The Practice of Mathematics Education

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Over the years mathematics education has been examined through a range of theoretical perspectives, and these have predominately been epistemological in nature. However, despite this rich history of research, it still seems some perennial issues with mathematics education remain, with many learners floundering and large numbers disliking mathematics or viewing it as irrelevant. Therefore, here a different perspective on mathematics education is presented – an ontological practice approach, which complements existing epistemological understandings. Considering mathematics education as practice foregrounds two key factors—the "site ontological" nature of mathematics learning and teaching that highlights its "happeningness", and the sociality of mathematics education. There are two reasons for considering a practice approach to understanding mathematics education. First, mathematics itself is a practice, and reciprocally mathematics education is concerned with enabling learners to *practice* mathematics. Second a practice approach to mathematics could offer insights into the perennial and intractable affective issues of mathematics. Considering these things, here practice philosophy and theory is employed to discuss mathematics education and mathematics education research.

Mathematics education *happens*, and regardless of what knowledge people may have about it, or what they may believe or feel, in the end, it happens. Furthermore, mathematics education occurs and unfolds in time and space in particular sites and at particular times. In addition, while there is rightly interest in things like educational leadership, curriculum, teacher standards, mandated external assessment regimes, etc, these things only exist and have meaning because they in some way impact what happens in classrooms[1]. The classroom is "always the existential; and ontological given in education" (Kemmis et al., 2014, p. 214). For these reasons, here a site-ontological practice approach is taken to understanding mathematics education. This practice approach does not necessarily usurp or supersede other epistemological understandings and conceptualisations of mathematics education, but rather it offers a complementary ontological perspective.

The Primacy of Practice

According to Schatzki (2002), we live our lives in practices, where we encounter one another as interlocuters in time and space. This ontological view gives primacy to the *happeningness*[2] of life, including mathematics, mathematics education, and mathematics education research. Indeed, as will be discussed later, a fundamental reason for considering mathematics education from a practice perspective is that mathematics itself is a practice and is comprised of practices. However, it is commonly assumed that the term "practice" has an unquestioned and unproblematic meaning, particularly given its common usage in everyday language, and so here an ontological conception of practice that sees it as site-based and social

[1] The term "classroom" here refers to any "learning site", not necessarily just a formal school classroom.

[2] Happeningness of practices refers to ways practices are observable acts unfolding temporally (in time), ontologically (in a particular place), discursively (communicatively in language) and relationally (intersubjectively between people) (Edwards-Groves & Grootenboer, 2015).

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will be first briefly outlined—specifically the "theory of practice architectures" (Kemmis & Grootenboer, 2008; Kemmis et al., 2014).

The Theory of Practice Architectures

The theory of practice architectures was developed by a group of colleagues through theoretical and philosophical discussions, and it draws upon data from a large-scale empirical project conducted over a number of years (see Kemmis et al., 2014). In brief, the theory posits that practices are comprised of characteristic sayings, doings, and relatings, that occur in semantic, physical, and social space respectively, and are held together in a project (see Figure 1). These sayings, doings, and relatings, are enabled and constrained by practice architectures—cultural discursive, material economic, and social political, arrangements and conditions that exist in the practice site.

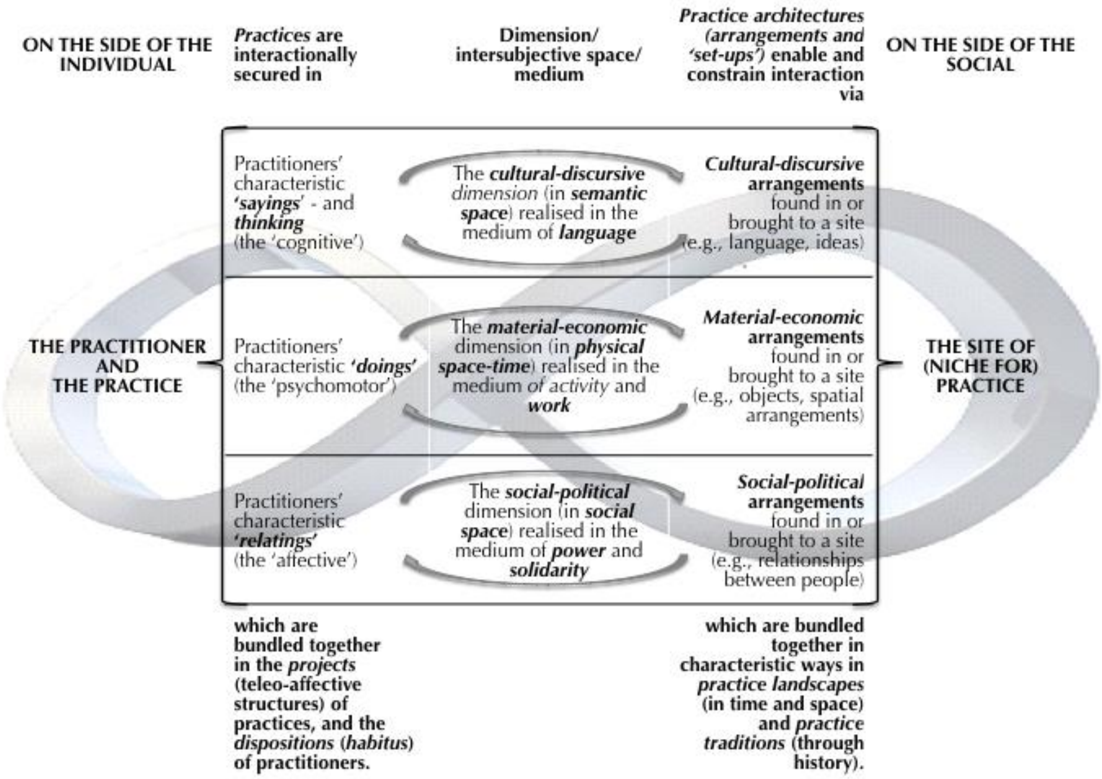

Figure 1: The theory of practice architectures (Kemmis et al., 2014, p. 38).

In this sense, practices are prefigured, but not predetermined, by the conditions and arrangements—the practice architectures, of the practice site, or,

> [People] make their own history, but they do not make it as they please; they do not make it under self-selected circumstances, but under circumstances existing already, given and transmitted from the past. The tradition of all dead generations weighs like a nightmare on the brains of the living. (Marx (1852/1999, p. 1)

While this outline is necessarily brief and lacking detail, it can be exemplified by considering a practice like teaching fractions to primary school students. To enable students to comprehend fractions, the teacher will talk in specific ways using regular and particular language (e.g., numerator, denominator), do certain things with resources (e.g., maybe cut a cake into equal pieces), and relate to the students in a professional but encouraging manner by using their position as a knowledgeable person in the room (e.g., to gently question or challenge students).

As noted earlier, what is important here is the site-ontological nature of practice.

> The site of a practice is the phenomenological reality that always and necessarily escapes standardisation in curricula, standards, assessments and policies. The site is not only a matter of happenstance (where practices happen to take place and where things happen to be arranged as they are), nor only because the site is the specific location in which participants' practical deliberation and their practical action takes place. The 'site' is also crucial *theoretically*—to be understood in existential and ontological terms as an actual and particular place where things happen, not just as a location in an abstract and universal matrix of space-time. (Kemmis et al., 2014, p. 215)

In other words, the site is more than a *context* in which practices take place—it is an integral part of the happening. This, amongst other things, means that the notion of "best practice" is, at best, an unhelpful myth, and at worst, a damaging misconception that sees all learners, sites, and communities as homogenous. The site-based nature of practices, including mathematics education practices, means one can only talk of "best practices *here* and *now*."

Also, an affordance of the theory of practice architectures is that it enables the individual (practices) and the communal (practice architectures) to be considered simultaneously and in a manner that sees them as symbiotic and complementary. The epistemological dilemmas related to individualistic and social understandings are not settled as such, but rather the ontological approach that focuses on the happening of practices enables these to be considered together.

Furthermore, and relatedly, the theory of practice architectures also allows for a critical understanding of education practices because it simultaneously requires consideration of practices, and the conditions and arrangements that enable and constrain them. Specifically, because practices are seen as prefigured (but not predetermined) by practice architectures, this means that to develop or change a practice there is also a need to reform the practice architectures. Critically, when educational reform or development is considered, it needs to be understood in a complex manner that concurrently addresses the relevant practices and the enabling and constraining conditions and arrangements.

Mathematics Education as Practice

Turning now specifically to mathematics education, the focus will be on mathematics teaching and learning as it is practiced in real time and space in classroom sites. This is focus on the "happeningness" of mathematics education—all the other phenomena including formal and informal mathematics curricula, mathematics education leadership, programs for teacher professional learning, education theory, and indeed, mathematics education research, only have significance and value because in everyday educational sites (e.g., schools) students and teachers meet as interlocutors around mathematical experiences in order to learn. However, first a brief discussion of mathematics practice is provided.

Mathematics Practice

Mathematics is a "coherent and complex form of socially established cooperative human activity" (MacIntyre, 1997, p. 187), and as such it can be understood as a practice. As such, mathematics is a human endeavour that is practiced in diverse ways across different sites. So, this means that the pure mathematician in the university, the engineer out in the field, the person shopping for groceries, or the child scoring a game, are all practicing mathematics. However, to illustrate here, the work of Burton (1999, 2001) will be briefly outlined.

Burton studied the mathematical practices of professional mathematicians in university sites. Not surprisingly, she found that they engaged in common practices (e.g., proof) to solve substantial and noteworthy problems and to expand discipline knowledge, and for those in this community of practice (Lave & Wenger, 1991), these were engaging, enthralling, and compelling. Furthermore, and far from the common perceptions, the community of mathematical practice provided significant collegial emotional and practical support as the mathematicians engaged in their intellectual and affective practices (Bass, 2011). So, for the professional mathematicians that Burton encountered, mathematical practice is wonderous, beautiful, and fascinating—something that is perhaps not shared with those learning mathematics at school. Also, it would seem that for professional mathematicians, learning mathematics is simply just an integral and routine aspect of their practice. Burton (2001) commented that "we have a responsibility to make the learning of mathematics more akin to how mathematicians learn and to be less obsessed with the necessity to teach 'the basics' in the absence of any student's need to know" (p. 598), so to this end perhaps there is a need for students to engage more in mathematical practices such as mathematical modelling (Stillman, Brown, & Czoches, 2020) and mathematical problem solving (Schoenfeld, 2020).

Learning Mathematics

First, in this practice perspective learning mathematics is understood as something more that acquiring mathematical content knowledge (including knowing how to do some "mathematical skills")—it is about learning how to practice mathematics. To be clear, learning how to practice mathematics does involve intellectual and skill growth (as is evident in the "sayings" and "doings" in the theory of practice architectures), but only as an integral part of learning how to go on in the particular mathematical practices concerned. To this end, Kemmis et al. (2014) proposed that:

> … learning is always and only a process of being stirred into practices, even when a learner is learning alone or from participation with others in shared activities. We learn not only knowledge, embodied in our minds, bodies and feelings, but also how to interact with others and the world; our learning is not only epistemologically secured (as cognitive knowledge) but also interactionally secured in sayings, doings and relatings that take place amid the cultural-discursive, material-economic and social-political arrangements that pertain in the settings we inhabit. Our learning is bigger than us; it always positions and orients us in a shared, three-dimensional—semantic, material and social—world. (p. 59; emphases original)

In this, they suggest that learning is not about somehow "obtaining" something (e.g., knowledge, skills), but rather about "being stirred into practices" or learning "how to go in practices" (Kemmis et al., 2017, p. 45). In other words, the goal (or project) of mathematics education is to enable students to continue with mathematical practices. In this sense, it is not that one can or cannot, do mathematics, it is about developing in competence and confidence in mathematical practices—becoming more engaged and proficient in the community of mathematical practice.

Second, learning as being "stirred into mathematical practices" provides a holistic view including cognitive, affective and conative dimensions. In this sense, emotions, actions, attitudes, and values are not separate aspects associated with learning mathematics—they are an integral part of learning mathematics. Thus, when someone is learning some mathematical practice, they are engaging with particular discipline knowledge, and at the same time and as part of the mathematical practice, they are coming to appreciate it as, for example, interesting and useful (or not). This is important for mathematics education because it places affective considerations as integral and central to the "learning"—not as something to be considered as an afterthought to the lesson. For many years mathematics education has been beset with affective issues that have regularly seen students indicating that mathematics is distasteful and to be avoided, and despite many attempts and ideas, this has largely remained unchanged. So, perhaps if learning mathematics is understood as learning how to *go in* mathematical practices, including the saying/knowing, doing and relating, then aspects such as affect will not be seen as an uncomfortable extra consideration, but as a fundamental inter-related aspect.

Finally, when learning is seen how to *go on* in the community of mathematical practice, then the role of the teacher as a fellow, albeit more experienced, member of the community, is crucial. Lave and Wenger (1991) see identity as a function of membership of a community of practice, and so here, teachers need to be themselves engaged in mathematical practices, and identify as “mathematicians” in this way. Of course, the somewhat artificial nature of schooling, where learning is mostly separated from participation in authentic communities of practice, means this can be difficult, and requires a reimagining of what a mathematician is, from a professional vocation to something that is undertaken in various forms and in various ways in a variety of different sites.

Mathematics Education and Practice Architectures

So, the possibilities for mathematics education practice are always enabled and constrained by the prevailing practice architectures. It follows then that the development of mathematics education practices involves more than just changing mathematics teachers’ practices—it demands a concomitant transformation of the practice architectures that shape the said practices.

Of course, the practice architectures that create possibilities for mathematics education are broader than just the disciplinary traditions of mathematics and mathematics education—the national and state level practices of educational administration, policy making, and assessment, are brought into school sites and act as powerful shapers. Indeed, Grootenboer et al. (2018) suggest that strong cultural-discursive arrangements can inhibit the possibility for education and leave us with mere “schooling.”

> If teachers are obliged to follow all the available advice too slavishly, if they take their eyes off the students in front of them because they are obliged to listen too closely to the voices of the advisors and administrators behind them, they may find themselves working on what the state intends – schooling – rather than for the good of their students and the society. The syllabus, instead of being a source of guidance and inspiration for teachers and students, may become a litany of imposed tasks to which teachers and students cannot do justice. (Kemmis, 2008, p. 14)

So together, these points make an argument for not having *mega conditions* (e.g., national or state curricula, external assessment regimes, policies) that are overly restrictive and controlling, because they limit the capacity for mathematics education that is responsive to the unique site-based needs and requirements and conditions. For example, if mathematics teachers are to practice in a reflective and responsive educational manner, then they require scope to develop and enact their pedagogy within the guidance of curricula, and not be slavishly required to follow a detailed prescription of teaching activity.

Learning Mathematics in Schools

As has been suggested, there is a difference between mathematics *education* and mathematics *schooling*, and the argument being presented now is that we need mathematics education that is educational, and indeed, mathematical. In dealing with the former, it is sufficient to highlight at this stage that mathematics education as it is realised in schools (and early childhood settings, and universities) needs to consistent with, and characterised by, mathematical practice (Burton, 2001).

It is the case that individuals and groups learn many things and practices, including mathematics, outside the formal settings of schools, and yet it seems that often the focus for understanding learning is restricted to the schooling context. This is not to say that mathematics learning in schools is necessarily a bad thing, but it does require some consideration about what this has done to mathematics education (and indeed, mathematics). Two related issues will be dealt with here: (1) the valorising of content knowledge over practice; and (2) the assumption of learning transfer.

The Valorising of Content Knowledge

While mathematics itself is a practice, and comprised of practices, school mathematics curricula tend to be dominated by discipline knowledge that is required to be taught and learned. This then leads to mathematics education (or schooling) that is dominated by trying to transfer the required knowledge from the curriculum into the heads of the students. Of course, it is not quite a simple as this, but this approach in general is fraught because, amongst other reasons, it is a misrepresentation of mathematics. This limits the curriculum to the *content* which tends to be abstract and general (and can be tested formally). Indeed, the inaccessibility of mathematical knowledge means the teacher can only know when the student has apprehended the required knowledge through implication from what they might display in practice.

This is not to say that mathematics curricula statements do not include some notions of mathematical practice, but they are often seen, whether by design or by assumption, as the separate add-on optional parts that need to fit in and around the content knowledge where possible. However, it is contended here that knowledge is an integral and entangled part of practice, and furthermore, is only ascertained and demonstrated "in practice." In other words, mathematical knowledge and skills are important, but alone are insufficient and inadequate, and a dangerous simplification of mathematics.

The Assumption of Learning Transfer

Schools, by design, see learning as "abstract" and removed from the relevant "communities of practice"[3]. Of course, as is clear from what has proceeded, learning is site-based, and so rather than being somehow *objective,* school mathematics learning is situated in the school site. Schools and school systems, to a greater or lesser degree, try to ameliorate this by the pedagogical practices employed, but nevertheless, schools as institutions are set up to have students learn general mathematical knowledge and skills. As noted above, there are benefits of this, but it is premised on an assumption that what is learned will be readily transferred to other practice contexts and settings as required. After researching the mathematical practices of people in "everyday life"(e.g., in the supermarket), Lave (1988) commented,

> Conventional academic and folk theory assumes that arithmetic is learned in school in the normative fashion in which it is taught, and is then literally carried away from school to be applied at will in any situation that calls for calculation. (p. 4)

Her clear finding from her large ethnographic study was that this was not the case, and, for example, some people who were quite poor at "school arithmetic," were actually very good at arithmetic in their everyday situations. Certainly, despite the common and widespread use of mathematics a large body of research indicates that most students see mathematics as useless—something they have apparently learned at school in their mathematics classrooms (Grootenboer & Marshman, 2017), and knowledge and skills that do not transfer to other sites outside and/or across the classroom, or other disciplinary practices.

Of course, these are complex and long-standing problems in mathematics education that defy simple quick-fix solutions, but perhaps it is timely to consider how school mathematics can be more "educational." Some possible approaches include:

[3] The communities of practice here are broader than just professional mathematicians, but all who practice mathematics

- Seeing mathematical practices as broader than just what happens at school or what "professional mathematicians" do. In other words, there is a need to develop and value a more comprehensive view of the mathematics community of practice.
- Considering a curriculum of mathematical practices that deliberately and overtly centres on the practicing of mathematics and the development of students' identities as mathematical practitioners (see Grootenboer et al., 2021).

Implications for Mathematics Education Research

To understand mathematics education from a practice perspective is to focus its site-ontological and social nature. Fundamental in this view is seeing mathematics education as equipping learners to 'go on' in mathematical practice—students are not so much learning mathematics, but rather becoming mathematicians (or becoming part of a mathematics community of practice. This view of mathematics and mathematics education has implications for mathematics education research.

First, there would seem to be an urgent and compelling case for examining the issue of learning transfer in mathematics education. When mathematics is learned in school classrooms, then the site becomes an integral part of the mathematics that is learned, and so the value and availability of mathematical practices gained through schooling seem to have limited influence on, or use for, mathematics outside the school setting. This is not to say that learning mathematics at school is a bad thing, but rather, the very nature of school mathematics learning requires that issues of learning transfer are central.

Second, it seems that individuals in a range of everyday settings are able to learn complicated mathematical practices (for example, see the work of Lave, 2019; Nunes et al., 1993; D'Ambrosio, 2006), and yet, when in more contrived formal learning site (e.g., a school), the *learning* was diminished, and even washed out. Although the common perception is that mathematics learning only happens in mathematics classrooms, this highlights that mathematics education happens in a range of sites—including powerfully in *informal* settings, and there is much to be learned for school learning from these everyday sites.

Third, a site-ontological practice conception demands methodologies and methods that are responsive to the *happeningness* of mathematics education practices, and the practice architectures that enable and constrain them. In essence, to be attentive to the practices, and the associated enabling and constraining arrangements, as they unfold in time and space, requires a phenomenological approach. This could include ethnographic observations and phenomenological interviews, where the focus is on collecting evidence about what actually happens in various mathematics education learning sites.

Conclusions

To take a practice approach is to pay attention to the *happeningness* of mathematics education, and this provides new insights and potential benefits for researchers and educators. These include a holistic and integrated perspective that incorporates knowledge, action, and affect as inherent aspects of mathematics and mathematics education practice. Understanding mathematics education as situated practice highlights, and perhaps offers some ways ahead, in considering the pervasive issues of learning transfer. It is a general assumption that school learning is readily transferred and available for use in other settings, including everyday life, but this seems problematic for mathematics education, and so developing mathematical practices may see them as more "transferrable."

Also, the site-ontological nature of mathematics education means that the development of broad universal versions of "best practice" is not possible, and that it unfolds in practice in each site uniquely everyday – mathematics teaching and learning is situated, and it demands to be understood and developed at the local level. The various "Lesson Study" projects (e.g., Hart, et al., 2011) that have been undertaken across the world have been seminal to this end, and while they are time consuming and labour intensive, they have realised some useful insights into mathematics education "as it happens." Finally, a practice perspective highlights the prefigured, but not pre-ordained, nature of mathematics learning and teaching. This is important because if mathematics education practices are to be developed, then there needs to be an allied commitment to developing the associated practice architectures that enable and constrain them. Perhaps then, a practice understanding could help ensure that mathematics *education* is possible rather than mere mathematics *schooling*.

References

Bass, H. (2011). A vignette of doing mathematics: A meta-cognitive tour of the production of some elementary mathematics, *TMME, 8*(1), 3–34.

Burton, L. (1999). The practice of mathematicians: What do they tell us about coming to know mathematics? *Educational Studies in Mathematics*, 37, 121–143.

Burton, L. (2001). Research Mathematicians as learners – and what mathematics education can learn from them. *British Educational Research Journal*, 27, 589–599.

D'Ambrosio, U. (2006). *Ethomathematics*. Sense.

Grootenboer, P., Edwards-Groves, C., & Kemmis, S. (2021). A curriculum of mathematical practices. *Pedagogy, Culture and Society*. https://doi.org/10.1080/14681366.2021.1937678

Grootenboer, P., Edwards-Groves, C., & Wilkinson, J. (2018). The possibilities for education in an era of schooling. In C. Edwards-Groves, P. Grootenboer, & J. Wilkinson. (Eds.) (2018). *Education in an era of schooling: Critical perspectives of educational practice and action research. A Festschrift for Stephen Kemmis* (pp. 1-16). Springer

Grootenboer, P., & Marshman, M. (2017). *Mathematics affect and learning: Middle school students' beliefs and attitudes about mathematics education*. Springer.

Hart, L. C., Alston, A. S., & Murata, A. (2011). *Lesson study research and practice in mathematics education: Learning together*. Springer

Kemmis, S. (2008). Praxis and practice Architectures in mathematics education. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia), pp. 17–28. Brisbane: MERGA.

Kemmis, S., Edwards-Groves, C., Lloyd, A., Grootenboer, P., Hardy, I., & Wilkinson, J. (2017). Learning as Being 'Stirred In' to Practices. In P. Grootenboer, C. Edwards-Groves, & S. Choy. (Eds.), *Practice theory perspectives on pedagogy and education: Praxis, diversity and contestat*ion (pp. 45–65). Springer

Kemmis, S., & Grootenboer, P. (2008). Situating practice. In S. Kemmis & T. Smith (Eds.), *Enabling praxis: Challenges for education* (pp. 37–62). Sense.

Kemmis, S., Wilkinson, J., Edwards-Groves, C., Hardy, I., Grootenboer, P. & Bristol, L. (2014). *Changing practices, changing education*. Springer.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life.* Cambridge University Press

Lave, J. (2019). *Learning in everyday life: Access, participation, and changing practice*. Cambridge University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge University Press.

MacIntyre, A. (1997). *After virtue* (2nd ed). Notre Dame University Press.

Marx, K. (1852/1999) *The Eighteenth Brumaire of Louis Napoleon*. Marx/Engels Internet Archive. http://www.marxists.org/archive/marx/works/1852/18th-brumaire/ch01.htm Retrieved May 13, 2022.

Nunes, T., Carraher, D. W., & Dias Schliemann, A. (1993). *Street mathematics and school mathematics*. Cambridge University Press

Schatzki, T. R. (2002). *The site of the social: A philosophical account of the constitution of social life and change*. Pennsylvania State University Press.

Schoenfeld, A. H. (2020). Mathematical practices, in theory and practice. *ZDM Mathematics Education, 52*, 1163–1175

Stillman, G., Brown, J., & Czoches, J. (2020). Yes, mathematicians do X so students should do X, but it's not the X you think! *ZDM Mathematics Education, 52*, 1211–1222