

Developing an Understanding of Shapes and their Representations in Multilingual Primary School Classes: A Systematic Literature Review

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The paper presents a systematic review synthesising and evaluating literature that focuses on developing a conceptual understanding of shapes and their representations. It maps out literature from 2010-2018 targeting students (5-13 years) based on a set of inclusion and exclusion criteria. The analysis revealed that studies fail to take account of complexities arising in multilingual contexts for developing geometry concepts. The paper calls for future studies to explore the potential offered by multilingual contexts for developing an understanding of shapes and their representations.

This systematic literature review concerns the geometry education literature that specifically focusses on the conceptual development of shapes and their representations at the level of primary school education in a multilingual context. Multilingualism accounts for “various forms of social, institutional and individual usage as well as individual and group competence, plus various contexts of contact and involvement with more than one language” (Franceschini, 2009, p. 29). This understanding of multilingualism, that celebrates language diversity and incorporates the sensitivity towards socio-cultural diversity, needs to be taken into account to understand how learners from diverse linguistic backgrounds develop an understanding of shapes and their representations. Therefore, the present analysis maps the relevant studies from 2010-2018, then synthesises and evaluates it to build our knowledge concerning the processes underlying the development of geometry concepts of shapes and their representations in multilingual primary classes. Presently, the theories that are specific to geometry education research are: (i) the van Hiele’s (1959/1985) theory, the theory of figural concepts by Fischbein (1993), and Duval’s (1995) theory of figural apprehension (Sinclair et al., 2016). The role of language in these theories is either limited to vocabulary, definitions of shapes and their properties (e.g., Duval, 1995; van Hiele, 1959/1985) or is absent (see Fischbein, 1993). Consequently, these theories do not provide us with insights to make sense of the processes that might underlie the negotiation of concepts of shapes by learners as they interact with others in a multilingual context. Thus, in light of the multilingual context of many primary classes, the present literature review aims to explore the extent to which existing geometry education literature explores multilingual issues in primary classes while developing an understanding of geometry shapes and their representations. To synthesise and evaluate the existing literature, the author explored the following research questions:

1. *What are the most influential theoretical frameworks that are employed in existing geometry education research to develop an understanding of shapes and their representations in primary school classes?*
2. *What different tools are employed in the existing research to support students’ understanding of shapes and their representations in primary school classes?*
3. *How does the existing literature address the issues of multilingualism regarding construction of shapes and their representations in primary classes?*

Method

Pickering, Grignon, Steven, Guitart, and Byrne (2015) argued that the systematic literature review approach enables researchers to be systematic about the methods they use to (i) search, (ii) survey, and (iii) select research papers for conducting a critical analysis of existing research body. The author used the review process as proposed by Pickering et al. (2015). This process enabled researcher firstly, to locate the relevant literature; secondly, to critically evaluate this; and thirdly, to succinctly present the review of relevant research. This section presents a brief summary of the key measures taken by the researcher to locate the pertinent literature and discard irrelevant research studies. These key measures involve the use of search procedures and inclusion-exclusion criteria.

Search Procedures

For this systematic literature review, peer-reviewed articles were searched across six different databases to develop a comprehensive set of articles using the same combination of keywords. The keywords were two-dimensional shapes, 2D shapes, three-dimensional shapes, 3D shapes, primary/elementary school, geometry concepts, geometry shapes, geometry figures, two-dimensional figures, and three-dimensional figures. The six databases were Springer, Wiley Online Library, Taylor n Francis, Education Research Complete EBSCO host, and ProQuest Education database. In total, 598 research papers were included.

Read and Assess Articles

In this stage, articles were assessed for their relevance keeping the inclusion and exclusion criteria in mind. The inclusion criteria were as follows:

- Participants were from primary/elementary school classes (5-13 years);
- Articles pertaining to the sorting, identifying, classifying, representing shapes and relationships among shapes and their representations;
- Peer-reviewed journal articles;
- Articles published in the English language; and
- Empirical studies.

In addition to this, an exclusion criterion was also applied:

- All conference proceedings, activity articles, literature reviews, theoretical papers, book chapters and book (e.g., Owens, 2015) were excluded.

Based on these inclusion and exclusion criteria, the author identified 36 articles for in-depth analysis.

Results

This section displays the findings with respect to each research question mentioned for this literature review. The findings inform us about the most influential theoretical framework used, different tools that are used to support students' understanding of geometry shapes, and the ways in which the complexities of multilingualism are addressed in the literature.

1. Most Influential Theoretical Frameworks in Geometry Education Research

The literature synthesis informs us that a variety of theoretical approaches have been used in geometry education research to develop an understanding of geometry concepts of shapes and their representations. These theoretical orientations include both geometry

specific theories/models and theories/models/approaches borrowed from other disciplinary perspectives. The most influential theoretical frameworks (see Table 1) are (i) the van Hiele's theory (n=11 out of 36), (ii) Vygotsky's approach to development (n=6), and (iii) Fischbein's theory of figural concepts (n=2).

Table 1
Major Theoretical Frameworks Used

S.no.	Theoretical framework	No. of studies	Studies
1.	Van Hiele's Theory	11	Abu, Ali, and Hock (2012); Chew Cheng and Lim Chap (2013); Erbas and Yenmez (2011); Forsythe (2015); Gunčaga, Tkacik, and Žilková (2017); Ismail and Rahman (2017); Lai and White (2012, 2014); Rizkianto, Zulkardi, and Darmawijaya (2013); Yanık and Ada (2013); Yi and Eu (2016)
2.	Vygotskian perspective	06	Hwang, Roth, and Kim (2010); Kobiela and Lehrer (2015) Kim, Roth, and Thom (2011); Roth (2014); Roth and Gardener (2012); Thom and McGarvey (2015)
3.	Fischbein's theory of figural concepts	02	Fujita, Kondo, Kumakura, and Kunimune (2017); Joglar Prieto, Sordo Juanena, and Star (2014)

It is interesting to note that besides these studies, the following theoretical models are used only in one study each. Likewise, van Hiele's theory and Fischbein's theory, geometry specific theoretical perspectives are Duval's theory of figural apprehension (see Hallowell, Okamoto, Romo, & La Joy, 2015) and Spatial Operational Capacity model (see Sack, 2013). Evidently, the studies also draw on spatial ability research literature. These include Pittalis and Christou's (2010) levels of 3D geometry reasoning (see Fujita et al., 2017), Bishop's (1983) visualization model (see Pittalis & Christou, 2013), and Lohman's (1988) model of spatial ability factors (see Pittalis & Christou, 2010). Semiotic approaches of Arzarello's (2006) concept of semiotic bundles (see Daher, 2014) and Bartolini Bussi and Mariotti's (2008) theory of semiotic mediation (see Bartolini Bussi & Baccaglioni-Frank, 2015) are also used. Language focussed theoretical approaches including Sfard's (2008) commognition approach (see Kaur, 2015), and Conversation analytic approach (see Mushin, Gardner, & Munro, 2013) are also used. In addition to these, other disciplinary approaches include Vergnaud's (1998) scheme characteristics (see Wright & Smith, 2017), Linn and Eylon's (2011) theory of Knowledge Integration pattern (see Vitale, Swart, & Black, 2014), Noss and Hoyle's (1996) notion of situated abstraction (see Panorkou & Pratt, 2016), Relational screening model (see Akarsu & Yilmaz, 2015), and Learning as Making approach (see Ng & Chan, 2018). Four research papers are found where the theoretical frameworks are not explicitly mentioned (see, Friedman, Kazerouni, Lax, & Weisdorf, 2011; Keşan, Vatansever, & Kaya, 2012; Ozkan & Bal, 2017; Turk & Akyuz, 2016).

2. *Different tools employed to support students' understanding of shapes and their representations in primary school.*

Table 2 indicates that a variety of tools have been employed to identify the ways through which primary class students develop an understanding of shapes and their representations. The majority of research studies have employed the use of technological variants (n=19 out of 36). These variants include Dynamic Geometry Education, Logo, GeoGebra, Google SketchUp, and Cabri among others. This shows that the present research trend favours the use of a variety of technological supports to foster the conceptual development of shapes and their representations in primary classes.

Table 2

Different Tools Employed to Teach Geometry Shapes and Their Representations

S.no.	Tools employed	No. of studies	See studies
1.	Technology variants (e.g. DGE, LOGO, Google SketchUp, GeoGebra, Cabri)	19	(Lai & White, 2014; Rizkianto et al., 2013; Yi & Eu, 2016) (Abu et al., 2012; Bartolini Bussi & Baccaglioni-Frank, 2015; Chew Cheng & Lim Chap, 2013; Erbas & Yenmez, 2011; Forsythe, 2015; Ismail & Rahman, 2017; Joglar Prieto et al., 2014; Kaur, 2015; Keşan et al., 2012; Lai & White, 2012; Ng & Chan, 2018; Panorkou & Pratt, 2016; Sack, 2013; Turk & Akyuz, 2016; Vitale et al., 2014; Yanik & Ada, 2013)
2.	Concrete manipulative (e.g. paper cut outs, wooden blocks etc.)	6	(Daher, 2014; Hallowell et al., 2015; Hwang et al., 2010; Kim et al., 2011; Roth, 2014; Wright & Smith, 2017)
3.	Language	5	(Akarsu & Yilmaz, 2015; Friedman et al., 2011; Kobiela & Lehrer, 2015; Mushin et al., 2013; Roth & Gardener, 2012)
4.	Standardised tests/ survey/ problem tasks	5	(Fujita et al., 2017; Gunčaga et al., 2017; Ozkan & Bal, 2017; Pittalis & Christou, 2010, 2013)
5.	Drawing	1	(Thom & McGarvey, 2015)

Only 5 out of 36 studies have explored language as a tool to develop an understanding of geometry shapes. Out of these five, three studies used a limited understanding of language for conceptual development. The focus of these studies was on defining geometry concepts using mathematical vocabulary (see Akarsu & Yilmaz, 2015; Friedman et al., 2011; Kobiela & Lehrer, 2015). The other two studies focused on how learners employ talk as a means to develop an understanding of shapes (see Mushin et al., 2013; Roth & Gardener, 2012). Mushin et al. (2013) examined the role of language and non-verbal behaviours in demonstrating an understanding of geometry concepts of shapes. They used Conversation Analysis to examine data and argued that language (non-) understanding may interfere with students' ability to perform in a geometry task (Mushin et al., 2013). An inability to follow teacher's instruction (often in a second language) is identified as a probable reason for the low performance of students rather than the task itself. Taking an ethnomethodological approach, Roth and Gardener (2012) argued that learners develop an understanding of shapes, as they talk about geometry shapes in everyday mathematics classes.


3. *Multilingualism in geometry education research*

It is thought-provoking that none of the studies explores the issues and complexities that may arise in the presence of multiple languages in geometry classes in spite of having participants from diverse linguistic backgrounds (e.g., Kaur, 2015; Kim et al., 2011; Sack, 2013; Vitale et al., 2014; Wright & Smith, 2017). Although Roth and Gardener (2012) and Mushin et al. (2013) underscored the importance of language in developing learners' conceptual knowledge of shapes, they failed to take account of the processes that influence this construction in the presence of multiple languages.

Discussion

In this section, the author presents her critical reflection (i) on the reasons for the major influence of van Hiele's theory in geometry education research, and (ii) on the widespread influence of technology in geometry education research.

Van Hiele's Theory as the Most Influential Theoretical Perspective

The literature review reveals that van Hiele's theory has been the most influential theory in geometry education research from 2010-2018. The reason for this influence is two-fold. Firstly, it has been argued that van Hiele's theory provides a framework that helps teachers and researchers to develop appropriate activities according to their students' thinking levels (e.g., Abu et al., 2012; Gunčaga et al., 2017; Ismail & Rahman, 2017; Lai & White, 2012, 2014). Secondly, it provides opportunities to incorporate the Digital Geometry Environment to promote students understanding of inclusion properties and the hierarchical relationships of shapes (Chew Cheng & Lim Chap, 2013; Lai & White, 2012, 2014; Yanik & Ada, 2013). With respect to the role of language, van Hiele (1999) argued that the function of language is to define the geometry concepts of sides and angles. This understanding is restricted to linguistic symbols and a system of relations that are particular to a thinking level. For example, at the thinking level 1, the learner is able to identify the shape  as a triangle, an enclosed figure having three sides and three angles. Thus, while van Hiele's theory emphasises the role of language in using geometry vocabulary, it neglects its communicational function that fosters meaning constructions of geometry concepts.

The Prominence of Technology Focussed Research in Existing Literature

It is evident from the present literature review that the existing trend of research studies favours the use of diverse technological variants (for example, DGE, Cabri geometry, Logo, Google SketchUp) to develop conceptual understanding of shapes and their representations in primary geometry classes. It has been argued that technological advancements, firstly, promote students' social interaction by providing a collaborative computing environment that enables them to develop richer mathematical discourses (Joglar Prieto et al., 2014; Kaur, 2015). Secondly, it has also been noted that the technologically-supported classroom environment enables students to develop a better understanding of hierarchical relationships among different shapes (Bartolini Bussi & Baccaglioni-Frank, 2015; Ng & Chan, 2018). Thirdly, the literature shows us that the strategies of dragging in a Dynamic Geometry Environment (DGE) enable students to overcome prototype phenomenon (Joglar Prieto et al., 2014; Kaur, 2015). However, while we acknowledge the role of technology in promoting conceptual development of shapes and their representations, it is crucial to note that technology is efficient in doing so only if there is careful instructional planning. Moreover, Keşan et al. (2012) noted that students having English as their second language display negative opinions about the use of DGE because the language for operating with the technological tools is English. An inability to understand the language of the DGE computer programs makes it inaccessible for the students to work with these. In addition to this, economic accessibility is another concern. It was noted that the studies could only be conducted in schools equipped with computer facilities (Erbaş & Yenmez, 2011). It can be argued that access to technologies for teaching and learning are not equitable (Oakes & Saunders, 2002); the question of equitable access and thus social justice for geometry education is inevitable.

The Significance of Multilingualism for Geometry Classes

Multilingual learners blend their multilingual competencies to work out their understanding of mathematical concepts as they successfully participate in the mathematical activity (Adler & Ronda, 2015; Moschkovich, 2015; Setati & Moschkovich, 2013). They contended that the understanding language as resource highlights the student's choice of language while engaging in the mathematical activity. Use of multiple languages in classes

also elucidates the issues of teaching dilemmas (Adler, 2002), power and dominance (Parra & Trinick, 2018), and access to meanings of mathematical constructs (Planas, 2014) along with its influence on a learner's self-concept and identity. However, the significance of language choice, power dynamics, and accessibility to geometry meanings in multilingual classes is often overlooked.

Conclusion

The present paper reports on a systematic literature review of the English language research literature in geometry education for primary students published from 2010-2018. It found that: (i) the most influential theory in the field of geometry education in this period is van Hiele's (1959/1985) theory, and (ii) the present research trend favours the role of technology variants for supporting conceptual development of shapes and their representations. However, the review also shows that there is a dearth of studies exploring the development of shapes and their representations within the multilingual context of geometry classes. It is crucial to recognise and explore the processes that enable multilingual learners to negotiate their understanding of the geometry concepts while interacting with others belonging to diverse linguistic backgrounds. Acknowledging the multilingual aspect of many contemporary classes has the potential to provide us with many research opportunities to gain valuable insights for geometry education.

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