Using interviews with non-examples to assess reasoning in F-2 classrooms

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The development of mathematical reasoning is a key proficiency for mathematics within the Australian Curriculum. However, reasoning can be difficult for teachers to assess, particularly with pen and paper tests. In this study, interview tasks were designed across three curriculum areas at three different levels to assess student reasoning through the use of examples and non-examples. Non-examples can be used to assist in building boundaries and deepening conceptual understanding. Through the interview, teacher and student dialogue can help students to demonstrate reasoning and clarify concepts through explanation and justification.

This paper examines the use of task-based clinical interviews to assess reasoning in the early years of school. The development of mathematical reasoning is considered a key proficiency within the *Mathematics Learning Area* of the *Australian Curriculum* and is described as a facility for “logical thought and actions” with “increasing sophistication” (Australian Curriculum Assessment and Reporting Authority, [ACARA] 2018a). This may be demonstrated, in part, through a student’s ability to compare and contrast ideas, explain their thinking and justify conclusions made. In partnership with and addressed through the learning area foci of the *Australian Curriculum* are the *General Capabilities*, including *Critical and Creative Thinking*. Within this capability, students develop capacity to “generate and evaluate knowledge” and “clarify concepts and ideas”, through “thinking broadly and deeply” and using reason and logic (ACARA, 2018b). These definitions are aligned to Kilpatrick’s (2001) description of adaptive reasoning, where students think logically about conceptual relationships, reflect on their learning and justify their work. As an essential part of the curriculum, responsibility for assessing reasoning and critical thinking lies with the teacher.

Assessing students’ capacity to demonstrate reasoning in mathematics can be challenging for teachers (Herbert et al., 2015). Formal, written pen-and-paper tests can be difficult for F-2 students (Foundation, the first year of school - Year 2) to complete. It has been established that this form of assessment may not accurately reflect students’ conceptual understanding (Clements & Ellerton, 1995) and presents challenges to students at this level due to the reading and writing skills required, in light of the students’ own developing literacy skills (Clarke, et al., 2006). One-to-one task-based interviews which are grounded in research are more effective at revealing students’ conceptual understanding as well as their thinking and reasoning. For the purposes of eliciting and demonstrating mathematical thinking, interviews are well suited to early-years students (Cheeseman & Clarke, 2007). It is through the dialogue that happens between the teacher and the student that the student’s reasoning becomes evident.

Task based interviews using non-examples, such as the ‘triangles task’ in the Early Numeracy Research Project, allow students to reason through justification (Horne, 2003). Similarly, Clements’ (1998) discussion of interview tasks using examples and non-examples of 2D shapes, demonstrated that they allow students, through comparing and contrasting, to focus on the essential attributes of the shapes and promote critical thinking. Examples in mathematics generally fall into two categories: examples of a concept; or examples of the

2021. In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & S. L. Chin (Eds.), *Excellence in Mathematics Education: Foundations and Pathways (Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia)*, pp. 163-170. Singapore: MERGA.

application of a procedure. Within these categories, examples can take the form of ‘generic example’, ‘counter-example’ or ‘non-example’. Non-examples can help to clarify understanding by sharpening distinctions and deepening understanding of mathematical ideas (Bills et al., 2006). They provide an opportunity to reveal student thinking, and for students to apply reasoning and formulate justifications for why an example is correct or incorrect (Cavey & Kinzel, 2015). Teachers using non examples can assess students’ conceptual understanding and reasoning using interview tasks designed to reveal misconceptions.

Methodology

Task-based clinical interviews were used to assess the reasoning of three students, at three different curriculum levels and in three different content areas. Task-based interviews were chosen for their utility as they are a valued tool for revealing student thinking, particularly for students in the early years of school (Clarke et al., 2006). Students are able to use discussion as a means of revealing understanding and therefore reading levels are not an issue (Bobis et al., 2005). Task-based interviews have developed from a background of Piagetian and Vygotskian theory, understanding that learning occurs in a social context. The interview process is centred around the dialogue which takes place between the child and the researcher, and the role of language is central to this. The researcher asks probing questions and the child clarifies meaning through explanation (Hunting, 1997).

Tasks were designed in consideration of research, including the development of conceptual understanding and common misconceptions, with one task for each level, at Foundation (number recognition, matching quantities and numerals to ‘seven’), Level 1 (Counting on and counting back for early addition), and Level 2 (fractions, identifying ‘quarters’, demonstrating understanding of equal parts in a continuous model and fractions in a discrete model). Tasks were created with examples and non-examples for each content area, to expose conflicts in understanding which can arise through misconceptions (Zazkis & Chernoff, 2008). With non-examples, students can dismiss concepts that do not fit with their conceptual understanding however the dialogue within an interview can challenge this notion. Non-examples were intentionally included because they can be used to clarify boundaries for a concept, or where a procedure may not be applied, or fails to get a correct answer (Bills et al., 2006).

“Kye”, aged five, “Cara”, aged seven, and “Oliver”, aged eight, (pseudonyms) attended an urban government school, where the need for assessing reasoning had been identified as an area for improvement within the school. The students were interviewed on site in a meeting room. Tasks were conducted with each student individually, and instructions, or questions were read to the students by the researcher. The students were then asked to explain their answers and why they had chosen (or not chosen) each answer. Each interview took approximately 10-15 minutes. The researcher recorded each answer and students’ use of reasoning and justification were analysed from their responses

Tasks

Task 1

Task 1 (Figure 1) is a Foundation level task about number recognition. The Australian Curriculum lists the content descriptor for this as: “Connect number names, numerals and quantities, including zero, initially up to 10 and then beyond (ACMNA002)” (ACARA, 2018a). Key concepts for this task include Gelman and Gallistel's Counting Principles (1978) which state that meaningful counting relies on children knowing how to count and what to count. How to count includes: the one-to-one principle, where each item is counted only once, and assigned to a number as it is counted; the stable-order principle, where the number names are always used in the same fixed order; and the cardinal principle, where the last number counted or named is the total of the collection. What to count, relies on understanding the abstraction principle where anything can be counted including where the items in a collection are different, and the order-irrelevance principle where objects can be counted in any order.

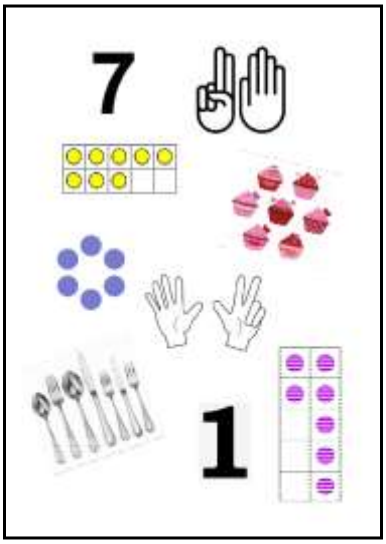

Figure 1. Foundation task

This task required the student to circle all the representations that showed 'seven'. Images chosen to represent familiar objects for Foundation students include: tens frames, counters, fingers, and common objects, as well as numerals. The types of images were chosen to reflect the counting principles, which are necessary for conceptual understanding. All items assess cardinality and the stable order principle. In addition, the cutlery assesses the abstraction principle, and the cupcakes and counters in a circle assess one-to one correspondence and order-irrelevance. The tens frames images assess order-irrelevance and could demonstrate knowledge of combining and partitioning (Clarke et al., 2006). Non-examples include the numeral '1', with extra 'tails' which could be mistaken by small children as the numeral '7'. The counters arranged in a circle represent 'six' but could be counted incorrectly by a student who is not able to create a start and end point for their counting. One set of tens frames and one set of hands are non-examples, displaying 'eight'.

Task 2

Task 2 (seen in Figure 3) is a Level 1 task about early addition and subtraction strategies of counting on and counting back. The Australian Curriculum (ACARA, 2018a) lists the content descriptor for this as: "Represent and solve simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts (ACMNA015)". Research used to construct the task focused on counting stages (Steffe et al., 1983), particularly those at the initial number sequence stage or counting in verbal unit items. Students at this level are able to hold a number in their head and have a conceptual understanding of the quantity that the number represents. Students are then able to count on a given amount of numbers to find a total. (See for example the top left column of Figure 3). This is a complex cognitive task requiring that the child understands the relationship between the symbolic representation of the task, as well as its relationship to process, numeration and quantity (Boulton-Lewis & Tait, 1994).

The first question demonstrates both a correct method, (top left column of Figure 3), and a common misconception for students who learn counting on as a process, (top right column of Figure 3). These students count on, but include the last number stated, lacking the conceptual understanding of the requirements of the task. Question two addresses counting back, which is often more challenging for children than counting forward (Steffe et al., 1988). A number line is provided for support, with the non-example showing a common misconception where the child counts marks on the number line, (top number line in Figure 3), and a correct example where a child draws 'jumps' on a number line, demonstrating counting back, (bottom number line in Figure 3).

Task 3

Task 3 (as seen in Figure 4) is a Level 2 task about fraction representations of quarters. The Australian Curriculum lists the content descriptor for this as: "Recognise and interpret common uses of halves, quarters and eighths of shapes and collections (ACMNA033)" (ACARA, 2018a). Key concepts for this task include the relationship between the numerical representation of a fraction and models to represent this. Due to the frequent use of 'pie' representations in the teaching of fractions, students can misunderstand the representation of a fraction in terms of a whole, particularly in a discrete model (Gould, 2005).

Representations of examples in the task include continuous and discrete models, equal parts, different shaped wholes, and an equivalent fraction. Common misconceptions for students include the understanding of equal parts in diagrams, and the relationship between wholes and parts of wholes, particularly in discrete items (Gould, 2005). Non-examples in this task include non-equal parts, images that represent one fifth in discrete and continuous models, and a whole that has been divided into quarters.

Results and Discussion

Task 1

Foundation student "Kye" completed the number identification task (Figure 2) and was immediately able to identify the numeral '7' as correct and the numeral '1' as incorrect, stating, "It's not seven, because it's a one". He then counted the seven fingers correctly, demonstrating one-to-one matching (Gelman & Gallistel, 1978) as he counted each finger. Kye counted the cupcakes as seven, again counting them with one-to-one deliberate matching, touching each cupcake as he counted. When drawing around the cupcakes, he recounted, drawing a line past each cupcake as he counted, resulting in an unusual 'circling' of the items. He then counted the bottom right ten frame (Figure 2) once and circled it. Kye once again relied on one-to-one matching, and did not demonstrate more complex understanding of number, which could perhaps have demonstrated part-part-whole number knowledge (Clarke et al., 2006), such as '5 and 2', or 'three empty spaces'. he then counted the second set of fingers as eight fingers and said he wasn't going to circle it, because it was eight fingers.

Kye had great difficulty counting the cutlery. He began counting and stopped halfway through and went back to the start twice. On the third attempt he said he was going to count them “carefully”. He proceeded to count each item very slowly, but again stopped. He then said, “I’m going to count them at the bottom, and use my pencil”. Kye counted the handle of each item, placing a pencil dot on the end of each cutlery item to count seven items. He then repeated the process before circling the items. The cutlery, demonstrating the abstraction principle (Gelman & Gallistel, 1978), were an obstacle that prevented Kye counting the items. His strategy was to count the items at the bottom, where the items were all the same. Kye also had difficulty counting the six dots in a circle and did not have a clearly identified beginning and end point for his counting. Kye counted seven dots, recounting his initial dot at the end, and immediately and confidently circled the group.

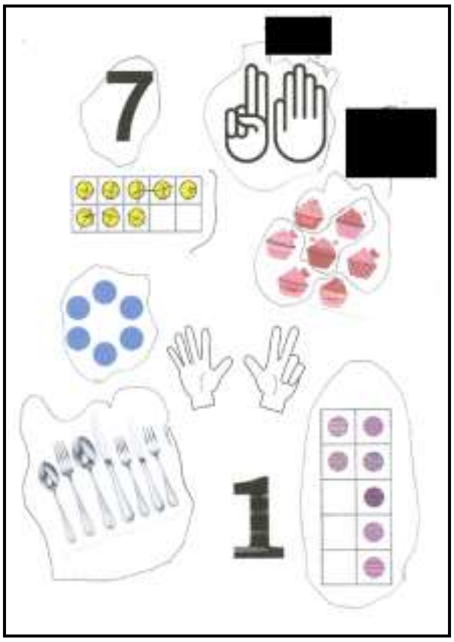

Figure 2. Kye’s response to Task 1

Kye counted the final ten frame as eight and then started to circle the ten frame. He was asked, “How many did you say there were?” and responded, “Eight”. He was then asked which ones he was circling, and he said, “the sevens” After discussion he decided he would recount the items. He recounted the dots, placing a cross on each to count eight and said he wouldn’t circle them because there were eight and not seven. Kye was able to demonstrate one-to-one counting and some of the counting principles. His reasoning demonstrated an ability to justify why he believed something was correct. His critical thinking skills were used in his ability to adapt his counting skills with the cutlery counting to enable him to effectively count the items.

Task 2

Year 1 student “Cara” completed the addition and subtraction task (Figure 3). Cara was able to correctly answer both questions in the task, but interestingly only able to demonstrate reasoning in one part of the interview. In the addition question, Cara wrote her answer clearly stating that the incorrect answer was wrong “...you don’t count the number your (sic) on.”

When questioned, Cara said, "you already have 7, you don't need to count it again, you have to count on the next number". Cara has demonstrated a correct understanding of the procedure for counting on (Boulton-Lewis & Tait, 1994). She has also demonstrated an ability to think logically about the relationship between the concept of addition and the example and non-example provided (Kilpatrick, 2001). Cara has justified why one answer was correct, and why another was incorrect.

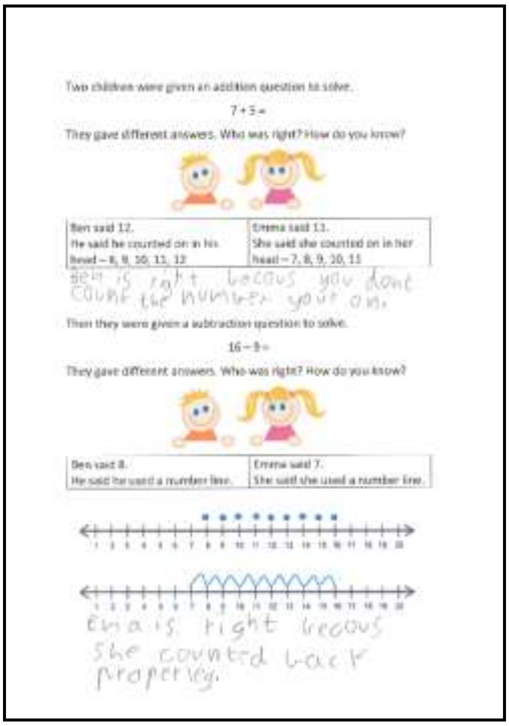

Figure 3. Cara's response to Task 2

In the subtraction question, although Cara was able to answer the problem correctly, she was unable to demonstrate reasoning. When questioned on what she meant by "counted back properley (sic)", she said, "That's the way you're supposed to do it." On further prompting, she continued to talk about the "right way". This was a procedural approach and her response demonstrated that Cara had a 'rule' for using a number line; however, she did not have a conceptual understanding of why this method was successful. Her inability to justify her response, or why the other answer was incorrect, revealed that although she could identify the correct solution, she could not articulate her mathematical reasoning.

Task 3

Year 2 student "Oliver" completed the fraction task (Figure 4). Oliver was able to demonstrate understanding of quarters in both a discrete and continuous fraction model (Gould, 2005). The task does not show discrete fractions with items of different sizes, which should be added to the task for future interviews. Oliver was able to articulate the reasons he provided to justify what was and what was not a representation of a quarter, including the need for equal sized parts in a continuous fraction model. He was able to clarify from the non-examples of fifths, what a quarter was: "This has five bits, but a quarter is one out of four", and "There's five people, not four, so it can't be a quarter. It's a fifth." The non-examples sharpened his interpretation of quarters (Cavey & Kinzel, 2015).

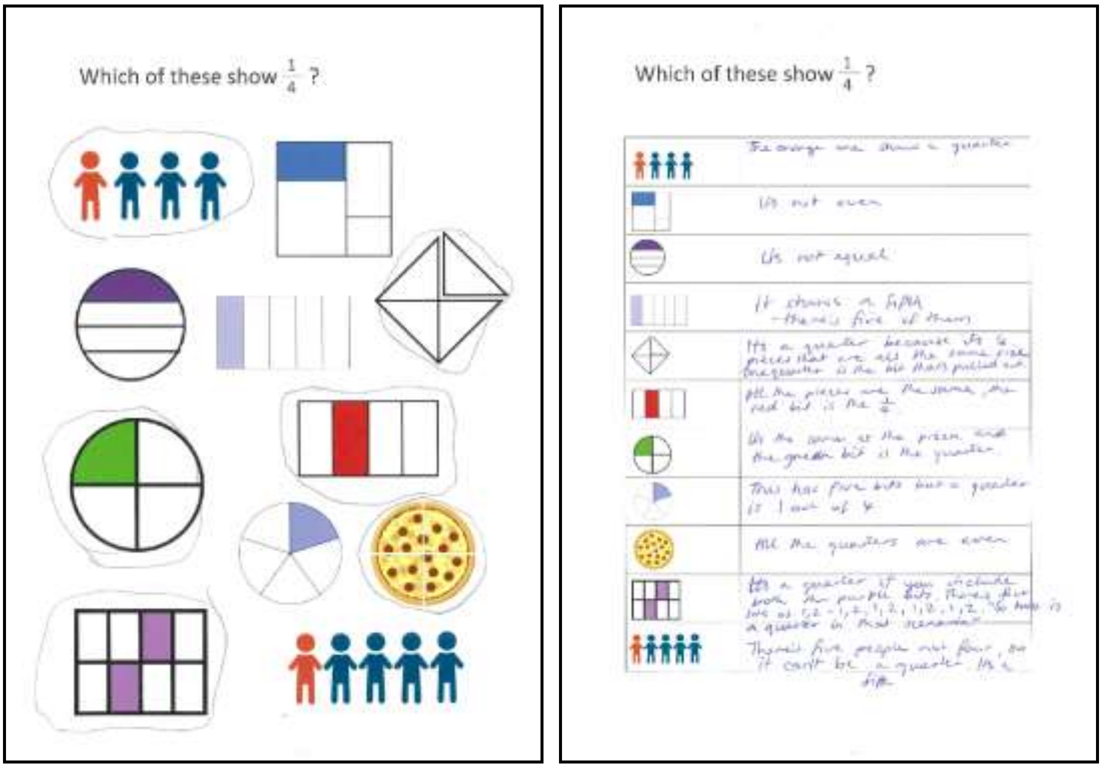

Figure 4. Oliver's response to Task 3 and interview notes

Logical reasoning is evident in Oliver's identification of two-eighths as a quarter. "It's a quarter if you include both the purple bits, there's four lots of 1, 2 – 1, 2, 1, 2, 1, 2, 1, 2. So, two is a quarter in that scenario". His justification and explanation of his ideas is a demonstration of clear reasoning and his current conceptual understanding (Kilpatrick, 2001). Interestingly, Oliver stated that the pizza showed a quarter as all the quarters were even. The interviewer said, "When I look at the pizza, I see four quarters, because they're all the same." Oliver responded, "You know, I think you're right, they are all the same." Initially the non-example had been dismissed by the student; however, the interview exposed this conflict in understanding, and enabled Oliver to more clearly clarify his understanding and create new boundaries for the concept of a quarter (Bills et al., 2006).

Conclusion

Although this was a small-scale study, only assessing one child within each identified concept, some conclusions can be drawn. Using examples and non-examples in a task-based interview situation allowed a teacher researcher to clarify conceptual understanding of three students, within specific topics of the mathematics curriculum area. The tasks required students to identify correct and incorrect examples of concepts and to justify their responses, in order to demonstrate mathematical reasoning. A task-based interview assessment allows for dialogue between the teacher and the student, to clarify the student's thinking, and provides an opportunity for the individual student to articulate conceptual understanding. Prompting questions from the interviewer can be used to seek explanations, with reasoning and justification from the student; however, this relies on the pedagogical content knowledge of the assessor. Therefore, the need for carefully planned, research-based tasks is essential to the effectiveness of an assessment such as this, and can be useful to teachers, promoting the assessment of reasoning, rather than just assessment of a procedure, or ability to follow a 'rule'. This enables the teacher, as the assessor, to gain a deeper knowledge of the conceptual understanding of the student. The potential for a larger study, with a wider range of students, could be considered to better understand the possibilities of using task-based interviews to assess reasoning in a wider range of mathematical concepts.

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