Learning Progressions for Students working within Level 1 of the New Zealand Mathematics Curriculum

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Learning trajectories/progressions are an emerging research focus in mathematics education. A set of descriptors of early progress in mathematics was developed for students with complex needs. Developers leveraged off existing research-based frameworks and meta-analyses, as well as integrating findings from individual research studies in some sub-constructs. Issues emerged during the process that are informative to others working on the development of learning trajectories/progressions.

Introduction

Research into learning trajectories in mathematics has momentum internationally though trajectories, or progressions, vary in grain size, scale, and the methodology used to create them (Siemon, Horne, Clements, Confrey, Maloney, Sarama, Tzur & Watson, 2017). In New Zealand learning progressions frameworks (LPFs) were developed recently. These frameworks provide high-level (or big-picture) illustrations of the typical pathways students take as they make progress in reading, writing, and mathematics (Ministry of Education, 2019).

This paper shares the results of a partnership between the New Zealand Council for Educational Research (NZCER) and the New Zealand Ministry of Education to develop an inclusive developmental mathematics framework for students who are learning within Level 1 of the *New Zealand Curriculum* (*NZC*) (Ministry of Education, 2007). The rationale for this development is that detailed descriptions of progress will support educators, particularly those working with learners with high and complex needs (learning, hearing, vision, mobility, language use, and social communication) of various ages, to provide needs-based opportunities to learn, monitor achievement, and inform students and parents.

This paper outlines how existing research is influencing the development of the progressions, and discusses the challenges faced by the team to develop descriptors that are inclusive of a diverse range of learners. The research question is: "*How can existing literature be used to create learning progressions for students at the early stages of their mathematical development?*"

Background

Establishing *growth paths* for student learning is an on-going research focus in mathematics education (Weber & Lockwood, 2014; Wright, 2014). Weber, Walkington, and McGalliard (2015) distinguish learning progressions from learning trajectories. They describe progressions as pre-determined, sequential checkpoints that are underpinned by a validation view of student learning. Learning trajectories, usually described as *hypothetical*, arise from an emergent view of student learning as it occurs in classrooms, interviews, and naturalistic settings. Curriculum statements are usually progressions while trajectories are the product of empirical research. The authors also trace types of learning trajectories back to epistemological positions of their authors; radical constructivist, cognitive science, or situated cognition.

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Siemon et al. (2017) position trajectories and progressions as synonymous. They document a panel discussion of prominent researchers in the field of learning trajectories. While the authors clearly outline their different epistemological perspectives, there is considerable consensus about the approaches they adopt. Development of learning trajectories begins with a conjectured *pathway* that is developed from experience and a detailed search and synthesis of the pertinent literature. This static trajectory is tested and iteratively revised with the aim of improving its usefulness for developing students' knowledge and understanding of the mathematical topic or field, within instructional settings. Usually trajectories include a framework for conceptual development, methods of evaluating students' thinking at points in time, instructional materials, and evidence for validation of the final trajectory. The research reported here adopts Siemon et al. (2017) perspective.

Research into learning trajectories looks for commonality among the relevant population of learners. The unique nature of the work reported here is that the intended learners are those with high and complex learning needs. Rankin and Regan (2004) identify the essence of complex needs as including both breadth (multiple needs that are interrelated or interconnected) and depth of need (profound, severe, serious, or intense needs). In New Zealand school settings, there is further definition by level of need for funding purposes—either very high or high needs. The areas of need are learning, hearing, vision, mobility, language use, and social communication.

The project team drew on the existing research literature, particularly that related to early childhood, to create a set of progressions relevant to students with high and complex needs. The most informative and readily applicable literature sources are established frameworks, learning trajectories, and meta-analyses of the known research. Few established research-informed frameworks exist for students with complex needs. A notable exception is the ABLES framework developed at the University of Melbourne (Strickland, Woods, & Pavlovic, 2016). Some detailed trajectories exist for a few mathematical topics. Clements, Wilson, and Sarama (2004) provided a detailed trajectory for young children's composition and decomposition of geometric shapes, as did Confrey, Maloney, Nguyen, Mojica, and Myers (2009) for equi-partitioning. Tzur (2019) developed a *reorganisation* conjecture into a trajectory for fractional number. Some meta-analyses of research on mathematical topics provide learning trajectories. For example, Baroody and Purpura (2017) include a table of significant developments in young children's development of number concepts. They list expected ages from birth to 7 years for key accomplishments, citing significant research in support of their claims.

With other topics, detailed research summaries of learners' development are general, partial, or non-existent. Classification of shapes is a good example. The prevailing theory of development—van Hiele's levels (van Hiele, 1986)—is too broad to provide a finer grained description of progression. A main criticism of the van Hiele levels is that students exhibit different levels for different tasks in the same mathematical domain. It is difficult to link trajectories with progressions for topics where the research is not well summarised.

A requirement of this project was that the descriptors connect with the early steps of the Learning Progressions Framework (LPF) (Ministry of Education, 2019). The developers created a set of hypothetical progressions based on research literature and their experience. The progressions were then validated by modelling data from teacher judgments about the sophistication of students' individual responses to tasks, as opposed to the modelling of student schemes most typical in trajectory research.

Method

Strategic planning for the research project began in 2017 with the establishment of principles, purposes, and audience for the set of progressions to be developed. The agreed principles were that the progressions contained clear descriptions of performance that were inclusive of all students working within Level 1, promoted and illustrated effective pedagogical practices, built on student strengths rather than deficits, and allowed for variability in progression. Important purposes were to provide data for educators to plan next learning goals and provide feedback to learners, to support monitoring of student progress, to help parents to support their student's learning at home, and to provide a focus for reflective discussions among educators. A range of stakeholders was considered, including teachers and leaders in schools, specialist support teachers, learners, and whānau (family members).

The exploratory study consisted of three phases over a period of 2 years.

- Phase One: Literature review investigated trends and developments nationally and internationally in assessment, descriptions of progress, and mechanisms for reporting.
- Phase Two: Exploratory study in mathematics focused on adapting existing approaches to reporting progress to include all learners, including those working long term within Level 1 of NZC due to high and complex needs.
- Phase Three: Development and exemplification of an inclusive mathematics progression framework occurred with the support of researchers and practitioners from the sector.

This paper reports on Phase Three. Work began with a comparison of existing frameworks for numeracy, either for students with complex needs or students in the years birth to 7 years. Progression frameworks from Australia, the United States, the United Kingdom, Ireland, and New Zealand were compared. Common sub-constructs were identified, from which a set of 18 indicators was created. The project team reviewed the set of indicators on three criteria; breadth of mathematical ideas, specificity, and functionality. *Specificity* meant a clear and discrete definition of the sub-construct—for example, counting as discrete from number sequence. *Functionality* meant the usability of the indicator for educators.

Indicators included *Forward and Backward Word Sequences, Subitising*, *Counting*, *Finding Difference*, *Equal Sharing*, *Repeating Patterns*, *Classifying and Structuring Shapes*, *Sequencing Events*, and *Organising Data*. A smaller set of eight indicators was selected for development, consultation, and trialling. It was anticipated that some indicators such as *Counting* would be supported by a broad literature, while others, such as *Sequencing Events*, would be more difficult in terms of locating relevant research.

Each indicator comprised descriptors of student schemes (i.e., action structures), and examples of situations in which students might demonstrate those schemes. Descriptors within each indicator were arranged vertically from less sophisticated behaviours at the bottom to more sophisticated at the top. Each individual descriptor described a scheme that was observably distinct from the one above and below it. The most sophisticated descriptors represented behaviours that are approximately aligned to the boundary between Level 1 and Level 2 of the NZC. Figure 1 shows the algorithm used by researchers to create descriptors.

Examples were developed to illustrate behaviour (schemes) that exemplify the descriptor. Situations were carefully selected to show a diverse range of contexts and learners (e.g., ages, ethnicities, modes of communication, school/home settings, learning areas), in consultation with a small working group of educators with considerable experience of working with students of complex needs.

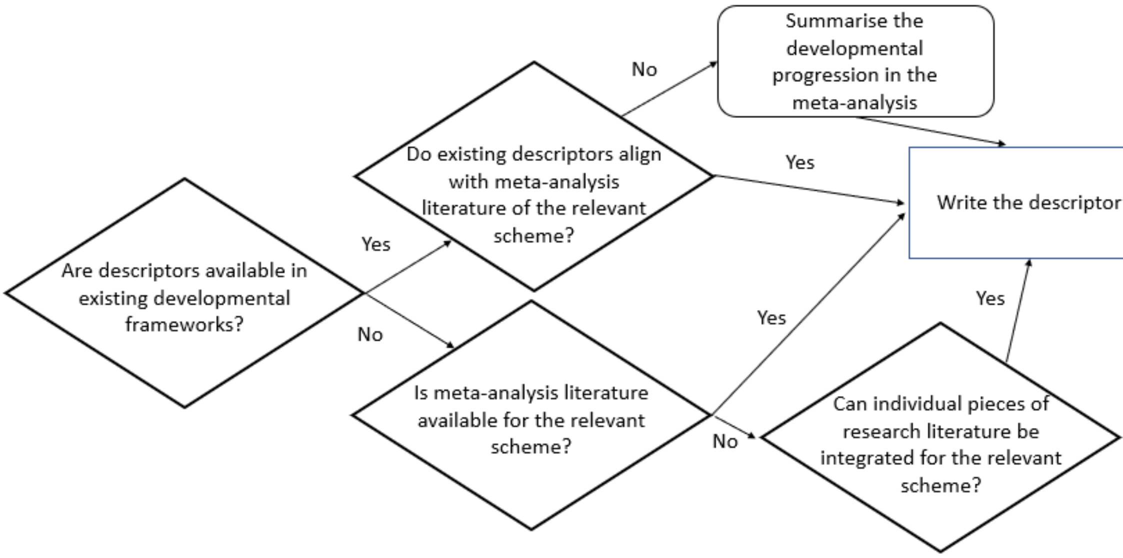

Figure 1. Flowchart of the descriptor creation algorithm.

Findings

Two indicators, *Counting* and *Sequencing Events*, are used below to illustrate key findings from the process of developing the set of descriptors. Below in the examples, the reader's attention is drawn to issues that emerged in the development process rather than to the artefact of the complete set of descriptors. The *Counting* indicator is chosen because it illustrates a sub-construct of mathematics for which a set of conceptual principles, and sequence of schemes, are extensively researched (Gelman & Gallistel, 1978; Steffe & Cobb, 1988; Wright, 1991). A meta-analysis (Baroody, Lai, & Mix, 2006) clearly described development from birth to 7 years or more. Other comprehensive meta-analyses were also available. Table 1 shows three of the six descriptors in the *Counting* indicator. The examples are a sample of those produced.

Creation of the *Counting* descriptors exposed two main issues— relationships among descriptors, and the use of specialised terminology. Creation of discrete indicators suggests that the learning progressions are independent of those in other indicators. Perceptions of independence can be an unintended consequence of creating discrete descriptors. Counting with understanding involves the integration of multiple schemes and concepts. For example, developing cardinality (count as quantity) involves using subitised images of intuitive sets (1–4) to create grouping knowledge for larger collections; matching of spoken nouns to these larger collections, across variable contexts; and connecting forward and backward word sequences with increasing and decreasing quantities. Separate indicators were created for *Subitising*, *Forward* and *Backward Number Sequences*.

Tension occurred with adapting research terminology for an educator audience. Researchers spend effort assigning terms to phenomena to capture nuances that are important to meaning making. For example, the term "intuitive numbers" describes numbers instantly recognised. The initial counting descriptors used the terminology "recognising intuitive numbers 1–4" to describe the observable actions of quantifying small sets of objects by appearance, rather than by counting. Feedback from the focus group indicated that teachers would not understand or engage with the term "intuitive". "Intuitive" was replaced with "instantly recognised numbers" or was removed entirely, as illustrated in descriptors two and three (refer to Table 1). Key feedback from the focus group was that teachers and parents needed to recognise the observable actions of their learners within the language of the descriptors. Finding 'middle ground' between commonly used language and the introduction of new vocabulary that has specific meaning in the research was a significant challenge. "Intuitive" conveys the fact that recognising quantities of one to four occurs innately, so some meaning is lost. Terminology from research was often adapted to make the descriptors accessible to the audience.

Table 1

Counting Indicator

Counting— Finding the number of items in a collection or set	
Descriptor progression	Examples (sample)
Counts and forms sets of up to ten objects by pointing to, or looking at, the objects one by one, and saying the whole number counting sequence correctly.	• Student nods and sub-vocalises, "One, two, three, four, five, six, seven, eight, nine, ten," as they count each biscuit when asked, "How many biscuits are in the packet?" • Student closes their eyes and listens to the beats on a drum. Student sub-vocalises as they count each beat and names the correct number of beats.
Correctly counts and forms sets of objects with instantly recognised numbers (1–4), and compares greater sets by global appearance.	• Student accurately counts pencils verbally, "One, two, three, four," when asked to get four pencils. They point to one pencil for one number word. • When asked, "Which bowl of apples has more?" (five and eight), the student indicates the bowl with eight because "The bowl is more full."
Attempts to count a set of objects 1–4 using number words, but without accurate one-to-one correspondence.	• Student attempts to count five cars given to them by the teacher. Student counts two cars as one and says "One, two, three, four." • Student counts out five when asked to collect four blocks. Student uses the number words to count but says two words for the same block.

The *Sequencing Events* indicator illustrates a descriptor for which the literature within mathematics education is sparse (Thomas, Clarke, McDonough, & Clarkson, 2017). Yet schemes for sequencing time are particularly important for student in their everyday life. The project team found no useful meta-analysis to create the *Sequencing Events* indicator, and existing frameworks tended to develop measurement through more tangible attributes, such as length or capacity. Time-focused research was sourced from the psychology and science education literature (e.g., Droit-Volet & Coull, 2016; McCormack & Hoerl, 2017). Although the articles described micro-developments, it was possible to combine the results of individual studies into a coherent progression, using data about average age bands for development. Four of the seven steps in the indicator are given in Table 2, for purposes of illustration.

Table 2

Sequencing Events Indicator

Sequencing Events—Ordering events chronologically including past and future	
Descriptor progression	Examples (sample)
Plans future sequences of events with causal awareness of both order and duration, based on past experiences.	• Student plans and executes getting ready to go home. They allow for different durations to complete activities, e.g., tidying the desk or tote tray takes longer than getting the homework bag.
Plans future sequences of events realising the causal significance of order.	• Student correctly orders pictures of actions to make pancakes, aware of the causal significance, e.g., "If you don't butter the pan, the pancakes stick." • Student correctly orders pictures of actions to make toast, aware of the causal significance, e.g., knows that the toaster button needs to be pushed down to cook the toast.
Realises that time is linear, directional and independent of events.	• Student acts on instructions that involve present and future actions, such as "Can you please brush your teeth and get your coat. We will tidy your room later." • Student accepts that two events can happen at the same time, e.g., "I went down the slide. Mere climbed the ladder."
Discriminates between past and present events.	• Student orders events that happened yesterday, e.g., "I got up, then went to rugby, then Dad picked me up." • Student relates today and tomorrow, e.g., "Today is Friday. It is Saturday tomorrow." Student can indicate this on a calendar.

The most significant issues that arose in the development of the *Sequencing Events* indicator were the importance of integration of related research studies, the power of clearly described progressions, and linkage to Levels 1 and 2 of the NZC. Meta-analyses play a key role in the development of research-based trajectories, a finding that became obvious to the project team when no such resource existed. Through their own reading of the literature team members developed their personal understanding of learners' development of sequencing events. For example, existing in the present generally precedes recalling of past events and predicting the future. Students' anticipation of order and duration are important to their planning for the future. This explicit knowledge guides educators in interacting with students. The process of creating examples helped team members better understand a stage, through describing students' actions and ways of communicating. For example, understanding that time is linear and one-directional can be shown through students distinguishing the past as unalterable, from the future which can be altered. Students with physical disabilities might use computer technologies to communicate with their carers about their preferences for what happens to them next.

The third issue of linkage occurred because schemes for *Sequencing Events* did not integrate easily with the achievement objectives given in the NZC. Level 1 in measurement begins with students' attendance to attributes through direct comparison, before transitive reasoning and the use of informal and formal units are taught. Time is less tangible than physical attributes, such as length. Non-synchronous events cannot be directly compared. Other attributes, such as speed, and emotions about a situation, distort students' perception of duration. Measure with units is needed to compare separate events. That is a higher degree of sophistication than normally expected with physical attributes at the beginning of schooling. Time is a complex sub-construct that involves connection of ideas about sequence, duration and measurement, including knowledge of devices. The current descriptor integrates sequence and duration but how measurement of time develops during early schooling and for students with complex needs remains unclear and poorly researched.

Discussion

The development of descriptors is a work in progress and field trials are needed to explore how useful the descriptions are for educators, students and families. It is difficult to create a set of descriptions that meet the needs of all educators and families irrespective of their situations.

Creation of the descriptors in this project was made significantly easier by reference to existing literature, particularly research-based developmental frameworks and meta-analyses. In many significant sub-constructs of early mathematical learning, such literature does not exist. This is particularly true in the areas of spatial and geometric thinking, measurement of time, and statistical investigation. This project showed that both learning trajectories and progressions can be supported by research but more work in the above areas is needed.

There were compromises made through the development of descriptors. Fragmentation of mathematics into discrete sub-constructs improves specificity but is balanced with loss of connection among the sub-constructs. In a similar way, organising progressions by steps creates clearly described growth paths, but is balanced by acknowledgement that learners vary considerably in their behaviour in the short term. Expert terminology was displaced by naturalistic language at times. Compromise occurred between conveying meaning and the usability of the descriptors for educators and parents. Inclusion of examples involved compromise between providing situations that clearly illustrate the meaning of wording of a step, and the need for representation of all students with complex needs. Examples can only be representative of the diverse situations and learners that educators encounter.

Other important issues emerged in the development process. The trajectories for early childhood and older learners with complex needs may not be the same. The project team made no assumption in that respect. Mostly, the progressions were developed from the early childhood literature. Similarity and difference in the way young children and learners with complex needs develop their schemes requires investigation. Development of the descriptors was a powerful learning experience for members of the project team. Educators would benefit from similar opportunities to hypothesise trajectories and compare their ideas with trustworthy research, as this is likely to be a more powerful experience than being given a completed artefact. However, compromises are always made between ideals for professional learning, and the constraints of resourcing. Learning trajectories are an important component of pedagogical-content knowledge and are noticeably absent for students with complex needs internationally. It is hoped that the set of research-informed descriptors can support educators and parents to provide students with targeted opportunities to learn.

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