Symposium: Research Methods Involving Children's Drawings in Mathematical Contexts

In this symposium we present and discuss some methodological issues and possible solutions that have been encountered during our research into children's mathematical thinking, behaviours and affective responses, as reflected, at least in part, through their drawings. It has been claimed that, "Drawing can be a window into the mind of a child" (Wolek, 2001, p. 215). Such a statement implies that a child's self-created drawing can provide an indication of his/her internalised mathematical perceptions and conceptions. Note that the word 'drawing' can be used as either a noun or a verb, and hence can refer to either a completed artefact or to the dynamic act of creation. Depending on the aims, theoretical perspective and context of the study, researchers may focus on one form of 'drawing' or explore both forms.

Although drawing has long been an expected component of children's mathematical activity, rigorous research methods utilising mathematical drawings have remained somewhat underdeveloped. In recent years, a number of researchers have grappled with the design and development of specific aspects of methodology in their separate projects. With few established research methods for guidance, researchers have been creating and refining task designs, interview protocols, data capturing strategies, analysis techniques and interpretation processes for their studies of children's mathematical drawing. Each of the symposium papers presents a different research tool or technique that has been developed within its own unique context, with the purpose of stimulating discussion and advancing the development of effective research methods in the field of children's mathematical drawing.

Wolek, K. (2001). Listen to their pictures: An investigation of children's mathematical drawings. In Cuoco, A. (Ed.), *The roles of representations in school mathematics*, NCTM 2001 Yearbook, (pp. 215-227). Reston VA: NCTM.

Chair/Discussant: Joanne Mulligan

Paper 1: Amy MacDonald & Steven Murphy *Using the drawing-telling approach to reveal young children's mathematical knowledge.*

Paper 2: Jennifer Way & Jennifer Thom. *Capturing the mathematical drawing process using a digital pen.*

Paper 3: Kate Quane, Mohan Chinnappan & Sven Trenholm. *The nature of young children's attitudes towards Mathematics.*

Paper 4: Jill Cheeseman & Andrea McDonough. *Coding young learners' pictorial responses to an open-ended assessment task.*

2019. In G. Hine, S. Blackley, & A. Cooke (Eds.). Mathematics Education Research: Impacting Practice *(Proceedings of the 42nd annual conference of the Mathematics Education Research Group of Australasia)* pp. 99-116. Perth: MERGA.

Using the Drawing-telling Approach to Reveal Young Children's Mathematical Knowledge

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This paper contributes to the symposium, '*Research methods involving children's drawings in mathematical contexts*' by exploring the "drawing-telling" approach to researching with young children. "Drawing-telling" is a methodological approach that encourages young children to represent their experiences and understandings through both drawings and accompanying narratives. The authors have used the drawing-telling approach to elicit young children's understandings about measurement within their first few weeks of starting primary school. This paper details the drawing-telling approach, and shares some insights and examples from the larger study of children's measurement knowledge at the start of school.

The "Drawing-telling" Approach

For many years, early childhood researchers have advocated for the use of drawings in researching with children. Research methodologies based on children's drawings are seen to attend to the communication strengths of young children (Wright, 2012), and empower children in the research process (Einarsdóttir, 2005). Children's drawings have been utilised by mathematics education researchers as a means of, for example, accessing children's mathematical knowledge (Lehrer, Jacobson, Kemeny, & Strom, 1999), investigating children's problem-solving strategies (Abu Bakar, Way, & Bobis, 2016), and exploring how children make "mathematical marks" (Carruthers & Worthington, 2006).

This paper reports on a drawings-based methodology known as the "drawing-telling approach". It was noted by van Oers (1997) that children often provide speech utterances when drawing as an attempt to ensure that all elements of the drawing are communicated. The notion of combining "drawing" with "telling" (a specific prompt to describe the drawing) has been explained by Wright (2007) as a process of seeking clarification from the child, as well as an extension of the child's images and stories. The use of both drawing and telling offers "an authentic kind of participation for the child" (Wright, 2012, p. 19) and ability for children to "bring to the surface what [they] already know, what they are grappling with and what they are motivated to explore further" (Wright, 2012, p. 214). Moreover, the "telling" component adds valuable information to the drawing because children's images can be selective, and on their own, tell an incomplete story (Einarsdóttir, 2005).

The drawing-telling approach has been adapted by Smith and MacDonald (2009), who encouraged young children to draw, and talk about, clocks as a means of discovering the knowledge about time possessed by young children upon entry to primary school. Smith and MacDonald highlighted the potential of drawing-telling as an open-ended task for finding out young children's mathematical knowledge. They noted the multimodality of drawing-telling as a particular strength of the methodology, as it provided children with the opportunity to "reveal their understandings in different but complementary ways" (p. 23).

This paper draws on data from a larger study carried out with 97 children who had just commenced primary school in NSW. As described in detail elsewhere (MacDonald, 2013), the study utilised six drawing-telling tasks to ascertain these children's experiences with, and understandings of, measurement concepts and processes as they commenced school. The children were invited to draw in response to a specific measurement-focused prompt (e.g. "Draw something tall and something short"), and while drawing, were encouraged to describe their drawing to the researcher. The children's comments were annotated on the drawing so that both the drawing and the children's narrative could be considered as a whole.

Insights and Illustrations

In this paper, we utilise data from two of the tasks from the larger study: 1. "Draw a ruler"; and 2. "Draw a clock". We have chosen to share these two tasks here because, in addition to offering insights into children's knowledge about measurement *concepts,* they reveal understandings about measurement *processes and tools*, as well as *structural* knowledge associated with partitioning and units. We offer a selection of examples that illustrate the contributions of the drawing-telling approach to a study of young children's mathematical understandings. Full analyses of these tasks are reported elsewhere (MacDonald & Murphy, 2018; MacDonald & Murphy, under review).

Revealing the Full Picture of Children's Knowledge

Drawing-telling allows children to offer a more complete picture of their understanding of measurement than a drawing alone. The clock drawn by Kyra (Figure 1) included only a round face and hands, whereas what she told indicated what she knew of the numbers of a clock and their relationship to the hands. Ella's ruler picture (Figure 2) shows number sequence and partitioning, but it is her narrative that reveals she has deliberately drawn a measurement of the house.

"The town clock. It has hands that tells the time. The hands point to the number that go up to 12."

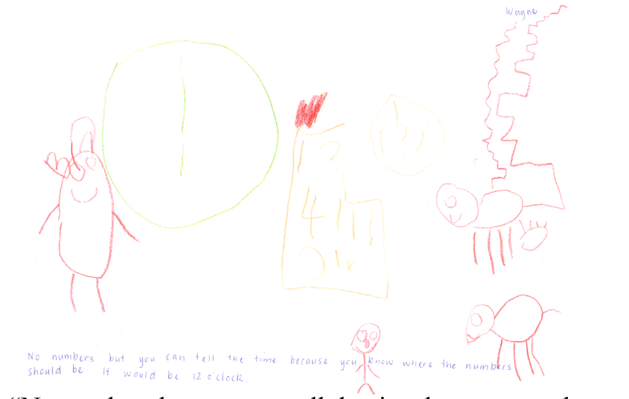
Figure 1. Kyra's clock.

"My ruler is from 1 to 7. I knew what to draw because I know the numbers 1 to 7. The numbers help you measure. The house is 6."

Figure 2. Ella's ruler.

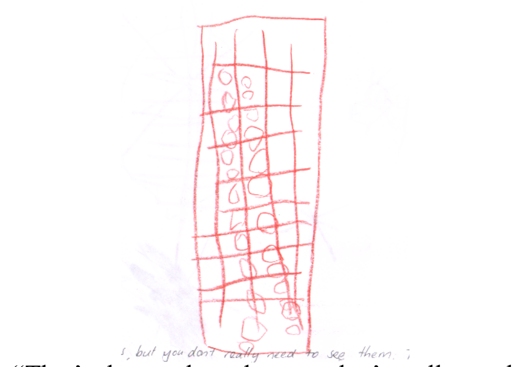
Revealing the Potential for Misinterpretation

Not only does limiting analysis to a child's drawings risk building an incomplete picture of their understanding, it may also lead to incorrect conclusions about a child's knowledge. Wayne's drawing (Figure 3) without his telling indicates only a rudimentary understanding of clock structure. However, his narrative exposes an awareness of the role of numbers and time-telling. Similarly, on first inspection of Tye's ruler drawing (Figure 4), it might be thought Tye knows little about rulers; however, his narrative suggests he has significant knowledge of ruler structure.



"No numbers but you can tell the time because you know where the numbers should be. It would be 12 o'clock."

Figure 3. Wayne's clock.



"That's the numbers, but you don't really need to see them. They're the lines. So you know how much they are."

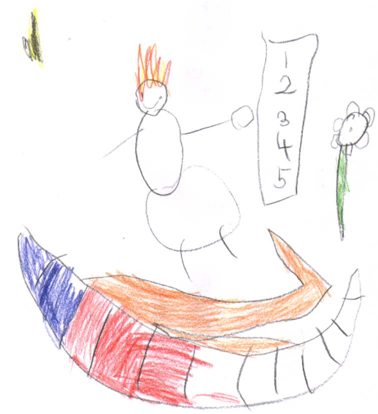
Figure 4. Tye's ruler.

Revealing how Experiences Shape Understandings

Drawing-telling offers insights into how children gained the understanding they display in their drawings. Makaylee drew a clock with differentiated hands and some numbers (Figure 5), and explained that she had been practicing with her dad. While it may not be clear what some of the objects in Willis' drawing are (Figure 6), his rich narrative offers insight into his understanding of ruler structure and application.

"My dad's been drawing clocks on a piece of paper for me, showing me how to put numbers on it and helping me tell the time."

Figure 5. Makaylee's clock.



"I put some numbers. They help us find how long and short things are. I've seen my daddy use a long ruler before, to measure the door. He took the front flyscreen door out to put new gauze in. He had to measure it to see how much gauze to use."

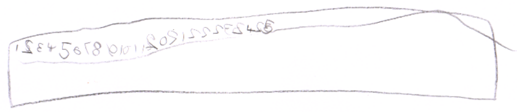
Figure 6. Willis' ruler.

Revealing the Utilitarian Value of Measurement

Finally, drawing-telling makes it possible for a child to share what they understand of the purpose of measurement. Phoebe used the invitation to describe her clock drawing (Figure 7) to explain that clocks help regulate aspects of her daily routine. Ethan used the invitation to discuss his ruler drawing (Figure 8) to reflect on the appropriateness of different measuring devices for different purposes.

"There are circle clocks, square clocks, love heart clocks, triangle clocks, and rectangle clocks. They go 'bing bing' when it is time to wake up. The numbers tell me when the time is breakfast or morning or play time."

Figure 7. Phoebe's clock.



"It's a ruler and it goes up to 25. I would measure with a ruler. I would measure wood, metal, my cubby house. My cubby house would be too long, but I'd still measure it. I'd measure my whole house, all the way around, even if the ruler's not long enough. I'd use a measuring tape instead. Maybe I'd join the ruler to the measuring tape to make it really, really long. I've seen a ruler in my dad's toolbox. And in my room. Three rulers in my room, and one in dad's toolbox."

Figure 8. Ethan's ruler.

Implications

As explained by Smith and MacDonald (2009), drawing-telling is a powerful tool for finding out the background knowledge and experiences of children before beginning formal instruction on a mathematical topic. It extends on the power of using drawings to explore the mathematical knowledge of young children, improving the accuracy and richness of the interpretations that can be made. Indeed, the ability to clarify information through describing the drawing enables children to ensure their knowledge has been recognised. For researchers, it offers the potential to explore not just what a child knows but also how the child knows it and why this knowledge is significant to them. For educators, drawing telling allows deeper insight into a child's mathematical understanding and provides the potential to personalise learning and connect prior learning to future learning experiences.

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Capturing the Mathematical Drawing Process Using a Digital Pen

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This paper contributes to the Symposium: *Research Methods Involving Children's Drawings in Mathematical Contexts* by exploring the use of digital pens as a data gathering tool. The availability of digital recording devices has been a boon to researchers wanting to capture the real-time dynamics of a research situation. When capturing a child's drawing process, an alternative to cumbersome video-recording equipment is a digital pen that records both the creation of the drawing and any nearby utterances. To highlight the affordances and limitations of the digital pen as a data collection tool for children's drawing we utilise examples from two different research projects, one with Australian children and the other with Canadian children.

The methodological issue addressed in this paper is the need to capture the dynamic process of drawing creation. This need resides in the theoretical perspective that the study of children's (re)presentational systems is essential to research that seeks to better understand children's mathematical learning (Goldin & Kapput, 1996). The distinction is often made between internal psychological representations and external physical representations, but it is the interplay between the two that is "… fundamental to effective teaching and learning" (Goldin and Shteingold, 2001, p2). As researchers we seek to infer the 'invisible' internal (re)presentations by interpreting observable external (re)presentations which are typically actions, such as gesture, speech, manipulation of objects and drawings (Bobis & Way, 2018). Drawings are often seen as products or static artefacts. However, the dynamic process of drawing itself enables further examination of cognitive functions and co-emerging understandings (Thom & McGarvey 2015), and so the process of creating a drawing can reveal other aspects about a child's thinking and learning than a completed artefact alone. Consideration of more than one form of (re)presentation enhances the meaning-making potential of the research. One (re)presentational form (in this case drawing) does not operate in isolation from other (re)presentational systems but rather, each is inseparable from the other(s). Thus, the focus of research is on examining events as they happen and to study the collective emergence that occurs amongst verbalisations, contextual influences, gestures and other movements (Depraz, Varela & Vermersch, 2003).

While in many qualitative research situations the ideal approach might be for the researcher to directly observe the class as a whole and each of the participants while capturing everything using multiple cameras and audio-recording for later analysis, the realities of research contexts such as classrooms, often prevent or limit such ideal data collection methods. Digital pens that capture mark-making and sound, and create 'pencasts' that can be replayed, can provide a useful alternative, or a supplement to, other data collection methods. The purpose of this paper is to share some experiences in using the digital pen as a data collection device and to reveal some of the affordances and constraints that it offers.

Example 1

Research Context

Example 1 is from a Year 4 child (Ethan, Age 10) participating in the *Thinking Tools: (re)presentations in Primary Science and Mathematics* project that is exploring primary school children's use of drawing(s) as thinking tools. In individual task-based interviews, students drew 'what happened' after rolling a toy car down a ramp that had been set at three different gradients. The interview was also captured by wide-angle video recording. Figure 1 presents Ethan's completed drawing, with comments describing the sequence of construction.

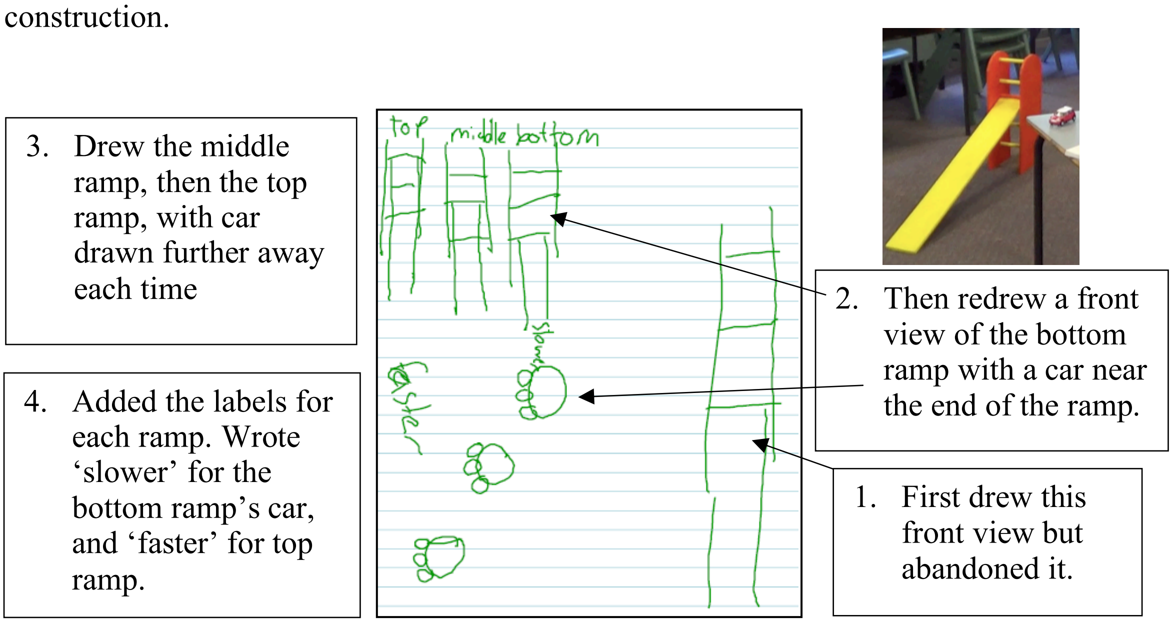

Figure 1: Ethan's drawing of the car ramp event

Commentary

Ethan started drawing on the right side of the paper and moved towards the left side (Figure 1). He produced a set of three drawing(s) from a *front view*, one for each ramp level The focus of the drawing(s) was the relationship between ramp height and the distance travelled by the car. When asked why the car went different distances, speed was spoken about, then 'faster' and 'slower' added as word labels. When asked why the car went faster on the top ramp he replied, "It could get more speed because it was going steeper". The connection between slope (steeper) and speed was made verbally (with arm gestures showing angles), but was not (could not be) depicted in front-view drawing(s).

Affordances and Constraints

An important feature of our analysis was the 'layered' approach. With our particular interest in what each child chose to draw when (re)presenting their thinking, we first replayed the pencast without sound to isolate the interpretation of the drawing itself. We found the clarity of drawing detail and sequence provided by the pencast to be very useful. Subsequent analysis included the synchronised sound-track, which enabled elaboration of the initial interpretations through listening to the child's verbal explanations. It also allowed identification of the effects of questioning by the researcher on the child's development of reasoning and the initial (re)presentations. The final layer of analysis drew on the video recording which revealed data beyond the pencast— what actually happened during the car experiment, as well as the child's movements, gestures and expressions.

Example 2

Research Context

The second example features the drawing(s) of one student—Sophia, who participated in a three-month study with her kindergarten and grade one class for 19 geometry and spatial reasoning lessons. In this excerpt, Sophia (7 years old), Emma (6 years old) and April (6 years old), were shown a photograph (Figure 2) and asked to express what they saw in a manner that was accessible to the others. The children then explored how what they saw in the photograph could be part of a 3D object and, what part(s) of the object might be seen when looking from different perspectives. Figures 2 also includes the sequence of key moments in which images and verbalizations by Sophia emerged and were dynamically captured by the digital pen:

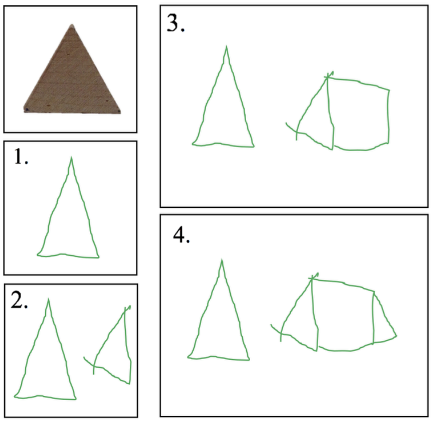

Figure 2. Digital photograph presented to the group and Sophia's sequence of drawing(s).

Commentary

Sophia's first drawing expressed what she saw in the photograph and verbally identified as "a triangle" (Figure 2.1). Later to the right of the paper, she drew what might be seen if the triangle was the 'front' of a 3D object and she was looking at it from the 'back' or opposite view (Figure 2.2), justifying "Because triangle, triangle, something else." Sophia then added to the drawing what the triangle in the photograph might be if it was a 3D block (Figure 2.3-4). Following this, she exclaimed, "Ha-ha! I just noticed something. That's so funny! It's the shape that we're drawing. The paper. See? Triangle, triangle, rectangles."

Affordances and Constraints

The digital pen captured Sophia's drawing(s) and dialogue in real time, providing a moment-to-moment account of how her thinking and reasoning emerged over the course of the lesson as drawing(s) and verbalisations; more specifically, the ways she moved conceptually back and forth as she considered the triangle from different perspectives and dimensions; the ongoing conversation of the small group; and how her drawing(s) as both act and artifact enabled further explorations of the triangle. Because the pen synchronously records drawing(s) and verbalizations, it was possible to replay any instance of the children's drawing activity (and conversation) by 'clicking' on a specific part of the drawing using the pen or on the computer after uploading the video and audio data from the pen. As the playback quality of sound and drawing(s) was very clear, certain parts of conversations and drawing(s) were replayed in later lessons to facilitate reflection and further inquiry with the teachers and children. It is also possible to amalgamate data from other pens to conduct comparative analyses across student work and to examine the collective thinking of a group or entire class; for example, according to temporal events or identified themes.

We mentioned in the first example that video data was necessary to capture other gestures, movements, and use of materials. The data was also required to examine how the different modes related to one another and with the study context at large. In a similar way, it was only by analysing the pencast of the second excerpt with the video data from the lesson that it became evident how Sophia continuously used— in addition to drawing(s) and verbalisations— found materials, the 3D space in front of her and distinct whole body and hand gestures to situate, draw, sculpt and connect triangles and rectangles on various planes.

Conclusion

As a data collection device, the digital pen affords an effective and practical means for capturing and reviewing the dynamic (re)presentation processes of drawing(s) and verbalisation. In data analysis, the pencasts facilitate a sharp focus on specific features of drawing(s) and intricate relationships between mark-making and verbalisations, which supports the researcher in making well-founded inferences about children's thinking and reasoning. The addition of video-recording to capture other interrelated (re)presentations, such as body movement and hand gestures, further enriches interpretations of the interplay between the child(ren)'s internal and external (re)presentations.

Acknowledgement

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The Nature of Young Children's Attitudes towards Mathematics

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This paper contributes to the Symposium: Research methods involving children's drawings in mathematical contexts by providing insight into young children's attitudes towards mathematics through the use of drawings. Children in Years 2 and 3 (n=25) participated in this study which aimed to identify a range of attitudes towards mathematics using the Three-dimensional Model of Attitude (Zan & Di Martino, 2007). The analyses illustrate drawings in conjunction with written responses, interviews and classroom observations offers authentic evidence of young children's attitudes towards mathematics. Further, findings indicate that children's Vision of Mathematics contributes significantly to their overall attitude.

The link between attitudes towards mathematics and achievement has been of considerable interest, as findings suggest negative student attitudes towards mathematics have a detrimental impact on their memory and achievement (Zan & Di Martino, 2007). However, the construct of attitude is nebulous and multi-dimensional (Di Martino & Zan, 2010; Hannula, 2002; Leder, 1987).

Traditionally, attitudes towards mathematics have been defined via the dichotomy of liking or disliking mathematics. While this simplistic view places emphasis on the emotional element of attitude, it fails to address the dynamic nature of attitude and how attitude influences, and impacts on behaviour (Ajisuksmo & Saputri, 2017). Further, such a narrow definition ignores the critical role of beliefs and values which may enact or inhibit behaviours influencing student learning (Goldin, Epstein, Schorr, & Warner, 2011; Skott, 2015).

In order to address this issue, Zan and Di Martino (2007) have conceptualised attitude towards mathematics in terms of three dimensions incorporating emotions, values and beliefs. The Three-dimensional Model of Attitude (TMA) comprises of the Emotional Dimension (ED; emotional responses to mathematics), Vision of Mathematics (VM; instrumental versus relational view, values and appreciation of the subject) and Perceived Competence (PC; perceived ability and self-concept). This model broadens the definition of attitude and has been successfully used in autobiographical research with older students (Di Martino & Zan, 2010). However, there is a paucity of research into affective aspects of mathematical learning and young children's attitudes towards mathematics (Grootenboer, Lomas & Ingram, 2008). The overall aim of our larger study is to address the issue by examining attitude via children's drawings. In this paper, we report phase one of the study.

Drawings as a data source for attitude

Children's drawings are the primary methodological tool for this research. Drawings provide a rich source of data that can convey subtle and multifaceted expressions of feelings and ideas that reflect children's understandings of real-world mathematical experiences (Cherney et al., 2006; Jolley, Fenn, & Jones, et al., 2004). Drawings are also an easy vehicle for communication, allowing children to express what is important to them. The review conducted to date indicates attitudes are voluntarily presented in children's drawings showing the complexity of children's attitudes, beliefs and values towards mathematics.

Method

This exploratory study was conducted in a South Australian State primary school. Four data collection techniques were used; drawings, written responses, interviews and classroom observations, providing both non-lesson and lesson contexts to gather data about children's attitudes towards mathematics. Twenty-five children were provided with 24 coloured textas, an A3 piece of paper and asked to draw themselves doing mathematics and write about their drawing. Each drawing was then labelled with a code indicating class, gender and age.

A prompt was read to the children followed by a series of question in a 10-15 minute semi-structured interview session Interview questions were designed to clarify what children had drawn and to help ascertain children's ED, VM and PC. For example: 'If mathematics was a food what food would it be and why?' Finally, classroom observations were conducted with a subset of children to examine how children's attitudes, depicted in their drawings, may be enacted during their mathematical learning experiences.

Code Development

Data were analysed using the TMA framework. The analysis involved deductive, anticipatory and inductive methods. A deductive approach was used to develop the main ideas and indicators using existing frameworks for analysing children's drawings, drawing upon the fields of mathematics and science education, education, and psychology. A rubric was then developed with the addition of two sub-dimensions for each dimension, with each sub-dimension measured on a scale from 0, ('cannot be categorised'), to 5, ('extremely positive'). Some indicators were anticipated, stemming from the researchers' own experience as a teacher. For example, the sub-dimension 'overall appearance' uses the anticipatory response 'messy hair' as an indicator. Additionally, an inductive approach was employed to analyse the children's drawings and determine the suitability of the categories.

Drawings were systematically analysed using the principles of atomism and holism. The six scores for each sub-dimension were then added to give an overall score out of 30. Taken together, this process provided a quantitative measure of a child's overall attitude towards mathematics, with intervals classified according to Table 1.

Table 1:
Interval Classifications for Attitudes towards Mathematics

Interval	Attitude Classification
< 5	Excluded from analysis
6 – 10	Extremely Negative
11 – 15	Negative
16 – 20	Neutral
21 – 25	Positive
26 – 30	Extremely Positive

Findings

A range of attitudes were identified in the 25 children. No child was classified as having an Extremely Negative attitude. One child (A10) was classified as having a negative attitude towards mathematics. To help illustrate the outcome of the analysis, her participation will now be discussed in more detail. At first, the drawing (Figure 1) suggested A10 enjoyed mathematics. However, asking the child to explain her drawing revealed the child felt a range of negative emotions including frustration, worry and animosity towards mathematics.

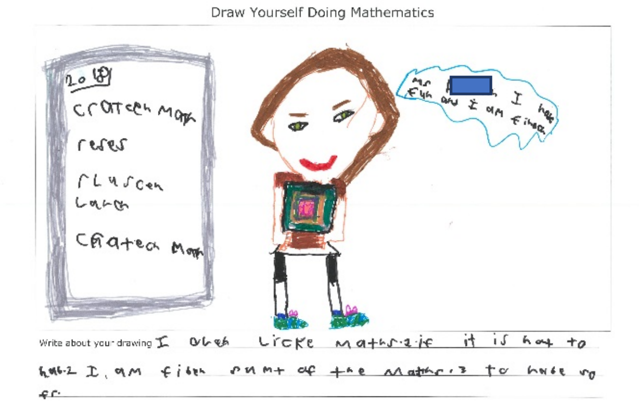

Figure 1: Drawing and written response from a child with a negative attitude (Yr 3, F, 8 Years ED = 5, VM = 3, PC = 3)

The excerpt below identifies a specific instance where her frustration was revealed.

A10: I like creating things a lot and I'm, well if I do too much maths, I don't enjoy it and also um if sometimes I find it easy and sometimes, I find it hard and if I find it hard, I start to hate it.

Interviewer: What do you find hard?

A10: Like if I don't know what that is, I don't really like it and I get really frustrating (sic) and then I get a headache.

Interviewer: Can you give me an example of when that happens?

A10: Like um if I don't know what a tens frame is which happened, and then I got really frustrated and I was worried that I got it wrong and Ms Teacher (edited for confidentiality reasons) looked at the rest of my work and then I started to get really frustrated.

In the case of A10, she could articulate a particular instance regarding not understanding a mathematical term that made her feel frustrated and worried, resulting in an animosity towards mathematics. Using gentle probing questions, the interview provided further insights into the child's attitude. The child proposed a schedule for learning mathematics that involved creative mathematics as she enjoys creating things. An example of this is the child's t-shirt which she described as an animal catcher that can catch insects. A10 was asked why she was smiling in her drawing. She responded with "because um I wanted to be in my relaxing time, I wanted to create a fun animal catcher." Relaxing time in this instance is the child's way of removing herself from reality. A10 articulated few mathematical concepts, reflecting an instrumental understanding of concepts.

When asked to rate herself as a maths student, she rated herself zero out of ten "because I am terrible at maths and it's too hard, cause sometimes I'm nearly in tears". The child's written statement below her drawing verifies how the child feels about mathematics. The child has written "I don't like maths. If it is not too hard. I am feeling some of the maths too hard, so frustrated." This is evidence of how the child is trying to make a statement regarding the perceived difficulty of the subject. It appears that the child is uncomfortable when challenged by cognitively demanding tasks. The child's written response contributes to all three dimensions of TMA. That is, emotionally she is feeling frustrated, is not liking the subject (ED) and her statement about the difficulty of mathematics provides an indication of her VM. Overall the statement contributes to understanding her negative self-concept (PC).

Child A10 was observed three times during class with the child partaking in maths rotations involved game-based tasks on telling the time and place value. Other lessons involved individual problem-solving using the four operations and an outdoor experience to collect data about animals in a natural habitat. During the observations negative talk and task avoidance were evident, including asking the teacher what jobs she could do, trying to find a particular coloured pencil and stating “I can’t do this”, “this is too hard” and “I don’t get it”. The child’s body language was observed providing further evidence of her feelings towards mathematics, including walking away from a task, shrugging shoulders and clenched lips. A10 appeared to be more engaged and willing to participate in the outdoor experience which had a science focus to the data collection. Altogether, drawings, written and interview responses provide a rich, relatively complete picture of A10’s attitude towards mathematics reflecting the authentic personal experience of the child.

Conclusion

Phase 1 of the study reported here, shows children’s drawings in conjunction with interviews and observations provide an effective means of ascertaining young children’s attitudes towards mathematics. Children were able to depict themselves doing mathematics and articulate their emotions, VM and PC. Children generalised a range of personal experiences having control over what they wanted to draw. A full range of attitudes towards mathematics were identified. Thus, drawings provided children with the tool to share not only their feelings about mathematics, but allowed insight into their values and beliefs about themselves doing mathematics. This paper reports a single child’s negative view of mathematics. A larger sample may find greater insights into children’s attitudes towards mathematics, in particular, alternative negative attitudes. Applying TMA to the four data types has been a challenge as a child’s response can be evidence for several sub-domains, indicating that the dimensions may overlap. The relationship between the three dimensions is currently being explored.

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Coding Young Learners' Pictorial Responses to an Open-ended Assessment Task

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Following the teaching of a unit on mass measurement to 274 children (6 to 8 years of age), each of 13 classroom teachers administered an open-ended assessment task. Children represented their perceived knowledge of mass measurement in response to an "Impress Me" prompt. Drawn and written recordings were complex and multi-dimensional, ranging from drawings and/or descriptions of activities children had undertaken or materials they had used, to the articulation of understandings related to foundational ideas of measure. The complexity of categorising and coding the pictorial responses of children is central to this paper. Interpreting children's drawings is not straightforward and this has implications for classroom teachers.

Often, young children possess informal knowledge of mathematics that is "surprisingly broad, complex, and sophisticated" (Clements & Sarama, 2007, p. 462). Ideas of measurement begin to develop early (Lee, 2012) and while direct measurement is relatively simple, complex mental accomplishments are involved in measuring (Wilson & Osborne, 1992).

With calls for increased focus on the assessment of measurement understandings, and the recognition of complexity in measuring, methods of assessment that better, or differently, allow complexity in children's understandings to be revealed are warranted. Children's perspectives on their learning of measurement can provide insights and can inform teachers' interactions with young learners (McDonough, 2002). Drawings are a familiar form of expression for most children and, for some, may be preferred over writing. Drawing can also be a "powerful medium for discovering and expressing meaning; for the young child, drawing brings ideas to the surface" (Woleck, 2001, p. 215). Our overarching research question investigated children's thinking about measuring mass. To provide insights into their thoughts, young learners responded to an open-ended assessment task by drawing and/or writing. Here we look at the processes we used to reliably interpret, categorise, and code learners' responses.

Method

Thirteen teachers from three schools taught a sequence of five lessons from our original teaching experiment (Cheeseman, McDonough & Ferguson, 2013) to 274 Year 1 and 2 students. The teachers then assessed the children's understandings by administering a task called "Impress Me", giving each child a sheet of A3 paper and reading the following prompt:

> We have been doing lots of weighing lately. I want you to show me on this piece of paper all you know about mass and weighing. You can write or draw or do both! Take your time and show your ideas and thinking as best you can. I want you to "impress me" with all you know about mass and weighing.

Children could choose to draw, write, or combine the two in conveying their understandings. This was seen as a flexible, responsive, sensitive approach to assessment that could benefit and liberate teachers and students (Woleck, 2001).

The data were analysed using a grounded theory approach. A central feature of this analytic approach is a general method of comparative analysis (Glaser & Strauss, 1967). In this methodology, theory may be generated initially from the data, or, appropriate theories may be elaborated and modified as incoming data are meticulously played against them. Researchers can also carry into current studies any theory based on their previous research, providing it seems relevant and theory matching is rigorously carried out (Glaser & Strauss, 1967). Each of these approaches was used to analyse the data discussed here.

The process of coding

Our focus here is describing the process of categorising and coding the data to summarise findings and describe patterns. All responses were read and each element - written, drawn, or both - was identified and clustered so that similar descriptions of students' responses were gathered. Patterns emerging from the data were sought. This initial reading of the data required modification as the classification was descriptive and lacked a measurement theoretical framework against which the data could be interpreted. In previous work in the Early Numeracy Research Project (ENRP), we developed and tested a research-based theoretical framework of "growth points" for mass measurement (Clarke et al., 2002). We adopted this framework to interpret the present data and each element of the responses was re-coded accordingly.

It is known that evidence may be coloured by prior knowledge when coding responses but it is enriched by the knowledge too as it provides some background to the thinking revealed by children. Questions about the reliability of our interpretation of the responses arose. To improve internal reliability, 25 percent of the children's work was re-coded to examine consistency between researchers and test whether similar conclusions could be reached independently about children's responses. Two teachers skilled at interpreting young children's written and pictorial ideas applied the defined categories and codes. The results produced an inter-rater reliability (traditionally measured as percent agreement, calculated as the number of agreement scores divided by the total number of scores) of 77 percent. All points of difference were discussed and an agreed understanding of the data was reached. New, tighter, definitions of the categories and codes were written. The inter-rater reliability test showed that: the categories that emerged from the data were applicable; some category descriptors required clarification; and the theoretical mass measurement framework was suitable although it required some elaboration.

The modified theoretical framework

For each of the five ENRP growth points (GP), two sub-categories were defined: Emerging and Deeper. Often there was not enough information or clear enough information for a response to be classified as meeting that growth point (Deeper), as illustrated in the following descriptors for GP3 Emerging: "Names or draws informal units (e.g., plastic teddies) with no, or unclear context. Draws balance scales with possible informal units but unclear or no labelling/explanation". To be categorised as "Deeper" a drawing/writing needed to show an awareness of the concepts underpinning the growth point. For example, for GP3, the child's response needs to show awareness of the principles of non-standard units (e.g., same mass unit). With these ideas in mind, we defined a 10-point scale (Fig. 1). The entire data set was re-coded applying the new protocols without any reference to the previous coding. Figure 2 is an illustration of the analysis of the coded data.

Results

In response to the Impress Me prompt each child produced an average of two and three drawings/writings (total n = 720). The distribution of all of the coded responses (Fig. 2) shows that very few drawn/written responses displayed no awareness of mass measurement (n = 5). Almost one third of responses (31%) were categorised as growth point (GP) 1 and the large majority (193) of them displayed a deeper understanding of the attribute of mass. Examination of the results reveals that the 6 to 8 year-old children drew representations displaying their knowledge of mass measurement that was beyond the intended curriculum (approx. GP1 - GP2). Space restrictions prevent further analysis of the findings here. However, one observation is pertinent to the coding processes that are our focus – the definition of the sub-categories (emerging and deeper). Figure 2 shows that at GP2 and beyond, more children demonstrated emerging ideas of measurement concepts specified by the growth point than those who had developed a deeper understanding of those concepts. Due to the use of these codes we could see evidence of children's emergent learning across the mathematical framework.

0 No apparent awareness of mass	
1 Awareness of the attribute of mass and use of descriptive language	
1A Emerging	Includes some mass terminology as single words or a list but with no or unclear context (e.g., mass, weigh, weighing, scales, measuring, hefting) Pictures and/or descriptions are unclear or incorrect
1B Deeper	Includes some correct explanation for mass terminology (e.g., mass or weighing is finding heavier/lighter/equal) Contexts are correct or appropriate (e.g., feather is light; brick is heavy; or reference to lessons where working with the attribute)
2 Comparing, ordering, & matching with the attribute of mass	
2A Emerging	Provides a description of the outcome of hefting or use of balances scales (e.g., finding heavier, lighter, the same, the answer, or as ordering) Comparison is shown through drawings of hefting or balance scales but with no or some incorrect labeling
2B Deeper	Describes/draws hefting or balance scales to compare mass of single objects with correct labelling (e.g., as heavier, lighter, heavy goes down)
3 Quantifying mass accurately, using units and attending to measurement principles.	
3A Emerging	Names or draws informal units (e.g., plastic teddies) but with no or unclear context Draws balance scales with possible informal units but unclear or no labelling/explanation
3B Deeper	Draws measuring using balance scale and informal units with correct labelling (e.g., one object measured with multiple same mass informal units) Describes process of weighing with informal units showing understanding
4 Choosing and using standard units for estimating and measuring mass, with accuracy	
4A Emerging	Implies formal mass units (e.g., grams, kilograms) without context or labelling or unclear Records terminology of grams and/or kilograms without context or incorrectly Records equivalence (e.g., 1 kilogram = 1000 grams)
4B Deeper	Draws or describes standard units of grams and/or kilograms (e.g., an object weighs x kilograms)
5 Applying knowledge, skills and concepts of mass	
5A Emerging	Refers to or draws weighing in an everyday, non-school context (e.g., people, food, post office parcels) or school context (e.g., playdough). Use of different scales (e.g., digital, kitchen, bathroom, spring) Shows some attention to relationships (e.g., size/mass; conservation of mass; gross/net) but complete understanding is not evident
5B Deeper	Provides an interesting insight about a big idea of mass measurement Clear articulation of relationships showing understandings (e.g., volume/mass; conservation of mass; gross/net)

Figure 1. Mass Measurement Expanded Analytic Framework

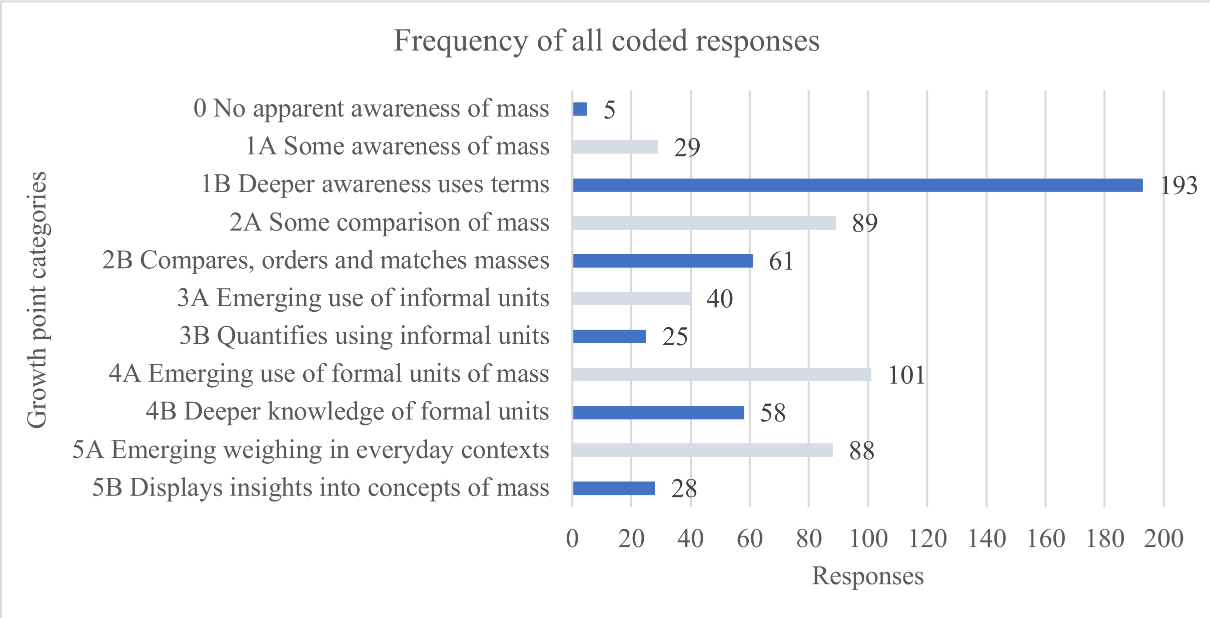

Figure 2. Distribution of all coded responses to Impress Me.

Discussion and conclusion

The Impress Me protocol was effective in eliciting children's drawings that were complex and revealing. It was possible to "read" the recordings as mathematical diagrams, stories of actions, and lists of ideas. Individual students' thinking could be summarised in general terms by studying the drawings/writings and teachers benefited by hearing additional explanations. However, as an assessment tool and for our research, the data needed to be analysed to provide an overview of responses. Categories and codes were assigned to the recordings using an iterative process and a refined framework for synthesis and analysis. Results showed patterns in the data and the range of concepts children had communicated.

To build an overview of students' thinking we believe teachers would find a rubric that defined a hierarchy of responses to be useful. However, there are some issues to consider. Is the student's score based on the demonstration of the "highest" level of sophistication of thinking shown? Does each work sample element receive a score (as we did for the research)? The answers to such questions are not straightforward. What we are convinced about is the potential of children's drawn representations of their thinking.

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