Re-thinking Fraction Instruction in Primary School: The Case for an Alternative Approach in the Early Years

Chelsea Cutting
Royal Melbourne Institute of Technology
< chelsea.cutting@student.rmit.edu.au>

There is a large body of research that suggests a sound understanding of rational number is vital for success in all areas of mathematics, at school and beyond. In the Australian Curriculum, fractions are formally introduced from Year 1, yet there is evidence to suggest that current approaches are not leading to deep understanding in later years. Consequently, this theoretical discussion will review the current literature on fraction instruction and propose an alternative approach to introducing fractions in the early years of primary school.

Research has revealed that many middle years students in Australia have significant misconceptions associated with fractions (Clarke, Roche & Mitchell, 2011; Thomson & Buckley, 2009), such as reading, renaming, ordering common fractions greater than '1', and the ability to recognise and apply ratio and proportions (Siemon, Virgona & Corneille, 2001). These problems are commonly derived from a child's inability to conceptualise the magnitude of a fraction in its given context (Fuchs et al., 2013; Gabriel, 2016; Moss & Case, 1999), and to recognise whole numbers as decomposable units when fraction instruction is introduced early in their formal education.

The development of numerical magnitude, through the creation of representations such as a mental number line, not only emerges very early in childhood (Siegler & Booth, 2004; Siegler & Opfer, 2003) but is largely spatial in nature. That is, children's mental number lines can develop based on spatial proportional judgment, integrating both whole and rational number ideas. For example, children can conceptualise where 4 may be placed on a 0 – 10 number line, by using informal proportional strategies such as visualising a halfway point, and 'a bit less'. Thus, the motivation for this literature review is to explore the current theories of young children's ideas about fraction magnitude, and to examine how 'spatialising' this area of mathematics could enhance the teaching and learning of fractions in early primary school for a deeper, more robust understanding of rational number.

Spatial Reasoning

Spatial reasoning is an overarching term that describes a set of spatial concepts, processes, and tools that learners engage with when processing a range of information, problem contexts and data sources (National Research Council [NRC], 2006). It can therefore often be defined ambiguously in literature and used interchangeably with terms such as spatial thinking, spatial abilities, and visual spatial reasoning. For the purposes of this discussion, the term spatial reasoning will be defined as a skill or ability that is underpinned by three core components: "concepts of space, tools of representation, and processes of reasoning" (NRC, 2006, p. 3). These three core components require a capacity to recognise and perform mental manipulations of visual stimuli; the ability to transform spatial forms of information (representations) into other visual arrangements; an awareness of the structural features of spatial information or objects (such as scale, orientation, perspective and proportion); and, critical thinking to find relationships, reason and hypothesise to solve problems (Mulligan, Woolcott, Mitchelmore & Davis, 2018).

The ability to utilise spatial reasoning skills has been found to contribute to student's general mathematics achievement (see Cheng & Mix, 2014; Lowrie, Logan & Ramful, 2017; Mulligan et al., 2018). Despite these findings and the positive impact spatial reasoning can

2019. In G. Hine, S. Blackley, & A. Cooke (Eds.). Mathematics Education Research: Impacting Practice *(Proceedings of the 42nd annual conference of the Mathematics Education Research Group of Australasia)* pp. 204-211. Perth: MERGA.

have on achievement in mathematics, there has been little evidence of spatial reasoning infiltrating traditional mathematics curricula (Mulligan et al., 2018). However, what is also discussed in many studies is the need to better understand the relationship between spatial skills and its impact on the different types of mathematics knowledge children develop – such as fraction understanding.

Theoretical Perspectives

Kieren (1976) was the first to identify five forms of fraction interpretations, known as part-whole, quotient, measure, operator and ratio – which are widely accepted in the educational field for instruction. His five-part model was later condensed into four interpretations and three underpinning concepts of partitioning, equivalence and unitising were added, as illustrated in Figure 1.

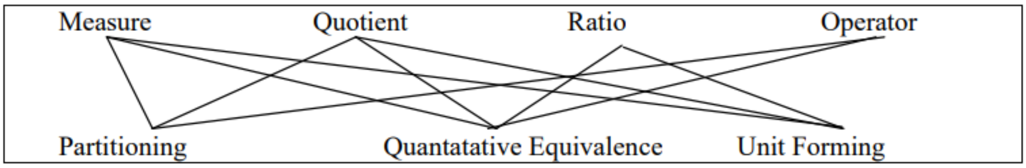

Figure 1. A section from Kieren's adapted framework for rational number knowing (1993).

In this model, the connecting line segments illustrate which concepts underpin each interpretation. For instance, the ratio interpretation requires unit forming and equivalence, the operator requires partitioning and equivalence, and the quotient and measure interpretations are underpinned by all three concepts.

The part-whole interpretation is 'embedded' within the four remaining interpretations of fractions, as it is said to form the basis for identifying the relationship of how each interpretation interacts with their parts and their wholes (and parts to their parts). Whilst important to building initial fraction knowledge, Lamon (2001) describes the *part-whole interpretation* as the least valuable path to understanding the structure of rational number. Gould (2011) suggests this is largely due to the premature introduction of symbolic notation, and an over emphasis on 'double-counting'. That is, children are taught to count the *number of shaded parts* (often of a regular shape) to record as the numerator, then taught to count the *total number of parts*, and record as the denominator. Each value is treated as a whole number, rather than a comparison of the parts to the relevant whole. This perpetuates a common stereotype in which fractions are considered *only* smaller than '1', and not perceived as numbers (Lamon, 2012). Despite its lack of value to developing sound rational number knowledge when over emphasised in instruction, fraction as "part-whole" is still the most common and relied upon interpretation that teachers present in primary school. Thus, embedding the part-whole interpretation into the remaining four fraction interpretations was intended to help alleviate these misconceptions.

Whilst the relationships are evident between the concepts and each individual interpretation in Kieren's (1993) framework, from a curricular perspective, there is a lack of connection evident between each of the interpretations or meanings. For example, in the Australian Curriculum: Mathematics, the content for Year 1 and 2 requires students to "Recognise and describe one-half as one of two equal parts of a whole" (Australian Curriculum Assessment and Reporting Authority [ACARA], n.d.); and "Recognise and interpret common uses of halves, quarters and eighths of shapes and collections" (ACARA, n.d.) respectively. Though the simplicity of these descriptors gives teachers the pedagogical freedom to explore this content as they see fit, it does not reflect the importance of the partitioning concept, or the role of each fraction interpretation. Despite the extensive research that has been conducted on partitioning (see Confrey & Maloney, 2010; Kieren, 1993; Mack, 1993; Steffe & Olive, 2010) little or no time is devoted to exploring this concept in junior and primary year levels (Bruce, Chang, Flynn & Yearley, 2013; Clarke, 2011).

What this illustrates is that within Australia at least, there is a divide between research, curricula and practice in fraction instruction (Clarke, 2011). The aim of this paper is not to diminish the content or pedagogical competence of our teachers. Rather, given the trends in understanding and competence children are currently exhibiting in this area of mathematics, it purports the need to discuss an alternative approach to fraction instruction.

Contemporary Theories for Teaching and Learning Fractions

There have been various attempts to refine Kieren's (1993) model for a more integrated approach, in order to address the identified problems of fraction knowledge construction. These refinements were intended to alleviate an emphasis on: procedural rather than conceptual approaches to instruction; the generalisation of whole number rules rather than the exploration on intuitive fraction solutions; and, the use of inappropriate or confusing representations and problems with notation (see Hiebert, Wayne & Tabor, 1991; Streefland, 1991; Moss & Case, 1999). Furthermore, Sowder (1995) strongly supported the case for a more integrated approach that considered the connections between the multiple fraction interpretations, which was explored in studies by Confrey (1995), Kieren (1995), Mack (1993), and Streefland (1991).

Following on from her earlier work, Confrey (2008) further refined Kieren's framework, suggesting that three fundamental meanings of fractions are required in the primary years of instruction. That is, *"a/b"* as a measure, *"a/b"* as a relation (ratio, proportion and rate); *"a/b of …",* for which *a/b* is an operator (Confrey, 2008). This proposed theory is built on empirical evidence which demonstrate that young children at school entry level are capable of engaging and reasoning with ideas related to all three fundamental meanings of fractions, which conflicts with traditional ideas of mathematics learning. The identified age groups are a unique aspect of Confrey's work, as much of the large-scale research on fractions, such as the Rational Number Project (Cramer, Behr, Post, & Lesh, 2009) focusses on students in Year 3 and above.

More importantly, a key finding from Confrey's work is that partitioning is fundamental to the development of division and multiplicative relationships of rational numbers in the introduction of fractions in early primary school – not counting, addition or subtraction with which children are more commonly engaged with at this age.

The Role of Partitioning

Partitioning, also known as *equipartitioning* or splitting, is described as the cognitive root to developing the multiplicative and division structures required for rational number knowledge (Confrey, 2008; Confrey & Maloney, 2010; Hurst & Hurrell, 2014). For simplicity, this paper will refer to this concept as "partitioning" throughout.

The goal of partitioning in a fraction context is to develop equal sized groups (from discrete collections), or equal sized parts (from continuous wholes), as *fair shares* (Confrey, 2008), and the ability to generalise from that experience to other contexts of fractions (Siemon, 2004). Partitioning in the context of fractions is in contrast to the notions of breaking, segmenting or fracturing (i.e., additive partitions), as these processes do not necessarily share the goal of creating equal portions.

Siemon (2004) states that many of the issues that middle school students exhibit with fractions stem from the absence of a sound understanding of partitioning. Furthermore, without a robust understanding of partitioning in a range of number contexts, the potential for developing complex proportional and algebraic reasoning ideas is limited.

In relation to Confrey's (2008) three fundamental meanings of fractions, partitioning allows children to go beyond the *out of* idea (e.g., 4 *out of* 5 parts or $\frac{4}{5}$), which only makes sense for proper fractions. That is, it is not logical to have 7 *out of* 5 parts, yet we can have 7 fifths, or 1 and 2 fifths. This distinction helps move children towards a *for each* idea, which draws attention to the role of the unit and supports a multiplicative structure, to think relationally about the parts derived from the whole, and the units that name these parts (e.g., *for each orange there are 4 quarters; therefore for 3 oranges there are 12 quarters)*. Siemon et.al (2015) describes the *for each* idea as the foundation to building understanding of: *times as many, rate*, the *Cartesian product* and *ratio* – that are necessary ideas for sound rational number knowledge and proportional reasoning.

Partitioning is vital to exploring Confrey's operator meaning or *a/b of x* in the early years of fraction instruction. For example, children can conceptualise that when half of the 20 children in the class go to music, there are 10 children left. Partitioning this quantity into two equal groups, allows children to identify the magnitude of each part.

The fraction meaning of *a/b as a measure* requires a flexible understanding of partitioning to utilise this interpretation. That is, the number of groups, the size of each group (unit), and the total (whole), illustrate the relationship between the quantities and size of the composite units involved, developing again, the *for each* idea. This enables students to understand that fractions can be determined by partitive division, aiding students to 'see fractions' as numbers and reason with numbers greater than '1' (Siemon, et al., 2011). This contrasts with how the measurement interpretation is often taught, as it can be emphasised as an additive approach of repeated addition. Working additively means students may count the number of units within a measure (Mitchell & Horn, 2008), or use repeated subtraction for division, rather than considering the multiplicative structure of units of the quantities and sizes involved.

Proposing an Alternative Approach to Fraction Instruction in the Early Years.

Research into exploring partitioning as the foundation to multiplicative and division relationships that underpin rational number knowledge instruction is not new. However, as aforementioned, the proposal of implementing Confrey's (2008) parallel approach to fraction instruction in Years 1 and 2 is in contrast with current curricula sequences. For instance, fraction as ratio and operator are traditionally considered as very complex meanings for young children. Currently, these ideas are not formally introduced until the middle and upper years of primary school in Australia – along with many other western curricula. Yet there are many contexts in which children playfully and intuitively engage with these concepts and ideas, such as, experimenting with proportion and ratio in the sandpit. That is, children can experiment with different sized containers, replicating the proportion of a water and sand mixture for a sandcastle. Similarly, children are also able to conceptualise what it means to split a deck of cards in half, or when a third of a garden bed has been planted.

An example of Confrey's parallel approach to exploring fractions that emphasises the concept of partitioning can be illustrated in the context of a 'tea party' – something with which many young children intuitively and playfully engage. When playing 'tea parties', children are dividing the tea set of cups, sources and food items equally between their guests. They are experimenting with the components of each *fair share* depending on the number of place settings, the type of fair sharing and partitioning that needs to occur (that is, discrete or continuous contexts). Moreover, children can also demonstrate an acute awareness of when there is an unequal sharing situation.

This context allows children to explore Confrey's three fundamental meanings of fractions, such as during an activity of making cupcakes for the party. The recipe may make 12 cupcakes, but there will only be 6 children coming to the party. An exploration of what a *fair share* of cupcakes for each child is, can develop a child's understanding of partitioning. For example, a child might explore what part (quantity) of the whole set each child receives if only half of the cupcakes were made. Children can be introduced to fraction as ratio ideas, by exploring how they may increase, or decrease, the quantity of cupcakes produced. For example, 12 cupcakes require 2 eggs, thus, for 6 cupcakes 1 egg is required. Fraction as measure may be explored by comparing the different size measuring cups, enabling children to discover the connection between the fractional language used, and the quantities and units involved. For example: *What would happen if the recipe called for 1* $\frac{1}{3}$ *cups of milk, but we only had a one third cup measure?* This problem introduces children to the idea of unit fractions and equivalence whilst exploring mixed and improper fractions.

As illustrated, children are naturally engaged in a range of play-based contexts that allow for the introduction and conceptual exploration of Confrey's three fundamental meanings of fractions. Recognising these situations as educators and explicitly drawing children's attention to these fraction ideas and concepts is not only possible, but pivotal to changing the way in which children currently perceive this important area of mathematics.

In light of the aforementioned evidence about the impact spatial reasoning can have on general mathematics achievement, *and* in addition to the 'spatial' nature of whole and rational number development, an alternative approach to the teaching and learning fractions also needs to consider the role of spatial reasoning. That is, how might the concepts of space, tools of representation, and processes of reasoning (NRC, 2006) be utilised to improve fraction understanding.

Spatial Visualisation. One possible spatial reasoning ability that may be beneficial is spatial visualisation, as this has been proven to be particularly important for general mathematics achievement (Ontario Ministry of Education [OME], 2013). Briefly, spatial visualisation involves generating, retaining, retrieving, and manipulating structured visual images (Hawes et al., 2017). Furthermore, Lamon (2001) contends that spatial visualising – and the concepts of partitioning, unitising and equivalence – are described as a *symbiotic relationship*. This is exemplified in the following task (Figure 2), that can be used to explore both fraction as measure and fraction as operator meanings at an early year level.

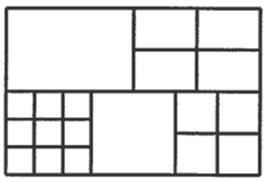

Figure 2. Where can you see $\frac{1}{8}$? (Lamon, 2012).

Spatial visualisation explores the concept of space by the child performing mental manipulations of the visual stimuli into other visual arrangements, whilst organising the structural features and abstracting these (such as the composition of the shape or object in Figure 2) to identify relationships within the information presented (Battista et al., 1998). That is, the child is required to mentally continue the partitioning, describing and reasoning where they can see different fraction magnitudes, and explore different unit fractions and their equivalencies. The child is required to generate and manipulate a mental image, which is a tool for representing the relevant mathematical concepts. The reorganisation of the mental activity of unit iterations is then engaged with to find relationships and develop conjectures (Tzur, 2018). Explicitly emphasising the use of spatial visualisation when partitioning a whole like the above diagram by repeated halving, thirding, and so on, may further help young children discover and conceptualise the idea that the magnitude of each identified part (unit) is reduced, as the number of parts increases (Kieren, 1993; Siemon, 2004).

Spatial visualisation can again develop the operator meaning of fractions in this task, by providing part of the above image, and posing the question: *If this is one quarter, what is the whole?* These types of questions are powerful ways to employ spatial visualisation when exploring equivalence and unit forming concepts in fraction contexts and may better help children develop rational number magnitude. Furthermore, spatial visualisation has been found to be a beneficial strategy to explore ratio and proportional contexts (Möhring, Newcombe & Frick, 2015) through imagining and exploring the equivalence of fraction magnitude in continuous proportions, such as the aforementioned sand and water mixture activity.

Conclusion

A rethink of the way we engage early years children with key fraction meanings and representations has been presented, in an attempt to mitigate the difficulties that students experience in the upper primary and middle years of school. Evidence suggests that introducing the concepts and ideas of fraction as measure, ratio and operator are not only appropriate for young children to explore, but the inclusion of spatial reasoning strategies may enhance the development of this essential area of mathematics that many students and teachers have struggled with for too long.

References

Australian Curriculum Assessment and Reporting Authority [ACARA], n.d. F-10 curriculum. Mathematics year levels. Retrieved from https://www.australiancurriculum.edu.au/f-10-curriculum/mathematics/?year

Battista, M. T., Clements, D. H., Arnoff, J., Battista, K., & Borrow, C. V. A. (1998). Students' spatial structuring of 2D arrays of squares. *Journal for Research in Mathematics Education*, 503-532.

Bruce, C., Chang, D., Flynn, T., & Yearley, S. (2013). Foundations to learning and teaching fractions: Addition and subtraction. Retrieved from http://www.edugains.ca/resourcesMath/CE/LessonsSupports/Fractions/FoundationstoLearningandTeachingFractions.pdf

Cheng, Y. L., & Mix, K. S. (2014). Spatial training improves children's mathematics ability. *Journal of Cognition and Development, 15*(1), 2-11. https://doi.org/10.1080/15248372.2012.725186

Clarke, D. (2011). Fractions as division: The forgotten notion? In J. Way, J. Bobis (Eds.). *Fractions: Teaching for Understanding* (pp. 33-41). Australia: Australian Association of Mathematics Teachers. Retrieved from https://www.aamt.edu.au/content/download/19933/273067/file/tdt_F_clarke2.pdf

Clarke, D., Roche, A. & Mitchell, A. (2011). One-to-one student interviews provide powerful insights and clear focus for the teaching of fractions in the middle years. In J. Way, J. Bobis (Eds.). *Fractions: Teaching for Understanding* (pp. 23-31). Australia: Australian Association of Mathematics Teachers. Retrieved from http://researchbank.acu.edu.au/fea_pub/33/

Confrey, J. (1995). Splitting re-examined: Results from a three-year longitudinal study of children in grades 3-5. In *Proceedings of the 17th Annual Meeting, International Group for the Psychology of Mathematics Education* (pp. 421-426). Columbus, Ohio: ERIC Clearinghouse for Science, Mathematics and Environmental Education.

Confrey, J. (2008). *Learning trajectories and rational number reasoning* [PowerPoint slides]. Retrieved from https://www.human.cornell.edu/sites/default/files/HD/nsfalw/Confrey-NSF.pdf

Confrey, J., & Maloney, A. (2010). The construction, refinement, and early validation of the equipartitioning learning trajectory. In *Proceedings of the 9th International Conference of the Learning Sciences, 1* (pp. 968-975). International Society of the Learning Sciences. Retrieved from https://dl.acm.org/ft_gateway.cfm?id=1854484&ftid=824864&dwn=1&CFID=106191748&CFTOKEN=e70e714620f15bd0-E8879DFE-E2F0-EF9C-D6FF73644FA6BC61

Cramer, K., Behr, M., Post T. & Lesh, R. (2009). *Rational number project: Initial fraction ideas*. Retrieved from http://wayback.archive-it.org/org-121/20190122152926/http://www.cehd.umn.edu/ci/rationalnumberproject/rnp1-09.html

Fuchs, L. S., Schumacher, R. F., Long, J., Namkung, J., Hamlett, C. L., Cirino, P. T., ... & Changas, P. (2013). Improving at-risk learners' understanding of fractions. *Journal of Educational Psychology, 105*(3), 683. http://dx.doi.org/10.1037/a0032446

Gabriel, F. (2016). Understanding magnitudes to understand fractions. *Australian Primary Mathematics Classroom, 21*(2), 36. Retrieved from https://files.eric.ed.gov/fulltext/EJ1106783.pdf

Gould, P. (2011). Developing an understanding of the size of fractions. *The Australian Association of Mathematics Teacher,* 63-70. Retrieved from https://www.aamt.edu.au/content/download/19931/273047/file/tdt_F_gould1.pdf

Hawes, Z., Moss, J., Caswell, B., Naqvi, S., & MacKinnon, S. (2017). Enhancing children's spatial and numerical skills through a dynamic spatial approach to early geometry instruction: Effects of a 32-week intervention. *Cognition and Instruction, 35*(3), 236–264. https://doi.org/10.1080/07370008.2017.1323902

Hurst, C., & Hurrell, D. (2014). Developing the big ideas of number. *International Journal of Educational Studies in Mathematics, 1*(2), 1-18. http://doi.org/10.17278/ijesim.2014.02.001

Kieren, T. E. (1976). On the mathematical, cognitive and instructional foundations of rational numbers. In R.A Lesh, D. A Bradbard (Eds.). *Number and measurement. Papers from a research workshop*. (pp. 101–144). Columbus, OH: ERIC/SMEAC. Retrieved from https://files.eric.ed.gov/fulltext/ED120027.pdf#page=108

Kieren, T. E. (1993). Rational and fractional numbers: From quotient fields to recursive understanding. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.). *Studies in mathematical thinking and learning. Rational numbers: An integration of research* (pp. 49-84). Hillsdale, NJ, US: Lawrence Erlbaum Associates, Inc.

Lamon, S. J. (2001). Presenting and representing: From fractions to rational numbers. In A. A. Cuoco & F. R. Curcio (Eds), *The roles of representation in school mathematics* (pp. 146–165). Reston, VA: National Council of Teachers of Mathematics.

Lamon, S. J. (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers* (3rd ed.). Routledge.

Lowrie, T., Logan, T., & Ramful, A. (2017). Visuospatial training improves elementary students' mathematics performance. *British Journal of Educational Psychology, 87*(2), 170-186. https://doi.org/10.1111/bjep.12142

Mack, N. K. (1993). Making connections to understand fractions. *Arithmetic Teacher, 40*(6), 362+. Retrieved from http://link.galegroup.com/apps/doc/A13770518/AONE?u=unisa&sid=AONE&xid=65a7003f

Mitchell, A., & Horne, M. (2008). *Fraction number line tasks and the additivity concept of length measurement.* Retrieved from http://researchbank.acu.edu.au/fea_pub/2077/

Möhring, W., Newcombe, N. S., & Frick, A. (2015). The relation between spatial thinking and proportional reasoning in preschoolers. *Journal of Experimental Child Psychology*, 132, 213-220. https://doi.org/10.1016/j.jecp.2015.01.005

Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal for Research in Mathematics Education*, 122-147. http://doi.org/10.2307/749607

Mulligan, J., Woolcott, G., Mitchelmore, M., & Davis, B. (2018). Connecting mathematics learning through spatial reasoning. *Mathematics Education Research Journal*, *30*(1), 77-87. https://doi.org/10.1007/s13394-017-0210-x

National Research Council, (2006). *Learning to think spatially*. Washington D.C.: The National Academy Press. Retrieved from https://www.nap.edu/read/11019/chapter/1

Ontario Ministry of Education. (2014). *Paying attention to spatial reasoning*. Toronto, ON: Queen's Printer for Ontario. Retrieved from http://www.edu.gov.on.ca/eng/literacynumeracy/lnspayingattention.pdf

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75*(2), 428-444. https://doi.org/10.1111/j.1467-8624.2004.00684.x

Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science, 14*(3), 237-250. https://doi.org/10.1111/1467-9280.02438

Siemon, D. (2004). Partitioning: The missing link in building fractional knowledge and confidence. In M. Goos & T. Spencer (Eds.). *Mathematics Making Waves: Proceedings of the Nineteenth Biennial Conference of the Australian Association of Mathematics Teachers Inc., 3,* (pp. 22-24). Australian Association of Mathematics Teachers Inc. Retrieved from http://www.aamt.edu.au/content/download/19062/252031/file/m~waves.pdf#page=411

Siemon, D., Beswick, K., Brady, K., Clark, J., Faragher, R., & Warren, E. (2015). *Teaching mathematics: Foundations to middle years.* (2nd ed). South Melbourne: Oxford.

Siemon, D., Virgona, J. & Corneille, K. (2001). *The middle years numeracy research project final report.* Bundoora: RMIT University. Retrieved from https://www.education.vic.gov.au/Documents/school/teachers/teachingresources/discipline/maths/mynumfreport.pdf

Sowder, J. (1995). Instructing for rational number sense. In J. T. Sowder & B. P. Schappelle, (Eds.). *Providing a Foundation for Teaching Mathematics in the Middle Grades,* (pp. 15-29). Albany, NY: State University of NY Press.

Sowder, J., Armstrong, B., Lamon, S., Simon, M., Sowder, L., & Thompson, A. (1998). Educating teachers to teach multiplicative structures in the middle grades. *Journal of Mathematics Teacher Education, 1,* 127–155. https://doi.org/10.1023/A:1009980419975

Steffe L. P. & Olive J. (2010) *Children's fractional knowledge*. Springer, New York.

Streefland, L. (1991). Fractions in realistic mathematics education: *A Paradigm of Developmental Research, 8*. Springer Science & Business Media.

Thomson, S., & Buckley, S. (2009). *Informing mathematics pedagogy: TIMSS 2007 Australia and the world.* Retrieved from https://research.acer.edu.au/cgi/viewcontent.cgi?article=1002&context=timss_2007

Tzur, R. (2018). Developing Fractions as Multiplicative Relations: A Model of Cognitive Reorganization. In A. Norton. *Constructing number*: *merging perspectives from psychology and mathematics education*. (pp. 163-191). Springer.