a factor that can influence CAS use, and by providing detail of specific problems that can prompt students to make different choices about the use of CAS.

This paper highlights some of the complexity behind students' choices regarding use of CAS or p&p for solving problems. While in many cases it appeared that a single factor influenced a student's choice, there were instances where a student identified multiple factors that influence their choice to use CAS for a problem. This result suggests that there may be a complex interplay between different factors when students are choosing between CAS and p&p.

A single factor can result in a different choice of CAS or p&p for different students or even different items for the same student. For example, where a student identifies an inability to use p&p to solve a problem, this may motivate some students to use CAS whereas some students might choose p&p for additional practice to improve their p&p facility.

If a teacher can understand the reasons behind a student's choice of CAS or p&p for a given problem, there is the potential to target instruction on the development of students' abilities to make informed choices. This is particularly important for students who will complete examinations where both CAS and p&p are allowed, but where the examinations are conducted under time constraints and students need to answer items quickly and correctly. This provides an impetus for teachers to promote classroom dialogue about the efficiency and usefulness of CAS or p&p for given problems.

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