"*I think it's 3D because it's not 2D*": Construing dimension as a mathematical construct in a New Zealand primary classroom

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The paper explores one episode from a larger study, where a multilingual student (10-year-old) described her understanding of what makes a shape 2D or 3D. Bakhtin's dialogic theory and Garfinkel's ethnomethodology inform the theoretical framework. Transcribed data of the episode is presented, which is analysed at micro-level and macro-level. The analysis revealed three major findings. First, the analogy of "flat vs fat" for describing shapes may not be useful. Second, the meanings of the terms such as fat are shaped by the interaction of unitary language and heteroglossia. Third, prosody embedded in utterances contribute to the meaning constructions during mathematical discussions. A few implications are presented.

The mathematical construct of dimension plays a crucial role in developing foundational skills in mathematics, more so for construing understanding of two-dimensional (2D) shapes, three-dimensional (3D) shapes and their properties. This paper presents an episode from a larger study from a New Zealand Year 5/6 geometry classroom, where a student described her understanding of what makes a shape 2D or 3D. In school geometry, 2D shapes are the plane shapes that have only two dimensions that are length and breadth. Whereas 3D shapes are the solid shapes that can be held, have thickness/depth along with length and breadth. These definitions of 2D and 3D shapes focus on the dimension as a measurement attribute of an object. In mathematics education research, very few studies have explored students and teachers' understandings of dimension. Lehrer et al. (1998) found that students construct dimension as a property of thickness of an object, a finding supported by Morgan (2005). Recently, Panorkou and Pratt (2016) investigated 10-year-old students' construction of ideas about the dimension, and provided additional understanding of dimension. They argued that children expressed that objects/spaces with lower dimensions can move within objects/spaces of higher dimension. Tossavainen et al. (2017) studied pre-service teachers' understandings of the area and its dimensional aspect. They argued that although the concept of area is central in elementary mathematics, the aspect of two-dimensionality is hardly considered in teaching and learning of shapes and their areas. In addition, they highlighted that the use of the same word for the boundary of the shape as well as the space within the shape might add to the difficulty in construing dimension as an important attribute for understanding shapes. For example, Bezgovšek Vodušek and Lipovec (2014) have shown that in Slovenian language, the boundary of circle is not considered as a 2D shape, and is called *krožnica*; whereas, a disk is a 2D shape of a circle, and is called *krog*. Interestingly, these two terms *krožnica* and *krog* highlight the understanding of shapes from Euclid's boundary notation perspective (Manin, 2006; Skordoulis et al., 2009). According to this perspective, the points are the boundaries of lines, lines are boundaries of the surface, and the surface accounts for the boundary of the solid object (Skordoulis et al., 2009). As a result, the dimension of, let us say, *krožnica* (circumference of a circle) and *krog (circular region)* would be different. However, in English, a "circle" is used to signal both the boundary and the area (as a disc) enclosed by the circle, which may complicate the process of understanding dimension as a mathematical construct.

Interestingly, research investigating students' understanding of dimension in multilingual classrooms are even rarer. Multilingualism research has often focussed on either

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mathematical terminology (e.g., Adler, 2002) or grammatical patterns of mathematical registers (see Kotsopoulos et al., 2015). This focus on mathematical terms and syntax negate the role of prosodic features of language use that might contribute to meaning-making. Ward (2019) argued that patterns of stress and intonation in language provide impact to words, their meanings, and their social significance. For example, Hay et al. (2008) have shown that New Zealand speakers often use a High Rising Terminal (HRT) intonation in their speech to show solidarity with the listener and to check if the listener is following the speaker, instead of asking a question. Thus, to explore students' understanding of dimension while acknowledging the superdiverse context of New Zealand, this paper aims to answer the following research question: *How do Year 5/6 multilingual students discursively construct and reconstruct their understanding of dimension during classroom interactions?*

Theoretical Framework

The theoretical framework for the larger study was informed by Bakhtin's (1981) dialogic theory and Garfinkel's (1967) ethnomethodological approach. Bakhtin (1981) argued that language provides us with a dialogic space, which opens shared space for all participants to generate meanings as they engage in dialogue, that is in this dynamic space, all possible meanings are considered in a continuum of meaning construction. The specification of meaning is dependent upon the preceding and succeeding dialogues. Bakhtin argued that this negotiation of meanings occurs in the realm of the constant struggle between unitary language and heteroglossia that operate concurrently at different levels of interaction. The *unitary language* (unifying language force) account for the system of norms that dictate the accurate use of language, with the aim to guarantee mutual understanding of the meanings of utterances by crystallising their meanings, thus, limiting the occurrence of divergent meanings of the utterances. At the same time, *heteroglossia* (diversifying language force) attempt to decentralise the already established meanings of the utterances by embedding the use of language with individualised meanings. It is the ongoing play of these unifying and diversifying language forces in a specific circumstantial context as well as the socio-cultural milieu that informs the specific meaning of an utterance within a sphere of communication (Barwell, 2018). Exploration of what is said, when it is said, and how it is said can enable access of these heteroglossic and unitary language forces. To explore what, when, and how an utterance is said, the paper made use of the ethnomethodological approach. The ethnomethodological description, therefore, aims to provide a detailed description of how members make sense of any activity as it unfolds in its everyday manner. Undertaking this theoretical approach helps us to acknowledge that knowing is construed as an ongoing action that takes place within the ongoing interactions. This paper aims to explore the processes through which a participant displayed and developed her conception of dimension from moment to moment as she participated in classroom interaction, on the one hand; while developing a critical understanding of dominant discourses that influenced this process of meaning-making in multilingual mathematics class, on the other.

Methodology

This paper presents one episode from a larger study. In this episode, a 10-year-old student discursively constructed her understanding of dimensions in a New Zealand primary classroom. Informed consent was sought from the participants. Six geometry lessons on shapes and their properties in one Year 5/6 New Zealand classroom were observed, and field notes were taken. Participants included 15 students (with nine multilingual students) and

their teacher. Data pertaining to students' languages were collected using a small questionnaire that was filled in by the parents. The observed six lessons were also audio- and video-recorded using two directional cameras, one eye-gear, and five audio-recorders. One camera was kept in the front of the classroom, and one at the back. The eye-gear/glasses with an inbuilt camera was used to record any moment of interaction that caught attention. Five audio-recorders were kept on the tabletops to records students' interaction as they worked on group tasks. Each lesson lasted for 45 to 50 minutes. In addition to audio-visual data from the lessons, three short (10-12 minutes) semi-structured interviews were conducted with the teacher, which were audio-recorded, transcribed, and sent back to the teacher for member checking. Each teacher interview focussed on seeking clarifications about the lessons or activities if there was any question. Four focus group interviews (with four students in each group) were also conducted once all six lessons were taught. Each focus group interview lasted for 18 to 20 minutes. Students' work samples were also collected.

The audio-video recorded data was the primary data set for the study, and participants' utterances as units of analysis. Field notes and repeated watching of six video-recorded lessons enabled the identification of the relevant moments, where participants displayed their understanding of dimension. This paper presents one such moment of classroom interaction when a student, Elie (pseudonym), and her teacher displayed their understanding of dimension as they engaged in classroom interaction. These relevant moments were then analysed at two levels: micro-level and macro-level (see Figure 1). The micro-level analysis formed the basis for macro-level analysis.

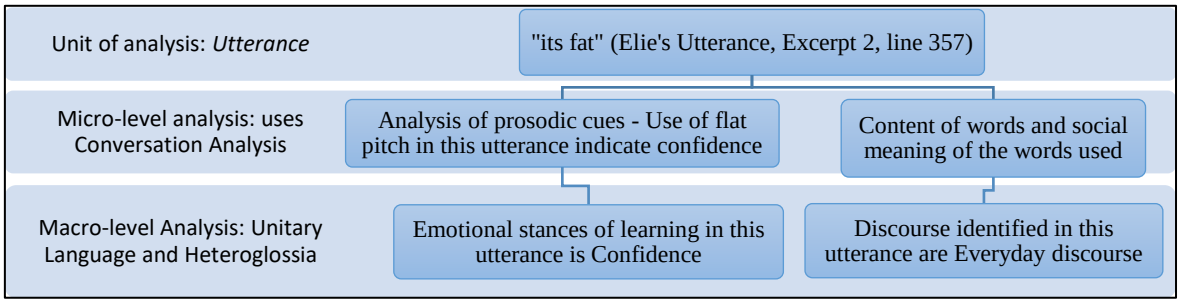

Figure 1. Example of data analysis procedure

At micro-level analysis, the selected moment was analysed using selected features of Conversation Analysis. The moment was transcribed using a simplified version of Jefferson's (2004) transcript convention. The analysis of participants' utterances explored how participants constructed their utterance using linguistic (including words and grammatical forms) and paralinguistic (that includes prosodic features of the pitch, silence along with gestures) features to convey their intended meaning and action (e.g., declaring their understanding, asking a question, seeking confirmation). Thus, the analysis at the micro-level focused on *what* and *how* discursive constructions were made. Following the micro-level analysis, the same moment was analysed at the macro-level. At this level, based on the prosodic analysis at the micro-level analysis, emotional stances embedded in participants' utterances were identified (Sullivan, 2012). In addition, the words used in the utterances with their intended socio-historical meanings within the utterance enabled the identification of the discourses (Sullivan, 2012).

Findings

During the classroom observations, it was noted that students often stated their understanding of dimension using a flat vs. fat analogy, as expressed in one of the students' utterance:

> "D is dimension. Two d is flat and three d is fat. Three d has a lot of stuff. Like a three d has some stuff in it. Two d is like flat and it has nothing. It's like his, his body was like he just, it's like squished over from the car" (Ozan, Focus Group Interview 1)

In this moment, the meanings that a student may imply when she distinguishes between 2D and 3D shapes as she participated in whole-class interaction are explored. During this lesson, the teacher organised students in groups and provided them with sticks and glue to make the shapes that they already knew. After the group work, the teacher invited one student at a time to describe the shape that they had made using "the language of geometry" (Teacher, Field notes, Lesson 2). In the moment presented in this paper, she invited Elie to describe the shape she made using sticks and glue. Questionnaire data revealed that Elie is a bilingual student with more proficiency in English than *Te Reo Māori* (an indigenous language of New Zealand, which gained the official status in 1987). She made a hexagonal shape (see Figure 2). The teacher then asked Elie if her shape was 2D or 3D (See transcript below, line351).

Figure 2. Elie making shapes with sticks and glue.

351	Teacher:	Elie just hang on a minute (.) is it three d: or two d: (1.0)
352	Elie:	um:: I think its three d because °its not (.) a two d°
353		((she was holding the shape and rolling it around her finger))
354	Teacher:	put it down on a on the grou:nd (1.0) is it (.) flat (.) or fat (0.5)
355	Elie:	its fat (1.5)
356	Teacher:	its fa:t (.) is it ↑coming ou:t towards you (1.0)
357	Elie:	((looks at the shape holding it near the eye level))
358	Teacher:	=okay lay it on the grou[nd (1.5)
359	Kimi:	[°no its flat°
360	Teacher:	its its okay. so: its not actually coming out of the ground or going through
361		the grou:nd (.)so we call so we call (.) ↑we call that a two
362		d? (0.5) okay [so:(.2)
363	Elie:	[↓uhm::

The teacher realised that Elie had not mentioned if the shape was 2D or 3D (line 351). To this question, Elie responded that the shape that she had made was 3D (line 352-353). Elie used a flat pitch for the first half of her utterance and a whispery voice (whispering voice is denoted by degree sign, ˚) for the second half. Research has shown that English speakers may use a flat pitch to display their authority or confidence (Ward, 2019). However, a whispery voice at the end of utterance may indicate a lack of confidence (Ward, 2019). Thus, the use of flat and whispery voice in the same utterance may indicate that Elie was partially confident of her claim. Field notes show that the teacher explained the difference between 2D and 3D shapes, as "*two d is flat. three d is fat. two d, straight onto the ground,*

three d, you can hold it, its fat, its solid" (Field notes, Lesson 1). The video-recorded data showed that Elie was holding the shape and spinning it around her fingers. It is possible that Elie understood the shape that she made was 3D as she could hold it. In the following utterance, the teacher asked Elie to put the shape on the ground (line 354). As the teacher did not repeat Elie's previous utterance or used markers like "good girl", it is probable that the teacher evaluated Elie's response as incorrect (line 354). Moreover, she stretched "ground" (stretching is denoted by a colon, :) to emphasise it, paused for one second (gap in utterances is denoted by (1.0)) probably to provide Elie with a cue. The field notes inform that during this activity, the teacher often stated that if the shape was coming out of the ground, it was 3D, otherwise, 2D. It seemed the teacher intended to use the same idea to help Elie to identify the shape as 2D. The teacher rephrased her question and asked Elie if the shape was flat or fat (line 354). The teacher did not emphasise 'fat' or 'flat' in her utterance. This lack of emphasis may imply that the teacher was expecting Elie to recall the 'flat vs. fat' distinction of shapes. It was noted in the field notes, and video-recorded data (Lessons 1 to 4) that the "flat vs fat" analogy was often used in this class to describe 2D and 3D shapes. To this question, Elie (line 355) responded that the shape was fat. Elie's flat pitch suggests that Elie was sure of her answer (Ward, 2019). The teacher waited for 1.5 seconds before constructing her turn, and then repeated Elie's response (line 355-356); however, she stretched "fat" for emphasis. Hellermann (2003) has shown that silence in between turns can be interpreted as the current speaker's (in this case, the teacher) orientation to the previous speaker's (in this case, Elie) utterance as a dispreferred response. Moreover, the teacher used different intonation patterns (line 356) with the same words (see fa:t) used by Elie (line 355). The use of different intonational patterns with the same words often imply contrast rather than agreement (Hellermann, 2003). Thus, it seems that the teacher again evaluated Elie's response as incorrect. Therefore, she again provided Elie with feedback to reconsider her response (line 356). The video-recorded data inform that Elie held the shape at her eye level instead of verbally responding (line 357) to the teacher's feedback in the previous turn. This may be interpreted as Elie's way of restating that the shape is 3D as she could hold the shape in her hand. Noticing this, the teacher (line 358) asked Elie to put it on the ground. As the teacher was talking to Elie, Kimi (a female, 10-year-old, Tongan student) self-selected and offered a repair on Elie's turn. Kimi structured her response in whispery voice (line 359), so that she did not interrupt the flow of conversation (Hay et al., 2008), a different way of using whispery voice than English speakers. The teacher attempted to build an understanding of the shape as 2D with Elie (line 360-362). She used the High Rising Terminal (HRT) (denoted by a question mark, ?) (line 362) as a way to overcome a barrier to comprehension and build solidarity (Hay et al., 2008). Therefore, through her utterances, the teacher attempted to develop a mutual understanding with Elie, as she explained that the shape was not "coming out of or going through the ground" (line 360-362). Interestingly, the teacher used "so we call" (line 361) twice in her utterance; the use of this phrase could be interpreted as her acknowledgement of the possibility of non-confirmation from Elie. Ward (2019) has noted that high onset (denoted by ↑) is often used in conversations to mark a change or draw attention to the topic of conversations. Thus, the teacher's use of high onset (line 361) with "we call" may be interpreted as intended action to change the topic of discussion.

It appears that the teacher realised that Elie was probably not convinced with her explanation; thus, the teacher attempted to change the topic of discussion. Elie picked up the cue in her utterance as in the following turn (line 363), Elie used 'uhm' as a hedging device probably to convey that she is not convinced (Drew, 2013). Ward (2019) has shown that low/falling pitch (denoted by ↓) may also be interpreted to show declining interest in

continuing a discussion. Thus, Elie's use of low pitch in this context may be interpreted as her way to indicate that she was not interested in carrying on with the conversation.

The micro-level analysis suggests, first, multilingual students may use prosodic features of repertoire of their multiple languages. Second, the micro-level analysis suggest that the analogies of "flat vs fat" shapes, and "shapes coming out of the ground" as 3D shapes may not be helpful for some students to understand dimension as a mathematical construct. Similar to Elie, another student also displayed his thinking about fat and flat while making shapes using play-dough. He stated that "it's like all three d you can't like make…not make a fat" (from another relevant moment of group interaction, Lesson 2).

During the presented moment, the teacher focussed on using geometry-specific language to describe the shapes that students made. However, what counted as the geometry-specific language for participants was constructed during the classroom interaction. The macro-level analysis focussed on the interaction of unitary language and heteroglossia to explore what discourses contributed to the meaning of geometry-specific language. Two dominant discourses can be identified in the moment analysed here: Eurocentric-Academic Discourse and Everyday Discourse. The use of terms like "2D", "3D" displays the use of Eurocentric-Academic Discourse, whereas the use of "coming out of ground" display the use of Everyday Discourse. Interestingly, the use of "flat vs fat" analogy may suggest the use of Eurocentric-academic Discourse as well as Everyday Discourse. The heteroglossia can be in located in the different meanings that can be drawn from these two terms. For example, "flat" can imply either smoothness of surface without any depth from the Everyday Discourse perspective; or a very thin object like paper cut-outs that are often used as resources in geometry classes for teaching 2D shapes from the Eurocentric-Academic Discourse. The use of term "fat" could mean thick, thin, or something that can be held in Everyday Discourse, and in case of geometry teaching within the use of Eurocentric-Academic Discourse may mean 3D shapes. In this moment, it is interesting to note that the unitary language force supported different discourses during different micro-moments within this interaction. The unitary language force supports the Eurocentric-Academic Discourse in teacher's utterance (line 354) as she asked Elie if the shape was flat or fat. Through this utterance, the teacher seemed to use the analogy of "flat vs fat" for identifying shapes as 2D and 3D. The Eurocentric-Academic Discourse supports the use of "flat" for 2D shapes and "fat" for 3D shapes. However, in the following utterance, Elie stated that the shape is fat (line 355). Elie's utterance highlights the heteroglossia embedded in her utterance. Based on the micro-level analysis, it seems that Elie construed the shape that she made as 3D as she could hold the shape and could see its slight thickness. It seems that at this moment, the unitary language force supported the use of Everyday Discourse instead of Eurocentric-Academic Discourse for keeping the flow of conversation. The interaction of unitary language and heteroglossia within the use of "flat vs fat" analogy highlights that these words are laden with geometric meanings as well as everyday meanings, which highlights the heteroglossia.

The micro-level and macro-level analysis of the moment presented here highlights three main findings. First, the analogies of "flat vs fat" shapes and "shapes coming out of the ground" may not be helpful for some students to understand dimension as a mathematical construct. Second, the macro-level analysis suggested that the meanings of terms "flat" and "fat" are constructed within the conversational moment, and the meaning of the term may be shaped by any of the discourses supported by the unitary language forces at a particular moment of interaction. Thirdly, the analysis suggest that prosodic features play a crucial role in meaning construction. Importantly, multilingual students may use prosodic features from their multiple languages, which may be differently used in English.

Discussion and Conclusion

In mathematics education research, the construct of "dimensions" has rarely been explored, even when the studies have focussed on geometric shapes and their properties (e.g., Lowrie et al., 2017; Seah & Horne, 2019). It appears that the mathematical understanding of dimension is often taken to be understood without explicit teaching. This study adds to the research literature exploring students' understanding of dimension at the Primary level. The analysis revealed that the student may use a "flat vs fat" analogy for explaining what D stands for in 2D and 3D. This finding is consistent with the research done by Morgan (2005) and Lehrer et al. (1998). Both these studies reported that students and teachers often describe "dimension" as one of the mathematical words that concern with the thickness of the shape. However, the analysis presented here suggests that the use of "flat vs fat" may not be useful for describing the dimensional property of shapes. The difficulty may be attributed to the two different understanding of dimension embedded in definitions of 2D and 3D shapes. First, defining 2D shapes as planar shapes and 3D shapes as solid shapes underscore the understanding of dimension from Euclid's boundary notation perspective (Manin, 2006). Second, defining 2D shapes as having length and breadth; and 3D shapes with length, breadth, and height underscore the understanding of dimension from a measurement perspective. This construction emphasises dimension as a measurement attribute and does not underscore the need for the "planes" to understand dimension. The analysis presented here underscore a need for developing a comprehensive understanding of what dimensions imply, as a mathematical construct, in curriculum documents; so that its understanding can be translated into teaching and learning of shapes including dimensions. Moreover, a comprehensive understanding of dimension may also help teachers and students to acknowledge the context within which the idea of dimension is used, and for what purposes. Hence, the study suggests future opportunities for further research in this area.

The second finding revealed that the meanings of the terms such as fat are shaped by the interaction of unitary language and heteroglossia. From a Bakhtinian perspective using unitary language and heteroglossia, it can be argued that the meaning of the utterances is dependent upon the discourse supported by the unitary language force within the milieu of discourses available in any particular interactional moment. The unitary language is a *theoretical* language force that tends to homogenise the meaning of the utterance to facilitate the flow of interaction (Barwell, 2018). It was evident in the data that the unitary language force may support either of the discourses depending on the interactional context. Therefore, providing different meanings to the same word as and when embedded in different discourses. Barwell (2013) claimed the participants may treat an everyday term mathematically during a particular interaction. Thus, the students engaged in using their everyday language in the form of mathematical language, embedding everyday words with mathematical ideas.

The third finding from this study highlights the role of prosody that contributes to the meaning constructions as participants engage in mathematical discussions. For example, the study supports the findings that of the use of low, high, flat pitch are some of the interactional devices that participants use to draw listeners' attention to the focus of their utterances (Reed & Michaud, 2015). Also, the study suggests that multilingual speakers engage in the practice of languaging that involves the use of prosodic features conventions from the linguistic repertoire of different languages along with the words used; instead of just engaging in a practice of code-switching to meet their needs to communicate their understandings in a particular interactional context. The research focussed on language as a resource perspective (Adler, 2002; Moschkovich, 2007) often ignores the role played by the prosodic repertoire

in contributing to the meanings conveyed in the utterance. Therefore, this study calls for further research on interactive practices that may support teaching and learning of geometry. The study also suggests that it is important for teachers to become aware of the subtleties of prosodic features of language that have an impact on the meaning-making process and learning of mathematical ideas.

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