Mathematics Education Research Group of Australasia

MERGA 44

Mathematical Confluences and Journeys

Launceston, 3-7 July 2022

Proceedings of the 44th Annual Conference of the
Mathematics Education Research Group of Australasia

Noleine Fitzallen, Carol Murphy, Vesife Hatisaru, & Nicole Maher
(Editors)

University of Tasmania
Launceston

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ISBN 978-1-920846-33-6

Published by:

The Mathematics Education Research Group of Australasia Inc.

GPO Box 2747

Adelaide, SA 5001

https://merga.net.au/

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Preface

This is a record of the Proceedings of the 44th annual conference of the Mathematics Education Research Group of Australasia (MERGA). The conference was hosted by colleagues at the University of Tasmania. It was a hybrid conference as there remained restrictions in travel due to the COVID-19 pandemic, which started in 2019. The proceedings were published online at the MERGA website.

The theme of the conference was *Mathematical Confluences and Journeys*, symbolic of the city of Launceston, which sits at the confluence of the North and South Esk Rivers, where they meet to form the Tamar River, known as Kanamaluka by the local Palawa people. This theme was chosen to recognise the nature of the conference as a mixing of our educational experiences and research, and the journey on which we all embark, both in coming to the conference, and our continued studies. Also a key focus of the conference was the confluence, bringing together of research and practice. This gathering provided various platforms for participants to come together to consider the ways in which we can bring our research together to influence the field of mathematics education and educational policy throughout Australasia.

The opening plenary lecture by Professor Peter Liljedahl introduced the theme, with a presentation, *Teaching to Learning and Research to Practice: Bridging the Fractal Divides*. The Clements/Foyster lecture was delivered by Professor Peter Grootenboer, employing practice philosophy and theory to discuss the confluence of mathematics education and mathematics education research. The second plenary lecture by Professor Elena Nardi continued the theme by considering the pedagogical potential of rapprochement and synergy amongst communities of mathematics teachers. An additional feature of the conference was the inaugural inclusion in the official program of a *Teacher's Day*, which showcased research applicable to classroom practice. The conference included presentations of symposia, research papers, short communications, round tables, and workshops that covered a wide range of topics related to mathematics education in Australasia and other countries. All symposia and research papers were double-blind reviewed by panels of mathematics educators with expertise in the field and accepted for publication and presentation or presentation only. All the short communications were reviewed by the Editorial Panel and were accepted for presentation if research focused. The published proceedings include the plenary papers, symposia papers, research papers, and abstracts of short communications and round tables.

We wish to thank the authors for sharing their research, and all the delegates from many countries (Australia, Canada, Chile, Greece, Hong Kong, India, Mexico, New Zealand, Singapore, South Africa, and United Kingdom), who participated in the conference via online platforms or in person, in somewhat difficult travel conditions and time zones. Gratitude goes to the Review Panel Chairs and all the reviewers for their professionalism and effort in reviewing the papers and providing constructive feedback. The review process ensured that the high academic standards of the MERGA community were upheld. The proceedings illustrate the breadth of mathematics education research undertaken in the region and beyond by the MERGA research community.

Greg Oates (Conference Convenor)

Noleine Fitzallen, Carol Murphy, Vesife Hatisaru & Nicole Maher (Editors)

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CLEMENTS-FOYSTER LECTURE

The Clements-Foyster Lecture acknowledges an eminent mathematics education researcher from Australia, New Zealand or a South East Asian rim country, who is invited to present a keynote address at the annual MERGA conference. This annual keynote address is named in honour of Ken Clements and John Foyster who initiated and organised the first Mathematics Education Research Group of Australia Conference at Monash University in 1977. This led to the establishment of the organisation now known as MERGA.

The Practice of Mathematics Education

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Over the years mathematics education has been examined through a range of theoretical perspectives, and these have predominately been epistemological in nature. However, despite this rich history of research, it still seems some perennial issues with mathematics education remain, with many learners floundering and large numbers disliking mathematics or viewing it as irrelevant. Therefore, here a different perspective on mathematics education is presented – an ontological practice approach, which complements existing epistemological understandings. Considering mathematics education as practice foregrounds two key factors—the "site ontological" nature of mathematics learning and teaching that highlights its "happeningness", and the sociality of mathematics education. There are two reasons for considering a practice approach to understanding mathematics education. First, mathematics itself is a practice, and reciprocally mathematics education is concerned with enabling learners to *practice* mathematics. Second a practice approach to mathematics could offer insights into the perennial and intractable affective issues of mathematics. Considering these things, here practice philosophy and theory is employed to discuss mathematics education and mathematics education research.

Mathematics education *happens*, and regardless of what knowledge people may have about it, or what they may believe or feel, in the end, it happens. Furthermore, mathematics education occurs and unfolds in time and space in particular sites and at particular times. In addition, while there is rightly interest in things like educational leadership, curriculum, teacher standards, mandated external assessment regimes, etc, these things only exist and have meaning because they in some way impact what happens in classrooms[1]. The classroom is "always the existential; and ontological given in education" (Kemmis et al., 2014, p. 214). For these reasons, here a site-ontological practice approach is taken to understanding mathematics education. This practice approach does not necessarily usurp or supersede other epistemological understandings and conceptualisations of mathematics education, but rather it offers a complementary ontological perspective.

The Primacy of Practice

According to Schatzki (2002), we live our lives in practices, where we encounter one another as interlocuters in time and space. This ontological view gives primacy to the *happeningness*[2] of life, including mathematics, mathematics education, and mathematics education research. Indeed, as will be discussed later, a fundamental reason for considering mathematics education from a practice perspective is that mathematics itself is a practice and is comprised of practices. However, it is commonly assumed that the term "practice" has an unquestioned and unproblematic meaning, particularly given its common usage in everyday language, and so here an ontological conception of practice that sees it as site-based and social will be first briefly outlined—specifically the "theory of practice architectures" (Kemmis & Grootenboer, 2008; Kemmis et al., 2014).

[1] The term "classroom" here refers to any "learning site", not necessarily just a formal school classroom.
[2] Happeningness of practices refers to ways practices are observable acts unfolding temporally (in time), ontologically (in a particular place), discursively (communicatively in language) and relationally (intersubjectively between people) (Edwards-Groves & Grootenboer, 2015)

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 1–8. Launceston: MERGA.

The Theory of Practice Architectures

The theory of practice architectures was developed by a group of colleagues through theoretical and philosophical discussions, and it draws upon data from a large-scale empirical project conducted over a number of years (see Kemmis et al., 2014). In brief, the theory posits that practices are comprised of characteristic sayings, doings, and relatings, that occur in semantic, physical, and social space respectively, and are held together in a project (see Figure 1). These sayings, doings, and relatings, are enabled and constrained by practice architectures—cultural discursive, material economic, and social political, arrangements and conditions that exist in the practice site.

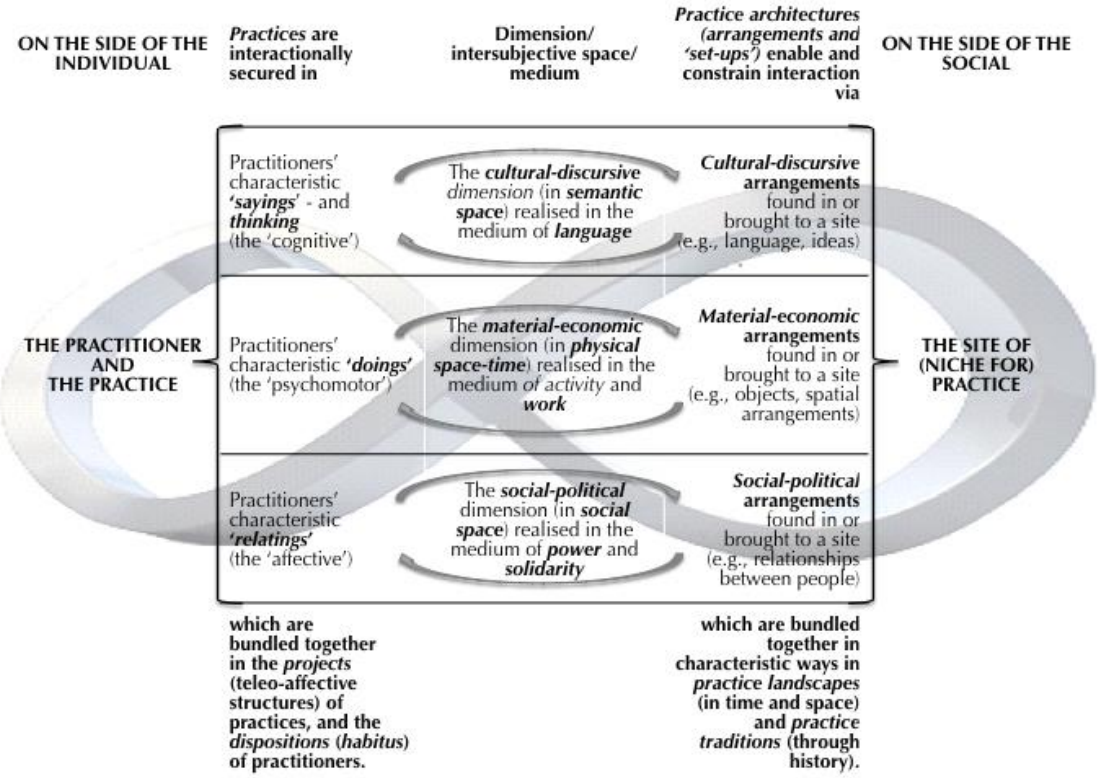

Figure 1: The theory of practice architectures (Kemmis et al., 2014, p. 38).

In this sense, practices are prefigured, but not predetermined, by the conditions and arrangements—the practice architectures, of the practice site, or,

> [People] make their own history, but they do not make it as they please; they do not make it under self-selected circumstances, but under circumstances existing already, given and transmitted from the past. The tradition of all dead generations weighs like a nightmare on the brains of the living. (Marx (1852/1999, p. 1)

While this outline is necessarily brief and lacking detail, it can be exemplified by considering a practice like teaching fractions to primary school students. To enable students to comprehend fractions, the teacher will talk in specific ways using regular and particular language (e.g., numerator, denominator), do certain things with resources (e.g., maybe cut a cake into equal pieces), and relate to the students in a professional but encouraging manner by using their position as a knowledgeable person in the room (e.g., to gently question or challenge students).

As noted earlier, what is important here is the site-ontological nature of practice.

> The site of a practice is the phenomenological reality that always and necessarily escapes standardisation in curricula, standards, assessments and policies. The site is not only a matter of happenstance (where practices happen to take place and where things happen to be arranged as they are), nor only because the site is the specific location in which participants' practical deliberation and their practical action takes place. The 'site' is also crucial *theoretically*—to be understood in existential and ontological terms as an actual and particular place where things happen, not just as a location in an abstract and universal matrix of space-time. (Kemmis et al., 2014, p. 215)

In other words, the site is more than a *context* in which practices take place—it is an integral part of the happening. This, amongst other things, means that the notion of "best practice" is, at best, an unhelpful myth, and at worst, a damaging misconception that sees all learners, sites, and communities as homogenous. The site-based nature of practices, including mathematics education practices, means one can only talk of "best practices *here* and *now*."

Also, an affordance of the theory of practice architectures is that it enables the individual (practices) and the communal (practice architectures) to be considered simultaneously and in a manner that sees them as symbiotic and complementary. The epistemological dilemmas related to individualistic and social understandings are not settled as such, but rather the ontological approach that focuses on the happening of practices enables these to be considered together.

Furthermore, and relatedly, the theory of practice architectures also allows for a critical understanding of education practices because it simultaneously requires consideration of practices, and the conditions and arrangements that enable and constrain them. Specifically, because practices are seen as prefigured (but not predetermined) by practice architectures, this means that to develop or change a practice there is also a need to reform the practice architectures. Critically, when educational reform or development is considered, it needs to be understood in a complex manner that concurrently addresses the relevant practices and the enabling and constraining conditions and arrangements.

Mathematics Education as Practice

Turning now specifically to mathematics education, the focus will be on mathematics teaching and learning as it is practiced in real time and space in classroom sites. This is focus on the "happeningness" of mathematics education—all the other phenomena including formal and informal mathematics curricula, mathematics education leadership, programs for teacher professional learning, education theory, and indeed, mathematics education research, only have significance and value because in everyday educational sites (e.g., schools) students and teachers meet as interlocutors around mathematical experiences in order to learn. However, first a brief discussion of mathematics practice is provided.

Mathematics Practice

Mathematics is a "coherent and complex form of socially established cooperative human activity" (MacIntyre, 1997, p. 187), and as such it can be understood as a practice. As such, mathematics is a human endeavour that is practiced in diverse ways across different sites. So, this means that the pure mathematician in the university, the engineer out in the field, the person shopping for groceries, or the child scoring a game, are all practicing mathematics. However, to illustrate here, the work of Burton (1999, 2001) will be briefly outlined.

Burton studied the mathematical practices of professional mathematicians in university sites. Not surprisingly, she found that they engaged in common practices (e.g., proof) to solve substantial and noteworthy problems and to expand discipline knowledge, and for those in this community of practice (Lave & Wenger, 1991), these were engaging, enthralling, and compelling. Furthermore, and far from the common perceptions, the community of mathematical practice provided significant collegial emotional and practical support as the mathematicians engaged in their intellectual and affective practices (Bass, 2011). So, for the professional mathematicians that Burton encountered, mathematical practice is wonderous, beautiful, and fascinating—something that is perhaps not shared with those learning mathematics at school. Also, it would seem that for professional mathematicians, learning mathematics is simply just an integral and routine aspect of their practice. Burton (2001) commented that "we have a responsibility to make the learning of mathematics more akin to how mathematicians learn and to be less obsessed with the necessity to teach 'the basics' in the absence of any student's need to know" (p. 598), so to this end perhaps there is a need for students to engage more in mathematical practices such as mathematical modelling (Stillman, Brown, & Czoches, 2020) and mathematical problem solving (Schoenfeld, 2020).

Learning Mathematics

First, in this practice perspective learning mathematics is understood as something more that acquiring mathematical content knowledge (including knowing how to do some "mathematical skills")—it is about learning how to practice mathematics. To be clear, learning how to practice mathematics does involve intellectual and skill growth (as is evident in the "sayings" and "doings" in the theory of practice architectures), but only as an integral part of learning how to go on in the particular mathematical practices concerned. To this end, Kemmis et al. (2014) proposed that:

> … learning is always and only a process of being stirred into practices, even when a learner is learning alone or from participation with others in shared activities. We learn not only knowledge, embodied in our minds, bodies and feelings, but also how to interact with others and the world; our learning is not only epistemologically secured (as cognitive knowledge) but also interactionally secured in sayings, doings and relatings that take place amid the cultural-discursive, material-economic and social-political arrangements that pertain in the settings we inhabit. Our learning is bigger than us; it always positions and orients us in a shared, three-dimensional—semantic, material and social—world. (p. 59; emphases original)

In this, they suggest that learning is not about somehow "obtaining" something (e.g., knowledge, skills), but rather about "being stirred into practices" or learning "how to go in practices" (Kemmis et al., 2017, p. 45). In other words, the goal (or project) of mathematics education is to enable students to continue with mathematical practices. In this sense, it is not that one can or cannot, do mathematics, it is about developing in competence and confidence in mathematical practices—becoming more engaged and proficient in the community of mathematical practice.

Second, learning as being "stirred into mathematical practices" provides a holistic view including cognitive, affective and conative dimensions. In this sense, emotions, actions, attitudes, and values are not separate aspects associated with learning mathematics—they are an integral part of learning mathematics. Thus, when someone is learning some mathematical practice, they are engaging with particular discipline knowledge, and at the same time and as part of the mathematical practice, they are coming to appreciate it as, for example, interesting and useful (or not). This is important for mathematics education because it places affective considerations as integral and central to the "learning"—not as something to be considered as an afterthought to the lesson. For many years mathematics education has been beset with affective issues that have regularly seen students indicating that mathematics is distasteful and to be avoided, and despite many attempts and ideas, this has largely remained unchanged. So, perhaps if learning mathematics is understood as learning how to *go in* mathematical practices, including the saying/knowing, doing and relating, then aspects such as affect will not be seen as an uncomfortable extra consideration, but as a fundamental inter-related aspect.

Finally, when learning is seen how to *go on* in the community of mathematical practice, then the role of the teacher as a fellow, albeit more experienced, member of the community, is crucial. Lave and Wenger (1991) see identity as a function of membership of a community of practice, and so here, teachers need to be themselves engaged in mathematical practices, and identify as "mathematicians" in this way. Of course, the somewhat artificial nature of schooling, where learning is mostly separated from participation in authentic communities of practice, means this can be difficult, and requires a reimagining of what a mathematician is, from a professional vocation to something that is undertaken in various forms and in various ways in a variety of different sites.

Mathematics Education and Practice Architectures

So, the possibilities for mathematics education practice are always enabled and constrained by the prevailing practice architectures. It follows then that the development of mathematics education practices involves more than just changing mathematics teachers' practices—it demands a concomitant transformation of the practice architectures that shape the said practices.

Of course, the practice architectures that create possibilities for mathematics education are broader than just the disciplinary traditions of mathematics and mathematics education—the national and state level practices of educational administration, policy making, and assessment, are brought into school sites and act as powerful shapers. Indeed, Grootenboer et al. (2018) suggest that strong cultural-discursive arrangements can inhibit the possibility for education and leave us with mere "schooling."

> If teachers are obliged to follow all the available advice too slavishly, if they take their eyes off the students in front of them because they are obliged to listen too closely to the voices of the advisors and administrators behind them, they may find themselves working on what the state intends – schooling – rather than for the good of their students and the society. The syllabus, instead of being a source of guidance and inspiration for teachers and students, may become a litany of imposed tasks to which teachers and students cannot do justice. (Kemmis, 2008, p. 14)

So together, these points make an argument for not having *mega conditions* (e.g., national or state curricula, external assessment regimes, policies) that are overly restrictive and controlling, because they limit the capacity for mathematics education that is responsive to the unique site-based needs and requirements and conditions. For example, if mathematics teachers are to practice in a reflective and responsive educational manner, then they require scope to develop and enact their pedagogy within the guidance of curricula, and not be slavishly required to follow a detailed prescription of teaching activity.

Learning Mathematics in Schools

As has been suggested, there is a difference between mathematics *education* and mathematics *schooling*, and the argument being presented now is that we need mathematics education that is educational, and indeed, mathematical. In dealing with the former, it is sufficient to highlight at this stage that mathematics education as it is realised in schools (and early childhood settings, and universities) needs to consistent with, and characterised by, mathematical practice (Burton, 2001).

It is the case that individuals and groups learn many things and practices, including mathematics, outside the formal settings of schools, and yet it seems that often the focus for understanding learning is restricted to the schooling context. This is not to say that mathematics learning in schools is necessarily a bad thing, but it does require some consideration about what this has done to mathematics education (and indeed, mathematics). Two related issues will be dealt with here: (1) the valorising of content knowledge over practice; and (2) the assumption of learning transfer.

The Valorising of Content Knowledge

While mathematics itself is a practice, and comprised of practices, school mathematics curricula tend to be dominated by discipline knowledge that is required to be taught and learned. This then leads to mathematics education (or schooling) that is dominated by trying to transfer the required knowledge from the curriculum into the heads of the students. Of course, it is not quite a simple as this, but this approach in general is fraught because, amongst other reasons, it is a misrepresentation of mathematics. This limits the curriculum to the *content* which tends to be abstract and general (and can be tested formally). Indeed, the inaccessibility of mathematical knowledge means the teacher can only know when the student has apprehended the required knowledge through implication from what they might display in practice.

This is not to say that mathematics curricula statements do not include some notions of mathematical practice, but they are often seen, whether by design or by assumption, as the separate add-on optional parts that need to fit in and around the content knowledge where possible. However, it is contended here that knowledge is an integral and entangled part of practice, and furthermore, is only ascertained and demonstrated "in practice." In other words, mathematical knowledge and skills are important, but alone are insufficient and inadequate, and a dangerous simplification of mathematics.

The Assumption of Learning Transfer

Schools, by design, see learning as "abstract" and removed from the relevant "communities of practice"[3]. Of course, as is clear from what has proceeded, learning is site-based, and so rather than being somehow *objective*, school mathematics learning is situated in the school site. Schools and school systems, to a greater or lesser degree, try to ameliorate this by the pedagogical practices employed, but nevertheless, schools as institutions are set up to have students learn general mathematical knowledge and skills. As noted above, there are benefits of this, but it is premised on an assumption that what is learned will be readily transferred to other practice contexts and settings as required. After researching the mathematical practices of people in "everyday life"(e.g., in the supermarket), Lave (1988) commented,

> Conventional academic and folk theory assumes that arithmetic is learned in school in the normative fashion in which it is taught, and is then literally carried away from school to be applied at will in any situation that calls for calculation. (p. 4)

Her clear finding from her large ethnographic study was that this was not the case, and, for example, some people who were quite poor at "school arithmetic," were actually very good at arithmetic in their everyday situations. Certainly, despite the common and widespread use of mathematics a large body of research indicates that most students see mathematics as useless—something they have apparently learned at school in their mathematics classrooms (Grootenboer & Marshman, 2017), and knowledge and skills that do not transfer to other sites outside and/or across the classroom, or other disciplinary practices.

Of course, these are complex and long-standing problems in mathematics education that defy simple quick-fix solutions, but perhaps it is timely to consider how school mathematics can be more "educational." Some possible approaches include:

- Seeing mathematical practices as broader than just what happens at school or what "professional mathematicians" do. In other words, there is a need to develop and value a more comprehensive view of the mathematics community of practice.

[3] The communities of practice here are broader than just professional mathematicians, but all who practice mathematics

- Considering a curriculum of mathematical practices that deliberately and overtly centres on the practicing of mathematics and the development of students' identities as mathematical practitioners (see Grootenboer et al., 2021).

Implications for Mathematics Education Research

To understand mathematics education from a practice perspective is to focus its site-ontological and social nature. Fundamental in this view is seeing mathematics education as equipping learners to 'go on' in mathematical practice—students are not so much learning mathematics, but rather becoming mathematicians (or becoming part of a mathematics community of practice. This view of mathematics and mathematics education has implications for mathematics education research.

First, there would seem to be an urgent and compelling case for examining the issue of learning transfer in mathematics education. When mathematics is learned in school classrooms, then the site becomes an integral part of the mathematics that is learned, and so the value and availability of mathematical practices gained through schooling seem to have limited influence on, or use for, mathematics outside the school setting. This is not to say that learning mathematics at school is a bad thing, but rather, the very nature of school mathematics learning requires that issues of learning transfer are central.

Second, it seems that individuals in a range of everyday settings are able to learn complicated mathematical practices (for example, see the work of Lave, 2019; Nunes et al., 1993; D'Ambrosio, 2006), and yet, when in more contrived formal learning site (e.g., a school), the *learning* was diminished, and even washed out. Although the common perception is that mathematics learning only happens in mathematics classrooms, this highlights that mathematics education happens in a range of sites—including powerfully in *informal* settings, and there is much to be learned for school learning from these everyday sites.

Third, a site-ontological practice conception demands methodologies and methods that are responsive to the *happeningness* of mathematics education practices, and the practice architectures that enable and constrain them. In essence, to be attentive to the practices, and the associated enabling and constraining arrangements, as they unfold in time and space, requires a phenomenological approach. This could include ethnographic observations and phenomenological interviews, where the focus is on collecting evidence about what actually happens in various mathematics education learning sites.

Conclusions

To take a practice approach is to pay attention to the *happeningness* of mathematics education, and this provides new insights and potential benefits for researchers and educators. These include a holistic and integrated perspective that incorporates knowledge, action, and affect as inherent aspects of mathematics and mathematics education practice. Understanding mathematics education as situated practice highlights, and perhaps offers some ways ahead, in considering the pervasive issues of learning transfer. It is a general assumption that school learning is readily transferred and available for use in other settings, including everyday life, but this seems problematic for mathematics education, and so developing mathematical practices may see them as more "transferrable."

Also, the site-ontological nature of mathematics education means that the development of broad universal versions of "best practice" is not possible, and that it unfolds in practice in each site uniquely everyday – mathematics teaching and learning is situated, and it demands to be understood and developed at the local level. The various "Lesson Study" projects (e.g., Hart, et al., 2011) that have been undertaken across the world have been seminal to this end, and while they are time consuming and labour intensive, they have realised some useful insights into mathematics education "as it happens." Finally, a practice perspective highlights the prefigured, but not pre-ordained, nature of mathematics learning and teaching. This is important because if mathematics education practices are to be developed, then there needs to be an allied commitment to developing the associated practice architectures that enable and constrain them. Perhaps then, a practice understanding could help ensure that mathematics *education* is possible rather than mere mathematics *schooling*.

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Appreciating the Intra-/Extra-mathematical Importance of Mathematics: Added Pedagogical Value Through Rapprochement and Synergy of Primary, Secondary and Tertiary Mathematics Teachers

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Although mathematics is seen as a *sine qua non* of curricula around the world, emphasis on appreciating the importance of the topics that constitute the backbone of said curricula within—and, crucially, beyond—mathematics is often limited. I draw on my experiences as mathematics education researcher and (non-research) mathematician to discuss the rarely tapped-into pedagogical potential of rapprochement and synergy amongst communities of mathematics teachers (primary; secondary; and tertiary, from within mathematics and in other disciplines).

Throughout the pandemic, public discourse about COVID-19—in the UK, led primarily by daily, televised conferences of the Government's Chief Medical Officer and his Deputy as well as the Chief Scientific Adviser—brimmed with mathematical references. As debates raged about whether and how to convince the public of the utter necessity for the personal, social and economic sacrifices that tackling the virus required, a stark realisation started to emerge: that many of these references may not have the impact that the scientists who were making them were hoping to achieve (Skovsmose, 2021). In tandem with findings from research that indicated how invisible mathematicians and mathematics often seem to be (Yeoman et al., 2017; Nardi, 2017), I conjecture that relentless exposure of the public to said mathematical references may make some difference—but for this to become the case, rapprochement and synergy between communities of mathematics teachers across educational levels and across disciplines is utterly necessary. To explore this conjecture, I draw on research, teaching and professional development initiatives that illustrate elements of this rapprochement and focus on boosting the intra- and extra-mathematical visibility of mathematics. The underpinnings of the analyses I present are largely commognitive (Sfard, 2008), with a focus particularly on constructs such as *literate* (as in, e.g., mathematical) and *colloquial* (as in everyday, public) *discourses* (p. 118). I conclude with reflections on what this analysis may imply for school and university mathematics pedagogy (Nardi & Biza, in press; Herbst et al., 2021), especially for students who traditionally experience alienation from mathematics and/or are on the cusp of deciding whether to include mathematics in the pursuit of employment or further studies.

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2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), p. 9. Launceston: MERGA.

Teaching to Learning and Research to Practice: Bridging the Fractal Divides

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In mathematics education it is taken as shared that we need to perpetually work on bridging research and practice. In this talk I look at some of the results of the *Building Thinking Classrooms* project, which has been shown to do just to—bridge research and practice. More than this, however, this project has also been shown to bridge teaching and learning—or more specifically, teaching and studenting—the bridge is in desperate need of repair.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), p. 10. Launceston: MERGA.

Mathematical Sequences of Connected, Cumulative and Challenging Tasks in the Early Years

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This symposium reports on a project that focused on *Exploring the Use of Mathematical Sequences of Connected, Cumulative and Challenging Tasks (EMC^3*) with students in the early years (Foundation Level to Year 2). The project was funded by the Australian Research Council, Catholic Education Diocese of Parramatta and Melbourne Archdiocese Catholic Schools (LP180100600). Together with industry partners the EMC^3 project was designed to enhance the cognitive and affective experiences of students when learning mathematics by researching teaching approaches that utilise sequences of cognitively challenging tasks.

Paper 1: *Exploring the Potential of Sequences of Connected, Cumulative and Challenging Tasks in the Early Years* [Peter Sullivan, Melody McCormick]

This paper outlines the rationale for the teaching approach the EMC^3 project aimed at studying an approach to teaching and learning mathematics in the early years (students aged 5–9).

Paper 2: *Differentiating Mathematics Instruction through Sequences of Challenging Tasks in the Early Primary Years* [James Russo, Jane Hubbard]

This paper reports on post-program questionnaire data collected from 100 teachers who express their views about the effectiveness of various instructional approaches to support differentiation in mathematics.

Paper 3: *Changing Teacher Practices: A "Slow Burn" or Rapid with "Big Shifts."* [Sharyn Livy, Janette Bobis, Ellen Corovic, Maggie Feng]

This paper reports on interview data collected from five teacher educators who provided support to the teachers when trialing the EMC^3 resources. The focus of this presentation will be on the notable changes to teacher practices.

Paper 4: *The Nature of Leadership and Other Support that Facilitate Innovation and Improvement in Teacher Practice.* [Ann Downton, Janette Bobis]

The final paper reports on survey data collected from 70 teachers about the forms of support that assisted implementation of project resources—in-class support and facilitation of planning.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), p. 11–27. Launceston: MERGA.

Exploring the Potential of Sequences of Connected, Cumulative and Challenging Tasks in the Early Years

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This paper outlines the rationale for, and some elements of, a particular approach to teaching and learning mathematics in the early years. The researchers worked with two school systems to offer both centrally delivered and school-based teacher professional learning, which included the application of illustrative teaching resources. The project gathered a range of data from teachers and leaders on their dispositions and knowledge, as well as the opportunities and constraints they experienced, and the influence these variables had on planning, teaching and student learning outcomes.

The following outlines the rationale for, and some elements of, an Australian Research Council funded project aiming to study a particular approach to teaching and learning mathematics in the early years (students aged 5–9). This contribution provides background information relevant for the other presentations in the symposium. Fundamental to this approach to teaching was the use of sequences of connected, cumulative, and challenging tasks that focused on mathematical content and proficiencies represented in the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2020).

Even though it is common for teachers to develop understanding and foster mathematical fluency associated with particular concepts before problem solving and reasoning, termed teaching *for* problem solving (Schroeder & Lester, 1989), the project explored the potential of the reverse. That is, we considered the impact on student learning and engagement when teachers pose problems that allow for student reasoning to occur first, with the intention of building understanding leading to fluency subsequently; this is termed teaching *through* problem solving (Schroeder & Lester, 1989).

The project task design and pedagogical emphasis were informed by two characteristics articulated by the Organisation for Economic Co-operation and Development (OECD) (2019; 2021). The first, agency, relates to students having the ability and will to make active decisions to positively influence their own and others' learning. This implies that students see themselves as not only capable of thinking for themselves but also having the confidence and aspirations to learn. In order to exercise such agency and realise their potential, learners require time initially to struggle productively (Sinha & Kapur, 2021) with problems or tasks without being told what to do by the teacher/educator or other students. During this uninterrupted time students are able to choose their own strategy and form of representation. This pedagogical focus aligns with the teaching through problem solving approach. In terms of emphasising student agency, we encouraged the project teachers to plan experiences that were productively challenging for students. Sullivan et al. (2020) explained that:

> Challenge comes when students do not know how to solve the task and work on the task prior to teacher instruction. Other characteristics of such tasks are that they: build on what students already know; take time; are engaging for students in that they are interested in, and see value persisting with a task; focus on important aspects of mathematics (hopefully as identified or implied in relevant curriculum documents); are simply posed using a relatable narrative; foster connections within mathematics and across domains …. (pp. 32–33)

The second characteristic, inclusion, involves identifying learning experiences and associated pedagogies that maximise opportunities of all learners. As far as possible, in the

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approach we are exploring, all learners are given opportunities to think for themselves and, especially for students experiencing difficulty, are provided support to access the full curriculum. This is elaborated further in Russo and Hubbard (Paper 2) and includes learning experiences in which the activities and tasks are accessible, while still being productively challenging, and with explicit teacher attention to actions that address the needs of individual learners. There are three aspects of the recommended pedagogies that are intended to foster inclusion. First, teachers are encouraged to choose learning experiences that are not only readily accessible for all students but also have the potential for further exploration. Second, teachers prepare specific enabling prompts for students experiencing difficulty and extending prompts for students who complete the set work quickly (see Sullivan et al., 2006). Third, teachers use a particular lesson structure, as summarised below, consistently to provide students with confidence of the ways the lessons develop.

The specific aims of the project were to:

- explore the potential of sequences of connected, cumulative, and challenging tasks that build a trajectory of consolidated learning of mathematics;
- explore responses from teachers, leaders and students when this approach to teaching mathematics is enacted;
- make recommendations for resource developers, curriculum designers and providers of teacher professional learning.

An Instructional Model for Student-centred Structured Inquiry

The EMC[3] project described this approach to instruction as *Student-centred Structured Inquiry* (the key elements of this are elaborated in Sullivan et al., 2020). The approach is also described as cognitive activation. Caro et al. (2016) analysed results of PISA 2012 involving over 500,000 students and provided compelling evidence of the effectiveness of this perspective. Characteristics of cognitive activation include posing problems that require students to think for an extended time, to choose their own solution procedures, to learn from mistakes, to explain their solution strategies and to solve problems in different ways.

To communicate the various associated teacher actions, the project participants and researchers developed an instructional model with four phases: *Anticipate, Launch, Explore, and Summarise/review*. The language of the instructional model draws heavily on Smith and Stein (2011) who focus on orchestrating classroom discussions, an essential element of creating opportunities for fostering student agency and inclusion. The aim is to make it obvious to students they have a role to play in creating new knowledge.

Anticipate phase. This phase is central to all planning. It includes identifying the intended learning outcomes (what, why and how); developing helpful resources; predicting students' solutions, strategies and possible misconceptions; and considering pre-requisite and new language, as well as other aspects of planning.

Launch phase. This phase addresses language and representation associated with the intended learning experiences. It includes providing opportunities for students to develop fluency in the mathematical processes and procedures relevant to the experiences. It also involves posing tasks without informing students on how to solve the problem, an essential aspect of fostering agency.

Explore phase. In this phase teachers interact with students, encouraging persistence, posing prompts, and identifying interesting and perhaps unanticipated solutions, selecting some for later presentation.

Summarise/review phase. This phase involves the teachers selecting and sequencing student solutions to be shared. Engagement is promoted by supporting students while they present their solutions and encouraging active participation of others. A key element of this phase is the teacher synthesising the essential ideas that represent the learning intentions of the experience.

Importantly, the launch-explore-summarise/review process happens more than once for each learning experience, with the tasks for the subsequent cycles based on Variation Theory (Kullberg et al., 2013). The variations, as represented by this theory, are intended to draw the attention of students to key elements of concepts by varying some aspects while keeping other aspects invariant. In other words, task design involves creating new tasks from existing tasks by keeping some aspects the same but varying other aspects. The variant might be the context, with the concept(s) staying the same. Alternatively, the variant might be the sophistication of the concept (or even the concept itself), with the context staying the same. The explicit intention of the subsequent iterations of the model is to consolidate thinking activated by the initial experience (Dooley, 2012). This consolidation involves repeating the preceding three phases, noting that consolidation can be in a subsequent lesson.

An important feature of the instructional model is that, when consistently applied, it is argued to help students to moderate their anxiety by normalising uncertainty. Buckley and Sullivan (2021) argued that students who are anxious can manage the threat to their learning opportunity by specific behavioural strategies and through familiarity with this lesson structure.

Project Resources

The project team and participating teachers developed coherent and connected sequences, representing the content descriptions and proficiencies of the *Australian Curriculum: Mathematics* (ACARA, 2020). The sequences were intended to make the mathematical ideas central to the learning obvious to the students. Participating teachers were provided with illustrative resources to support the implementation of the pedagogical approach. An example of a low floor/high ceiling task, focusing on making and naming polygons, that is intended to be productively challenging for students aged 6–8 is as follows.

Making polygons out of trapeziums

> Using some or all of four trapeziums (all the same), what polygons can you make?
> Draw the new polygons on isometric dot paper and name them.
> How are your new polygons the same? How are they different?

Students are provided with sets of trapeziums such as those in Pattern Blocks and isometric dot paper. The "floor" is when students make and draw one polygon. The "ceiling" is the possibility of making and drawing multiple different shapes (there are many). An example of an enabling prompt is "what shapes can you make with two trapeziums?" An example of an extending prompt is "draw a triangle made out of three trapeziums without using the materials". An example of a consolidating task is as follows:

Making polygons out of rhombuses

> Using some or all of four rhombuses (all the same), what polygons can you make?
> Draw the new polygons on isometric dot paper and name them.
> How are your new polygons the same? How are they different?

Even though acknowledging individual students' thinking as paramount, both the mathematical focus and the pedagogical approach are intentional and go beyond unstructured inquiry or play (Bruner, 1961; Mayer, 2004). At the same time, the approach rejects the notion that the optimal way to teach mathematics is by explicitly telling students what to do, followed by practice. The teacher has an active role, but this happens after students have had the opportunity to engage in the mathematics and the contexts of the tasks. Likewise, students are exposed to illustrative worked examples, some of which can come from the students themselves.

By proposing carefully constructed and effectively trialled sequences supported by related professional learning, teachers can experience not only ways in which learning can be sequenced but also how sequences enhance learning opportunities for students. The goal of offering suggestions for teachers was to free up energy for them to engage with the complexity of converting tasks, lessons and sequences into learning experiences for their students. The aim was to support the development of manageable and sustainable teaching practices. Part of the professional learning for participating teachers was illustration of ways of adapting the contextual stories and including the level of challenge to suit their particular class and student context. Participating teachers took an active role in the adaptation of the tasks, lessons and sequences, not only improving on the initial designs but also gaining insight into the process of sequence creation.

The project partnered with two school systems that invited schools to participate. In each of three years, participating teachers were offered an initial day of professional learning on the goals and resources of the project, were supported in their schools by researchers and system educators and offered further professional learning. Resources were made available in both hard copy and electronically. There were two sets of participants for each partner over the three years (due to COVID challenges). Data were collected from teachers, school-based leaders and system educators through surveys in each year of the project. There were also interviews with teachers and educators, classroom observations, and assessment of student learning. The findings of the project are in the process of publication.

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Differentiating Mathematics Instruction through Sequences of Challenging Tasks in the Early Primary Years

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We report on questionnaire data gathered from teacher participants (n = 100) following their participation in the project, Exploring Mathematical Sequences of Connected, Cumulative and Challenging Tasks. Teachers shared their views about the effectiveness of various instructional approaches to support differentiation in mathematics, including those illuminated through the project, and a description of a lesson involving effective differentiation.

Differentiating instruction in the context of mathematics teaching refers to the suite of strategies that teachers draw on to cater adaptively to the learning needs of heterogeneous groups of students, with the explicit aim of improving mathematical learning outcomes (Russo et al., 2021). Effective differentiation is acknowledged as a particularly demanding aspect of classroom teaching, with Shernoff et al. (2011) finding that "teaching large heterogeneous groups of learners" (p. 65) was the most notable student-related source of job stress for teachers, alongside "managing disruptive behaviour" (p. 64). There is further evidence that identifying and accessing appropriate learning tasks to meet the range of student learning needs is particularly challenging for teachers. For example, Gaitas and Alves Martins (2017) found that primary school teachers view the matching of activities and materials to the diversity of student characteristics, in relation to their academic readiness, interests and learning profiles, as the most difficult aspect of differentiating instruction effectively.

There is also evidence that without opportunities to develop further their pedagogical content knowledge, teachers may struggle to realise the differentiation potential of a given task. Bardy et al. (2021) found that German secondary mathematics teachers tended to be more focused on the surface structure of tasks (such as their layout) and less focused on the deeper design features (such as the adaptive features of the task), compared with mathematics task design experts. The authors concluded by noting that realising the full potential of a task, or sequence of tasks, to effectively support differentiation requires specific expertise and therefore targeted professional learning support for teachers. Despite its relevance to practitioners and implications for equity, the beliefs and practices about how teachers attempt to differentiate instruction by providing rich learning opportunities for all students remains insufficiently researched in the early years of schooling (Bobis et al., 2021).

We are currently involved in a research project, *Exploring Mathematical Sequences of Connected, Cumulative and Challenging Tasks (EMC^3)* (Sullivan et al., 2020). A key component of the EMC^3 project is a consideration of the extent to which teaching with sequences of challenging mathematical tasks supports differentiation in the mathematics classroom, through both low floor/high ceiling tasks, enabling and extending prompts, and purposeful tasks designed to consolidate learning after productive classroom dialogue about solution strategies. Importantly, given what we know about what teachers find particularly difficult in relation to differentiation (Gaitas & Alves Martins, 2017), participating teachers were provided with illustrative resources to support the implementation of the pedagogical approach, whilst also being encouraged to take an active role in the adaptation of the tasks, lessons and sequences to their particular context. The pedagogical approach presented to teachers in the EMC^3 project, including how it supports differentiated instruction, is elaborated on in Sullivan and McCormick (Paper 1, this symposium). The research questions we will briefly explore in this paper include:

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(1) To what extent do teachers view pedagogies promoted through the EMC^3 project as an effective means of differentiating mathematics instruction relative to other instructional approaches?

(2) How do EMC^3 project teachers describe their approach for differentiating mathematics instruction effectively?

Method

Participants in this study were Foundation to Year 2 (F–2) generalist Australian primary teachers who were involved in the EMC3 professional learning program during 2019 (n = 100). Teachers were introduced to the EMC3 approach during a full day of professional learning with the research team at the start of the school year. They were provided with sequences of challenging tasks and suggestions for their implementation. Support for enactment of the approach was provided to teachers through school visits from members of the project team. Participants also engaged in a second professional learning day in November of 2019. The purpose of day two was to provide teachers with an opportunity to share their post-program learnings and insights with teachers from other schools, as well as to consolidate their understanding around the instructional approach. Teachers were then invited to complete a questionnaire, including questions focussing on their beliefs and approaches for differentiating mathematics instruction. These data form the focus of the current paper.

Results

Perceived Usefulness of Instructional Approaches for Supporting Differentiation

Teachers were asked to indicate the degree to which they considered various approaches useful for differentiating mathematics instruction by responding to the prompt: *The following teaching approaches are useful for catering to students of different performance levels in the mathematics classroom*. For each of the approaches listed, participants recorded their response on a 7-point Likert-type scale, presented with two anchors (1-not at all useful; 7-extremely useful). Mean scores and the percentage of teachers' responses were calculated for each approach. See Bobis et al. (2021) for a more elaborate discussion of this data.

The data from the post-program questionnaire (Table 1) revealed that three teaching approaches were viewed by the majority of teachers as useful for catering to students of different performance levels in the mathematics classroom: problem solving—prompts; problem solving—low floor, high ceiling; and mixed game. These three approaches have important similarities. Most notably, they are the only three approaches of the eight listed that do not involve some form of a priori grouping of students according to perceived mathematical performance, whether such groupings take place within the classroom (grouped game; grouped rotations; grouped online; grouped worksheets) or between classrooms (fluid groupings). It is particularly encouraging that as many as 90% of teachers believed that differentiating problem solving tasks through students accessing enabling and extending prompts was a useful means of catering to different performance levels, with half of the teachers describing this approach as extremely useful. Such tasks formed the core of the learning sequences that teachers accessed as part of EMC^3, and the implication is that this approach was effective at allowing students of all levels to access the tasks. Our second research question focuses on how the teachers described effective differentiation.

Table 1
Usefulness of Approaches for Catering to Students of Different Performance Levels (n = 100)

Instructional Approach	Mean score	positive (5, 6, 7)	extremely useful (7)	not at all useful (1)
Presenting the whole class with the same core problem-solving task, differentiated through students accessing enabling and extending prompts*	6.12	90%	50%	0%
Presenting the whole class with the same core problem-solving task, differentiated through the task having a "low floor, high ceiling"*	5.65	83%	33%	1%
Playing the same mathematical game with the whole class in mixed-performing groups, with the game "naturally" differentiated through students using strategies of choice	5.55	83%	23%	1%
Playing the same mathematical game with the whole class in similar-performing groups, with the game differentiated through groups using resources matched to their performance level	4.36	46%	11%	3%
Between class performance grouping ("fluid groupings"), where similar-performing students are grouped together across classes and undertake activities that match their performance level	3.41	35%	3%	21%
Within class performance grouping, where similar-performing groups rotate through workstations undertaking activities matched to their performance level	3.27	28%	1%	21%
Allowing students to work through on-line activities/Apps at different levels of challenge, depending on their performance level	3.11	27%	4%	21%
Allowing students to work through worksheets at different levels of challenge, depending on their performance level	2.65	24%	4%	44%

*Instructional approach promoted through the EMC3 project

Teacher Descriptions of Effective Differentiation

Participating teachers (n = 94) responded to the following open-ended item post-program questionnaire prompt: *Think of a time in which you feel like you effectively catered to students of different performance levels in your mathematics classroom. Describe the lesson in as much detail as possible, including the structure of the lesson, the tasks and activities, your role as a teacher and what your students were doing.* Teacher responses were detailed, varied and extensive. From a mathematical content perspective, 69% of teachers specifically described a number lesson, 14% as a measurement lesson, and 9% as a geometry lesson. Some responses did not provide details for a particular content area but explained strategies for differentiation (9%). The majority of responses referred explicitly to lessons that were part of the EMC^3 project resources. Although all teachers implicitly or explicitly referred to using something that could be construed as an enabling and/or extending prompt to support their effectively differentiated lesson, three other notable themes that supported teachers to effectively differentiate instruction emerged: *the role of the teacher* (64%); *provisions to establish student agency* (53%); and *opportunities for peer learning* (48%).

The role of the teacher. The comments teachers made about their role in differentiated instruction reflect the active nature of teaching when supporting effective differentiation. To ensure learning remained student-centred, teachers described the ongoing adaptations and pedagogical actions deployed both in planning and during lessons to meet student learning needs. Many of these teacher actions to support individual student learning needs paradoxically occurred at a whole class level. For example, teachers described how they made sure the task was set within a familiar context to support all students in comprehending the task and accessing the mathematics more readily. Other adaptations that supported all students included sharing students' work and prompting class discussions. Many teachers acknowledged that the use of open, prompting questions was helpful in initiating mathematically focussed discussions around the task. This included both general questions posed at a whole class level, as well as specific questions to target individual students.

Provisions to establish student agency. Teachers reported the different ways that students were afforded agency in how they approached the task, represented their thinking, and organised their solutions. The use of concrete materials, visual representations and recording templates were frequently mentioned as intentionally provided to support students in making choices as to how they communicated their thinking in meaningful ways. A different perspective on student agency encompassed comments that referred to class norms and consistent expectations that students persist when solving challenging tasks. Student friendly phrases such as “sweaty brain time” indicated a shared expectation that students needed to think for themselves and be willing to work hard to make sense of the mathematics.

Opportunities for peer learning. The data reflected widespread recognition of the role of peer learning when supporting differentiated instruction. Although reference to small groups and paired work featured throughout, orchestrating opportunities for class discussions were the more prevalent examples of peer learning. One frequently mentioned strategy was the use of “spotlights” to present student work for collective discussion during the explore phase of the lesson. Creating opportunities to share student work highlights how teachers draw on specific examples of student thinking to scaffold learning for the rest of the class. Sharing alternative strategies can support other students in considering alternative solutions, maintaining motivation, and/or consolidating learning.

Summary

Teachers viewed the pedagogical approaches emphasised through the EMC^3 project resources, particularly tasks differentiated through enabling and extending prompts, as more effective for differentiating instruction than other approaches. Moreover, teachers viewed effective differentiation in mathematics as being supported by several factors including: providing students with enabling and extending prompts; the teacher being in an active role during the lesson, facilitating adaptations to meet learning needs; providing opportunities for students to exercise agency; and adopting structures to support peer learning.

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Changing Teacher Practices: A "Slow Burn" or Rapid with "Big Shifts"

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This paper focuses on the time teachers take to adapt to a student-centred inquiry approach to teaching mathematics, the nature of those changes and the reasons for variations in both these aspects among teachers within and across schools. Five numeracy experts, who worked with 200 Foundation to Year 2 teachers to facilitate the implementation of the approach, were individually interviewed at the end of the year to obtain their perspectives on teachers' adaptation. Thematic analysis of the data revealed two overarching themes—context and teacher agency. The findings reinforce recommendations that professional learning providers acknowledge and take account of individual teacher learning trajectories to maximise potential change in practices.

Most experts in the field of teacher professional learning consider achieving significant change to teachers' practices to be a difficult and slow process (e.g., Guskey, 2003). However, there are examples constantly emerging of teachers who have made profound changes to their beliefs and practices in relatively short periods of time (e.g., Bobis et al., 2016). Those studying the reasons for the differential impact of professional learning interventions on teachers have taken different perspectives. For instance, Huberman (1993) and Brunetti and Marston (2018) identified numerous phases or career stages that teachers characteristically experience as part of their professional development journeys. Huberman also developed several models representing the potential reasons for variations in teacher developmental trajectories. Similarly, Gregoire (2003) proposed a model of teacher conceptual change accounting for affective and cognitive teacher characteristics that could potentially determine variations in teacher readiness to change their beliefs and practices. Together, these studies reveal commonalities of teacher learning and suggest potential reasons for variations in the time individual teachers take and the degree to which change occurs. In particular, the findings highlight the importance of context (physical and social environment) and teacher agency (their sense of empowerment and willingness to act) to help facilitate change. Brunetti and Marston (2018) explain that the influence of context on teacher development needs to be studied in terms of both *space* (e.g., school resources and leadership) and *time* (e.g., teacher prior learning). Importantly, a connection between context and teacher agency exists. In exploring this connection, Beauchamp and Thomas (2009) suggested that a strong sense of teacher agency has the power to "transform the context" (p. 183). This means that even in the face of contextual constraints, teachers with a strong sense of agency can still experience rapid and big shifts in their approaches to teaching.

The focus of this paper is on the time teachers take to adapt to a student-centred inquiry approach to teaching mathematics, the nature of those changes and the reasons for variations in both these aspects across schools and among teachers within schools. An understanding of such development is important to school and system leaders, and designers of professional learning (PL) to ensure they appropriately respond to the professional needs of teachers.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 20–23. Launceston: MERGA.

Background to the Study and Setting

This study was conducted as part of a large, funded research project, *Exploring Mathematical Sequences of Connected, Cumulative and Challenging Tasks (EMC^3).* This project involved working with approximately 200 Foundation to Year 2 (F-2) teachers from 19 different schools across two Australian states of New South Wales and Victoria each year for three years in a PL intervention. Following PL days conducted at the start of each school year, teachers implemented multiple sequences of challenging tasks utilising a student-centred inquiry approach that is described in detail by Sullivan et al. (2020) and outlined in Sullivan and McCormick (Paper 1) as part of this symposium. To support implementation of the approach, one school system participating in the project, engaged five system-level numeracy content-specific experts referred to as Teaching Educators (TEs), to assist individual schools and teachers. The nature of their support is detailed in Downton et al. (Paper 4) as part of this symposium, but in brief, TEs assisted individuals and teams of teachers to plan and teach the sequences. They also regularly observed lessons and facilitated post-lesson debriefing sessions where teacher practices and student responses were unpacked. TEs are highly experienced primary teachers and leaders with a mathematical subject and pedagogic expertise beyond the norm of primary classroom teachers. TEs received additional PL from the research team and remained involved in the project implementation for at least three years. Every year of the project implementation, each TE was allocated between two and four school teams of F-2 teachers (approx. 3-16 teachers per school depending on the school size) to assist in their implementation of the sequences and the associated EMC^3 teaching approach.

The research questions were: (1) What were the most notable changes to teacher practices resulting from their involvement in the project? (2) Why did teachers vary in the degree of change and the time it took to adapt to a student-centred inquiry approach?

Method

Participants and Data Collection Process

Participants in this study were five TEs. Working closely with teams of teachers to plan and implement the project in classrooms, they were ideally placed to comment on teachers' adaptation to the new practices inherent in the approach. At the end of the first year of the project's implementation, each TE was individually interviewed for approximately one hour to gain their perspectives on the project's strengths and shortcomings. One aspect of this semi-structured interview focused on changes to teacher practices that the TEs perceived. For example, TEs were asked to comment on practices teachers used to launch challenging tasks, to elicit student thinking and conduct class discussions as they supported student learning. They were also asked to: (a) describe how these practices had changed over the time of the project's implementation; and (b) provide their perspective on the reasons for variations in teacher adaption to the approach. All interviews were audio recorded and transcribed for later analysis.

Data Analysis

Interview data were analysed thematically using an adaptation of Braun and Clarke's (2006) approach. A process of reading for familiarity, followed by coding using both deductive and inductive means before identifying themes that helped capture the notable features of the data that were considered most relevant to addressing the research questions. For instance, we approached coding the interviews knowing that we were interested in the reasons for variations in time and intensity of teacher adaption to the approach advocated during the PL component of the project. However, we did not know which practices or aspects of the approach teachers would find more challenging to adapt to and why.

Results

In this section, we briefly identify the most notable changes to teacher practices that TEs perceived took place during the project. We then identify the themes and sub-themes that emerged from the analysis of interview data that helped to explain the variations in the degree of change and in the amount of time individual teachers or teams of teachers took to adapt to the EMC^3 approach. Pseudonyms are used when reporting TEs responses.

All five of the TEs identified "the biggest shift … has been the pedagogy of launch, explore, summarise and holding back from telling" [Athena]. Elise estimated that "80 per cent of the teachers I am working with are launching without telling or at least trying" and Athena thought that "75% of them are on board and doing a great job with the launch … holding back from telling." Although the biggest shift in practice had been teaching "without telling," Diane felt that it was "different for all the teachers" and that "the launch phase" without telling "was still a challenge for some." Nancy considered that even though the teachers at one of her schools were already familiar with the lesson structure, the "not telling" was new.

> They've changed their practice in terms of less teacher talk … and not doing too much telling. [Nancy]

Thematic analysis of the data revealed two overarching themes in the TEs' explanations for the variation in time teachers took to adapt to the EMC^3 approach—context and teacher agency. Context in space and in time emerged as important reasons why the process of adaptation was "a slow burn" [Athena] or rapid with "big shifts" [Elise] in teacher practices.

Context in space. Variation in the availability of support from their numeracy leadership team members was regularly highlighted as a reason why some teachers found it easier to adapt in their school spaces than others.

> They plan their program together; so, there's three teachers in each space. They've got two class teachers and one diversity support teacher, plus the numeracy leader plus the instructional leader; so, it's many heads, they are very focused. But at School R, which just has the instructional leader, there was several staff who didn't have much buy-in. there's been a bit of a staff turnover…Their numeracy leader only has one day [a week], so she's trying really hard to catch them up to speed. [Athena]

In terms of physical resources, Megan considered that "it's been really helpful to have the tasks there so they can focus" on the pedagogical approach. However, she realised that sometimes the classroom space itself acted as a constraint for teachers to adapt to new practices when "the room can be noisy, … it's a shared space."

Context in time. Regardless of the rate and extent to which schools and teachers adapted to the approach, the five TEs agreed that individual teachers commenced the project at "different starting points" [Nancy] in terms of their knowledge and practices. Prior PL associated with challenging tasks meant that some teachers could "go deeper with the maths" [Megan] from the start. The variation in teacher readiness to adapt to certain aspects of the teaching approach meant that TEs had to be flexible in how they worked with individuals and groups of teachers.

> At School M they were already on this trajectory of deepening teachers' understanding about math tasks. They had done a lot of professional learning around mathematics already … At my other school, staff who didn't have much buy-in had missed out on the professional learning. [Athena]

> At my first school … the kids are used to talking about the maths and used to explaining their thinking and turning and talking to their learning partner. My other school, we're not at that point yet. We tend to give them some little props sentence starters to get the kids talking … but they've gotten better. [Nancy]

Teacher agency. Teacher agency was characterised by examples of teacher resilience to work (or not work) hard in the face of challenge. Elise remarked that at one of her "slow burn" schools, the teachers showed little agency at the beginning as they "thought all of the tasks … were too challenging" and "there was no encouragement to try" for many months. Most teachers, however, were described by TEs as "really working hard" [Megan, Athena and Elise] despite the challenges of adapting to a new teaching approach.

Agency was also exemplified by increases in teacher efficacy. Teachers perceived to be slower in adapting to the approach were described by TEs as initially "hesitant" or "afraid … of the challenge." However, "in the doing there has become believing" and growth in the belief that they "could make the approach work." [Athena]

> … it's been a bit of a slow burn for them. But they are on board; they are positive, and they are thinking that they're doing a good job. [Elise]

Discussion and Conclusion

In accordance with prior research (e.g., Brunetti & Marston, 2018; Beauchamp & Thomas, 2009), our findings show that context (in space and in time) and teacher agency were central to explaining variations in the time teachers take to adapt to change. Teachers with a strong sense of agency were perceived by TEs to be more willing to work hard and try new practices despite contextual constraints. As the results of this study show, the important point is that teachers must be active in the process of professional learning for any form of change to occur. The findings reinforce recommendations initially expressed by Huberman (1993) and reinforced by Brunetti and Marston that providers of PL need to acknowledge and take account of individual teacher trajectories of learning (context in time) to maximise the potential of teachers adapting to new approaches. In the current study, TEs could adapt to the nature and extent of support individual teachers required. An understanding of school contexts and a sense of individual teacher agency are important to school and system leaders, and to designers of PL to ensure they appropriately respond to the professional needs of teachers.

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The Nature of Leadership and Other Support that Facilitate Innovation and Improvement in Teacher Practice

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In the context of a professional learning research project we investigated the nature of support offered to classroom teachers and school mathematics leaders to facilitate teachers' implementation of sequences of challenging tasks. End of year questionnaire data were collected from 70 Foundation to Year 2 teachers and ten numeracy leaders who participated in the project. Thematic analysis was used to analyse the open response questionnaire items. Findings reveal that two forms of support were helpful: in-class support, such as co-teaching, observation, followed by co-debriefing; and facilitated planning prior to instruction.

It is widely acknowledged that professional learning assists teachers to implement innovative pedagogical practices to enhance the teaching and learning of mathematics and ultimately improve students' learning outcomes (Akiba & Liang, 2016; Bobis et al., 2005). For professional learning to be effective there needs to be a bridge between research and classroom practice (Kretlow et al., 2012). Such a bridge is provided by external coaches or experts associated with an education system, who are educators with specialist expertise. These external experts are considered critical to the effective implementation of the new learning in schools (e.g., Cobb et al., 2018; Timperley et. al., 2007). They work in partnership with school leadership teams and their role includes collaborative professional support, mentoring school-based mathematics leaders, in-classroom instructional support, and leading professional learning within schools (e.g., Cobb et al., 2018). Critical to their work is the engagement of teachers in dialogue about the mathematical content, pedagogical practices and student learning (Campbell & Griffin, 2017). In this paper, these specialised coaches are referred to as Teaching Educators (TEs). These educators possess content-specific expertise and are employed by a school system to support school-based leaders and teachers to improve the quality of teaching and learning of mathematics.

Studies have highlighted the important role school-based mathematics leaders play in supporting teachers to implement new practices as part of a professional learning project (e.g., Sexton & Downton, 2014). Unlike the external coaches, these school-based leaders often have classroom responsibilities as well as their leadership responsibilities outside the classroom (Wenner & Campbell, 2017). Support by both the coaches and school-based numeracy leaders is provided alongside the daily work of teachers in the classroom, and is characterised by a cycle of planning, practice and reflection (Bruce et al., 2010). These studies highlight the importance of having both external and school-based support for teachers as they implement new learning.

The study reported in this symposium paper is part of the *Exploring Mathematical Sequences of Connected, Cumulative and Challenging Tasks* (EMC^3) funded research project (Sullivan et al., 2020). Details of the project are provided in Sullivan and McCormick as part of this symposium (Paper 1). The focus of this study was the nature of leadership support provided by TEs and school-based leaders to classroom teachers to facilitate the implementation of innovative practices. The research question was:

What leadership supports are provided for classroom teachers during the implementation of innovative pedagogical practices involving challenging mathematics tasks?

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 24–27. Launceston: MERGA.

Method

Participants in the study were Foundation to Year 2 (F–2) teachers involved in the EMC[3] professional learning (PL) program during 2019 (n = 70); five Lead Numeracy teachers (LNT); and five TEs employed by the school system who worked in project schools to support the LNT and teachers implement the new learning in the classroom. Some schools had an Instructional Leader (IL) who supported the LNT and classroom teachers. TEs followed up the main PL delivered by the researchers at the start of the year by facilitating PL in project schools at a point of need; implementing the co-teaching cycle (co-planning, co-teaching and co-debriefing) with LNT and classroom teachers, and observing and acting as a "guide on the side" (Morrison, 2014) for both LNT and teachers. At the second PL day in November participants completed a questionnaire that included open response questions focused on how they were supported in the classroom and in planning. We adapted Braun and Clarke's (2006) thematic approach to analyse the open response items. Responses were collated, then categorised according to themes that emerged from the data. Where participants had written multiple ideas in one response, each was categorised and coded.

Results

Seven main themes emerged from the analysis of the two open response items—one relating to how teachers were supported in the classroom and the other to how they were supported in planning. The results are presented in Tables 1 and 2 respectively.

Table 1
Themes Relating to Classroom Support Frequency of Responses and Illustrative Quotes

Theme	Illustrative quotes
1. Team teaching or co-teaching (n = 38)	TE taught in-situ with us and gave us an opportunity to reflect on our teaching and work with us to know where to take the students with their learning TE visited the classroom and co-taught with me so I could learn some new and different questioning techniques and enabling & extending prompts.
2. Providing feedback (n = 22)	[TE] feedback/feed forward time afterwards allowed me to get a better picture of the students' success and where to next.
3. Pre and post lesson discussions (n = 20)	Meeting with both [TE and LNT] to talk out the lessons, and time to prepare lessons was always helpful. We were given time to plan with our TE and lead numeracy teacher.
4. Modelling lessons (including reflections) (n = 19)	TEs modelled how the lesson structure should be like in the classroom. [TE] was also happy to run some parts of lessons (especially the reflections), which enhanced my learning and student learning.
5. Leaders helped them feel comfortable and supported (n = 8)	TE gave me confidence to not explicitly teacher rather let students explore. Both the TE and LNT were AMAZING support during the sequences TE supported me so much as a new LNT - I couldn't have done it without her!
6. Assisted with the reflection stage of the lesson (n = 8)	Our TE demonstrating reflection time throughout the lesson to see how probing questions facilitate the learning of the students. Suggestions given by [the TE] about students thinking to capture and share with the class as well as ways to reflect at the end of the session.
7. Observations by leaders (n = 4)	TE - visited weekly to observe a sequence in action. LNT and TE would come into the room and observe.

Note: TE (Teaching Educator), LNT (Lead Numeracy Teacher).

Table 2

Themes Relating to Planning Support, Frequency of Responses and Illustrative Quotes

Theme	Illustrative quotes
1 Planning (*n* = 58)	The most effective support from [TE] was when we were able to plan together prior to the lesson and anticipate the possible problems or modifications TE also helped with planning of where the students were at and where to begin our sequences as we didn't want to begin where kinder would start etc.
2. Professional learning (*n* = 10)	As someone who hadn't completed the beginning sequences courses it was really great having a release day to sit with the TE and IL [Instructional Leader] to go through tasks. Our TE was also there for our PL and assisted us to notice the maths content of tasks. We went through the tasks with [TE] and tried them out for ourselves. This helped us anticipate possible answer we would of received from the students.
3. In-class support (*n* = 10)	TE would often come into the classroom and work closely with me as a teacher to help see what students' needs were and how best to support them. TE was able to model what this looked like and how to use Talk Moves for more student talk rather than teacher talk.
4. Time (*n* = 16)	Leadership provided us time to plan together as a Stage and with our TE who guided us. Sequence planning time with TE and LNT allowed us to plan for the week.
5. Resources (*n* = 4)	The school were fantastic at giving us …. resources to use during the sequences. Resources were sourced, organised in preparation to teach the sequence by the IL.
6. Data analysis (*n* = 4)	TE and LNT answered all of my questions, in particular to tracking student's development in number through a variety of tasks. Met with the IL and TE to …plan the next steps using evidence of data collected.
7. Feel supported (*n* = 2)	[The TE] was advocating for me as a part-time teacher. We all felt very supported and comfortable to ask our TE questions. It was incredible to have [TE] with us as well. She assisted us to drive the learning.

While some overlap was evident in the data across the two tables, the themes that emerged from the analysis revealed the specific nature of the support provided by TEs in the classroom and in planning. Within the classroom, co-teaching, modelling of the lesson structure, and how to orchestrate the reflection part of a lesson featured prominently in the teachers' responses. The value teachers placed on the feedback they received and suggestions TEs had to progress students' learning, indicate that these teachers respected their advice and were committed to embracing this new pedagogical approach.

TE support with the planning prior to the enactment in the classroom was also recognised as being an important factor in the implementation. The teachers realised the benefit of engaging with the task before instruction, anticipating how the students might respond and the types of prompts that they could employ during the lesson. Some teachers also recognised the planning support as a form of ongoing professional learning for them. Time was a consideration in planning, post-lesson reflection and analysis of the data. Some teachers commented that leadership recognised the need to maximise the learning opportunities when the TEs were in the school and provided additional release time.

Theme 5 (Table 1) and theme 7 (Table 2) highlight the affective aspect of the support the teachers received. Teachers' felt comfortable with the TEs and were very supported when exploring this new learning. Comments related to these and other themes suggest the rapport the TEs had developed with the teachers was critical to the effectiveness of the implementation. Acknowledgment must be given to the support offered by the school-based leaders—LNT and IL who provided ongoing support on a daily basis.

Conclusion

These results highlight the nature of the leadership support that the teachers found beneficial when implementing this innovative pedagogical approach, in particular the support offered by the TEs. Three findings are evident from these results. First, that the support of an external knowledgeable other (TEs), who has an understanding of the philosophy underpinning the project and of the pedagogical approach was essential when expecting teachers to embrace new learning. Second, schools needed to factor in additional time for collaborative planning, debriefing and reflecting on the new learning with TEs and school-based leaders (LNTs). Third, teachers valued and respected the expertise of the TEs and developed a rapport and positive working relationship with them. Building mutual respect and trust was a contributing factor to teachers' willingness to embrace the new learning.

These findings resonate with earlier research related to the importance of an external expert with specialised expertise who works in unison with school leadership teams to support the implementation of new learning (e.g., Cobb et al., 2018; Kretlow et al., 2012; Timperley et. al., 2007). A key difference is that the TEs have a long-standing relationship with the school leadership team, LNTs and ILs had an understanding of school contexts, so they are considered a "guide on the side" (Morrison, 2014) to teachers and a critical friend or mentor to school-based leaders, rather than an expert who provides additional support from time to time. We acknowledge the following limitations of the study. First, the results reflect a small sample, which is not generalisable to the whole population. Second, these results present the responses of teachers only. Future papers will report the TEs perspective.

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Supporting the Leadership of Mathematics in Schools

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This MERGA symposium addresses three aspects of the Numeracy Suite professional development program for leaders of mathematics in schools. The papers include: a description of online courses offered in the program and an analysis of their effectiveness, a report of action research projects conducted by leaders as short “teaching sprints”, and an analysis of leaders’ thinking about their role in improving mathematical outcomes for students stimulated by a one-day workshop.

The Numeracy Suite (2019–2022) was initiated by the Department of Education and Training in Victoria and implemented though the former Bastow Institute of Leadership now the Victorian Academy of Learning and Teaching. A team of mathematics educators from Monash University developed and delivered the program, which was designed to facilitate the professional learning of leaders of mathematics and numeracy in primary and secondary schools in Victoria. To establish leaders perceived professional development needs, a state-wide survey was conducted online, and the leaders’ responses were analysed to inform the program design. The purpose of the Numeracy Suite was to challenge numeracy and mathematics leaders to develop a deeper understanding of themselves as leaders and teachers of mathematics and numeracy. The Numeracy Suite supported the leaders to create conditions for effective teacher professional learning and strategic planning for whole-school improvement in mathematics teaching and learning. It also supported the leaders to improve the learning experiences, mathematical dispositions, and achievement of all learners. In analysing the results of the professional learning our purpose was to understand the current practices, views and aspirations of leaders of mathematics and numeracy in primary and secondary schools and to evaluate the professional learning opportunities we offered to the leaders.

Chair & Discussant: Jill Cheeseman

Paper 1: *Online Courses for Leaders of Mathematics and Numeracy in Primary and Secondary Schools: Overview and Effectiveness*
[Ann Gervasoni, Aylie Davidson, Ann Downton, A., Sharyn Livy, & James Russo]

Paper 2: *Teaching Sprints: Action Research Led by School Mathematics Teacher Leaders*
[Colleen Vale & Carmel Delahunty]

Paper 3: *Ways in Which a Workshop Stimulated Leaders’ Thinking*
[Jill Cheeseman, Penelope Kalogeropoulos, Marj Horne, & Michele Klooger]

Acknowledgement. The Numeracy Suite discussed in each of the three symposium papers was funded by the former Bastow Institute of Educational Leadership, Department of Education and Training, Victoria.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 28-40. Launceston: MERGA.

Online Courses for Leaders of Mathematics and Numeracy in Primary and Secondary Schools: Overview and Effectiveness

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Professional learning for school mathematics leaders is a key aspect of the Victorian Government's strategy for improving mathematics for Victorian students. This is because middle leaders in schools play a vital role in designing and leading school improvement. As part of the Numeracy Suite, four online courses were designed in 2020 to support the professional learning of mathematics leaders. The courses were implemented and evaluated across 2020-2021. The evaluation showed that each online course was effective in meeting the professional learning needs of primary and secondary mathematics leaders.

As part of a five-year initiative to improve the mathematics learning of students in the State of Victoria, the former Institute of Educational Leadership launched the *Numeracy Suite* (https://www.academy.vic.gov.au/initiatives/numeracy-suite) to build the capacity of mathematics leaders in primary and secondary schools. Monash University academics were awarded the contract to design and deliver four 15-week online courses as part of the *Numeracy Suite*. This paper provides an overview of the four courses and insights from the course evaluations about their effectiveness.

Development and Overview of the Online Courses

Prior to developing the online courses, the Monash University team conducted a Needs Analysis survey of Victorian mathematics leaders in 2019 to inform the design and focus of the online courses (Vale et al., 2020, 2021). Two items addressed leaders' professional learning needs. Question 7 invited leaders to select four priorities for their *mathematics leadership professional learning* from a list of nine topics. Five topics were selected by approximately half of all participants (n = 196). These were:

1. Facilitating effective mathematics planning (60%).
2. Leading teacher professional learning in mathematics/numeracy teaching (56%).
3. Encouraging staff to take risks and trial different teaching strategies and tasks (53%).
4. Supporting, mentoring and coaching colleagues (50%).
5. Enhancing positive dispositions of students and teachers (49%).

These five topics were selected by leaders in both primary and secondary schools, regardless of their location or region, and were consistent with previous research about the challenges middle leaders face in leading change (Grootenboer, 2018).

The mathematics leaders were also invited to select four priorities for their professional learning in mathematics teaching practice from a list of ten topics (Question 8). Five professional learning topics were selected by about half of all leaders. These were:

1. Effective assessment of content, proficiencies, and dispositions (54%).
2. Differentiating instruction to cater for the needs of all students (53%).
3. Using strategies to improve student proficiency in understanding, fluency, problem solving, or reasoning (53%).

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4. Using data, including artefacts and work samples, to measure learning growth over time (49%); and
5. Including student voice and providing opportunities for students to negotiate their learning (47%).

Informed by these identified priority areas and the current mathematics and leadership literature, four online courses for mathematics leaders were developed. Each 15-week course was organised into four inquiry cycles so that content and leadership approaches could be adequately explored, trialled, and critiqued. Each inquiry cycle included a virtual workshop, optional online synchronous discussion groups, asynchronous learning activities and professional readings via the Bastow 307 learning management system (LMS), a weekly school-based investigation, and a weekly online discussion post to share insights about the school-based investigation. In learning cycle four, participants undertook a project relevant to their leadership context. Participants were invited to complete mid and end-of-course evaluations that inform course improvement. Outlines of the four online courses (OCs) follow.

Online Course 1: Leading Differentiated Teaching in Mathematics

Effectively differentiating learning for students with diverse abilities, backgrounds, and performance levels is a challenging aspect of teaching mathematics. This course enables school mathematics leaders to explore and critique several inclusive pedagogical approaches that cater for diverse students. Leaders focus on how attending to specific learning design characteristics when developing (or sourcing) tasks enables the whole class to undertake the same core mathematical activity, at a level of challenge, appropriate for each student.

Online Course 2: Leading Mathematics Planning

Collaborative planning is a critical part of the learning and teaching cycle. In this course, leaders explore key features of planning that underpin and enhance student-centred learning. Course content explores planning documentation that focuses on student-centred learning, embedding professional reading to support teachers' mathematical knowledge for teaching, and a model that guides leaders through the complexities of mathematics planning. Participants examine their school's planning approaches and develop a plan to lead teachers towards placing student-centred learning at the forefront of mathematics planning decisions.

Online Course 3: Leading Student-centred Assessment in Mathematics

Assessment is often viewed and practiced as a periodic externally imposed event or as an individual teacher-conducted activity that interrupts instruction—both practices treat assessment as something completed by students. The course aims to assist mathematics leaders to understand assessment practices and ensure that assessment is an integral part of instruction. Overall, the content focus of the course is to investigate how collaborative assessment practices can provide new "eyes" for understanding learners' mathematical thinking and dispositions, thereby guiding more effective teaching responses. A range of assessment strategies are explored and analysed across both cognitive and affective domains.

Online Course 4: Leading Improvement in Mathematics Teaching

Improving mathematics teachers' knowledge, confidence, attitudes, dispositions, and mindsets are important goals for professional learning. This course enables mathematics leaders to explore and critique approaches to leading professional learning in mathematics/numeracy teaching, and for supporting teachers to take risks and trial different teaching strategies and tasks. Participants use protocols and approaches to collect and analyse data with their teachers in order to trial and enact evidence-based teaching practice in classrooms.

Model for School-Based Professional Learning/Improvement Cycles

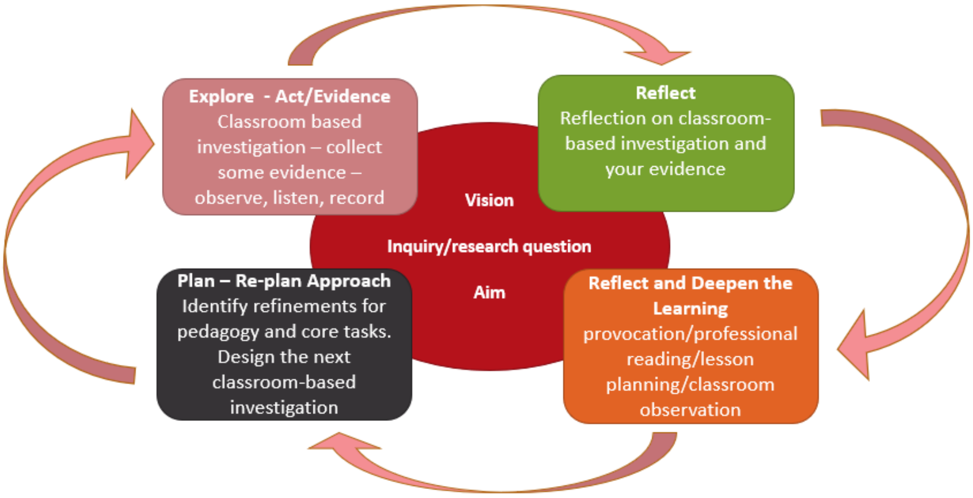

Figure 1. Model for school-based professional learning/improvement cycles.

The overarching goal of the Numeracy Suite is building leadership capacity to create and shape the conditions for whole school improvement in mathematics and numeracy learning and teaching. Thus, the online courses need to prepare leaders to enact school-based professional learning cycles/improvement cycles in partnership with colleagues (e.g., see Grootenboer, 2018). Figure 1 shows the model for school-based professional learning/ improvement cycles developed for the online courses to guide leaders in this endeavour. This model is informed by critical participatory action research (Kemmis et al., 2014). Course activities and LMS content support the model and provide material for leaders to use when leading school teams.

Effectiveness of the Online Courses

Participants in each of the Numeracy Suite online courses were invited to complete online mid-course and end-of-course evaluations that consider the knowledge and skills gained, the effectiveness of the course design, learning environments, facilitation, structure, and discussion groups, and participants' views of their overall experience of the course, including the most positive aspects, and aspects that could be improved. The evaluation included a mix of 5-point Likert-scale items, and open response questions. Mean-responses were calculated for each Likert-scale item, and the open responses were examined to identify key themes, using a grounded theory approach (Charmaz, 2008). Analysis of data from Semester 2 2021 end-of-course evaluation was used to provide illustrative examples of the course evaluation findings. Of interest for this paper was whether courses were viewed by mathematics leaders as effective, aspects of courses that were viewed positively, and aspects that might be improved. These findings can inform the design and content of other online courses.

Overall, the evaluation findings for Semester 2 2021 provide strong endorsement of the relevance and quality of the online courses. The mean responses (n = 62) for the 5-point Likert-scale items for the 6 evaluation categories, averaged across the four courses, were: knowledge gained (4.5); skills developed (4.4); online learning environment (4.1); virtual workshop facilitation (4.7); course design (4.6); and course structure (4.0). These positive results were amplified by nearly 100% of respondents across all courses indicating that they would recommend their course to colleagues and were satisfied with the quality of the course.

Positive aspects of the courses identified by participants in the two open-response questions included course content, opportunities for discussions and collaborations with other mathematics leaders, course design, facilitators, and the readings and resources. For example, one participant in OC2 commented, "*I found the structure of the course great, it built on skills and knowledge each week and prepared you well for the final project."*

Participants also indicated that they valued the practical nature of the *tasks* and being able to put into practice many of the leadership strategies about which they were learning. For example, "*I have adapted our assessment schedule to fit the new learning"* (OC3) and "*The Planning Model allowed me to lead my team through improving our current planning practices. I really valued the Enabling and Planning prompts to support learning success for all students*" (OC2). Participants also highlighted the opportunities to work with and learn from other leaders as one of the most positive aspects of the course. For example, "*talking with other Numeracy Leaders"* (OC4) and *"also enjoyed the numerous opportunities to interact with staff from other schools to gain new perspective and ideas"* (OC1). Participants valued the facilitators' expertise as one of the most effective aspects of the course, consistent with literature indicating that access to experts is important for leading and sustaining change in mathematics (Clarke, 1997; Goos et al., 2018). For example, *"[The facilitators] were able to provide us with on-the-spot resources based on the discussions we were involved in"* (OC4), and *"Facilitators were engaging and extremely knowledgeable"* (OC2).

Suggestions for course improvements varied across the four courses. Common themes related to difficulties with workshops being scheduled after school hours and competing workload expectations, clarity of course requirements, challenges with using the Bastow 307 LMS, and one request for more specific secondary content.

Conclusion

Overall, the findings provide confidence about the quality and positive benefits of the Numeracy Suite online courses for leaders of mathematics. It was clear that participation in the courses was having positive impact and assisting mathematics leaders to create conditions for effective teacher professional learning and strategic planning for whole-school improvement in mathematics teaching and learning. However, many of the mathematics leaders had little time during school hours to support their professional learning, or to implement their initiatives. (Vale et al., 2020). These time constraints limit the potential impact of leaders' work.

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Teaching Sprints: Action Research Led by School Mathematics Teacher Leaders

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Action research is a means for teachers and researchers to develop evidence-based practices. This paper reports the process and outcomes of *teaching sprints*, an approach to action research, conducted by secondary school mathematics leaders as part of a professional learning program. Mathematics leaders consistently reported the value of developing collaborative practices throughout the planning, enacting and reflection of the teaching sprint.

The roles of school mathematics leaders are varied and depend on the school and individual context (Driscoll, 2017; Grootenboer et al., 2015). Kemmis et al. (2014) described mathematics leaders as middle leaders, whose responsibilities sit between the classroom and the school principal. They are often engaged in complex interactions with students, teachers, and the school leadership team. Middle leaders are likely to have the greatest impact on student achievement when they focus their role on improving teacher practice (Robinson et al., 2008; York-Barr & Duke 2004). Grootenboer et al. (2020) reported action-orientated professional learning where middle leaders worked collegially with small teams of colleagues in an "… ongoing and sustainable way to develop educational practice collaboratively in response to local needs and conditions based on evidence. It is a way of developing pedagogy and curriculum *from the classroom out*" (p. 39). They did not, however, provide examples of mathematics leaders' action-oriented projects. In this paper, we report on a qualitative study of *teaching sprints* (Breakspear & Jones, 2020), that is, short, targeted action research projects conducted by secondary school mathematics leaders as part of an online professional learning course entitled, *Leading Mathematics for Improvement in Teaching and Learning*.

Action research is a form of practitioner research. Kemmis and McTaggert (1988) described it as both a process and practice used by teachers, often collaborating with other teachers that involves a cycle of planning, observing, reflecting, revising the plan, and continuing the spiral of investigation. More recently, Kemmis (2008) defined action research as "a practice that … transforms the sayings, doings and relating that compose those other practices" (p. 463). The sayings (what is said), doings (activities and work) and relating (ways of relating or interacting) of mathematics leaders are part of the framework of "practice architectures" of middle leadership (Kemmis et al., 2014, p. 31).

Investigating teaching practices to improve student learning is promoted by the Department of Education and Training in Victoria (2010); however, there is no specific advice for leading action research within schools. Breakspear and Jones (2020) proposed three phases for action research: prepare, teaching sprint, and review. In the prepare phase, they emphasised collaborating with the teaching team to identify the focus of practice for improvement. McNiff (2010) recommended this phase should identify a question for investigation, and the gathering and collaborative analysis of data. Findings and implications of the data analysis are used to identify a goal for changing practice that they then enact as a "teaching sprint." The teaching sprint is enacted in a short period of time, such as 2–3 weeks. Further data, including observations, are collected and used in the final phase of review to reflect on the effectiveness of the teaching sprint and determine the implications for future practice. In this paper, we report on a qualitative study of teaching sprints conducted by secondary mathematics leaders (MLs) to identify the influence of these teaching sprints on the sayings, doings and relating of the MLs and their understanding of evidence-based practice.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 33–36. Launceston: MERGA.

The Study

The *Leading Mathematics for Improvement in Teaching and Learning* course was designed for primary and secondary mathematics leaders (MLs). It was conducted over 15 weeks and involved five cycles, including an online virtual workshop and school investigations for each cycle. The themes for each cycle were: (1) The role of mathematics leaders; (2) Developing trusting relationships; (3) Effective practices in mathematics professional learning; (4) Enacting an action research cycle–Teaching Sprint. Having conducted other school-based activities to learn about their teachers and students and to trial leading various professional learning activities in their school, the final cycle involved the leaders completing a co-constructed action research project over 4 weeks with the teacher(s). This involved: choosing an aspect of teaching mathematics (Week 10); formulating a question and collecting data about their question (Week 11); co-constructing implication statements from the data analysis (Week 12); designing and conducting a teaching sprint around one implication statement (Weeks 13 & 14); sharing the teaching sprint with the group and critiquing a colleague's teaching sprint (Week 14).

Both primary and secondary mathematics leaders participated in the Leading Improvement in Mathematics Teaching course in 2020 or 2021. Fifteen secondary MLs and 45 primary MLs completed the reports for their teaching sprint. For this paper we collected the written reports of the teaching sprints that the secondary MLs shared with other participants in Week 14 and conducted a qualitative analysis of these teaching sprint reports. These secondary MLs were from metropolitan, regional, and rural schools. Thematic analysis (Bryman, 2016) of these reports was organised according to the sayings, doings, and relating (Kemmis et al., 2014) that occurred during each stage of their Teaching Sprint. Pseudonyms are used when quoting from the teaching sprint reports of the secondary MLs.

Findings

Focus of the Action Research

Sayings. There were a range of foci, or areas of practice to make sense of and improve identified in the initial step. These included: student engagement, student achievement, teachers' pedagogical content knowledge, problem solving, reasoning, student disposition, and differentiated learning.

> I had noticed in my year 10 students were eager to learn ... but really struggled to explain their thinking ... I had also ... heard other staff's frustrations at student's poor results on our tiered ALTS (Assessed Learning Tasks) [with] three exit points … (Bec)

> … Can we improve our students' disposition to Maths? (Chris)

> The class teacher is primary trained and finding it difficult to manage the Year 8 class and to explain mathematical concepts to the students … Year 8 students ... were disengaged and behaviour was poor. (Faye)

> How do we assess student understanding throughout a lesson? (Indira)

> … recent data suggests that many of our students are "cruising" .... How can we change our practice to enhance every student's opportunities to achieve at least one year's worth of growth in a year? (Narelle)

Relating. When analysing their reports, we found that all but four of the secondary MLs identified the focus for the teaching sprint without consulting their staff. These four MLs used a team meeting or meeting with one other teacher to identify the focus.

Data Analysis and Planning the Teaching Sprint

Doings. MLs reported using a range of data to analyse, identify, and set the goal for their teaching sprint. The data that the MLs collected and analysed included NAPLAN (https://www.nap.edu.au/) and other assessment data, formal and informal surveys of students or teachers, feedback from students, interviews of teachers, observation of lessons and teacher team meetings, which were used to discuss the focus issue.

> We conducted a Learning Design walk. Whilst the teacher explained .... We observed when the students talked to peers, looked around the room, or opened games on their laptops and calculated an approximate time that they were engaged. (Andy)

> In one of my PLCs, I placed the word "mathematical thinking" onto a Padlet and asked staff to write down their thoughts on how we were currently approaching teaching this and how they thought our students were at doing it. (Bec)

> Staff Opinion Survey shows that 47% of staff are not confident in using data to inform practice. (Jackie)

> I grabbed these [NAPLAN] questions [with low scores] and presented these to a small team of Year 8 teachers. We discussed the features of these questions to see if there were any commonalities. (Narelle)

At each school the teachers gathered, and analysed data collected during the teaching sprint.

> Throughout the sprint teachers collected anecdotal evidence from their classes and I observed some classes. (Bec)

> We surveyed students before and after the 'teaching sprint' to determine the students' dispositions to Maths. (Chris)

> Students were given the same survey post the mathematics experience as a means of assessing their "soft skill" development. Teacher observation of the development of student's team working skills also formed part of this assessment. (Faye)

> We developed a range of tasks that involved some form of reasoning …. Finally, the process of moderation would be used to develop our ability to make consistent judgements on progress and growth. (Narelle)

Relating. In the majority of the cases, the MLs collected and analysed the data. They then held a team meeting to analyse or discuss the findings of the data analysis. In almost all cases the teaching sprint was co-planned by the MLs with the other teacher(s) at that the year level(s) to be involved.

Reflecting on the Teaching Sprint

Sayings. The MLs reported on the mathematics teachers' new understandings of their students, pedagogical practices such as strategies for developing a growth mindset or student responsibility and engagement, planning to address student learning needs and teacher questioning.

> … with us continually modelling mathematical thinking but by the end of the two weeks cycle, we had most students being able to explain why they thought something didn't belong .... Mathematical thinking is something that the team is now seeing as important and something that we need to explicitly teach. (Bec)

> … both the teacher and the Learning Specialist noted that students were more willing to work in their teams and were more willing to persist when challenges arose .... The classroom teacher was challenged … with the questioning needed to direct student thinking .... (Faye)

> All staff have access to PAT-M Data and know how to interpret Group reports …. Maths teachers can identify misconceptions and address these. (Jackie)

> The moderation process allowed us to share ideas as to what we were looking for in the work to represent each level on the rubric. This … also gave us the opportunity to think about what specific skills, ideas, and concepts we should focus on with our students. (Narelle)

Relating. Following the teaching sprint, the MLs reflected on their relationship with colleagues and their collaborative practices:

> Collection of data and sharing of data was super important at getting the team on board to change practice. It is important that I value all of the team's opinions and that I listen and reflect on their opinions. (Bec)
>
> … I need to encourage and remind teachers to develop these skills in students. (Chris)
>
> Year 8 Mathematics team meetings will focus on developing the teacher's capacity to plan and deliver rich tasks. (Faye)
>
> We wanted to celebrate the growth that had been achieved in this area. (Narelle)

When reflecting on the teaching sprint, some of the MLs explicitly identified the value of continuing to promote and provide opportunity to collaborate, collect and analyse various data, plan lessons, and reflect on student students' proficiencies and engagement. Other leaders commented that they need to lead the professional learning of their colleagues.

Conclusion

The teaching sprints provided MLs with a collaboration and consultation process that supported them to relate with teams of teachers to explore teaching practices to improve student learning, engagement, and dispositions. Whilst MLs attempted to keep the focus small, their reports showed that they tackled significant curriculum and pedagogical challenges. Similar to that noted by Grootenboer et al. (2020), the small-scale action research projects (teaching sprints) reported in this paper provided the MLs with evidence of practices that were effective for their students and worthy of both celebrating and continuing.

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Ways in Which a Workshop Stimulated Leaders' Thinking

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Leading mathematics teaching and learning in schools is a complex job that requires the development of specialised knowledge and skills. Opportunities to learn in professional workshops can help to build leadership knowledge and skills. The results presented here describe 262 leaders' prompted reflections at the end of a professional development workshop. Leaders thought deeply about their role in transforming school mathematics. They considered the context of their schools, what was important, and what they valued in mathematics. Leaders thought about the tasks and pedagogies teachers select and the impact teachers' choices have on students' learning. Most of all, leaders reflected on ways that they could act to inspire changes that would lead to improved mathematical outcomes for students. In addition to the reflections of leaders, we present features of the workshop that stimulated leaders' thinking.

There is an increasing appreciation by leaders in educational sectors that the role of mathematics leaders in schools is important in improving student learning outcomes (Department of Education and Training [DET], 2022). To maximise gains by students, leaders of mathematics teachers need to develop both their capacity to lead their colleagues, and their pedagogical content knowledge (Driscoll, 2021). One model of professional learning designed with this purpose in mind is reported here. The professional development program was structured as a single whole day format. A single one-off professional development is usually not recommended (DET, 2005). Yet, in reality, one day of replacement of senior staff is all that is available and feasible in many schools. The challenge was to design a workshop that had the potential to act as a valuable stimulus for thinking for school mathematics leaders and require them to carefully consider ways to improve mathematics teaching and learning in their schools.

The Numeracy/Mathematics Leader Area Workshop 2021 was delivered to 386 participants as 14 single day professional development events. It was designed as part of the Numeracy Suite to lead a cultural shift in thinking about mathematics, develop shared values and passion for mathematics, build confidence of teachers, and positive dispositions for all learners (DET, 2021). It was delivered as an online workshop as the COVID-19 pandemic at the time prevented teachers meeting face-to-face. Every attempt was made to deliver the workshop with Clarke's (1994) inspiring principles of effective professional development in mind. The features of the workshop included:

- pre-workshop tasks to collect interview data from teachers and students to be used in discussions about positive dispositions towards mathematics,
- collaborative small "breakout" groups,
- information sessions connecting research-based theoretical perspectives to the lived classroom experience of the participants, and
- reflection by participants on their thinking at the end of the day.

The participants were all leaders of mathematics in their schools. Each workshop was designed to focus on a specific geographical area of the state to enable participants to network and to build knowledge of their local schools. Participants were allocated to either primary or secondary school groups when collaborating in small groups to enable them to share their expert knowledge about leading the improvement of mathematics learning.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 37–40. Launceston: MERGA.

Method

The data reported here are the participants' (n = 262) end-of-day reflections in response to the prompt: "Today I thought quite deeply about …." Although the responses were written in the online chat box and were visible to others, there was little evidence of repetition of ideas in the data. We contend that the responses provided insights into the meaning the participants made of the workshop.

Findings

Data collected from the online chat were uploaded and coded in NVivo software. As a first pass of the text a word cloud (Figure 1) was produced to look at an overview of the participants' thinking. As can be seen from the central words in the largest font size, students' learning of mathematics together with tasks and teachers in schools were the main foci of the reflections. These words accurately described the main emphases of the workshop which centred on improving students' learning of mathematics, the importance of teachers' productive pedagogies and the student engagement generated by well-selected tasks. The next "ring" of words reflects the content of the workshop day—specifically including dispositions towards mathematics, and staff and student collaboration around mathematics to build engagement and improve thinking and reasoning. Although this representation was affirming in terms of the purpose of the workshop, it offered limited insight into participants' thinking.

Figure 1. Word frequency in the data represented as a Word Cloud in NVivo.

Each written response was then coded using decisions made in the context of this study (Elliot, 2018). Where a leader mentioned more than one thing they had been thinking deeply about, a second categorisation of the response was recorded. For example, "how to develop mathematical thinking in the students at school and how to engender a positive attitude to mathematics at my school". This response was coded *improving mathematical thinking* and *creating positive dispositions.* Therefore, as can be seen in Table 1, there were 371 ideas from 262 respondents. The categories of response revealed the leaders were thinking deeply about; querying their personal leadership, creating positive dispositions and motivating aspirational thinking of their teachers, improving the selection of tasks for better teaching of mathematics, focussing on effective mathematics pedagogies, especially those eliciting students' thinking, reasoning and learning, working as a team with resources for teaching and professional learning, and prioritising mathematics and thinking about the future.

Table 1
Frequency of categories of response

Category	Frequency (%) (*n* =371)
Querying personal leadership	89 (24%)
Creating positive dispositions and motivation aspirations of teachers	76 (20%)
Task for better teaching of mathematics	43 (12%)
Teaching pedagogy	42 (11%)
Working as a team with resources for teaching and learning	40 (11%)
Teachers discussing students' thinking and learning	36 (10%)
Developing students' mathematical reasoning	23 (6%)
Prioritising mathematics and thinking about the future	22 (6%)

The reflections of the leaders are illustrated by several quotes. Many leaders (*n* = 89) queried how they could implement ideas, for example, "How I can best support my teachers to try something new in mathematics—try a challenging, open-ended task and to enjoy teaching mathematics?" Leaders (*n* = 40) reflected on their teams saying, for example, "Today I thought quite deeply about how we are working as individuals and not as a team—and how we should share student thinking [and take] the next step of learning with each other." Many leaders reported their deep consideration of the importance of positive dispositions (*n* = 76) writing comments such as,

> Today I thought deeply about how our school can come together more regularly to foster a more positive disposition. To provide opportunities as a team of mathematics teachers to encourage one another and support one another. To take risks in the classroom.

Table 2
Personal Leadership Sub-categories of Response to "Today I thought quite deeply about…"

Sub-category	Frequency (%)	Illustrative examples
Questioning action needed by them	33 (37%)	How to deliver learning for peers that drives forward their appreciation for mathematics.
Intended actions described	17 (19%)	Working with teachers to start more discussions around students' mindsets.
Personal behaviours required	15 (17%)	Listening more to both teachers and students to understand their perceptions and dispositions and to act upon that information.
What is important/valued	12 (13%)	What success in mathematics looks like, skills and dispositions we value.
Reflecting of school issues	12 (13%)	How we can make changes at our school to encourage teachers to grow their content knowledge, trust their judgements and explore other avenues of assessment. Also, how we can change teachers' mindsets away from 'hating mathematics' themselves and passing those feelings onto students.

The most frequent category of responses termed, Querying Personal Leadership (24%) dealt with applying workshop ideas to the leader's personal setting. The finding echoes Jackson and her colleagues' statement (2015) that leaders needed to apply professional learning to the reorganisation of school practices (Table 1).

Sub-categories of personal leadership considerations were defined (Patton, 2002) to determine what leaders considered important (see Table 2). Almost one quarter of responses (24%) involved participants questioning how to act on ideas raised in the workshop. However, a further 19% of leaders had made up their minds about how to act and stated their intentions as mathematics leaders in their schools. Some leaders (17%) reflected on the personal behaviours they would adopt. Other leaders (13%) thought deeply about what was important in their schools. A further 13 percent of leaders considered broad school issues of leadership. Examples of each sub-category are found in Table 2.

Conclusion

We found that mathematics leaders were stimulated to think deeply about their role in transforming school mathematics during one day of professional development. Leaders considered the context of their schools, what was important and what they valued in mathematics. Further, leaders thought about the tasks and pedagogies teachers use and their impact on students' learning. Most of all, they reflected on ways that they could act to inspire changes that would lead to improved mathematical outcomes.

While there is explicit advice that one day professional development workshops are ineffective (e.g., Campbell, 1997), we argue that it is not the duration of a professional development workshop that is critical (Adey, 2004). It appeared that the participants came to the workshop as producers of knowledge, not as consumers of knowledge. Setting participants' expectations by asking them to collect interview data from teachers and students to use in discussions about positive dispositions towards mathematics sends a strong message about valuing and using their knowledge. By encouraging collaborative small "breakout" groups the learning is personalised and shared and opportunities are made for future networking. Also, providing research-based information connects theoretical perspectives to participants' lived experience affirms their knowledge. Finally, requiring reflection by participants on their thinking at the end of the day gives participants time to consider how to use their learning to initiate changes in mathematics teaching and learning.

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Key Shifts in Thinking in the Development of Mathematical Reasoning

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This symposium will draw on the evidenced-based learning progressions for multiplicative thinking, algebraic reasoning, geometrical reasoning, and statistical reasoning presented at previous MERGA conferences (see references by symposium authors in the papers that follow). The four papers will consider key shifts in thinking identified within each progression, without which students' progress may be seriously constrained.

Paper 1: *A Disposition to Attend to Relationships: A Key Shift in the Development of Multiplicative Thinking*
[Dianne Siemon]

This paper draws on multiple data sources to better understand the shift from additive to multiplicative thinking, which is crucial to all further participation in school mathematics.

Paper 2: *Key Shifts in Students' Capacity to Generalise: A Fundamental Aspect of Algebraic Reasoning*
[Max Stephens, Lorraine Day, & Marj Horne]

This paper will elaborate five levels of algebraic generalisation and two key understandings based on an analysis of students' responses to RMFII algebraic reasoning tasks.

Paper 3: *Cognitive Flexibility and the Coordination of Multiple Information in Geometry and Measurement*
[Rebecca Seah & Marj Horne]

This paper analyses students' solutions to problems in geometry and measurement situations in order to identify key components needed to nurture reasoning.

Paper 4: *Facilitating the Shift to Higher-order Thinking in Statistics and Probability*
[Rosemary Callingham, Jane Watson, & Greg Oates]

Students have difficulty moving from concrete representations and procedural mathematical statistics to context-based appreciation of data. This paper examines the barriers to this shift to higher-order thinking based on the Statistical Reasoning Learning Progression.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), p. 41–57. Launceston: MERGA.

A Disposition to Attend to Relationships: A Key Shift in the Development of Multiplicative Thinking

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This paper draws on numerous data sources to better understand the shift from additive to multiplicative thinking in years 4 to 9. Research studies that have used the Scaffolding Numeracy in the Middle Years assessment tasks have found that while students can be supported to move through the early and upper zones of the Learning and Assessment Framework for multiplicative thinking, it has been difficult to move students through Zone 4 at the same rate. A closer examination of item responses at this level reveal that a disposition to notice and work with relationships between quantities may explain this phenomenon.

Access to multiplicative thinking has long been recognised as critical to success in school mathematics in the middle years and beyond (e.g., Harel & Confrey, 1994; Hilton et al., 2016; Lamon, 1993; Siemon et al., 2006). However, many students at this level do not have access to this critical capacity (Brown et al., 2010; Siemon, 2019) suggesting that the transition from additive to multiplicative thinking is more complex than previously recognised (e.g., Clark & Kamii, 1996; Van Dooren et al., 2010; Vergnaud, 1983).

Research studies that have used the *Scaffolding Numeracy in the Middle Years* (SNMY) assessment tasks have found that while students can be supported to move through the early and upper zones of the Learning and Assessment Framework (LAF) for multiplicative thinking (Siemon, 2016, 2019), this appears not to be the case for Zone 4, which is where students are starting to use multiplicative thinking on a more consistent basis (see Figure 1 for examples). This and the fact that the proportion of students in Zone 4 is typically higher than in any other zone confirms the difficulty of acquiring multiplicative thinking, but it also prompts the question, "What can be learnt about the barriers to multiplicative thinking from a closer analysis of student responses to tasks that span Zone 4?"

Solves more familiar multiplication and division problems involving two-digit numbers (e.g., *Butterfly House c* and *d, Packing Pots c, Speedy Snail a*).

Tend to rely on additive thinking, drawings and/or informal strategies to tackle problems involving larger numbers and/or decimals and less familiar situations (e.g., *Packing Pots d*, *Filling the Buses a* and *b*, *Tables & Chairs g* and *h*, *Butterfly House h* and *g, Speedy Snail c, Computer Game a, Stained Glass Windows a* and *b*). Tends not to explain their thinking or indicate working.

Able to partition given number or quantity into equal parts and describe part formally (e.g., *Pizza Party a* and *b*), and locate familiar fractions (e.g., *Missing Numbers a*).

Beginning to work with simple proportion, for example make a start, represent problem, but unable to complete successfully or justify their thinking (e.g., *How Far a*, *School Fair a* and *b*).

Figure 1. Rich text description of Zone 4 (Siemon et al., 2006).

Approach

The *Stained Glass Windows* task (Figure 2) was selected for analysis as the item difficulties ranged from Zone 3 to Zone 7 and the setting, while accessible, did not conform with the more familiar multiplicative models implicit in problems such as *Packing Pots* (i.e., equal groups or arrays). It was also selected because the context invited additive thinking, which tested the extent to which students could see past that to the underlying multiplicative structure (e.g., Vergnaud, 1983), which was hinted at in the task stem. These same criteria were met by another

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 42–45. Launceston: MERGA.

task, *Canteen Capers*, which involved lunch order options given two choices of rolls, four choices of filling, and three choices of drink. The first item required students to identify the number of options for a roll with a specified filling and a drink item (2 x 3). The second item required them to determine if everyone in a class of 26 children could have a different lunch order made up of a roll, filling, and drink. In both cases students were asked to explain their reasoning using as much mathematics as they could.

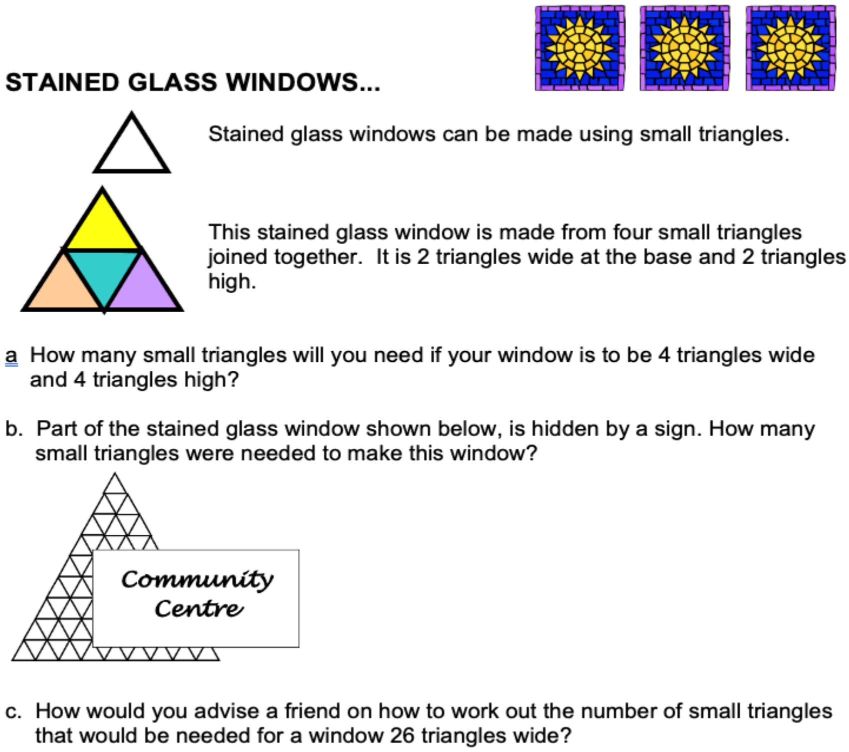

Figure 2. Stained Glass Windows task from SNMY Assessment Option 1 (Siemon et al., 2006).

Data sets from four different projects are used in the analysis reported here. That is, the SNMY project (Siemon et al., 2006a), the *Reframing Mathematical Futures Priority* project (Siemon, 2016), the *Reframing Mathematical Futures II* project (Siemon et al., 2018), and the *Growing Mathematically—Multiplicative Thinking* project (Callingham & Siemon, 2021). The student populations across the four projects ranged from Year 4 to Year 9 of whom approximately 65% were from low socio-economic backgrounds.

A total of 11,775 students (67% in Years 7 or 8) responded to the Stained-Glass Windows task and 4985 students (83% in Years 7 or 8) to the Canteen Capers task. Student responses were marked by project schoolteachers using partial credit scoring rubrics and entered into a deidentified spreadsheet which was forwarded to the research team for analysis.

Analysis and Discussion

Table 1 shows the proportion of students scoring a 1, 2, or 3 on items a, b, and c of the two tasks with the last entry for each item indicating the proportion of students providing a multiplicative response. The very low proportion of students evidencing either an additive or a multiplicative response to both problems is at odds with the suggestion that strategy usage is impacted by the numbers involved or the extent of the challenge (Downton & Sullivan, 2017; Larsson et al., 2017). It is undoubtedly the case that "some students use strategies that are only as complex as they need" (Downton & Sullivan, 2017, p. 303). However, the proportion of students providing a correct answer supported by additive reasoning (i.e., a score of 2 on items a and b of Stained Glass Windows and item a of Canteen Capers) is surprisingly low, given that the majority of the students were from Years 7 or 8.

Table 1
Proportion of Students Scoring a 1, 2, or 3 on Each Item of Each Task

	Stained Glass Windows (*n* = 11,775)			Canteen Capers (*n* = 4985)	
Score	A	b	c	a	B
1	22.9%	29.5%	11.3%	22.2%	29.8%
2	28.5%	13.8%	22.4%	22.6%	24.9%
3	13.7%	18.5%		17.8%	

An insight into why this might be the case is afforded by the item difficulties shown in Table 2 for the Stained Glass Windows task. On the ordered list of item difficulties produced by the Rasch analysis a score of 3 on item a (sgwa3) was located towards the top of the scale in Zone 7. However, the item difficulties associated with recognising and using the same relationship in items b and c (i.e., sgwb3 and sgwc2) were located in Zone 6, which suggests that noticing the rule is harder than applying the rule despite the strong suggestion of the rule in the stem (2 x 2) and the likelihood that 4 and 16 would be recognised as square numbers.

Table 2
Scoring Rubrics for Stained Glass Windows by Item Difficulty (LAF location)

Item	Rubric (item difficulty code)	Score	Zone
A	Incorrect based on inaccurate drawing and/or counting of triangles, or correct with little/no explanation (sgwa1)	1	3
	Correct (16 triangles), with evidence of additive reasoning based on drawing and counting (sgwa2)	2	4
	Correct (16 triangles), with evidence of multiplicative reasoning based on 4 x 4 (sgwa3)	3	7
B	Incorrect based on inaccurate drawing and/or counting of triangles, or correct (81 triangles) with little/no explanation (sgwb1)	1	3
	Correct (81 triangles), with evidence of additive reasoning based on drawing and counting, or inappropriate use of area formula (e.g., L x W) (sgwb2)	2	4
	Correct (81 triangles), with evidence of multiplicative reasoning based on pattern (e.g., 9 by 9) (sgwb3)	3	6
C	Advice based on additive thinking (e.g., “2 less each time you go up”) (sgwc1)	1	5
	Correct, advice based on rule (e.g., 26 x 26) (sgwc2)	2	6

A similar phenomenon is observed for the Canteen Capers task where the item difficulties ranged from Zone 2 to Zone 8. Recognising and providing a multiplicative explanation for part a (e.g., “It’s 6 because for each roll she could have one of the 3 drinks”) was located in Zone 8. For item b, determining that there were enough different options for each child in a class of 26 on a systematic basis that suggested use of 2 x 4 x 3, was located in Zone 6. Again, this suggests that noticing the relationship was harder than applying it.

There are a number of possible explanations for the difficulty of these items that warrant further investigation. One is the absence of a familiar multiplicative model, which is known to facilitate multiplicative understanding and calculation (Larsson et al., 2017). However, the fact that multiplicative thinking is elicited by these tasks despite this suggests that something more is needed to support the shift from additive to multiplicative thinking, particularly as models connected to solution strategies can invoke instrumental responses (Skemp, 1976) making it difficult to discern multiplicative thinking.

Apart from the obvious need to offer a broader range of multiplicative tasks and contexts that are not readily connected to students' existing models of multiplication (e.g., Downton & Sullivan, 2017), the analysis here suggests that the "something more" is a disposition to attend to relationships between quantities in ways that look for generalities rather than particulars. In other words, it is about an alertness to and appreciation of mathematical structure (e.g., Mason et al., 2009) and multiplicative structure in particular (e.g., Mulligan, 2002; Vergnaud, 1983).

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Key Shifts in Students' Capacity to Generalise: A Fundamental Aspect of Algebraic Reasoning

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This paper will elaborate five levels of algebraic generalisation based on an analysis of students' responses to Reframing Mathematical Futures II (RMFII) tasks designed to assess algebraic reasoning. The five levels of algebraic generalisation will be elaborated and illustrated using selected tasks from the RMFII study. The five levels will be matched against the eight zones identified in the RMFII study supported by its Rasch analysis. We identify two shifts where students' capacity to generalise appear difficult to navigate. The first being where students move from noticing and describing regularities to formalising these regularities into verbal or symbolic expressions. The second is where students use their understanding of equivalence based on relational thinking to write and recognise equivalent algebraic expressions.

Key ideas implicit in the idea of generalisation as they relate to the algebraic reasoning tasks of RMFII have been presented by authors such as Love (1986) and Mason (1996), who suggested that the generalisation of a pattern, at its core, rests on the capability of noticing something *general in the particular*. Kieran (2007), however, noted that this feature alone may not be sufficient to characterise the *algebraic generalisation* of patterns, arguing that, in addition to seeing the general in the particular, students need to be able to express their generalisation algebraically, drawing on *explicit* reasoning in terms of justification and explanation. These points are directly relevant to the tasks used by RMFII to assess algebraic reasoning in which students were invited to explain their reasoning. Kieran's ideas will feature clearly in the third, fourth and fifth levels of a progression for algebraic generalisation advanced in this paper.

These five levels were enumerated in a previous paper (Stephens et al., 2021). They are: Working with particular instances; Noticing and describing regularities and patterns; Forming expressions—either verbal or symbolic; Using equivalence to examine different expressions of the same relationships and expressions; and Explicit generalised reasoning where students move between the particular to the general and vice versa, are able to identify and describe what varies and what stays the same, and work confidently with generalised expressions including their representation in different forms.

The research in RMFII developed an effective evidence-based learning progression with associated tasks for students' algebraic reasoning (Day et al., 2017). Nearly all tasks are graduated (multi-part) and designed to elicit progressive levels of students' algebraic generalisation, which is a key element of algebraic reasoning. Assessment tasks of this kind are helpful for classroom teachers to focus on the key shifts in students' thinking in order to foster their capability in this area. This paper will firstly show how the existing RMFII tasks, supported by Rasch modelling, align with and illustrate our five-level categorisation of algebraic generalisation. Secondly, the paper will show teachers of mathematics in the middle school years the importance of having *all* students progress at least to the third level of algebraic reasoning.

Drawing on the Rasch modelling (Bond & Fox, 2015) that was used in RMFII to rank the task item difficulty of scored responses across eight zones of algebraic reasoning, the Learning Progression for Algebraic Reasoning (LPAR) is related to the five levels of algebraic

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generalisation. In our recent paper (Stephens et al., 2021), several of the RMFII tasks were used to illustrate and validate the five levels of algebraic generalisation and in this paper one task, the Relational Thinking task, is used to exemplify how the LPAR zones relate to the levels of generalisation (Table 1). The Relational Thinking task (ARELS) is comprised of seven task items (ARELS1-ARELS7). The coding in the right column refers to the task items enumerated in Table 1, and to the score obtained for that item. For example, in Table 2, ARELS4.3 refers to the fourth relational thinking task item for which a score of 3 has been obtained.

Table 1
Relational Thinking Task Items and Rubrics

Item no.	Task item	Task item rubric Score
ARELS1	What numbers would go in these boxes to make a true number sentence (the numbers may be different). Explain your reasoning. $\square + 521 = 527 + \square$	0 No response or irrelevant response 1 Incorrect response but suggest the difference of 6 is recognised in some way (e.g., *add 6 to the right hand side)* 2 Two correct numbers given (e.g., 13 and 7; 527 and 521) but little/no reasoning. 3 Two correct numbers given where the number on the left is 6 more than the number on the right (e.g., 100 and 94) with reasoning that reflects the relationship between 521 and 527 (difference of 6).
ARELS2	Find a different pair of numbers that would make the number sentence above true.	0 No response or irrelevant response 1 A different and correct pair.
ARELS3	Describe how you could find **all** possible pairs of numbers that would make this a true number sentence.	0 No response or irrelevant response 1 Incomplete attempt based on previous answers (e.g., *add 2 more to both*). 2 Statement regarding the difference of 6 (e.g., *number on the left must be six more than the number on the right)* or expression showing the difference (e.g., a + 6, and a)
ARELS4	What numbers would go in these boxes to make a true number sentence (the numbers may be different). $\square - 521 = \square - 527$ Explain how you worked it out.	0 No response or irrelevant response 1 Incorrect answer (possibly due to errors in calculation) but recognises relationship between 521 and 527 (difference of 6). 2 Two correct numbers given (e.g., 613 and 619) but little/no reasoning, may include some calculations. 3 A pair of correct numbers given where the number on the right is 6 more than the number on the left (e.g., 600 and 606) with reasoning that reflects the relationship between 521 and 527 (difference of 6).
ARELS5	Find another pair of numbers that would make the number sentence above true.	0 No response or irrelevant response 1 A different and correct pair.
ARELS6	Describe how you could find all possible pairs of numbers that would make this a true number sentence.	0 No response or irrelevant response 1 Incomplete attempt based on previous answers (e.g., *add 10 to both*). 2 Statement regarding the difference of 6 (e.g., *number on the right must be six more than the number on the left*) or an expression showing the difference (e.g., a and a + 6)
ARELS7	What can you say about the relationship between *c* and *d* in this equation? $c \times 2 = d \times 14$	0 No response or irrelevant response 1 Specific solution provided (e.g., *c must be 7 and d must be 1 to make it a true number sentence)* or a general statement *(*e.g., *c is bigger than d*) 2 Statement correctly describes relationship (e.g., *c is 7 times the number d)*

The first level of our classification of algebraic generalisation is working with particular instances, where students find solutions to simple equivalence situations or extending simple growing patterns. For example, in the Relational Thinking task the first part of the task asks students to find two numbers that make the number sentence true (ARELS1), and the second part of the task asks the students to identify a second pair of numbers that also make the statement true (ARELS2).

The second level of our classification of algebraic generalisation is noticing and describing regularities, where students are asked to notice regularities among a sequence of particular cases. In these cases, students attend to quantities that stay fixed and those that vary (Radford, 2006; Rivera, 2013) within the context of the task. This is an important level as the next three algebraic generalisation levels rely upon being able to notice regularities.

Forming expressions, either verbal or symbolic is the third level of algebraic generalisation, which extends the noticing of regularities to expressing these regularities, as constants and variables in formulae that may be articulated verbally or using symbolic language. To obtain all three marks for the ARELS1 task item, students have to provide two correct numbers as well as demonstrate reasoning that showed the difference of six relationship.

Establishing and using equivalence enables students to be able to recognise that generalisations may be represented by different symbolic expressions. Students should be able to show that different expressions can generate the same number where the same variables are used and/or algebraic simplification can be used to show equivalence. It is important for students to be able to distinguish situations where although two expressions may look different from each other, they are in fact equivalent.

The final level of our classification of algebraic generalisation is explicit generalised reasoning. This is where students can move flexibly between the particular and the general and vice versa. Students at this level can identify and describe variables and constants and work confidently with generalised expressions. The Relational Thinking task item (ARELS7) asked students to comment on the relationship between c and d in the equation c x 2 = d x 14. To answer this successfully, students need to understand the equivalent relationship between two product expressions, and to generalise a relationship explicitly between the two variables c and d, using appropriate mathematical language.

Table 2
RMFII Zones and Levels of Generalisation Reported in Stephens et al. (2021)

Item no.	RMFII Zone	Level of algebraic generalisation
ARELS1.1	Zone 1	Level 1: Working with particular instances.
ARELS1.2	Zone 2	Level 1: Working with particular instances.
ARELS1.3	Zone 6	Level 3: Forming expressions – verbally or symbolically.
ARELS2.1	Zone 3	Level 2: Noticing and describing regularities.
ARELS3.1	Zone 5	Level 2: Noticing and describing regularities.
ARELS3.2	Zone 6	Level 4: Using equivalence.
ARELS4.1	Zone 3	Level 2: Noticing and describing regularities.
ARELS4.2	Zone 4	Level 2: Noticing and describing regularities.
ARELS4.3	Zone 7	Level 4: Using equivalence.
ARELS5.1	Zone 5	Level 2: Noticing and describing regularities.
ARELS6.1	Zone 6	Level 3: Forming expressions – verbally or symbolically.
ARELS6.2	Zone 6	Level 4: Using equivalence.
ARELS7.1	Zone 5	Level 2: Noticing and describing regularities.
ARELS7.2	Zone 7	Level 5: Explicit generalised reasoning.

From the examination of the Relational Thinking task, coupled with the analysis of three other RMFII tasks (Stephens et al., 2021) where several responses were located in Zone 8, it appeared that two of the key shifts in students' ability to generalise are difficult for students to navigate. The first of these key shifts is where students move from Level 2 noticing and describing regularities to Level 3 where they formalise this noticing and describing to correctly form algebraic expressions, either verbally or symbolically. This is demonstrated by noticing and describing regularities appearing in Zones 3 and 4 and the beginning of Zone 5 in the LPAR (Table 2), while Level 3, which formalises this in verbal and symbolic algebraic expressions does not appear until Zone 6. The second of the key shifts, which students find difficult to negotiate, is moving from Level 3 to Level 4 drawing on students' understanding of equivalence based on relational thinking and the writing and recognition of equivalent algebraic expressions. This level is evident in Zones 5, 6 and 7 of the LPAR (Table 2).

As these two key shifts are somewhat problematic for students, it is important that teachers provide multiple opportunities for students to identify regularities, identify variables and constants, form and communicate expressions, and use equivalence. One way for teachers to do this is to utilise rich tasks, such as Garden Beds from maths300 (maths300.com), that provide opportunities for students to demonstrate all forms of generalisation. By using several rich tasks within different contexts, teachers can ensure that students are being exposed to these critical steppingstones in the generalisation process. The RMFII Teaching Advice (Day et al., 2018) includes references to rich tasks from well-known sources such as maths300, reSolve (resolve.edu.au) and nrich (nrich.maths.org) at each of the LPAR Zones, which provide teachers with tasks that will assist them to progress students in their algebraic learning journeys.

The algebraic generalisations exemplified in this paper require students to become proficient in using appropriate combinations of language, algebraic representation, and mathematical justification. These forms of reasoning and proof are applicable across many problem-solving situations and explicitly generalised algebraic reasoning will be necessary for students' continuing study of mathematics. Just as important, this paper has drawn attention to assisting all students to navigate successfully Levels 3 and 4 where they learn to form correct algebraic expressions either verbally or symbolically, and subsequently become able to recognise and work with equivalent expressions. Navigating these two key shifts appears essential for students to be able to reason algebraically.

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Cognitive Flexibility and the Coordination of Multiple Information in Geometry and Measurement

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Building from the evidence-based learning progression in geometric reasoning from the RMFII project, this paper presents data from students' solutions to three problems in geometry and measurement situations to identify key components needed to nurture reasoning. To show emerging analytical reasoning students must coordinate multiple pieces of information and demonstrate cognitive flexibility in their use of visualisation, diagrams, language, and symbols.

Understanding, fluency, problem-solving and reasoning are an integral part of becoming numerate. Good problem solvers exhibit cognitive flexibility, the ability to coordinate number skills, visual-spatial and other cognitive processes such as organising multiple pieces of information (Ionescu, 2012). Given the considerable difficulty Australian students face with solving problems and justifying their mathematical thinking (Thomson et al., 2017), we seek to identify key components needed to nurture reasoning. Geometric reasoning is the ability to critically analyse axiomatic properties, formulate logical arguments, identify new relationships, prove propositions, and used geometric knowledge in solving measurement problem situations (Seah & Horne, 2021b). A draft learning progression was developed based on Battista's (2007) exposition of Van Hiele levels of geometric thinking. Analysis of student data produced an evidenced based learning progression comprise of eight thinking zones: Zone 1: Pre-cognition; Zone 2: Recognition; Zone 3: Emerging informal reasoning; Zone 4: Informal and insufficient reasoning; Zone 5: Emerging analytical reasoning; Zone 6: Property-based analytical reasoning; Zone 7: Emerging deductive reasoning; Zone 8: Logical inference-based reasoning. We analyse student work in depth to determine how to nurture increasingly sophisticated reasoning from informal (Zone 3) through to emerging deductive reasoning (Zone 7).

Method

The data source used for this analysis is taken from the Reframing Mathematical Future II project. The results of these findings have been published elsewhere. Our aim here is to identify significant changes in student thinking by finding factors that cause a shift from Zone 3 to Zone 7. We do this by analysing students' responses to three tasks: 1) reasoning about nets (Seah & Horne, 2020), 2) making deductions of angle magnitudes (Seah & Horne, 2021a), and 3) enlarging a logo and determining its area (Seah & Horne, 2021b) (Figure 1). The geometric contexts of these tasks allow students to demonstrate their knowledge and understanding. The reasoning required for the net task is Zones 2, 3 and 5. The angle magnitudes task is Zones 2, 5, 6, and 8. The logo drawing task is Zone 3 and 5. The logo area task is Zones 4 and 7.

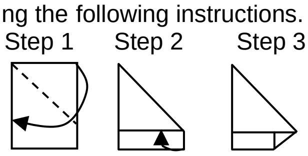
In designing the rubric, we determined that a zero score is given for no response or irrelevant responses. A '1' score denoted some recognition of the concepts but not full application. A maximum score would be given for a correct response with sound reasoning. Scores in between, the number of which depended on the complexity and the context of the task, would be given for partially correct answer and reasoning. For example, GCRD1 requires either a correct or incorrect enlargement logo drawn so the ceiling score is 2. Conversely, it was possible to get some of the angle magnitudes (GANG4) correct and give partial reasoning, thus requiring more gradation with a score of 4 being the ceiling. The data analysed came from students in 12 trial schools and 32 project schools.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 50–53. Launceston: MERGA.

GNET 4. Sam thinks he has drawn a net of a cube using six squares but it does not fold up to make a cube. What might Sam's drawing look like? Explain how you know.

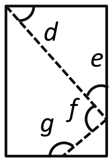
Geometric Angles 2

A four-sided shape is folded from a sheet of A4 paper using the following instructions.



a [GANG3]
What is the name of this shape?
____
Explain your reasoning.

b [GANG4]
Unfold the paper and find the size of each marked angle.
Angle *d* = ____________ Angle *e* = ____________
Angle *f* = ____________ Angle *g* = ____________



Explain your reasoning.

LOGO

A designer draws a triangular logo on grid paper. He wants to enlarge the logo so the sides are twice as long.

a. [GCRD1] Draw his enlarged logo on the graph.

b. [GCRD2]. Write the coordinates of the corners A', B', and C' of the new large triangle**:**

c. [GCRD3] If the area of the original logo is $2.25m^2$, what will the area of the new logo be? Explain how you know?

Figure 1. Sample of assessment tasks on geometric reasoning.

Findings

Overall Results

Students' responses to the tasks reflected not only their ability to reason, but the extent of the task requirement and the exposure they had with the concepts. As shown in Table 1, by the number of no responses and correct responses received, the GNET task was the easiest whereas GCRD3, which required students to find the area of the enlarged shape, was the hardest.

Table 1
Breakdown of Student Responses for Each of the Questions (percentage)

	GNET4		GANG4		GCRD1		GCRD3	
	Trial	Project	Trial	Project	Trial	Project	Trial	Project
Score	*n = 233*	*n = 566*	*n = 157*	*n = 270*	*n = 118*	*n = 328*	*n = 118*	*n = 328*
0	13.2	9.1	38.5	16.3	17.8	30.8	37.3	47
1	11.4	10.7	28.9	27.4	53.4	38.4	52.5	47
2	36.5	30.4	18.6	19.6	28.8	30.8	1.7	2.1
3	38.9	49.8	10.9	22.6			8.5	4
4			3.9	14.1				

In GNET4, 48% of the students used just the information in the question by either drawing six squares that would fold into a cube or drew a correct shape but did not provide a reason. In the trial data, 39% of students gave a correct response. This improved in the project data. Students who gave a correct reason went beyond the information given in the question and called on other knowledge, such as visualising the nets from different perspectives. Compared to GNET task, the number of no response or irrelevant responses was higher in the GANG4. Around 29% of the students showed partial angle knowledge by providing a label (e.g., acute, or obtuse) or recognising one angle magnitude; 19% showed emerging analytical reasoning giving two angles correctly, with some explanation; and 11% trial and 23% project students correctly calculated the angles giving some reasons though often sparce. Logical inference-based reasoning, albeit about a simple situation, was shown by 4% and 14% of trial and project students respectively who reasoned correctly and deduced all angle magnitudes.

In the logo task, 18% of the trial students did not draw an enlarged logo and 37% did not attempt to calculate the area. More than half of the students (53%) operated within the information given in the question by drawing a larger logo in some form although incorrectly either by enlarging one dimension only or a larger logo with no attention to the magnitude of the enlargement. A similar number (52%) gave a response to the area question that was incorrect, often just using the numbers given in the question by doubling 2.25 and did not provide units or gave little reasoning. Around 29% correctly enlarged the logo and just over 2% were able to give a correct area measurement, often using a procedural explanation. Just over 8% were able to reason correctly, giving an explanation that recognised that doubling the length of all the sides quadrupled the area, thus showing emerging deductive reasoning.

Types of Reasoning

Table 2 shows the responses to the three questions. The questions are shown with the score given following the dot so that GANG4.1 means a sore of 1 on the question GANG4.

Table 2
RMFII Zones of Geometric Thinking

Zone 2. Recognition	GNET4.1	GANG4.1		
Zone 3. Emerging informal reasoning	GNET4.2		GRD1.1	
Zone 4. Informal and insufficient reasoning			GRD3.1	
Zone 5. Emerging analytical reasoning	GNET4.3	GANG4.2	GRD1.2	
Zone 6. Property-based analytical reasoning		GANG4.3		
Zone 7. Emerging deductive reasoning			GRD3.2	GRD3.3
Zone 8. Logical inference-based reasoning		GANG4.4		

We can see that student responses to these three questions spread across the zones of reasoning. For GNET, the move to analytical reasoning appeared to occur with a response scored of 3. The two student responses in Figure 2 demonstrate this. Student A used recognition of a taught prototype. Student B used visualisation and then used a combination of diagram and language to explain the image in their mind and hence their reasoning.

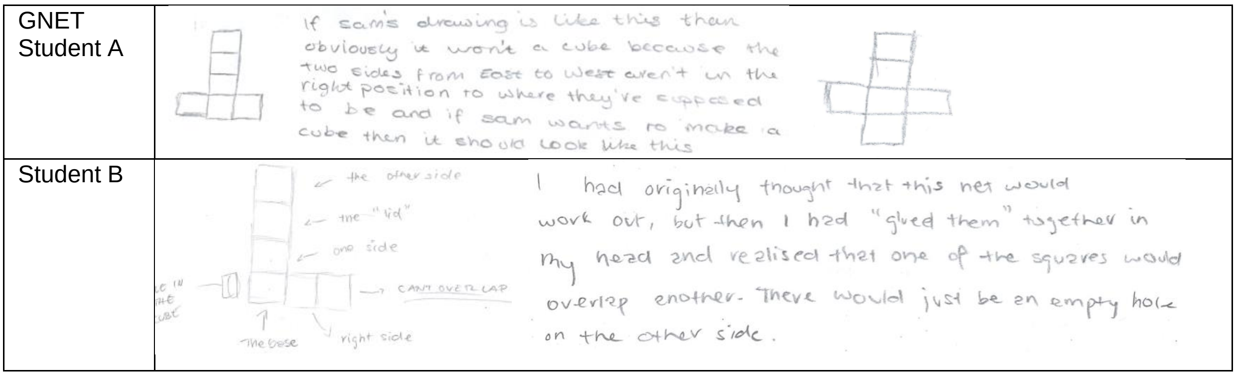


Figure 2. Students' responses on the GNET4 task.

In GANG4, analytical reasoning emerged with a response score of 2 where students gave partially correct answers (usually 45° with no explanation). Some students were starting to make connections but tended to explain using benchmarks such as 90°, as demonstrated here by student C who used no diagrams.

Student C: *d* and *e* has the same size angle as you can see, *f* as everyone knows that it is 90° because it’s a right angle and *g* is an obtuse, which is 180° (wrote 45°, 45°, 90°, 180°).

Limited ability to explain, use diagram effectively and present a sequential argument show clearly in the attempts of the students. The few students who were able to reason deductively justified 45° as half of the corner right angle and calculated the 135° either by using the interior angles or the straight angle with 45°. For GCRD3, student 10JW27701 shows an attempt to calculate area but is just using the numbers given in the question rather than demonstrating analytical reasoning in the solution (Figure 3). Meanwhile, student 10YL4700 demonstrates sound deductive reasoning showing explanations both algebraically and in words.

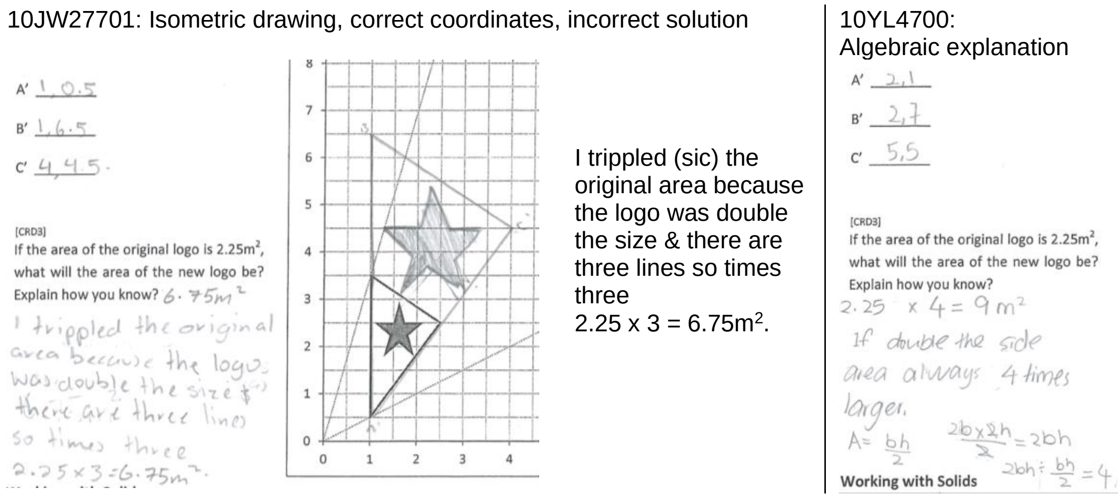


Figure 3. Students’ responses on the GCRD task.

To reason analytically or deductively, coordination between the information presented in the question with the network of one’s own conceptual understanding is needed. While knowing the mathematical concepts is important, the results here demonstrate that students needed to visualise the problem in situ, coordinate the information in the question with their prior knowledge to obtain a solution and present their argument using diagrams, language, and symbols flexibly. Finally, they need to be able to check that their reasoning is sound. In short, they need cognitive flexibility. These things need to be explicitly in the curriculum. At the moment, visualisation and the flexible use of communication tools is absent.

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Facilitating the Shift to Higher-order Thinking in Statistics and Probability

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It is increasingly recognised that to be informed citizens and to participate fully in the workforce requires an understanding of statistical data and risk. Such understanding is underpinned by statistical reasoning. It has been shown, however, that students have difficulty moving from concrete representations and procedural mathematical statistics to the context-based appreciation of data drawing on proportional reasoning that is becoming increasingly necessary. Based on the Statistical Reasoning Learning Progression (SRLP), this paper examines the barriers to shifting to higher-order thinking.

Introduction

As statistics and probability began to be acknowledged as a fundamental part of the mathematics curriculum towards the end of the 20th century (Australian Education Council [AEC], 1991), it became important to consider the new challenges for students in mastering this part of the curriculum. Although traditionally the other parts of the mathematics curriculum have claimed to have applications across other school subjects and outside of the classroom, two aspects of statistics and probability add even more potential to the application of the curriculum outside of the mathematics classroom: *uncertainty* and *context* (Callingham et al., 2021). At this point in time, the combination of uncertainty and context is seen starkly in society's experience of the COVID-19 pandemic (Watson & Callingham, 2020). The uncertainty associated with chance events and the confidence associated with decisions in contexts where statistics have been collected, is different from the rest of the mathematics curriculum, which is based on undisputed facts and proved theorems. Further, context is essential to any meaningful data that are collected (Cobb & Moore, 1997), and the entire statistical problem-solving process is based on anticipating, acknowledging, accounting for, and allowing for variability in these data (Bargagliotti et al., 2020). At each step in this process, particular skills and understandings need to be applied and combined to reach the answer to the statistical problem posed.

Students' progress in developing their statistical understanding and reasoning has been described in terms of an 8-zone Statistical Reasoning Learning Progression (SRLP) (Callingham et al., 2019). A question has arisen, however, as to why, as students progress through the middle school years (aged 11 to 16 years), many have difficulty moving to the highest zones in the learning progression but remain around the middle zones, particularly in Zone 4 (Callingham et al., 2019).

Approach

The Statistical Reasoning Learning Progression (SRLP) was developed during the Reframing Mathematical Futures (RMFII) project (Siemon et al., 2018). The SRLP describes an increasingly sophisticated hierarchy in which procedural mathematical statistics, such as calculation of an average or quantifying outcomes from a probability experiment, interact with an understanding of the context of the problem. In Zones 1 and 2, skills are limited to, for

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 54–57. Launceston: MERGA.

example, reading a value from a graph or offering an opinion about a context with no reference to data. At the higher levels (Zones 7 and 8), students call on proportional reasoning with data integrated with contextual understanding to make decisions and draw informal statistical inferences. Of particular interest here are the middle levels of the 8-zone hierarchy (See handout).

Students in Years 7 to 10 undertook a series of assessments based on statistical reasoning tasks. The student data reported here are taken from the third round of RMFII assessment, (Callingham et al., 2019) and have not been previously reported. Two tasks are used to exemplify the shifts observed in moving across zones, particularly in respect of the difficulties observed in moving up from of Zone 4: one based on probability (STATS) in the context of the interpretation and implications of winning Tattslotto; and the second based in a statistical context (STWN) with students contrasting two different graphical representations of the same data set to tell a story in the context of how long families have lived in a town. Both tasks are based in social contexts with which students are likely to be familiar. The abbreviated titles were used to identify tasks during the analysis and are used here for consistency. The tasks were marked by teachers based on the rubrics provided. The tasks and rubrics are shown in Figure 1.

Findings and Discussion

Table 1 presents the findings from a sample of 581 students in Years 7 to 9 (aged 13 to 15 years) who undertook at least four statistical reasoning tasks (not just the tasks reported here) during the third round (MR3) of assessment. Student responses were Rasch analysed, and the person measures used to determine the distribution of students across the zones.

Table 1
Number and Proportion of Students across SRLP Zones

	n	Zone 1 (%)	Zone 2 (%)	Zone 3 (%)	Zone 4 (%)	Zone 5 (%)	Zone 6 (%)	Zone 7 (%)	Zone 8 (%)
Yr 7	165	19 (11.52)	19 (11.52))	51 (30.91)	48 (29.09)	20 (12.12)	6 (3.64)	2 (1.21)	0 (0.00)
Yr 8	215	13 (6.05)	16 (7.44)	43 (20.00)	56 (26.05)	41 (19.07)	25 (11.63)	19 (8.84)	2 (0.93)
Yr 9	201	26 (12.94)	29 (14.43)	49 (24.38)	38 (18.91)	38 (18.91)	6 (2.99)	14 (6.97)	1 (0.50)
Total	581	58 (9.98)	64 (11.02)	143 (24.61)	142 (24.44)	99 (17.04)	37 (6.37)	35 (6.02)	3 (0.52)

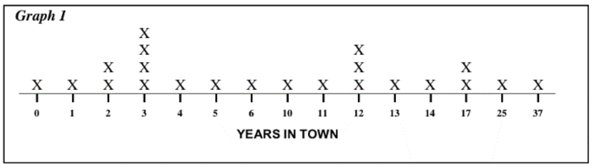
The proportion of students in each zone is very similar to that reported elsewhere (Callingham et al., 2019), and in previous similar studies (Callingham & Watson, 2017). It should be emphasised that this analysis is based on a new and different group of RMFII students, and that the nature of the analysis allows for skewed distributions and is not based on a normal distribution. The very similar patterns shown to previous analyses suggest that the sticking points in the middle zones are not environmental but related to cognitive development.

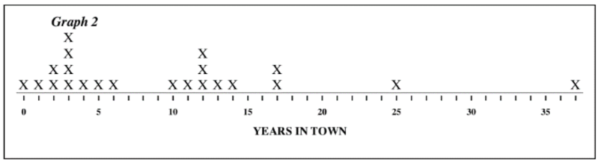
Shifts to Higher Zones

As shown in Figure 1, the rubrics reflect an increasing sophistication and quality of response and their position along the SRLP is based on the Rasch analysis. Across these two different tasks, to reach higher levels of response students need to bring together multiple aspects of reasoning.

STATS	One day Claire won Tattslotto with the numbers 1, 7, 13, 21, 22, 36. So she said she would always play the same group of numbers, because they were lucky. What do you think about this?
Code 1 **Zone 3**	Affirms a belief in being lucky (e.g., *I think it would be lucky I will pick the same number's too; I don't think many numbers are lucky. But I think 4, 7 & 9 are, so I guess I'd agree in a way you can have lucky numbers*).
Code 2 **Zone 4**	Rejects 'luck' (e.g., *There is no such thing as lucky numbers*) or states that numbers were unlikely to occur again, or less likely to occur than other numbers (e.g., *I think she shouldn't go for the group of numbers again because you can't get the same numbers after numbers, you always get different numbers all the time*).
Code 3 **Zone 6**	Implicitly recognises that all combinations of numbers have the same chance of occurring on any draw (e.g., *It was just a stroke of luck because any number could of come up; There is no such thing as a lucky number, things like Tattslotto are picked at random*).
Code 4 **Zone 7**	Explicit recognition that all numbers or combinations of numbers are equally likely, may/may not offer an opinion (e.g., *There is an equal chance for all combinations, but she's already won once, so why keep gambling, why not invest the money, you would get more out of it*).
Code 5 **Zone 8**	Reasoning that recognizes equal chance and interprets Claire's comments relative to context (e.g., *It is a good idea to use the same numbers all the time but there is as much chance as getting any other six numbers*).

A class of students recorded the number of years their families had lived in their town. Here are two graphs that students drew to tell the story.





STWN1: What can you tell by looking at Graph 1?
STWN2: What differences do you notice between Graph 1 and Graph 2?
STWN3: Which graph is better at presenting information and "telling the story"? Explain your answer.

	STWN1
Code1 **Zone 3**	Tautological response (e.g., *The numbers along the bottom tell you how many years; How long people lived in that town).*
Code 2 **Zone 5**	Response refers to one or more specific aspects (e.g., *3 and 12 have the most; 1 family had lived there 37 years, There are 22 kids).*
Code 3 **Zone 8**	Summative or comparative response that reflects some appreciation of information overall (e.g., *They range from all years; Not many families have stayed there for the same time*).
	STWN2
Code 1 **Zone 4**	Incorrect (e.g., *Less people live in the town in Graph 2 than Graph 1; There are more Xs in Graph 2*) or superficial comments related to the appearance of the graph (e.g., *Graph 2 is harder to read because numbers are together, Graph 1 is easier to read because numbers are spread out).*
Code 2 **Zone 5**	Some indication that difference recognised in terms of spread and accuracy (e.g., *Graph 2 goes up in fives and Graph 1 doesn't*).
Code 3 **Zone 7**	Acknowledges that graphs show the same data and describes the difference in terms of the scales used (e.g. *There is no difference from graph 1 to graph 2 except that graph 2 shows the spaces where graph 1 doesn't; graph 2 says all the years between 0 and 37 – while graph 1 only tells the relevant ones*).
	STWN3
Code 1 **Zone 3**	Statistically inappropriate choice (Graph 1) with reasoning that ignores spread (e.g., *Graph 1 because it only has the time it needs*)
Code 2 **Zone 5**	Statistically appropriate choice (Graph 2) with reasoning based on personal preferences (e.g., *Graph 2 because they have set it out better*) or indicates both the same (e.g., *Neither – they tell the same amount of information*).
Code 3 **Zone 7**	Statistically appropriate choice (Graph 2) with reasoning that recognises the importance of seeing all the years (e.g., *Graph 2 because you can see the difference between the years more clearly and the graph is more spaced out; Graph 2 because it has all the years*).

Figure 1. Exemplar items and rubrics.

In the probability item (STATS), it appears that developing the complex concept of random, and the necessity for appreciating the probability of groups of numbers occurring rather than single values, as appropriate for the context of the question, is important in moving responses to the higher zones. The emerging recognition of randomness and the application to groups of numbers is evident in the Code 2 (Zone 4) response but this loses coherence and falls back on individual numbers ("you always get different numbers all the time"). The Code 4 (Zone 7) response, however, is confident about working with groups of numbers but falls back on opinion ("but she's already won once so why keep gambling") to justify the thinking.

The other item (STWN), in its two-part structure (presenting two graphs and requiring a comparison rather than a single description), requires several components of the context, both visual and textual, to be integrated for a higher-level response. Students need to recognise the subtlety of the comparison needed between the graphs and to bring together understanding of the nature of the graphs and the context of the question to reach higher zones. That the lowest levels of the responses (Code 1) appear in Zones 3 and 4 rather than lower down the SRLP indicates that comparing two graphs creates some difficulty for students. The reasoning demonstrated to obtain a Code 1 is procedural, (e.g., Graph 1 is easier to read because numbers are spread out) focussing on aspects of the graph alone, rather than the information each graph conveys. To reach a Zone 7 response, students have to explicitly reason by integrating both the visual appearance of the graph and the nature of the information conveyed (e.g., Graph 2 says all the years between 0 and 37—while Graph 1 only tells the relevant ones).

Conclusion

It appears that coordinating different types of information and bringing together diverse aspects of mathematics and context are critical to shift reasoning to more sophisticated levels of response. This capacity to bring together two or more aspects of knowledge and understanding is important in other areas of mathematics, including in the shift from additive to multiplicative thinking. The inclusion of Statistics and Probability in the Mathematics Curriculum (AEC, 1991) has extended the appreciation of the structure of the multiple understandings required when data and context need to be combined rather than considered separately. It is appreciating this combination that moves reasoning to higher zones.

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RESEARCH PAPER

Comparative Effectiveness of Example-based Instruction and van Hiele Teaching Phases on Mathematics Learning

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This article explores the effectiveness of the pedagogical practices associated with the cognitive load theory and the van Hiele theory, which are two theories from cognitivism and constructivism perspectives, respectively. Following a quasi-experiment, the quantitative analysis of 157 high school students' responses to pre, post, and retention tests revealed that the students taught with the van Hiele teaching phases performed significantly better at the post and retention tests. While the cognitive load theory intervention bridges the gap between low and high ability students, the van Hiele teaching phases is beneficial to both low and high performing students. These results have implications for mathematics teaching practices and learning.

The concern for higher mathematics students' outcomes and the application of mathematical knowledge to real-world scenarios is increasing across the globe. While some 15-year-old students' performance in mathematics is below the average competency level at the Programme for International Student Assessment (PISA), others at an average level or above could not transfer mathematical knowledge to solve practical problems around them (Organisation for Economic Co-operation and Development, 2019). This is similar to reports from African countries, particularly Nigeria (Omobude, 2014). However, one of the main facilitators of students' learning outcomes is teachers' pedagogical approach (Bolstad, 2021; Li & Schoenfeld, 2019), which is the focus of this investigation.

There are numerous studies that have applied several pedagogies generated from different learning theories (Ginga & Zakariya, 2022; Schneider et al., 2022). Of interest to this study are the worked example instruction – a popular instructional design, following the principles of the cognitive load theory (CLT) (Sweller, 2011)—and the van Hiele teaching phases (VHTP) (van Hiele, 1986)—which align with the constructivist approach and an element of the van Hiele theory. While available empirical findings have reported that both the CLT worked example instruction and VHTP is effective (Centre for Education Statistics and Evaluation, 2017), no study, either empirical or theoretical, has compared the effectiveness of the approaches. As several studies around the world have indicated that students struggle to solve complex algebraic equations with rich knowledge of concepts and procedures (Johari & Shahrill, 2020), which means students cannot transfer the acquired mathematical knowledge to solve real-life problems (Bolstad, 2021; Li & Schoenfeld, 2019), this study explores the CLT worked example instruction and the VHTP to determine their effectiveness for solving complex mathematical problems using the example of simultaneous equations. A predictor of students' levels of mathematical understandings is their mathematical ability (Ayebale et al., 2020). Consequently, this study examines the influence of students' ability levels on the effectiveness of these pedagogies. Specifically, this article answers the following research questions:

1. *How do students' learning outcomes in the cognitive load group differ from the van Hiele group?*
2. *What are the effects of each of the cognitive load theory intervention and the van Hiele teaching phases across the three time-points?*
3. *Are there differences in the learning outcomes of students in the cognitive load group and van Hiele group based on ability levels?*

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 58–65. Launceston: MERGA.

Cognitive Load Theory

The CLT (Sweller, 2011) aims to improve mathematics and science teaching and learning by focusing on human cognitive architecture, which is characterised by a limited working memory and unlimited long-term memory. The working memory processes information while the long-term memory stores the processed information. CLT contends that for learning to occur, the cognitive resources required to learn a task must not exceed the available working memory resources (Sweller et al., 2019). Moreover, the working memory can only process four to five pieces of new information at a time, and such information may be missing if not properly rehearsed after 20 seconds (Miller, 1956). Based on the understanding of the principles for processing information in humans, CLT recommends several instructional designs that could facilitate effective learning. One popular instructional design that manages the working memory resources is worked example instruction. In this design, students are provided with worked examples to study and transfer their understanding to similarly structured problems. Several studies have established that the use of worked examples is effective because it imposes a relatively low cognitive load and does not interfere with learning (Ngu et al., 2019; Renkl, 2017; Richey & Nokes-Malach, 2013). However, these studies mainly focused on simple mathematical topics such as one-step and two-step equations. Moreover, most of the studies assessed students' responses based on the procedural steps leading to the final answer, without considering the quality of the students' responses. Furthermore, the long-term effect of this pedagogy has not been widely investigated, and no study has reported the effectiveness of the pedagogy in relation to students in Africa.

The van Hiele Theory

The van Hiele theory, formulated by Pierre van Hiele, originated from the difficulties students encountered in learning geometry. He proposed a developmental framework that requires teachers to understand how students' geometric thinking progresses in levels, known as the van Hiele levels of thinking. In a joint effort with his wife, Dina van Hiele, they prescribed five sequential teaching phases for developing students' cognitive reasoning through the levels, called the van Hiele teaching phases (VHTP) (van Hiele, 1986). This was based on their belief that students' cognitive progression from one level to the next is dependent on instruction rather than maturity and age. They claimed that learning from real-life scenarios enhances life-long learning, and they recommended student-centred activities for learning. The five teaching phases are information, directed orientation, explication, free orientation, and integration. Upon successful completion of these teaching phases, students' thinking is moved to the next level and the phases are repeated. The van Hiele teaching phases align with the constructivist perspective and emphasise that students construct their own mathematical knowledge in their own unique way by exploring the learning environment, seeking clarification, and developing initiatives for problem-solving (Serow et al., 2019). This pedagogy acknowledges the changing roles of teachers and students during learning and emphasises language development and building new knowledge on pre-existing information. Moreover, this pedagogical lens serves as a tool for guiding teachers in designing relevant activities for a lesson (for further information see van Hiele [1986] and Serow et al. [2019]).

As the van Hiele theory was formulated to improve performance in geometry, several studies across the world have reported the effectiveness of the phase-based pedagogy for geometry teaching, learning, and curriculum (Alex & Mammen, 2016; Machisi & Feza, 2021; Serow & Inglis, 2010). There is, however, need to transfer the lens of van Hiele theory to other areas of mathematics (Colignatus, 2014; Vojkuvkova, 2012). Since then, some attempts have been made to investigate the effectiveness of the van Hiele teaching phases in other aspects of mathematics, but results have been inconsistent (Nisawa, 2018; Walsh, 2015). Thus, there is the need for further research in this area.

Notably, both pedagogies considered in this paper rely on schema from other people to learn and emphasise the contribution of prior knowledge to learning. However, while VHTP stresses that students are to construct their knowledge by exploring their environment and developing crisis in thinking, CLT claims that these activities may overload students' working memory and thus result in no learning. Furthermore, unlike the VHTP, the CLT-associated pedagogy does not encourage social interaction such as peer discussion.

Method

This quasi-experimental design followed a pre-, intervention, post-, and retention test sequence, which involved two experimental groups—one for the CLT and the other for VHTP. Each group comprised one intact class of first-year senior school students (ages 14 to 15 years) from two government schools in Nigeria. A total of 157 male and female students was involved: CLT group *(n* = 72) and VHTP group (*n* = 85). The groups were equivalent in terms of mathematical content coverage, access to materials and human resources, English language competencies, and geographical locations. Due to the limitations in contact occasioned by the COVID-19 pandemic, the regular teachers of each group implemented the interventions after undergoing training from the researchers. The students completed three similar tests and were exposed to the interventions across eight weeks. Initially, students completed an open-ended pre-test to determine their current knowledge about solving simultaneous equations. The groups were then exposed to eight (40-minute) carefully sequenced lessons, with one group receiving the worked examples instruction and the other receiving the VHTP instruction. The students then completed a post-test. Three weeks after the post-test, a retention test was administered to the students to establish the lasting effects of the interventions. Students were required to solve the mathematical problems in the tests and provide an explanation for their responses. Generally, the study followed the research ethics standard and was approved by the University of New England (Approval number HE20-224). Rasch analysis was employed to ascertain the degree to which the data (items and persons) fit the model. The Rasch model is suitable because of its significant role in considering both items and persons as connected constructs, the acknowledgment of unequal intervals within the functioning of the items, and the non-assumption that all items are of equal difficulty (Bond & Fox, 2013). The model fitness is reported by four statistical parameters: outfit, infit, separation index, and reliability. According to Linacre (2013), infit and outfit values ideally range between 0.5 and 1.5. Hence, when the infit mean square estimate is close to 1, it indicates that the set of items and persons perfectly fit the Rasch model.

Scoring

Students' responses (procedural steps and explanation of the procedures) to the tests were classified into increasing levels of thinking and scored following the rubrics of the structure of the observed learning outcomes (SOLO) model (Biggs & Collis, 2014). SOLO is considered appropriate because it examines both the quality and quantity of students' responses in the evaluation process. Six levels of response were identified: prestructural = 0, unistructural = 1, multistructural = 2, relational = 3, formal mode 1 = 4, and formal mode 2 = 5.

Results

Table 1 presents the Rasch results. The item reliability (I) indices (> 0.9) across time indicate that a large range of item measures are adequate for stable item estimates, which implies that the sample size can be used to establish a reproducible item difficulty hierarchy.

Table 1
Rasch Summary Statistics for Items (I) and Persons (P) estimates

Tests		Separation index (I)	Separation index (P)	Infit (I)	Infit (P)	Outfit (I)	Outfit (P)	Reliability (I)	Reliability (P)
Test1	CLT	7.08	1.31	0.93	0.83	1.70	1.23	0.98	0.63
	VHTP	5.87	1.46	1.01	0.87	0.95	0.95	0.97	0.68
Test2	CLT	5.79	1.52	1.04	0.98	0.99	0.99	0.97	0.70
	VHTP	4.57	1.30	1.02	1.07	0.94	0.94	0.95	0.63
Test3	CLT	4.98	0.92	1.02	1.04	0.95	0.95	0.96	0.46
	VHTP	4.25	1.15	0.97	0.83	0.86	0.86	0.95	0.58

With regard to the person estimates, most of the person reliability (P) and separation indices for both groups were greater than 0.5 and 1, respectively. This means that the Rasch model identified more than one level of ability within the participants. Correspondingly, the participants were classified into low and high ability levels. The infit and outfit for both items and persons ranged between 0.5 and 1.5, except for the item outfit of CLT, which was 1.70. The outfit measure of 1.70 may be a result of a few random responses by the low-performing students. Furthermore, the high item separation indices (> 3) for the two groups indicate that the samples for each group were large enough to identify the item difficulty of the test instrument.

Additionally, the Wright (variable) map in Figure 1 indicates the relationship between the ranking of person abilities and item difficulties before the intervention. The figure shows that the persons' abilities range between -5 and 3 logits while the item difficulties range between -3 (easiest) and 1.5 (most difficult) logits. The most difficult items were Questions 5, 8, and 9, which were located between 1 and 2 logits, while Question 1, the easiest question, was located at -3 logits. Since the question difficulties ranged between the logits of persons' abilities, the items of the test instrument were adequate for the targeted students. Thus, it was concluded that the test items fit the Rasch model, have a good range of difficulty, have high reliabilities, and are appropriate for the cohort of participants for whom it is targeted. This has the potential for significant productive measurement and results.

The data analysed and presented here were part of a robust investigation that sought to explore two pedagogical practices. The person estimates, measured in logits, from the Rasch measurement of 157 participants were exported to the Statistical Package for Social Sciences (SPSS). An independent *t*-test was performed to test the equality of the effectiveness of the CLT and VHTP pedagogical interventions. Initially, there was a weak difference between the two groups at the pre-test, in favour of the CLT. A further analysis of the immediate and long-term effects of the interventions yielded statistically significant differences between the two groups. For both the post-test and retention test, the van Hiele group significantly outperformed the CLT group with large effect sizes ($t_{(138.63)} = -6.15$, $p = 0.00$, $d = 1.01$ and $t_{(154)} = -9.76$, $p = 0.00$, $d = 1.57$ at 95% confidence interval), as shown in Table 2.

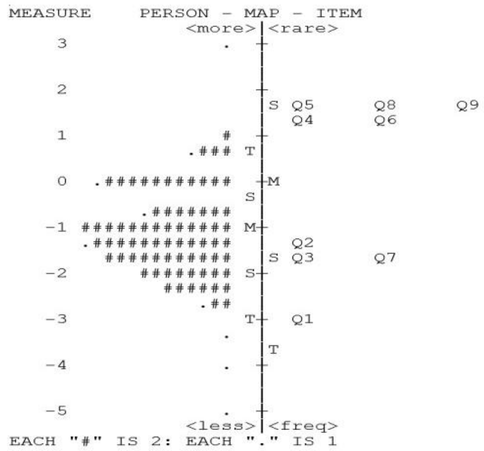

Figure 1. A Wright map showing the person abilities and item difficulties.

Table 2
Independent t-test of Students' Learning Outcomes Across the Three Time-points

Tests	Intervention	*N*	*M*	*SD*	*t*	*df*	Sig	*D*
Pre-test	CLT	72	-0.75	1.15	2.72	155	0.00*	0.44
	VHTP	85	-1.26	1.16				
Post-test	CLT	72	-0.14	1.19	-6.15	138.63	0.00*	1.01
	VHTP	85	0.95	0.99				
Retention test	CLT	72	-0.31	0.87	-9.76	154	0.00*	1.57
	VHTP	84	1.23	1.06				

*$p < 0.05$

A summary of the mean scores of the two groups from pre-test to post-test then to retention test is shown in Figure 2. The data indicate that while the learning outcomes of students in both groups at the pre-test were relatively close, they slightly favoured the CLT group. However, after the intervention, post-test scores of the VHTP group were significantly better than those of the CLT group, and at the retention test, the difference in the sizes of the effect was larger.

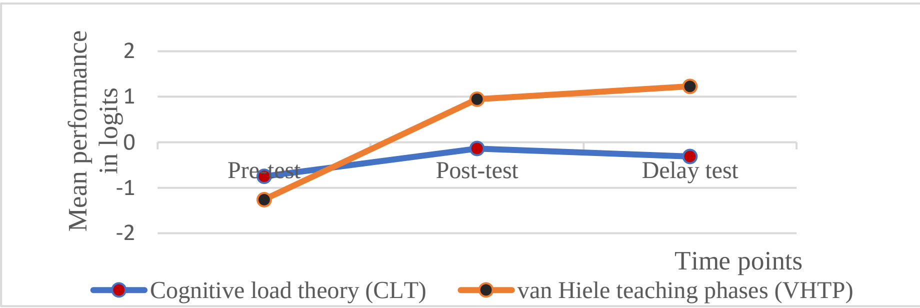

Figure 2. Line graph showing a summary of students' learning outcomes at the pre, post, and retention test.

To test the hypothesis of equal mean across the three tests for each interventional group, a repeated measure analysis of variance was conducted to explore the within-subject effects on students' learning outcomes. The Mauchly's test of sphericity was significant [χ^2 (2) = 6.50,

p = 0.04] for CLT group and not significant for VHTP group [χ^2 (2) = 0.24, p = 0.89]. Hence, the assumption on sphericity was considered differently. The results indicated that there was a significant medium effect of the CLT on students' learning outcomes [$F_{(1.84, 130.45)} = 8.88$, p = 0.00, η_p^2 = 0.11] and a significant large effect of VHTP on student learning outcomes [$F_{(2, 160)}$ = 260.93, p = 0.00, η_p^2 = 0.76]. A Post hoc comparison using the Bonferroni adjustment revealed that for the CLT group, significant difference existed between the pre-test and post-test but no significant difference in the means of the post-test and retention test. Similarly, there is significant difference between the pre-test and post of the VHTP group, however, the significant difference between the post-test and retention test only existed at 90% confidence interval.

Table 3
Mean, Standard Deviation and Repeated Measures Analysis of Variance of Students' Learning Outcomes

	Pre-test		Post-test		Retention test			
Intervention	*M*	*SD*	*M*	*SD*	*M*	*SD*	F	η_p^2
CLT	-0.75	1.15	-0.14	1.19	-0.31	0.87	(1.84,130.45) = 8.88*	0.11
VHTP	-1.26	1.16	0.95	1.00	1.23	1.06	(2, 166) = 260.93*	0.76

*$p < 0.05$

An analysis of the influence of students' ability levels on the effectiveness of the interventions is shown in Table 4. The result of the CLT group indicated a large significant difference between the learning outcomes of low and high ability students at the pre-test ($t_{(70)}$ = -11.90, p = 0.00, d = 2.86). After the intervention, the difference between low and high ability students was still significant but with moderate size ($t_{(69.93)}$ = -3.20, p = 0.00, d = 0.72). During the retention test, no significant difference was found between low and high students ($t_{(70)}$ = -0.09, p = 0.93, d = 0.02), suggesting that the CLT intervention favours the low ability students than the high ability students. For the VHTP group, the strong differences observed between the low and high ability students at the pre-test ($t_{(39.55)}$ = -7.26, p = 0.00, d = 1.92) continues at the post-test ($t_{(37.65)}$ = -3.91, p = 0.00, d = 1.05) and the retention test ($t_{(82)}$ = -4.94, p = 0.00, d = 1.12). These results suggest that the VHTP is beneficial to both the low and high ability students.

Table 4
Analysis of Students' Learning Outcomes Based on Ability Levels Across the Three Time-points

Groups	Tests	Level	*N*	*M*	*SD*	*t*	*df*	*Sig*	*D*
CLT	Pre-test	Low	29	-1.89	0.81	-11.9	70	0.00*	2.86
		High	43	0.01	0.54				
	Post-test	Low	29	-0.62	0.88	-3.20	69.93	0.00*	0.72
		High	43	0.19	1.27				
	Retention test	Low	29	-0.32	0.81	-0.09	70	0.93	0.02
		High	43	-0.30	0.92				
VHTP	Pre-test	Low	54	-1.85	0.61	-7.26	39.55	0.00*	1.92
		High	31	-0.22	1.17				
	Post-test	Low	54	0.61	0.59	-3.91	37.65	0.00*	1.05
		High	31	1.55	1.26				
	Retention test	Low	53	0.84	0.90	-4.94	82	0.00*	1.12
		High	31	1.89	1.01				

*$p < 0.05$

Discussion

As it is often claimed that pedagogical practices are essential in achieving the key goals of mathematics curriculum across the globe, this study examined the effectiveness of two pedagogical practices that have their roots in cognitivism and constructivism approaches. The results indicated that both the CLT and VHTP interventions were observed to have short-term effects on students' learning outcomes; however, the learning outcomes of students in the VHTP continued to increase at the retention test, while the CLT group experienced a waning effect after the post-test. This pattern of the results may be attributed to many factors, including the nature of the instruction and forgetfulness. Specifically, CLT recommends instructional designs that require students to acquire schema with minimal cognitive effort (Sweller, 2011). The worked examples instruction utilised in this study provided a step-by-step guide to solving a problem, emphasising more procedural knowledge than conceptual knowledge, and students do not experience interference with learning. Conversely, one of the main principles underlying the movement of students' thinking from one level to the next in VHTP is the crisis in thinking during learning (Serow et al., 2019), that is often experienced by students in the fourth teaching phase and allows them to investigate various thinking paths, identify correct reasoning for the domain of thought, and develop a strong perception of the mathematical ideas, which is observed to last for a long time. Therefore, the VHTP seems to offer more conceptual mathematical knowledge than procedural knowledge.

Furthermore, students' learning outcomes in the VHTP group were observed to be significantly better than the CLT group regardless of their ability levels. While the CLT is more favourable to the low ability students by bridging the gap between the low and high ability students, the VHTP appears to improve both the low and high ability students in similar magnitude. These findings seem to support and advocate that the major attributes of VHTP—an exploration of learning materials, sequential development of students' thinking, thinking crisis, students' active participation, language development, and discourse—are essential for learning complex mathematics topics. Another practical implication of these findings could be that pedagogies that strongly focus more on conceptual knowledge tend to have more lasting effect than the reverse. The results of this study, which was conducted in an African context, are consistent with several other studies around the globe (Kalyuga et al., 2001; Machisi & Feza, 2021; Renkl, 2017; Walsh, 2015). However, the appropriate use of VHTP requires extra commitment from teachers. Lastly, the researchers acknowledge the interference of noise from the natural setting, where this experiment was carried out, as a limitation of this study.

Conclusion

The findings presented in this paper suggest that the pedagogical practices employed by teachers significantly affect students' learning outcomes and long-term knowledge retention. This study highlights that VHTP allows students to demonstrate ownership of mathematical ideas, and the crisis in thinking has a significant effect on students' achievement in mathematics. The VHTP students demonstrated higher achievement in the short and long term than their peers who learned through the CLT worked example instruction. The findings from this study extend existing evidence on the application of CLT and VHTP in mathematics learning. It also contributes to the growing evidence on effective teaching practices in mathematics education. Lastly, the study is significant for its methodological (SOLO model and Rasch model for scoring and analysis), empirical, and contextual contribution to the improvement of mathematics learning and retention.

Acknowledgements. The researchers thank the students, mathematics teachers and principals of the schools involved in this study. The researchers acknowledge that this article is part of the doctoral thesis of the first author.

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Senior High School Students' Perceptions of Mathematics Teachers' Assessment Practices in Ghana

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This study examines mathematics teachers' assessment practices for senior high school students in Ghana. Formative assessment has been identified in the literature as having a significant impact on students' learning. However, less attention has been given to students' perceptions of teachers' assessment practices in Ghana. Data involved questionnaires for 420 senior high school students and 308 senior high school teachers in the Ashanti Region of Ghana. The results showed that students and teachers hold different perceptions of assessment practices, and suggest teachers should pay more attention to questions, homework, student observation, student demonstration and group work to support students' progress and better examination attainment.

Introduction

This study examines mathematics teachers' assessment practices for senior high school students in Ghana. Classroom assessment is a component of all teacher's activities which should serve as the basis for the decisions made about students' learning in the classroom (Moss & Brookhart, 2019). Teachers have a professional responsibility to assess students' learning progress. Recent research suggests that teachers should regularly review and check students' progress to ensure that students achieve planned learning outcomes (Birenbaum et al., 2015; Klenowski, 2009). Some teachers focus more on the end of term assessment (summative assessment) rather than checking students' learning regularly and throughout the students' learning experiences (formative assessment) (Andersson & Palm, 2017). Summative assessment is a judgement to determine students' overall performance in a program (Moss & Brookhart, 2019) and is commonly referred to as assessment of learning. In Ghana, the annual examination conducted by the West African Examination Council for students at the end of their third year of senior high school education is an example of high-stakes summative assessment (Okyere & Larbi, 2019). The purpose of this assessment is for certification and selection into tertiary institutions (Dogbey & Dogbey, 2018). Summative assessment provides less support for student learning because its emphasis is on achievement at the end of the program (Kippers et al., 2018). In contrast, formative assessment is the process by which teachers use assessment practices to elicit information throughout a learning program so teaching and learning can be adjusted to improve students' outcomes (Popham, 2009). This is also known as assessment for learning (Black & Wiliam, 1998; Moss & Brookhart, 2019).

Assessment for learning helps teachers to identify gaps in students' learning (Kippers et al., 2018), to monitor students' progress (Moss & Brookhart, 2019), and to enhance students' performance through feedback. Researchers are of the view that assessment for learning methods that focus on day-to-day progress enable students to think creatively and develop their analytical skills (Thompson & Goe, 2009). Assessment for learning practices include observation, questioning, homework, group work, portfolios, self-assessment, peer assessment, student demonstration, student observation, classroom discussion and quizzes (Moss & Brookhart, 2019). These instructional practices necessitate students' involvement in the formative assessment process, and allow teachers to identify specific student errors, and provide feedback to help correct these errors (McMillan et al., 2013).

The assessment paradigm has shifted from the assessment of learning to assessment for learning. Formative assessment has been strongly advocated by organizations such as the World Bank (2013) and the Assessment Reform Group (2002). The World Bank "links high

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quality, formative assessment to better outcomes on standardised tests, and links better learning outcomes to increased national prosperity" and "directly links improvements in reading and mathematics to increased GDP" (Browne, 2016, p. 4). Many developed countries such as Australia, Canada, England, Singapore and USA have already embedded formative assessment into their education systems (Organisation for Economic Co-operation and Development, 2005). This has shifted the attention of Sub-Saharan African countries such as South Africa, Malawi, Uganda, Ghana, Nigeria and Ethiopia to practice formative assessment in their education systems (Browne, 2016).

Ghana has a "vision … to transform its economy and society into a stable, united, and prosperous country with opportunities for all" (Government of Ghana, 2014), to increase the average national income. A major driver of this vision is seen to be in the transformation of educational practices. Ghana's pre-tertiary framework (2018), developed for basic and senior high schools, has a focus on mathematics and science as the fundamental building blocks for success in the era of technological advancement in Science, Technology, Engineering and Mathematics (STEM) subjects. This framework also recommends that schools should "shift from an emphasis on summative assessment to the formative, a philosophy that espouses the need to employ multiple sources of evidence about learning, which will guide instructional decisions and support each learner's learning trajectory" (p. 19).

Despite these recommendations, there is little use of Assessment for Learning (AfL) in Ghanian schools which is believed to be due to lack of experience and knowledge by teachers, and also limited availability of resources for teachers to use. The syllabi give detailed information on what the students learn, but no information on tools and approaches teachers can use, scoring criteria rubrics and suitable assessment tasks for AfL (Ghana's pre-tertiary curriculum framework, 2018, p. 11).

There have been many studies conducted on the assessment for learning practices of teachers in Ghana and elsewhere (e.g., Amoako et al., 2019; Amua-Sekyi, 2016; Martínez et al., 2009; Pat-El et al., 2015; Yan & Cheng, 2015), but there are fewer studies that include students' perspectives even though the students' perceptions can play a vital role in the choice of assessment practices of the teachers (Drew, 2001; Struyven et al., 2005). MacLellan's (2001) study of tutors and students' perceptions of assessment for learning in a higher education context showed that whereas the tutors perceived their assessment as formative, the students perceived their assessment to be mainly summative. With these factors in mind, the research questions for this study were.

- RQ1. What assessment practices do senior high school mathematics teachers use to assess senior high school students' learning?
- RQ2. What assessment practices do senior high school mathematics students report their teachers use to assess their learning?
- RQ3. To what extent do senior high school students perceive these practices are helpful in their learning and their examination attainment?

Teachers' Assessment Practices

Internationally, teachers use a variety of different practices to gather information about students' ongoing learning in the classroom (e.g., Kippers et al., 2018; Okyere & Larbi, 2019; Wiliam et al., 2004). Kippers et al. (2018) investigated AfL classroom assessments in the Dutch education system and found that the most frequently used assessment practices were paper-and-pencil exercises, asking questions, classroom observations, and homework assignments. Other less often used assessment practices included digital tests, oral tests, portfolios, practical tasks, presentations, and questionnaires. Wiliam et al. (2004) studied the use of AfL practices of secondary mathematics teachers and found that they used questioning, feedback, sharing criteria with learners, self-assessment, posters and presentations. A study of formative assessment practices of teachers in Ghana (Asare, 2015; Bekoe et al., 2013) showed that teachers used varied assessment practices. For example, Bekoe et al. (2013) examined the formative assessment practices tutors used to assess pre-service teachers' learning in Social Studies in three Colleges of Education in Central Region of Ghana. The findings indicated that diagnostic assessment, portfolio assessment, self-assessment and peer assessment were the major formative assessment practices used in the Colleges of Education in Ghana. Also, Asare (2015) investigated kindergarten teachers' classroom formative assessment practices based on two subscales: (a) teachers' modes of assessment frequently used, and (b) their reasons for selecting a particular mode of assessment. The results of the study indicated that paper- and-pencil test, standardized test, interviewing, observation portfolios and performance task were the formative assessment practices used by teachers to assess kindergarten pupils. However, Okyere and Larbi's (2019) research of mathematics teachers' assessment practices revealed that the assessment practices used by these teachers included written tests, quizzes, classroom discussions, teacher questioning, answering responding to questions from students, observations, student journal writing, classroom discussion, student demonstration and role play/presentation.

Students' Perceptions of Teachers' Assessment Practices

Teachers' use of assessment practices to support students' learning requires students' participation (Cachia et al., 2018). However, as noted earlier, students and teachers may have different perceptions of teachers' assessment practices (Crook et al., 2006). For example, Gao (2012) examined 248 American high school students' perceptions of mathematics classroom assessment. The study found that the students perceived the tasks as less authentic (featuring real-life situations) and less transparent (student knowledge of the purpose of the assessment) than the teachers believed they were providing. Dhindsa et al. (2007) evaluated 1,028 upper secondary science student perceptions of their assessments. Whereas the students perceived that their assessment matched what they had covered in class, they also thought that there was little connection between their assessment tasks and what they used in daily life. A study of high school students and teachers in the Netherlands also found that the teachers perceived they were using a higher level of AfL than the students (Pat-El et al., 2015).

Iannone and Simpson (2013) investigated undergraduate mathematics students' perceptions of which assessment methods are better discriminators of mathematical ability. Forty-eight undergraduate students took part in this research. The assessment practices used were multiple choice tests, closed-book exams, oral exams, presentations, example sheets, dissertations, open-book exams and projects. The findings of the study showed that while closed-book examinations are a common practice of teachers, the undergraduate students perceived they inhibited their mathematical ability. This demonstrates how students can have different perceptions of assessment from their teachers.

Methods

Study Participants

A total of 728 students and teachers participated in the study, of which 420 were senior high school students and 308 were senior high school mathematics teachers. Both teachers and students were purposively recruited from 20 public schools in six districts in the Ashanti region of Ghana. The researcher contacted the principals of the various senior high schools who facilitated the researcher's access to the assistant academic principals, heads of mathematics departments and the mathematics teachers in the selected schools. In consultation with the heads of the mathematics departments, the mathematics teachers selected the students from different programmes offered by the schools and invited them to participate.

Instruments

Two sets of questionnaires, one for teachers, and one for the students, were developed based on the gaps and emerging themes identified in the literature. Both teachers' and students' questionnaires were adapted from instruments previously developed by Lysaght and O'Leary (2013), and Pat-El et al. (2013), both with large cohorts. The questionnaires were piloted with a smaller sample of students and experienced senior mathematics teachers in selected schools before administering to the wider study participants.

The teachers' questionnaire included both open-ended and closed-ended questions categorized into two sections of 33 items in total. The open-ended questions in the first section (Part A) gathered teachers' demographic information with seven items in respect of gender, age, academic qualifications, category of the school, teaching experience and professional qualification. The second section (Part B) comprised 26 items on the frequency of teachers' given assessment practices for formative and summative purposes (See Table 1). The students' questionnaire was in three sections. Section A gathered participants' demographic data in respect of gender, age, the course offered, category of school and year level in the school. Section B (13 items) sought students' perceptions about teachers' frequency of use of the listed assessment practices (See Table 2). These items were measured on a five-point Likert scale, ranging from *seldom used* to *used very often*. Section C sought students' perceptions of how these teachers' assessment practices supported their learning and better examination attainment (See Table 3). These 13 items were measured on a 5-point Likert scale ranging from *not at all helpful* to *extremely helpful*.

Both teachers' and students' questionnaires were administered personally by the first author at the same time in all the selected schools, during the COVID-19 pandemic period, with attention to COVID-19 protocols. Both questionnaires were in the English language as this is the medium of instruction used in senior high schools in Ghana.

Reliability of the questionnaires was initially checked using the results of the pilot test with fifty students and ten teachers in a senior high school that was not part of main study. calculated Cronbach's alpha was calculated to check the internal consistency of the instruments (DeVellis, 2012), with both teachers' assessment practices and students' perception of teachers' assessment practices showing high correlations for reliability (0.89 and 0.87 respectively) for the whole study. Out of the of the 520 questionnaires distributed to senior high school students, 420 were returned, representing a response rate of 82.6% for the data analysis. For the teachers, out of the 400 questionnaires, 308 were returned representing a 77 % response rate for the data analysis.

Ethical Considerations

The wider study from which this paper is drawn received institutional approval from the University of Tasmania Social Sciences Human Resources Ethics Committee, and the selected Districts and Municipals Directors of Education in Ghana, before the data collection.

Data Analysis

The data were entered and analysed using the Statistical Package for the Social Sciences (SPSS), Version 27. The quantitative data gathered from the questionnaire of both teachers and students were analysed using descriptive statistics such as means and standard deviations. Because of the large sample size, it was assumed that the data were normality distributed (Field, 2013).

Results

In addressing RQ1, the mean and standard deviation of the frequency of mathematics teachers' formative and summative assessment practices were calculated. The means and standard deviations of the respondents' responses are summarized as follows in Table 1. From this table it is observed that the most frequently used forms of formative and summative assessments were questioning, traditional assessment such as multiple-choice tests, and classroom discussion. It was observed that the least practised formative and summative assessment strategies were peer-assessment, student demonstration and portfolios.

Table 1
Teachers' Reported Use of Various Forms of Assessment. Higher Means Reflect Higher Use.

No.	Assessment Practices	Mean	Standard Deviation
1	Questions	4.42	0.74
2	Traditional Assessment (e.g., multiple choice, true-false, and short answers)	4.08	1.15
3	Students Observation	3.86	0.83
4	Classroom Discussion	3.77	1.25
5	Quizzes	3.62	0.91
6	Paper and Pencil Test	3.51	1.21
7	Role play/Presentation	3.37	1.36
8	Self -Assessment	3.29	1.04
9	Group work	3.28	1.81
10	Homework/Assignment	3.03	1.12
11	Peer Assessment	3.03	1.12
12	Student demonstration	2.82	1.36
13	Portfolios	2.78	1.18

In addressing RQ2, the mean and standard deviation were calculated for how the students perceived the frequency of their teachers' assessment practices, and these are summarised in Table 2. The most dominant formative and summative strategies that students saw used by teachers were questioning and homework/assignments. The students perceived that portfolios, student demonstration and peer assessment were used least often. Table 3 shows the means and standard deviations students' perceptions of the helpfulness of the formative assessment used by their mathematics teachers in their learning and attributed success in examinations. According to the students, the most helpful formative strategies identified by students were questions and homework/assignments. The least helpful was role play/presentation.

Table 2
Students' Perception of Teachers Use of Assessment Practices. Higher Means Reflect Higher Use.

No.	Assessment Practices	Mean	Standard Deviation
1	Questions	4.32	.94
2	Homework/Assignment	3.88	1.16
3	Classroom Discussion	3.77	1.25
4	Student Observation	3.67	1.24
5	Role play/Presentation	3.37	1.36
6	Paper and Pencil Test	3.34	1.31
7	Traditional Assessment (e.g., multiple choice, true-false, and short answers)	3.30	1.34
8	Self -Assessment	3.24	1.42
9	Quizzes	3.14	1.26
10	Group Work	3.03	1.33
11	Peer Assessment	2.98	1.33
12	Student Demonstration	2.82	1.36
13	Portfolios	2.53	1.31

Table 3
Showing the Mean and Standard Deviation of Students' Perceptions of the Helpfulness of the Assessment Practices for Learning and Better Examination Attainment

No.	Assessment Practices	Mean	Standard Deviation
1	Questions	4.31	0.97
2	Homework/Assignment	4.10	1.05
3	Students Observation	4.06	1.04
4	Student Demonstration	4.04	1.06
5	Group Work	4.01	1.07
6	Paper and Pencil Test	3.93	1.06
7	Peer Assessment	3.91	1.11
8	Quizzes	3.90	1.09
9	Traditional Assessment (e.g., multiple choice, true-false and short answers)	3.89	1.14
10	Self- Assessment	3.78	1.11
11	Portfolios	3.71	1.08
12	Classroom Discussion	3.62	1.26
13	Role play/Presentation	3.31	1.22

Discussion

The results of this research showed that Ghanaian teachers used varied assessment practices to monitor student's progress and improve learning performance, of a mixed formative and summative nature. The students and teachers both perceived that questions, student observations and classroom discussion were used frequently. However, the students perceived homework (second in frequency) as used more often than the teachers did (tenth in frequency. They also perceived traditional assessment to be used less often than the teachers did. These teacher findings are similar to the study by Kippers et al. (2018) who found that the teachers most often used paper-and-pencil tests, questioning, classroom conversations and homework assignments. These are, however, very different from the findings of Wiliam et al. (2004) who found that the teachers used feedback, sharing the criteria with students, and self-assessment more frequently than these teachers in Ghana. This may be owing to a combination of large class sizes in Ghana and a lack of knowledge of AfL practices by the teachers in Ghana (Amoako et al., 2019; Amua-Sekyi, 2016).

When it came to how the students regarded the helpfulness of the different assessment practices for their learning and examination attainment, the students listed questions first, followed by homework, then student observations, student demonstrations and classroom discussion. However, on the teachers' list in terms of frequency, question was first, homework was tenth, student observation was third, student demonstration was twelve and class discussion was fourth. This implied that questions, student observation and classroom discussion were perceived by both teachers and students as the most helpful practices of teachers' assessment practices to support learning and better examination practices. However, homework and student demonstration which were perceived by students as very helpful were not frequently used by teachers. This add a new knowledge to existing literature which suggests that mathematics teachers should pay attention to homework and student demonstration which are perceived by students as very helpful to support learning and better examination attainment. Further, this research adds new knowledge to existing literature where students' perception of teachers' assessment practices is less studied in the developing countries. It was noted earlier that whereas the Ghanaian Ministry of Education recommends a shift in emphasis from summative to formative assessment, Ghanaian teachers do not have an extensive knowledge of these practices and there are few resources available for them to develop these practices. This research illustrates that students do not perceive that their teachers use formative assessment practices such as self-assessment, peer-assessment and portfolios, even though the students believe these to be helpful to them. This knowledge may assist in motivating teachers to learn more about, and more widely apply, assessment for learning practices.

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Pre-service Teachers' Re-constructed Geometry Disposition Scale: A Validity and Reliability Study in the Ghanaian Context

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This study reports on the development, validation, and reliability of a geometry disposition scale (GDS) to measure pre-service teachers' (PSTs') attitudes to geometry learning. PSTs from two Colleges of Education (CoEs) in Ghana volunteered to participate in the study (N = 153). A principal component analysis (PCA) extracted four factors: deep affect (positivity expressed towards geometry learning), working privately, collaborative working and technology or calculator use. The final GDS contained 15 items. While validation is still not fully tested, the psychometric properties to-date suggest the GDS has promising benefits in measuring PSTs' attitudes to geometry learning, which may enable the adjustment of the teaching of geometry accordingly.

Background

The current trend of educational systems in most countries around the world shows an overriding objective of investing in Science, Technology, Engineering, and Mathematics (STEM). This is true of Ghana, where it is believed that future technological advancement, development, and innovation is vital to the future of the country (Keaveney et al., 2018). Of particular importance to the context of technological advancement is the contribution of geometry knowledge (Wang, 2016). Geometrical ideas and concepts remain a formidable force in this respect as they are increasingly utilised in pursuits such as architectural design, engineering, building construction and packaging. It is, therefore, not surprising that geometry has been described as the "tool for understanding and interacting with the space in which we live, [and] is perhaps the most intuitive, concrete and reality-linked part of mathematics" (Wang, 2016, p. 1). Thus, Jones (2002, p. 122) said, "geometry [as an appeal] to our aesthetic, visual and intuitive senses." In addition, Kundu (2018, p. 212) described it as a lively and stimulating strand of mathematics, which he argued offers the key needed to understand our world as we experience "geometric figures and [identifying their] relationships." It is "grasping space … that space in which the child lives, breathes, and moves. The space that the child must learn to know, explore, conquer, in order to live, breathe and move better in it" (Freudenthal, 1973, as cited in National Council of Teachers of Mathematics [NCTM], 1989, p. 48). If this sense of geometry as a "grasping space" is to be adopted, a conscious effort must be made to develop children's geometric knowledge and concepts, and to engage them in geometric phenomena. It then becomes important to be deliberate in planning learning activities intended to empower children to build connections and identify relationships and develop spatial sense.

Through the study of geometry, learners are imbued with the requisite mathematical tools and skills that are catalytic to develop the complex reasoning and problem-solving skills used in STEM and its many related skilled trades and professions. It is important, therefore, that school students' capacity to develop the understanding of geometry is enhanced, so that they can participate in a technological world. This requires geometry to be taught well.

A key to this success is an intentional investment of developing such knowledge in our PSTs who will be teaching these school students. The place of geometry in teacher education cannot therefore be overemphasised. It is a crucial strand of mathematics education, and as

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 74–81. Launceston: MERGA.

such PSTs are expected to connect geometric concepts to geometric phenomena (Salifu et al., 2018; University of Education Winneba [UEW)], 2018). Notwithstanding this critical importance, multiple studies have identified how PSTs' attitudes to geometry can negatively affect and influence their geometry performance, as discussed next. This study therefore sought to develop, validate, and estimate the reliability of a geometry disposition scale, which can be used to measure the attitude of PSTs to geometry learning and hence inform effective instruction.

Literature

Despite the importance of geometry as suggested, previous research has indicated that the achievement of both PSTs and school students is lower in geometry than in other domains of mathematics. For instance, it was reported that US students showed progress both in national and international organised assessment in other areas of mathematics but failed to sustain that improvement in geometry (Casa et al., 2017). Duatepe (2013) also indicated that students' performance in geometry was adjudged lower than in other areas of the mathematics discipline. This assertion confirmed the reports from earlier studies (e.g., Clements & Battista, 1992) that students were not studying geometry as they should, and is echoed by Dimla (2018).

Research indicates that PSTs have mostly a procedural understanding rather than a relational understanding of geometry. For example, Patkin and Plaksin (2019) conducted a study involving PSTs (n = 16), who were taught the geometry of solids during an academic year. The students solved problems based on three dimensional shapes including pyramids. They engaged in other activities to develop their spatial perception and were examined at the end of the year. The findings illustrated that PSTs attained higher in procedural understanding related problems than that of relational understanding, leading the authors to hypothesise that understanding a learned material relationally comes by using special teaching methods. Their limited relational understanding, with high procedural understanding, suggested that the PSTs may have lacked the necessary deep understanding of concepts taught, and this could limit their ability to transfer acquired knowledge fully to their future students.

Research has increasingly identified the under-performance of PSTs in geometry, both internationally (e.g., Aslan-Tutak, 2009), and in Ghana (Salifu et al., 2018). This is worrying, as the trend could hinder progress in STEM related endeavours in the future both globally, and in the context of this study, Ghanaian students, if teacher education does not empower PSTs with sound geometrical ideas and concepts.

Attitude and Its Importance to Learning

The trends in under-performance in geometry have been building for some time. Betiku (2001), for example, observed that students shy away from geometry studies and argued strongly that this behaviour was indicative of negative attitudes when it comes to geometry learning. It was suggested that PSTs' attitudes impair their effective learning of geometry, which contributed to lower geometry performance (Adolphus, 2011). Geometry is not alone in this regard, the learning of mathematics has been associated with many variables, of which attitude towards mathematics learning is one (Mazana et al., 2019).

Affect variables (e.g., attitude) influence the level of personal effort expressed by an individual student to learn mathematics (Fennema & Sherman, 1976). The influence of attitude can either be positive or negative. It can be argued that if students have a positive attitude, they will become active in the knowledge construction process to develop a conceptual understanding of mathematics. In contrast, a negative attitude may cause them to lose interest, and not put in the effort that is needed. It is noted that PSTs' attitudes to teaching methods courses are at times at odds to their knowledge and engagement, but learner-focused teaching relates positively with their course attitudes (Rios, 2017; White et al., 2006). For example, PSTs could be keen about doing mathematics at one time but not have the requisite background to succeed; could be insecure in an aspect of mathematics but feel confident to do mathematics; or show positive attitudes but could not have sufficient mathematical content knowledge. Thus, they suggest support is needed to assist PSTs to become aware of their own attitudes, which can then be made clear and observable.

Extending White et al.'s (2006) and Rios' (2017) arguments, it is suggested that, although students should ideally be independently responsible for knowledge acquisition and engage interactively with their peers (Ontario Education, 2005), teachers should mediate between the learners and the knowledge to be acquired (Arpin & Capra, 2004). The reason is simple: "effective teaching goes further: creating an environment that not only makes learning possible now, but also teaches attitudes and behaviours that enhance learning and success in later life" (Goss & Sonnemann, 2017, p. 7); and teachers' attitudes influence students' attitudes (Tsao, 2017). Thus, if a PST learns geometry in an environment where the lecturer shows a negative disposition towards teaching the subject, it is likely to generate PSTs' dislike towards the subject as well.

Multiple research studies have investigated PSTs' attitudes to the learning of mathematics, of which geometry was a part (Tsao, 2017). Enu et al. (2015) completed a study on the factors influencing PST students' mathematics performance in some selected colleges of education [CoEs] in Ghana. Their study found that 66% disagreed that "they are always under a terrible strain in the mathematics class" (p. 71), and the PST students' mean responses to the survey that explored their attitude towards mathematics indicated a positive attitude towards mathematics. However, the other 34% reported to be under a "terrible strain" when learning mathematics. This was consistent with other research evidence, which suggested a substantial proportion of PSTs held negative attitudes towards mathematics (Burton, 2012). However, attitudes of PSTs towards the learning of geometry in the context of Ghana is an area that has been less explored.

It is argued that if learners are to be encouraged to develop positive attitudes and behaviours to a discipline, it is important that their initial attitudes are measured (Esikci et al., 2017; Tavşancıl, 2006). This was the motivation for the development of the Geometry Disposition Scale (GDS) scale used in this study, and the objectives reported here, namely, to test the factor-structure of the GDS instrument using responses of PSTs sampled from the colleges of education in Ghana, and to interrogate the reliability and the validity of the GDS in the Ghanaian context.

Earlier Geometry Attitude Scales

A search of the literature indicated that there were fewer geometry attitude scales than that for mathematics overall, particularly for studies involving PSTs. Most existing geometry attitudes scales measured only two to three components of attitudes (Avcu & Avcu, 2015); for example *motivation and self-confidence* (Duatepe & Ubuz, 2007); *enjoyment, value, and motivation* (Mogari, 2004); and *usefulness, confidence, and enjoyment* (Utley, 2007). In addition to measuring almost the same dimensions, scales thus far have mostly dealt with senior high school students. This limited attention to attitude dimensions may overlook other dimensions of geometry attitudes that may equally impact students' geometry learning. A Turkish adaptation of Utley's scale to assess undergraduate attitudes to geometry introduced two new dimensions: *future use* and e*veryday use* (see Avcu & Avcu, 2015). This extension of dimensions agreed with earlier authors (Fennema & Sherman, 1976; Utley, 2007) that there were more dimensions of attitudes that could be explored. Further, it has been suggested that the geometry attitude construct may be seen as analogous to that of mathematics attitudes dimensions (Avcu & Avcu, 2015, p. 16). Thus, an adaptation of Brookstein et al.'s (2011) mathematics Students Attitude Survey (SAS) seems appropriate for use in the development of a geometry disposition scale (GDS), as an instrument to explore and measure other unexplored dimensions of PSTs geometry attitude constructs.

This paper therefore reports on one component of a wider ongoing study into the teaching of geometry in teacher education courses, namely the creation and validation of a geometry-specific instrument (GDS) to measure PSTs' attitude to geometry learning.

Methodology

Participants

The participants were first year PSTs in three Ghanaian Colleges of Education. Ethics approval was gained from the Tasmania Social Science Human Research Ethics Committee and subsequently from the president of the Principal Conference of Colleges of Education in Ghana. The PSTs were invited to participate by three geometry educators, who agreed to inform the participants through their various WhatsApp platforms. The questionnaire was completed online, hosted on an online survey platform. Respondents' login to complete the questionnaire indicated explicit consent for participation in the study. In all, 153 PSTs (from 302 invitations) volunteered to participate in the study.

The Instrument

The GDS is a survey questionnaire for the PSTs designed to explore their attitudes towards geometry learning. As introduced in the literature review, the GDS was primarily adapted from the existing mathematics attitude scale developed and validated by Brookstein et al. (2011). In general, the adaptation was to change mathematics terms and references to more specific ones to reflect the context and objectives of the study, which focusses on geometry. For instance, "I think mathematics is important in life" was changed to "I think geometry is important in life."

Ultimately, the GDS contained 15 Likert-type items (as shown in Table 1), with five possible alternatives (from strongly disagree to strongly agree) to collect data directly from the participating PSTs. During the analysis, four factors were identified suggesting four subscales, which are described under results and discussion.

Analysis

An exploratory factor analysis (EFA) with Principal Component Analysis (PCA) technique (IBM SPSS Version 27) was performed to investigate the factor structure of the data and to validate the measuring tool. The strength among the variables was checked by generating and inspecting the correlation matrix, which revealed several coefficient values greater than 0.3 (Tabachnick & Fidell, 2013), signifying healthy strength of the intercorrelation among the variables. Cronbach's alpha tests were employed to check on the internal consistency of the subscales that were created. As a result of this process, 15 items were rejected from an initial bank of 30 items, as described next.

Results and Discussion

Factor Analysis (FA)

During the analysis, each negative item was reverse-scored. Each of the items numbered 12 (My geometry tutor was not friendly and patient with us); 16 (The presence of the geometry tutor in the class puts me off); 27 (Our geometry tutor does not review our assignment/homework); and 30 (My geometry tutor encourages us in class) had low coefficients (less than 0.3) with some other items. This implied they may not factor well and so they were subsequently deleted.

To assess the factorability of the data set, two precautions were observed as we employed factor analysis. The first was to ensure we used an appropriate sampling size. The other was to ensure the sampled data were not an identity matrix. Therefore, Kaiser-Meyer-Olkin (KMO) and the Bartlett's Test of Sphericity were conducted. The KMO measure of sample adequacy was 0.883, which exceeds the recommended 0.6 value (Kaiser, 1974; Tabachnick & Fidell, 2013). A KMO measure close to 1.00 identifies that the sum of partial correlations is large relative to the sum of correlations, which suggests the pattern of correlations is compact and hence factor analysis would result in distinct and reliable factor extraction (Field, 2005). Thus, the KMO statistic here of 0.883 is an endorsement of effective sample size. The Bartlett's Test of Sphericity was found to be significant [(x^2 (325) = 2014.088, $p < 0.000$)], testifying that the sample was not an identity matrix. These statistics show that the data set was appropriate for factor analysis (Field, 2005).

Principal component analysis, with Oblimin rotation, extracted six components with the components having Eigen values of 1 or more. A communality cut-off point of 0.40 was used as recommended by Pituch and Stevens (2016). Thus, Item 7 (In secondary school, I learned geometry more from talking to my friends than from listening to my mathematics teacher) was discarded as its communality (0.386) was under the cut-off point. The process was then re-run with no further changes.

The six component-solution accounted for a total of 67.020% of the variance. Because of the insufficient number of items that loaded into the fifth component (i.e., Item 25 *loaded: 0.768* and Item 6, *loaded: 0.745*) and the sixth component (i.e., Item 11 *loading 0.518*), they were removed and, thus, four components were retained (accounting for 55.063 % of the variance). Subsequently, Items: 25 (I sometimes feel nervous talking out loud in front of my classmates in geometry class); 6 (I get anxious in geometry class); and 11(I enjoy being part of groups learning geometry outside school) were removed from the instrument.

Retained Components (Factors) and Associated Items

The internal consistency using the Cronbach's alpha was determined for each component. The first component explained 36.872 % of the total variance and had the maximum loadings of eight variables: 19, 15, 24, 22, 17, 3, 1, and 8. However, Item 3 (I am able to learn more about geometry when working on my own), as well as Item 8 (Technology/calculator can make geometry easier to understand) were eliminated. This was done to improve factor reliability and to enhance factor meaningfulness, the first component was named *Deep affect* (D) *($\alpha = 0.908$)*. The items in this factor reflect the positivity that was expressed towards the learning of geometry such as receiving good grades on a geometry test, liking geometry, and geometry being interesting. This factor, similarly, may be seen to reflect one's feeling of confidence in their ability to solve geometry problems, and thinking of geometry being important in life. Thus, "affect is a significant factor in the learning process" (Chamberlin, 2010, p. 175).

The second component, *Working privately* **(**Wp**)** *($\alpha = 0.676$)* had five loadings (Items 4, 5, 14, 21 & 29) and accounted for 8.520 % of the variance. The removal of Item 21 (I enjoy working in groups better than alone in geometry class/lecture) improved the alpha coefficient (0.789), but factor interpretation was still difficult. Thus Item 29 (my geometry tutor encourages us in class) was also eliminated, gaining a moderate alpha coefficient ($\alpha = 0.676$)) which was believed appropriate enough, as it ensured factor meaningfulness consistent with the literature (Brookstein et al., 2011). The items in this component reflect working on geometry privately such as preferring to work alone instead of being in group to do geometry, not liking to speak in class or group discussion and disliking geometry lecture attendance.

The third component, *Working collaboratively* **(**Wc**)** *($\alpha = 0.536$)***,** was loaded with three items (13, 18, & 20), and explained 5.190% of the total variance. Wc measures the extent to which PSTs support each other in learning geometry, either in class or outside the classroom.

For instance, whether they enjoy geometry class, participate in geometry discussions, and participate in group activities involving geometry. Although the Cronbach alpha seemed a little low, the variables loaded highly, and the value of learners working co-operatively is upheld in studies that show it fosters learners' learning (Seidouvy & Schindler, 2020); leads to improved attainment (Oner, 2016); and promotes desirable attitudes (Edwards & Jones, 2003). It was, thus, adjudged appropriate for inclusion in the instrument.

Finally, the fourth component, *Technology or calculator use* (Tc) ($\alpha = 0.785$), accounted for 4.481% of the variation. Two items, Item 2 (In secondary school, my mathematics teachers listened carefully to what I had to say) and Item 10 (I like my own space outside school the majority of the time) made factor meaningfulness difficult. Suggestions to improve factor meaningfulness by Pituch and Stevens (2016) were accepted; and thus these two items were removed. This component shows that PSTs feel good any time they must use technology/calculators to learn geometry, enjoy using computers and/or calculators to learn geometry, and that cell phones are important part of their learning engagement.

Table 1
Summary of the Factor Loadings and Communalities

	Component					
ID	Item	D	Wp	Wc	Tc	Com.
19	I receive good grades on geometry tests and quizzes	.853				.677
15	I like geometry.	.811				.829
24	Geometry interests me.	.761				.745
22	I like to go to the board or share my answers with peers in geometry class/lecture.	.751				.663
17	I feel confident in my abilities to solve geometry problems.	.732				.719
1	I think geometry is important in life.	.491				.793
5	I prefer working alone rather than in groups when doing geometry.		.751			.663
4	I do not like to speak in geometry class or group discussion.		.709			.643
14	I do not like attending geometry lectures.		.603			.761
18	In the past, I have not enjoyed geometry class.			.763		.640
20	I am not eager to participate in discussions that involve geometry.			.691		.740
13	I do not participate in many geometry group activities outside school.			.452		.523
28	When using technology/calculator for learning geometry, I feel like I am in my own private world.				-.836	.586
9	Cell phones are an important technology in my life.				-.703	.632
26	I enjoy using a calculator/computer when learning geometry.				-.434	.745

Summary

This paper reports on a preliminary component of a wider study to create and validate geometry specific instrument to measure PSTs' attitude towards geometry learning. The potential value of such an instrument arose from the literature review which identified two gaps:

a) The few existing geometry instruments mostly measure attitudes of secondary school students towards geometry and tended to assess limited dimensions of attitude such as motivation, liking, enjoyment, and usefulness of geometry.
b) The research literature in mathematics education (e.g., Brookstein et al., 2011; Fennema & Sherman, 1976; Utley, 2007) identified and supported the need for exploration of more attitude dimensions.

Given these gaps, the work of Brookstein et al. (2011) was therefore instrumental in devising the GDS scale, with most of their items adapted for this geometry version.

The large sample size of 153 PSTs who volunteered to participate in this study has provided effective support for the development of the scale. An exploratory factor analysis with PCA technique retained four factors that accounted for a total variance of 55.063% with the first explaining 36.872%. After testing of the value of each item, 15 items were subsequently removed from the initial scale, with the GDS ultimately retaining 15 items that indicated strong correlations and construct validity. These items were further composed of four sub-scales:

Deep affect—positivity expressed towards geometry learning ($\alpha = 0.908$)
Working privately (α = 0.676)
Collaborative working ($\alpha = 0.536$)
Technology or calculator use ($\alpha = 0.785$)

Based on the psychometric properties of this instrument, the GDS was adjudged to be an effective tool, which should prove useful in measuring PSTs' attitudes to geometry learning and help inform instructors in the design of the geometry programs for our PSTs. We thus recommend the GDS for other researchers interested in examining the attitudinal profile of PSTs in learning geometry, which in turn may contribute further to the value of the instrument.

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Developing Pre-service Teachers' Understanding of Numeracy

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In Australia, all teachers are expected to explicitly attend to numeracy in the subjects they teach. Pre-service teachers, therefore, need to begin to develop effective numeracy teaching strategies but there is a limited research base to inform the design of courses that address this need. This paper reports on findings from a study investigating the impact of one such course. The paper draws on data from two pre-service teachers: two course tasks completed at the beginning of the course and interviews conducted at the end of the course. Data were analysed using the 21st Century Numeracy Model. Findings suggest there were changes to the pre-service teachers' understanding of numeracy and their capacity to embed numeracy in the subjects they will teach.

Numeracy (or mathematical literacy) encompasses the capacity to use mathematics effectively across a wide range of contexts—in private and public life situations and for civic participation (Geiger, Goos, & Forgasz, 2015). The question of how to best provide opportunities for students' numeracy development, however, remains unanswered with a range of approaches being adopted internationally (e.g., European Commission, 2011; South African Department of Basic Education, 2011). Two forms of curriculum integration that appear to be showing promise are interdisciplinary enquiry and across the curriculum approaches (Geiger, Goos, & Forgasz, 2015). The latter approach takes advantage of numeracy learning opportunities inherent in all subjects and is the approach that has been adopted in Australia.

Numeracy is identified in the *Australian Curriculum* (Australian Curriculum, Assessment and Reporting Authority, n.d.) as a one of seven General Capabilities to be developed in all learning areas. Consequently, all teachers have a responsibility to explicitly address numeracy inherent in the subjects they teach. The place of numeracy in the Australian Curriculum is supported by the Australian Professional Standards for Teachers (APSTs) (Australian Institute of Teaching and School Leadership [AITSL], 2011). Graduates of Initial Teacher Education (ITE) programs, for example, must be able to demonstrate knowledge and understanding of "numeracy teaching strategies and their application in teaching areas" (p. 13). As part of accreditation requirements, ITE providers must provide evidence of how programs support pre-service teachers to meet this standard through whole of program approaches that may include courses with a specific focus on numeracy. The research base to inform the design of courses that build pre-service teachers' capacity to explicitly address numeracy in the subjects they will teach is, however, just beginning to emerge (e.g., Bennison, 2019; Forgasz & Hall, 2019). The aim of the study reported on in this paper is to add to this research base.

The study builds on earlier work with practising teachers in which teacher identity was used as a theoretical lens to investigate the learning and development of teachers participating a larger study that supported them to embed numeracy into subjects across the curriculum. The earlier work developed a framework for teacher identity as an embedder-of-numeracy and provided insights into the development and trajectory of this identity (Bennison, 2022). The present study shifts the focus to pre-service teachers and investigates how a course in an ITE program—*Literacy and Numeracy Across the Curriculum*—contributes to shaping a future teacher's initial identity as an embedder-of-numeracy.

The framework for teacher identity as an embedder-of-numeracy (Bennison, 2022) is organised by five Domains of Influence: Life History, Knowledge, Affective, Social and Context. Each Domain includes factors particularly relevant for teachers embedding numeracy across the curriculum. The Life History Domain, for example, encompasses a teacher's past experiences of mathematics, pre-service teacher education, and initial teaching experiences.

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These factors will contribute to forming a pre-service teacher' s attitudes towards mathematics, personal conception of numeracy and influence their beliefs about the place of numeracy in the subjects they will teach (factors in the Affective Domain). Factors in the Life History Domain also play a role in developing pre-service teachers' knowledge to design tasks that develop students' numeracy capabilities while enhancing subject learning (factors in the Knowledge Domain). While it is difficult to isolate the impact of a single course within an ITE program on a pre-service teacher's identity as an embedder-of-numeracy, the aim of the present study is to investigate how the *Literacy and Numeracy Across the Curriculum* course contributes to a pre-service teacher's initial identity as an embedder-of-numeracy. Specifically, the research question addressed in this paper is: What impact does the *Literacy and Numeracy Across the Curriculum* course have on secondary pre-service teachers' understanding of numeracy and their capacity to embed in the subjects they will teach?

Theoretical Framework

Because of the number of terms used and the range of perspectives taken in research on numeracy, researchers need to be explicit about the interpretation of numeracy that underpins any study (Geiger, Goos & Forgasz, 2015). The conceptualisation of numeracy articulated though the *21st Century Numeracy Model* developed by Goos, Geiger and Dole (2014) underpins this study and is the theoretical framework used to analyse participating teachers' conceptualisations of numeracy. This model was chosen because (1) it is consistent with widely accepted definitions of numeracy (e.g., OECD, 2019) and (2) Goos and colleagues have demonstrated the utility of the model for analysing classroom activities, interview transcripts, and other data (e.g., Geiger, Goos, 2015).

The 21st Century Numeracy Model comprises five dimensions: *context*, *mathematical knowledge*, *tools,* and *dispositions* which are embedded in a *critical orientation* (for a full description of each of the dimensions of the model see Goos et al., 2014). The use of the model as a theoretical framework is illustrated by Geiger, Goos and Dole's (2015) analysis of a task developed by a teacher participating in one of their research and development projects. The teacher designed a task for a physical education class in which students used a pedometer to investigate their level of physical activity. Students recorded the number of paces they took over a week and converted the total number of paces to the distance travelled. Individual and class data were analysed using the graphing tool in Excel. Geiger, Goos and Dole identified the following dimensions of numeracy in the activity: a problem in a life-related *context*; *mathematical knowledge*, including measurement and ratio, and selection of an appropriate graphical representation of the data; *tools*, including pedometers, tape measures, calculators, and an Excel spreadsheet; *dispositions* were addressed through the use of digital tools to enhance student engagement and requiring students to think flexibly about how to represent their data; and students were required to adopt a *critical orientation* when comparing their results and speculating on possible differences between themselves and others. A similar approach can be used to utilise the 21st Century Numeracy Model to analyse text such as interview transcripts.

Research Design

The study commenced in 2018 and was funded by a School of Education Pilot Research Grant. It was conducted with successive cohorts of pre-service teachers enrolled in the *Literacy and Numeracy Across the Curriculum* course at a university located in Queensland. The course was offered in Semester 2 and during the Summer Semester. Ethics approval was granted for the study by the university's Ethics Committee (A181104) and all participants gave informed consent. Some findings from the 2018/19 Summer Semester cohort are reported in this paper.

Participants

All pre-service teachers enrolled in the *Literacy and Numeracy Across the Curriculum* course in the 2018/2019 Summer Semester were provided with information about the study and invited to participate. Thirty-four of the 66 pre-service teachers enrolled in the course gave informed consent to participate in the study. Five of these pre-service teachers also gave additional informed consent to be interviewed.

The Course: Literacy and Numeracy Across the Curriculum

The *Literacy and Numeracy Across the Curriculum* course was a compulsory course for all pre-service teachers enrolled the Master of Teaching (Secondary) program, which could be completed over 2 years or in eighteen months if an accelerated pathway was chosen, and dual degree programs (e.g., Bachelor of Arts/Bachelor of Education), which were completed over four years. The *Literacy and Numeracy Across the Curriculum* course was co-taught to postgraduate and undergraduate cohorts with differences in course learning outcomes and assessment. There were no prerequisites so pre-service teachers could enrol in the course in any year of their program. The course comprised a Numeracy Module taught by the author of this paper and a Literacy Module taught by a literacy specialist. The main aim of the Numeracy Module was to assist pre-service teachers to identify and plan for numeracy demands and opportunities in the subjects they will teach, thereby assisting them to develop the knowledge and understanding of numeracy appropriate for their intended teaching area as required by the APSTs (AITSL, 2011). The module promoted engagement with the 21st Century Numeracy Model (Goos et al., 2014) as a tool for planning and analysis of tasks. Pre-service teachers were given opportunities to explore and analyse tasks developed by practising teachers in earlier projects, such as the use of timelines in science to understand geological time (Bennison, 2015) and the use of pedometers in Health and Physical Education to monitor physical activity (Geiger, Goos & Dole, 2015). The course content for this part of the course was very similar for both postgraduate and undergraduate offerings as it was assumed that the pre-service teachers had limited previous knowledge of how numeracy can be embedded in subjects across the curriculum.

The Summer Semester offering of the course was delivered using an intensive teaching mode. All teaching took place during Week 1 (mid-November) of the Summer Semester. Teaching was face-to-face and took place in form of lectorials that combined interactive activities with the presentation of course content. Students undertook independent study and completed assessment tasks over the remaining 6 weeks of the semester which concluded in mid-January.

Data Collection

All pre-service teachers enrolled in the *Literacy and Numeracy Across the Curriculum* course were asked to complete a Numeracy Confidence Survey and Understanding Numeracy Task during the first lectorial for the Numeracy Module in Week 1 of the Summer Semester. The purpose of these tasks, as well as serving as sources of data for the study, was to get pre-service teachers to reflect on their current understanding of numeracy and preparedness to embed numeracy in the subjects they were going to teach. Both data collection instruments have been used in previous research (e.g., Goos et al., 2014). The Numeracy Confidence Survey contained 22 items around the domains of professional knowledge, attributes and practice. Respondents were asked to rate their confidence on each of the items (e.g., Understand the meaning of numeracy within their curriculum area; Promote active engagement in numeracy learning within their own curriculum context) using a 5-point Likert-type scale (1 = very unconfident to 5 = very confident). For the Understanding Numeracy Task, respondents completed five numeracy stems (Numeracy involves ...; A numerate person knows …; A numerate person is …; A numerate person can …; An individual's numeracy can be improved by …). Only data from those pre-service teachers who had consented to participate in the study were analysed.

Semi-structured interviews were conducted in February 2019 after finalisation of grades for the *Literacy and Numeracy Across the Curriculum* course. Pre-service teachers were asked about their backgrounds (reasons for becoming a teacher, mathematics subjects studied at school, feelings about mathematics, intended curriculum areas), perceived role in developing the numeracy capabilities of their students, experiences in relation to numeracy during their Supervised Professional Experience (teaching practicum) and reflections on the Numeracy Module. Interviews lasted between 21 and 33 minutes, were audio recorded and transcribed.

Data Analysis

The Likert-type scale ratings for items on the Numeracy Confidence Survey are reported for the two pre-service teachers whose case studies are presented. No further analysis of these data is required for the purposes of this paper. Analysis of the Understanding Numeracy Task and interview data was through the lens of the 21st Century Numeracy Model (Goos et al., 2014). The five dimensions of the model were used to code written responses and transcripts, respectively. Comparing findings from data collected at the beginning of the course with findings from the interviews provides insights into changes in the pre-service teachers. Case studies of Sally and Jodie (pseudonyms) were chosen to represent different learning trajectories of participating pre-service teachers.

Findings

Sally

Sally was enrolled in the accelerated pathway of the Master of Teaching (Secondary). Her teaching areas were Biology and Agricultural Science. She had completed 12 months of the program, including two science curriculum courses, when she enrolled in the *Literacy and Numeracy Across the Curriculum* course. Sally's previous degree was in science, with a major in Agriculture. She had worked in agribusiness and other industries before deciding to become a teacher. Sally had completed two mathematics subjects involving the study of calculus in her final two years of schooling and in the interview described her previous experiences of mathematics as "very positive, I love maths".

Sally's responses to the Understanding Numeracy Task included that a numerate person knows "how to manipulate quantities to make decisions based on accurate information" and numeracy involves "interpreting, understanding and applying patterns and proportions in our everyday life". These responses indicates that her understanding of numeracy included *mathematical knowledge*, *context*, and *critical orientation*. Her lowest self-rating for any item in the Numeracy Confidence Survey was 3 (*unsure*). Sally rated herself as unsure for two items: *demonstrating knowledge of a range of appropriate resources and strategies to support students' numeracy learning* in her curriculum area and *recognising the knowledge and experiences that learners bring* to her classroom. Thus, Sally began the course with an understanding of numeracy that included three of the five dimensions in the 21st Century Numeracy Model and she was unsure how to effectively embed numeracy into science lessons.

In the interview following the completion of the course, Sally was asked if she thought her views about numeracy had changed in any way since she started the course. The response she gave was in the context of her teaching:

> Well, I definitely see the difference between numeracy and maths in that numeracy is the application to real life. Similarly, to literacy, I would incorporate it more and I think I was challenged to – by being challenged to incorporate it – forgot the name of the model [referring to the 21st Century Numeracy Model] … By, yeah, not just thinking, okay, what's the maths in it, the mathematical skills but how does it relate to the kids? How can you make it authentic and use critical numeracy thinking?

Sally did not express any concerns about addressing numeracy in science because "maths is sort of part of science". She was able to describe how explicit attention to numeracy can enhance understanding of size in science:

> Biology, the unit typically starts with the size of cells and looking at cells and this whole proportional – I mean it's maths, numeracy, no, more numeracy and getting them to think about the size of cells. That cells can be the size of a water molecule through to the size of a human egg. To be able to appreciate that size difference and then that – you can build on that to go to the next topic, which is membrane, the cell membrane. Appreciating the proportional size of things and making that real, relative to a millimetre. A millionth of a millimetre is what you're looking at.

Later in the interview she came back to the issue of size in science, relating the geological timeline example discussed in the course to understanding the vast distances between the planets and the very small units used for measuring the size of cells:

> Having those examples, so the timeline for example, for me that was like okay, well it's not just time, but space. So, the different sizes and that there's lots of little videos you can watch but pulling it away from a video and trying to make it hands on, even for something that's nanometres is a challenge. But I think you can get there, just as that exercise about how far the planets are. The standard model doesn't give a true representation of the space and so a simple exercise where you – this is the earth, now relative scale [pause] walk it out. I think you could do the same sort of thing at the nano size.

Sally was also keen to encourage students to adopt a *critical orientation* towards the results of experiments as she saw this as a skill that was transferrable beyond school:

> Do rough estimates in your head. Could that answer be roughly, right? If you're doing a water rocket balloon, so let's think through it roughly. So, this is a skill that they would be able to take into their daily life.

Sally's understanding of numeracy and her capacity to address numeracy in science seems to have been enriched by her participation in the *Literacy and Numeracy Across the Curriculum* course. Her responses to the Understanding Numeracy task included *mathematical knowledge*, *context*, and the need to adopt a *critical orientation*. Following her participation in the course, the examples Sally gave in the interview indicate that her understanding of numeracy now also included also included *tools* and *dispositions*. The use of *tools* is implicit in the need to "walk it out" to show relative sizes and distances and, it could be argued, that her desire to "make it authentic" and "relate to the kids" is evidence that she understands the importance of dispositions. Sally's examples and her descriptions of how explicit attention to numeracy helps understanding in science indicate that she can identify numeracy in this subject and is likely to have developed some strategies for embedding numeracy in science.

Jodie

Jodie was enrolled in a Bachelor of Arts/Bachelor of Education (Secondary) and had completed two years of her program when she enrolled in the *Literacy and Numeracy Across the Curriculum* course. Her teaching areas were English and History. Jodie's previous studies included two history courses and an English curriculum course, but she was yet to enrol in any history curriculum courses. In her final two years of schooling, Jodie completed a single mathematics subject that did not include the study of calculus. In the interview, she described her school experiences of mathematics in the following way:

> I hated maths. I won't lie. It was definitely English, History or the creative stuff were all of my strengths. I guess it was just because picking it up was hard and falling behind. I would just constantly get frustrated with it. So that's why. Yeah, I just grew a hate for it.

In the Understanding Numeracy Task, Jodie wrote that numeracy involves "problem solving, mathematical understanding (formulas etc)" but also indicated that a numerate person can "apply these problem-solving skills within other subject areas e.g., measurements of an archaeological site (History)". Her understanding of numeracy, therefore, included the use of *mathematical knowledge* in the *context* of history. Jodie rated herself as 2 (*unconfident*) on three items in the Numeracy Confidence Survey: her capacity to *demonstrate knowledge of appropriate resources and strategies to support students' numeracy learning*; *model ways of dealing with numeracy demands;* and *providing all students with opportunities to demonstrate numeracy knowledge* in her curriculum area. She rated herself as 3 (*unsure*) on ten additional items. Jodie, therefore, began the course with an understanding of numeracy that included two of the five dimensions in the 21st Century Numeracy Model and was unconfident or unsure about many aspects of how to effectively embed numeracy into history.

In the interview following the completion of the course, Jodie was asked if she had recognised numeracy in any of the history courses she had studied previously. Her response indicated that she saw the mathematical aspects she encountered in these studies "as a part of learning history" and that she had a growing awareness of the use of mathematics in the broader context of everyday life:

> I've noticed definitely since the literacy and numeracy course just how maths is so important for everyday things that we do. Yeah, you've definitely made me aware of that.

Jodie said that before completing the *Literacy and Numeracy Across the Curriculum* course, she had found it "very daunting that we actually have to do that [explicitly attend to numeracy in history]. Because I think it's a case of if I can't fully understand it myself how can they expect me to teach it to kids". When asked if she could now see how she could incorporate numeracy in history, she was able to provide several examples:

> Yeah, in history, population. How it's decreased and increased over time in certain civilization. Or, what's another good example? Yeah, timelines. Understanding timelines … Something to do with archaeological sites. So, if you compare the pyramids of Egypt to, I don't know, some other historical site to do with the Romans maybe, the ancient Greeks. You can compare how big the site there is in comparison to Egypt maybe. I'm just throwing an idea out there. Of course, it would have to align to the curriculum … Maybe even looking at the architecture, Roman architecture and comparing that to the South American architecture and how big it was. That sort of thing.

Jodie said that her ideas about numeracy in history had changed since completing the *Literacy and Numeracy Across the Curriculum* course, perhaps leading to increased confidence in her capacity to embed numeracy in this subject:

> Yeah, they have [her ideas about numeracy in history]. I still feel they need to be developed more … but I do feel I have more of an idea of how to approach it and what I could set as a task if I just really work hard at developing a few ideas. I think you've shown that such simple things, graphs and timelines, are considered numeracy. So, it's not all big and scary and not all about algebra and everything.

There were changes to Jodie's understanding of numeracy and her capacity to address numeracy in history following her completion of the *Literacy and Numeracy Across the Curriculum* course. At the beginning of the course, Jodie's understanding of numeracy encompassed the use of mathematical *knowledge* in *contexts*. Although she did not mention the use of *tools* explicitly, representational tools (tables of population data, graphs, and timelines) were evident in the examples of numeracy in history she provided in the interview. There is no evidence to suggest she saw *dispositions* and *critical orientation* as important aspects of embedding numeracy in history. Jodie's apprehension about addressing numeracy in history, something she initially described as "very daunting," seemed to have lessened. She seemed more aware of numeracy and was able to provide several examples of numeracy in history thereby suggesting that she could *see* numeracy in this subject and may have developed some strategies for embedding numeracy in history.

Discussion

A framework for identity as an embedder-of-numeracy was developed and used in earlier research as an analytic lens through which to view the identities of practising teachers as they developed the capacity to embed numeracy into the subjects they were teaching (Bennison, 2022). Teachers' past experiences of mathematics and their Initial Teacher Education program were seen as likely to influence their initial identity as an embedder-of-numeracy by shaping their attitudes towards mathematics, personal conception of numeracy and capacity to identify and design tasks that are effective in developing students' numeracy capabilities. Pre-service teachers enter their Initial Teacher Education programs with varied past experiences that result in different levels of mathematical knowledge, attitudes towards mathematics and understandings of numeracy. Sally and Jodie, for example, commenced the *Literacy and Numeracy Across the Curriculum* course with very different previous experiences of mathematics at school, resulting in different levels of mathematical knowledge and extreme attitudes towards mathematics—Sally's mathematical knowledge could be considered more comprehensive than Jodie's based on the subjects they had completed at school. Furthermore, Sally loved mathematics whereas Jodie hated it. There were also differences in the way the two pre-service teachers conceptualised numeracy.

The *Literacy and Numeracy Across the Curriculum* course appears to have had an impact on both pre-service teachers' understanding of numeracy and their capacity to embed numeracy in the subjects they will teach. Sally and Jodie were able to articulate a richer personal conception of numeracy following their participation in the course. Sally, it could be argued, had a much richer understanding of numeracy than Jodie both before and after the course. Her understanding initially encompassed adopting a critical orientation to the use of mathematical knowledge in real-world contexts and, following the course, she gave examples of numeracy in science that included references to tools and dispositions. Jodie, on the other hand, began with a narrower understanding of numeracy that seemed to encompass only the use of mathematical knowledge in contexts. The examples of numeracy in history she gave following the course included references to the use of tools but there was no evidence that she saw dispositions or a critical orientation as part of numeracy. Before completing the course, neither pre-service teacher was confident in her capacity to demonstrate appropriate strategies to support students' numeracy learning in their respective curriculum areas. Jodie was unconfident or unsure on many aspects of embedding numeracy in history whereas Sally's responses to the Numeracy Confidence Survey indicated greater confidence than Jodie on these aspects. Following the course, both pre-service teachers were able to provide several examples of how numeracy could be embedded in their respective teaching areas, indicating that they could see numeracy opportunities in these subjects. The *Literacy and Numeracy Across the Curriculum* course, which promoted the use of the 21st Century Numeracy Model (Goos et al., 2014), is likely to have contributed to the changes observed in both pre-service teachers. Sally referred to the model even though she could not remember its name, and Jodie now thought she "had more of an idea how to approach it [numeracy in history]."

Strong claims cannot be made about the impact of the *Literacy and Numeracy Across the Curriculum* course on pre-service teachers' understanding of numeracy and their capacity to embed numeracy in the subjects they will teach because of the small number of pre-service teachers who participated in the study. One of the challenges of conducting any form of research, particularly with pre-service teachers, is dealing with recruitment and attrition. Even though approximately half of the cohort agreed to participate in the study, only a small number completed the two course tasks during the Summer Semester and none, not even the five pre-service teachers who were interviewed, completed the Understanding Numeracy Task and Numeracy Confidence Survey that were emailed to them in February 2019. The findings presented in this paper do, however, add to those from an earlier offering of the course where increased levels of confidence in addressing numeracy were found among pre-service teachers who completed the course (Bennison, 2019).

Concluding Remarks

In Australia, all teachers are required to explicitly address numeracy inherent in the subjects they teach (ACARA, n.d.). Pre-service teachers, therefore, need to begin to develop the capacity to do so during their Initial Teacher Education program. The study reported in this paper investigated the impact of the *Literacy and Numeracy Across the Curriculum* course on pre-service teachers' initial identity as an embedder-of-numeracy (Bennison, 2022). The aim of the study was to address a gap in the literature by contributing to the research base informing the design of courses in Initial Teacher Education programs. Specifically, this paper focussed on the impact of the course on pre-service teachers' understanding of numeracy and their capacity to embed numeracy in the subjects they will teach Findings suggest that the course had some impact on Sally's and Jodie's understanding of numeracy and their capacity to identify how it could be embedded in science and history, respectively. The findings also highlight the very different starting points and learning trajectories for pre-service teachers completing such courses. The challenge remains to encourage participation in the study to further grow the research base underpinning the design of courses supporting pre-service teachers to embed numeracy in the subjects they will teach.

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RESEARCH PAPER

Exploring Visual Representations of Multiplication and Division in Early Years South African Mathematics Textbooks

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Early years mathematics textbooks are support material for teachers and learners as they contain visual representations which communicate and clarify mathematical concepts. In this paper, we report on the types and functions of visual representations of multiplication and division in South African early years mathematics textbooks. There seems to be a reliance on textbooks amongst teachers in South Africa as they assist in providing pedagogic content knowledge. The research from which this paper emerged was a document analysis of multiplication and division visual representations in nine textbooks (Grade 1–3). A Visual Representation Framework was used to analyse the textbooks. The findings indicated that the most dominant type of visual representations across all three textbook series and across the three grades were equal groups and the majority of visual representations had an exemplifying function.

Spaull and Taylor (2015) maintain that only 58.6% of South African Grade 6 learners are functionally numerate. The Southern and Eastern Africa Consortium for Monitoring Education Quality IV (SACMEQ) results attested to this and highlighted that South African learners had difficulties with multiplication and division (DBE, 2011). In response to the need to improve the South African learners' performance in mathematics, the Department of Basic Education (DBE) introduced a series of mathematics textbooks for Grades R to 7 learners in public schools. The textbooks were intended to standardise the content learning covered in each grade. This paper focuses on textbook research and asks the question: *What are the types and functions of visual representations of multiplication and division in early years (Grades 1–3) mathematics textbooks in South Africa?* Although the research was set within South Africa, textbooks are used internationally. We thus maintain that this research is of relevance to many countries worldwide and illustrates the application of the visual representation framework as an analytical tool.

Literature review

Ben-Peretz (1990) asserted that textbooks were the leading pedagogic tool used by teachers in trying to understand the intended curriculum. In many countries this is still the case. Textbooks have been used predominantly as a monitoring tool to inform teachers on what to teach, when to teach and how to teach (Nicol & Crespo, 2006). Textbooks provide a framework that informs teachers about possible ways to teach the required content (Nicol & Crespo, 2006) and assist teachers with learner assessment.

During the COVID-19 pandemic and ensuing lockdown, textbooks became an essential resource for learners during the closure of schools. In contexts such as South Africa, where most learners do not have access to modern technologies and data, the use of textbooks provided the only opportunity for learners to develop an understanding of multiplication and division concepts. This issued was raised more than 20 years ago by Harries and Spooner (2000).

Mathematics is a language rich in visual representations (Mudaly & Rampersad, 2010). Visual representations assist learners in developing an understanding of mathematics as they support and clarify mathematical concepts (Presmeg, 1986). The visual representations in a textbook may assist non-expert readers in understanding a mathematics concept (Fotakoupoulou & Spiliotopoulou, 2008; Mazumder et al., 2020). Presmeg (2006) asserted, a

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 90–97. Launceston: MERGA.

visual representation can also assist learners in making connections between and across mathematics concepts.

There are two broad views on the order in which to teach multiplication and division. First, Harries and Barmby (2007) asserted that mathematics is hierarchical and that the number operations should be taught sequentially. They proposed that number operations be taught in the following order: addition, subtraction, multiplication and division. In other words, they suggested that multiplication be taught before division. Second, Nunes and Bryant (1996) asserted that multiplication and division should be introduced to learners simultaneously to understand the inverse relationship between the two operations (multiplication and division). Although it is some time since these recommendations were published, the practices have endured in South African classrooms.

The South African Curriculum and Assessment Policy Statement (CAPS) for Mathematics in the early years (Grades R–3) introduces repeated addition leading to multiplication before repeated subtraction leading to division, grouping objects into groups and sharing a set of objects into groups. In the CAPS document, division is viewed as separate from multiplication and multiplication is taught first (Askew et al., 2019; DBE, 2011).

Theoretical Framework

This paper is underpinned by constructivism. Constructivism involves an active and meaningful learning process in the classroom (Kukla, 2000). Learners use *signs* and *tool*s to assist in making meaning and mediating the learning process (Daniels, 2008). Signs are defined as an internal psychological activity that involves mastering the tool used (Vygotsky, 1978). Veraksa (2013) suggested that a sign mediates the transitions between speech, actions, and concept formation. Tools are cultural artifacts that control behaviour from the outside (Daniels, 2008) and assist learners in accomplishing an activity. Signs and tools are mediated by actions (Werstch, 1997). In this study, the tools are the textbooks that learners use in the classroom.

Vygotsky (1978) argued that cognitive processing is still necessary for learning to occur and that signs and tools are auxiliary means of problem-solving. Learners construct meaning by using visual representations (Yackel, 2001). Thus, the visual representations used in textbooks are essential in supporting understanding of mathematics concepts as they are the signs and tools used to mediate the learning process of multiplication and division in the early years (Nghifimule, 2016).

Visual Representation Framework

A Visual Representation Framework developed by Fotakopoulou and Spiliotopoulou (2008) was used to analyse the nature of visual representation in textbooks. Fotakopoulou and Spiliotopoulou's (2008) framework examines the type of visual representation, its relation to content and reality, and the function of the visual representation. This paper focuses specifically on the type and function of the visual representation in the textbooks.

The visual representations *type* refers to the nature of the visual representations evident in the textbooks, such as equal groups, arrays, tables, function diagrams and number lines. The *function* of a visual representation refers to the purpose of the visual image and its relation to what is taught. The different functions include a decorative function, explanatory function, exemplifying function, complementary function, and organising function. A decorative function is a visual representation that serves aesthetic purposes. An explanatory function assists in expanding the concept to assist learners in completing the exercise. An exemplifying function provides an example of how to solve the problem. A complementary function is a visual representation accompanying the exercise to be completed. It provides the learner with additional information that might not be required to solve the problem. An organising function assists learners in structuring their thought processes (Fotakopoulou & Spiliotopoulou, 2008).

Methodology

An interpretivist orientation underpinned this research as the focus was meaning-making, interpreting, and understanding (Cohen et al., 2018) the types and functions of visual representations in textbooks. This research study has a subjective view of reality based on the researchers' interpretations of the textbooks analysed.

The research included both qualitative and quantitative data. Qualitative research allows for a thick description of the data, while quantitative research focuses on basic statistical methods (Apuke, 2017). A mixed-methods approach allows different areas of the research process to blend, for example, the research questions, orientations and methodologies (Cohen et al., 2018). By combining qualitative and quantitative data, the researcher can develop a more comprehensive understanding of the research question (Creswell, 2014). A mixed-method approach allows greater insights into the type and function of visual representations found in South African textbooks. The benefit of a mixed-method study is that it increases the credibility of the research results. Document analysis was used to generate the data. The sample included nine South African early years (Grades 1 to 3) textbooks. Three of the textbooks were the Department of Basic Education (DBE) workbooks and six of the textbooks were from two textbook series widely used in South Africa.

The textbooks were analysed using the Fotakoupoulou and Spiliotopoulou (2008) Visual Representation Framework. Initially, we analysed a Grade 4 textbook separately to ensure inter-rater reliability. After that, the first author focused on the Grades 1 to 3 textbooks. This involved identifying the types and functions of visual representations of the multiplication and division content present in the textbooks. The frequency of the different categories (i.e., the types of images and the function of each visual representation across the nine textbooks) was tallied using an excel spreadsheet.

Data Analysis and Findings

Types of Visual Representation

This section presents some of the Grade 1, 2 and 3 textbook analysis findings on the visual representations used to support learners' understanding of multiplication and division. The findings focus on the types and function of visual representations in the nine Grades 1 to 3 South African textbooks.

Across the three textbooks, Textbook A has the most visual images (n =282), followed by Textbook C (n = 76) and then Textbook B (n = 55). From Table 1 it is evident that the textbooks had more than double the number of multiplication visual representations (n = 209) compared to division visual representations (n = 83). There were 47 examples that consisted of both multiplication and division visual representations and 74 that were categorised as "other". This category included problems focused primarily on doubling and halving. This category was more evident in Textbook A.

There are a total of 122 images in the Grade 1 textbooks (A, B and C). As noted in Table1, Textbook A has the most visual images. Textbook A has 80 more visual representations than Textbook B.

The Grade 2 textbooks had a total of 128 images. Textbook A had 78 and 82 more images than Textbooks B and C, respectively (Table 1). There were 163 visual representations across the three Grade 3 books. Textbook A had 93 visual representations, Textbook B had 24 and

Textbook C had 46 visual representations (Table 1). Across all three grades, Textbook A has the most visual representations.

Table 1
Number of Visual Representations in Each Grade of Three Different Textbooks

Visual Representations of multiplication and division problems	Textbook A			Textbook B			Textbook C			Total
	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	
Number of visual representations	93	96	93	13	18	24	16	14	46	413
Multiplication	60	54	50	4	13	9	4	8	7	209
Division	12	17	11	6	5	8	0	1	23	83
Both Multiplication & Division	2	0	22	0	0	7	0	0	16	47
Other (Doubling, halving.)	19	25	10	3	0	0	12	5	0	74

A single exercise in a textbook, might consist of more than one type of visual representation (Figure 1). As noted in Table 2, the most prominent visual representations across the three textbooks are equal groups (256) followed by arrays (67). In Grade 1, Textbook B only has examples of equal groups and arrays while Textbook A includes number lines and Textbook C, a function diagram (Table 2). The Grade 2 textbooks have a wider variety of visual representation than Grade 1 (Table 2). Textbook A has more types of visual representations than Textbooks B and C. Textbook A is the only textbook that has multiplication grids, tables and unifix blocks in Grade 2 (Table 2). Textbook A and C have all the different types of visual representations listed in Table 2 except for multiplication grids and unifix cubes. In Grade 3, Textbooks A and C have number lines, equal groups, arrays, tables and function diagrams, Textbook B has no tables.

Table 2
Types of Visual Representations in Textbooks

Type of visual representations	Textbook A			Textbook B			Textbook C			Total
	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	
Number lines	4	12	10	0	3	3	0	1	9	42
Multiplication gird	0	2	0	0	0	0	0	0	0	2
Equal groups	105	57	26	9	7	10	10	10	22	256
Array representation	18	9	13	5	5	2	4	0	11	67
Tables	0	3	12	0	0	0	0	0	6	21
Function diagram	0	0	21	0	0	6	1	2	1	31
Unifix cubes	0	8	0	0	0	0	0	0	0	8

Figure 1 is an example of three different types of visual representations, that is, equal groups, arrays and number lines. This example was taken from Textbook A, Grade 2.

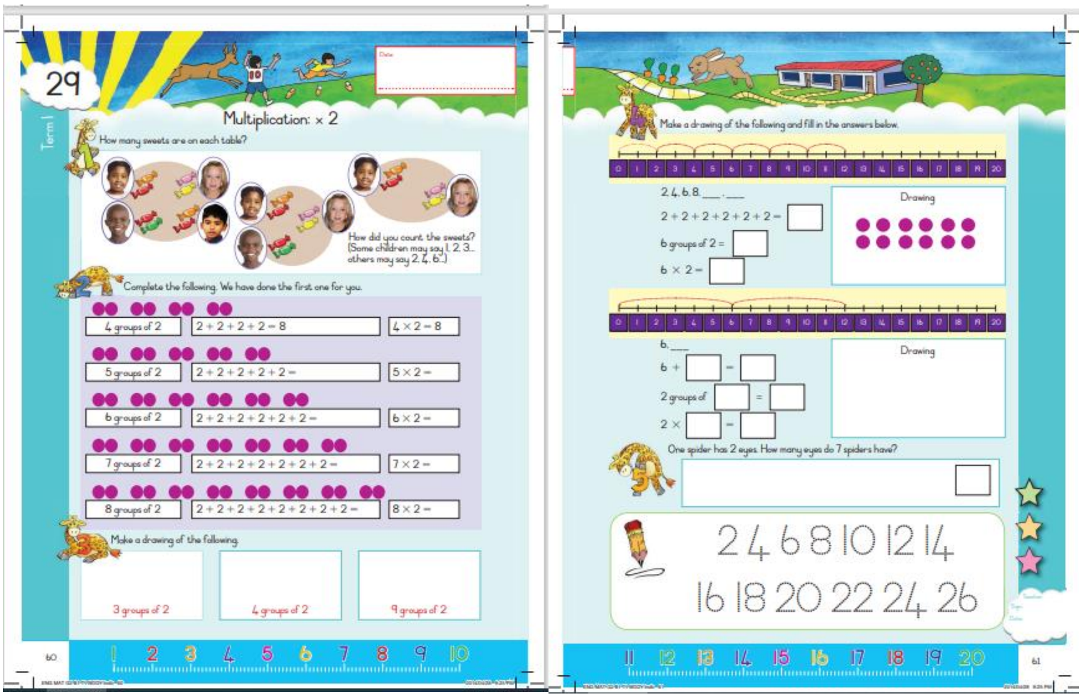

Figure 1. Equal groups and the number line.

In the exercise numbered 2 on page 60 of Figure 1, learners are expected to form a link between 4 groups of 2 as 2 + 2 + 2 + 2 (repeated addition) and 4 x 2 (multiplication). In the exercise numbered 4 on page 61 of Figure 1, the learners are required to make the link between skip counting, repeated addition, groups of and multiplication using a number line and drawing. The drawing is given in array form.

Function diagrams (n = 31) appear in all three textbooks but are most prominent in Textbook A (Figure 2). However, in Grade 1, Textbook C is the only textbook that exposes Grades 1 and 2 learners to function diagrams. A function diagram defines a relationship between the input number and the output number using a rule. For example, the function diagram provided in Figure 2 shows the relationship between the input (n = 11) and output (n = 55) by implementing the rule (x 5). After identifying the types of visual representations in the textbooks, analysis of the function (purpose) of each visual representation was undertaken.

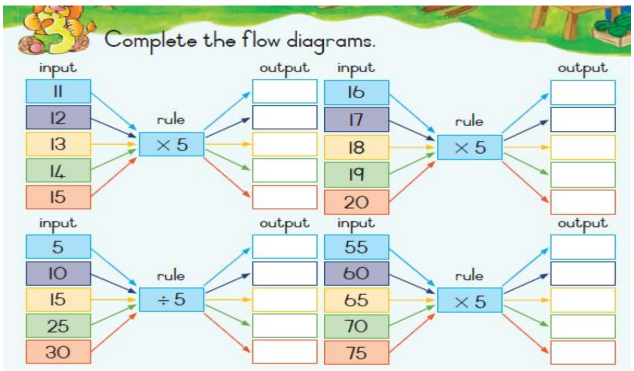

Figure 2. A function diagram.

The Function of the Visual Representations

Table 3 presents the functions of the multiplication and division visual representations across all Grade 3 textbooks. The most prominent visual representations had an exemplifying (n = 353) function followed by a complementary (n = 40) and explanatory function (n = 21). A visual representation with a decorative function (n = 2) was only evident in Textbook A.

Table 3
The function of visual representations in textbooks

Function of visual representations	Textbook A			Textbook B			Textbook C			Total
	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	
Decorative	0	0	2	0	0	0	0	0	0	2
Explanatory	4	3	3	0	0	4	0	0	4	21
Exemplifying	88	91	68	12	11	17	15	12	39	353
Complementary	0	2	22	0	7	3	1	2	3	40
Organising	0	0	0	0	0	0	0	0	0	0

Figure 3 is an example of a visual representation with an exemplifying function. The example shows that 5 x 4 is represented visually as 5 groups of 4 and 4 x 5 is represented visually as 4 groups of 5.

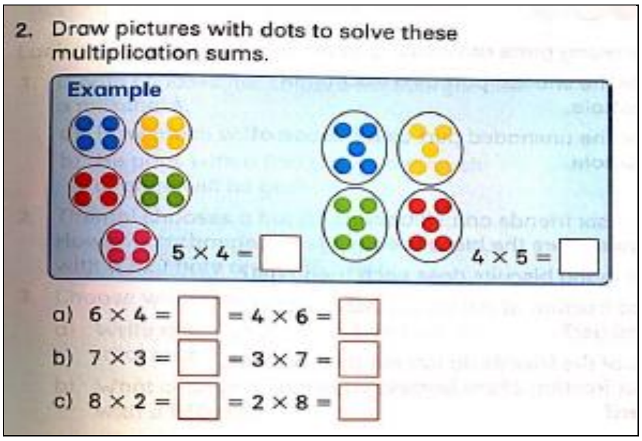

Figure 3. A visual representation with an exemplifying function (Textbook B).

Visual representations with a complementary function (n = 40) were evident across all the textbooks except for Grades 1 in Textbook A and B. Figure 4 is an example of a complementary function. The visual representation includes a stack of four books. Without the visual representation, the learners would still be able to solve the problem.

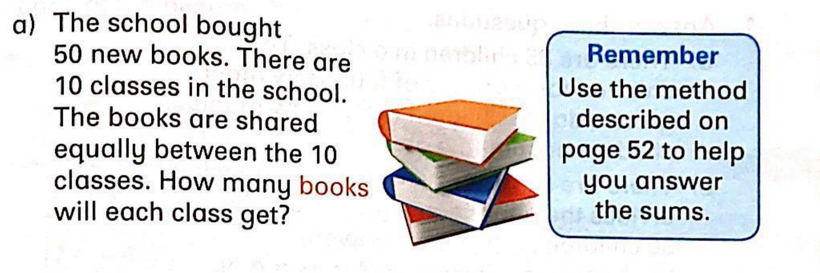

Figure 4. A visual representation with a complementary function (Textbook B).

Textbooks A, B and C all had examples, albeit very few, of visual representations with an explanatory function (Table 2). An example of a visual representation with an explanatory function is included in Figure 5. The speech bubbles show how the two learners solved each word problem.

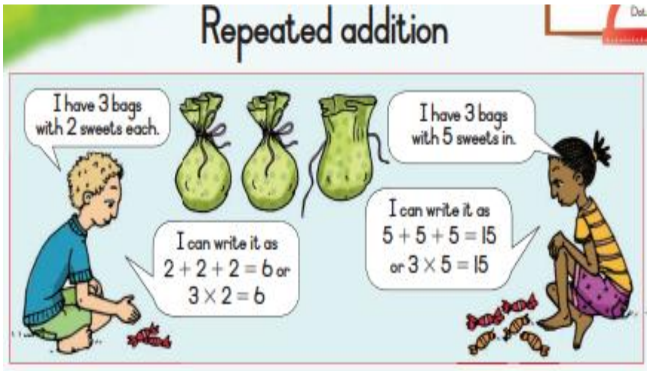

Figure 5. A visual representation with an explanatory function (Textbook A).

Discussion and Conclusion

Visual representations assist learners in understanding mathematical concepts (Kosko, 2019). In this research, equal groups were the most common visual representation type across all three textbooks. This is not surprising given the emphasis on multiplication and division as grouping in the curriculum document. CAPS (DBE, 2011) emphasised the different types of visual representations, namely equal groups, arrays, multiplication grids and number lines. However, there is a misalignment between the curriculum and the textbooks as the types and frequency of the visual representations differed. Textbook A in Grade 2 was the only textbook with a multiplication grid. Learners exposed to Textbook B and C were not exposed, through use of the textbook, to this type of visual representation as CAPS suggested.

Textbook B and C had fewer visual representations than Textbook A. This is concerning as learners use visual representations to make sense of the problem. Greeno and Hall (1997) ascertained that learners needed to be able to draw on multiple representations of a problem in order to develop their mathematical thinking fully.

The most common function of the visual representations across the grades in the three textbooks was an exemplifying one. An exemplifying function is crucial as it provides the learner with support in how to solve the problem. The second most prominent function, an explanatory function, was found in all grades in Textbook A, but only in Grade 3 in Textbooks B and C. An explanatory function elaborates on the thinking process necessary to solve the problem.

This study found that textbooks for early years contained written explanations for young learners who may or may not be proficient in reading and comprehending the explanation. The learners may depend on the teacher to explain the explanatory visual representations. The second most prominent function in Textbooks A, B and C in Grade 2 and 3 was a complementary function.

The government-provided textbook (Textbook A) had the most visual representations and reflected a greater variety of visual representations. Textbook B and C were the most popular textbooks in South Africa, yet they had very few visual representations to support learners. An implication of this study is the importance of quality textbooks with a variety of visual representations and their alignment with the intended curriculum.

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Mathematics and Coding: How Did Coding Facilitate Thinking?

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This paper reports on teachers' perceptions of students learning as part of a project examining teacher practice and student learning when using *ScratchMaths* in their classroom programmes. The project used design-based methodology, incorporating video-recorded classroom excerpts; teacher interviews; and teacher analysis and review of their practice. The teachers identified the students' problem solving, using unplugged activities, and collaborating using explicit mathematical and coding language as ways to facilitate mathematical thinking. They also recognised that their practice became more faciliatory, with their understanding of coding developing through learning with their students.

In 2017, the technology education curriculum in New Zealand (NZ) was altered to include coding as part of computational thinking, with the overall aim to develop core programming concepts. The new Digital Technology Curriculum (DTC) became mandatory for NZ primary-school aged children in 2020 as directed by the Ministry of Education ([MoE], 2017). As well as digital coding, it involves unplugged activities, using authentic contexts to develop precise, step-by-step instructions for non-digital activity, and debugging of errors that emerged as the instructions are enacted (MoE, 2017). However, research indicates that many NZ teachers would find implementing DTC challenging due to proficiencies such as coding (Crow et al., 2019; Education Review Office, 2019). Crow et al. (2019) indicated a gap in the availability of resources that are specifically situated in curriculum contexts, such as mathematics, that would practically assist engagement with coding. They also advocated that teachers and schools develop unique implementations, suitable for their school context.

This paper reports on aspects related to teachers' perceptions of student learning in a small two-year research project that examined teacher practice with coding through the use, evaluation and adaption of UCL's *ScratchMaths* resources, and the associated student learning. This two-year project aimed to support teacher learning in DTC through using, evaluating and modifying the University College London (UCL) *ScratchMaths* project resources to enhance teachers' computational thinking-based pedagogies. It also aims to address the limited resources available for teaching coding in NZ by evaluating and adapting the UCL *ScratchMaths* resources. The intention was that the project will impact positively on learners' computational and mathematical thinking. *ScratchMaths* is a highly effective resource to develop coding and computational thinking for primary-aged children in the UK school context, which is different to NZ. Hence, another aim was to evaluate and modify these effective UK resources to enhance their applicability in the NZ context.

While research has evaluated similar curriculum implementation at NZ high-school level (Johnson et al., 2017) and international research has examined some aspects of DTC (e.g., Falkner et al., 2014; Johnson et al., 2014) very little research has examined the implementation of DTC in the NZ primary-school context. There has been minimal research on the use of coding in NZ schools, hence the implementation of the DTC would benefit from being analysed by a collaborative partnership of teachers and researchers, as teachers consider its effectiveness when integrated into existing classroom practice.

Scratch is a free-to-use graphical programming environment that provides opportunities for creative problem-solving. It is a media-rich digital environment that utilises a building block command structure to manipulate graphic, audio, and video aspects (Peppler & Kafai, 2006). Studies have shown its potential for developing computational and mathematical thinking in an integrated way, particularly in geometry and algebraic thinking (Calder, 2018).

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 98–105. Launceston: MERGA.

ScratchMaths aims to integrate computing and mathematical thinking effectively. Coding, particularly with *ScratchMaths*, is identified as being influential on the development of mathematical thinking (Benton et al., 2018) and the understanding of mathematical ideas such as algorithms and the 360-degree turn (Benton et al., 2017). However, the *ScratchMaths* resources, while well-tested and effective resources, are designed to be undertaken by students individually, whereas in NZ learning is seen as a more collaborative, creative process (MoE, 2007). For instance, the development of collaborative student-led projects in *Scratch* (e.g., Calder, 2018), which might also emerge with *ScratchMaths*, would be conducive to collaborative problem solving. The paper reports on the teachers' perceptions of students learning through problem solving, including when debugging script, collaborating, and engaged in "unplugged" activities. It is primarily focussed on two classes of nine to eleven-year-old students, using *ScratchMaths* on laptops and iPads.

Problem Solving through Collaboration

Collaborative Problem Solving

While discussing collaborative problem solving, collaborative learning is first discussed, together with its potential to improve learning and understanding. Ways that collaboration supports learning when digital technologies are used, and the influence of both in facilitating problem solving are then briefly identified. The connection between collaborative problem solving and blending digital and unplugged experiences is then considered. Collaborative learning occurs when two or more students are engaged in an activity, interacting with each other and learning together (Dillenbourg, 1999). This perspective of learning in mathematics repositions learning more as participation in a social practice than as an acquisitional process (e.g., Sfard, 1998). Collaboration when problem solving in mathematics has been linked to enhanced understanding. For example, Mercer and Sams (2006) showed how students collaborating while engaged in tasks indicated enhanced learning outcomes in mathematics. Other studies contend that the collaborative use of digital technologies supports students in developing more flexible approaches to problem solving (e.g., Mercier & Higgins, 2013).

Mercer and Littleton's (2007) definition of collaborative learning goes beyond the sharing of ideas and task coordination to "reciprocity, mutuality and the continual (re)negotiation of meaning" (p. 23). Collaborative learning in line with this definition involves the utilization of individual understandings and expertise, with the collaborative interaction influencing the thinking of at least one participant in the interaction, even if there is only a minor adaption, coupled with a repositioning of the learners' perspective and understanding. When students work collaboratively on a task there is frequently a coordinated approach to the sense making and the approach taken when engaging with the task. The joint coordination of a task enables students to communicate and negotiate in order to support decision-making (Zurita & Nussbaum, 2004), and, as such, they are involved in "a coordinated joint commitment to a shared goal" (Mercer & Littleton, 2007, p. 23).

In general, digital technologies enable opportunities to explore and organise data to see patterns and trends more quickly in mathematical situations that might otherwise be too complex. With coding, they offer potential to learn through the interaction between the coding and the output that the coding generates. The coder can try something and instantaneously identify the effects of the new coding, enabling them to generalize coding attributes and refine their approach. With a visual environment such as *Scratch*, where the coding and output screen sit side by side, these relationships are easily identified (Calder, 2018).

Computational Thinking

Computational thinking can be considered a collection of problem-solving skills that relate to principles of computer science (Curzon et al., 2009). Abstraction, allied with logical thinking, innovation, and creativity, is considered central to the constitution of computational thinking (Wing, 2006). These elements also resonate with mathematical thinking and problem solving in mathematics. *ScratchMaths* is an interactive, intuitive space for problem solving.

Research has indicated that students become more engaged when using digital technologies, with enhanced mathematical learning also evident (e.g., Attard, 2018; Bray & Tangney, 2015; Pierce & Ball, 2009). With regards to using mobile technologies in the process of learning mathematics, Attard (2018) concluded that they improve student engagement at operative, cognitive, and affective levels. Additionally, studies have indicated that *Scratch* was an effective medium for encouraging communication and collaboration (e.g., Calder, 2018). This paper considers teachers' observations and perspectives of the students' problem solving, thinking and collaboration as they undertook coding tasks using *ScratchMaths*.

Research Methodology and Design

This research project used a design-based methodology, to examine teachers and their students' use of the *ScratchMaths* resources. The teachers were co-researchers in the process. This methodology, designed by and for educators, endeavours to enhance the implementation of educational research into improved classroom practice (e.g., Anderson & Shattuck, 2012). It can illuminate the challenges of implementation, the processes involved, and the associated pedagogical and administrative elements (Anderson & Shattuck, 2012). Design research involves multiple cycles, which include a number of different design and research activities. Nieveen and Folmer (2013) divide these activities into three distinct phases: the preliminary research phase; the prototyping or development phase; and the summative evaluation phase. These three phases, involving the teachers and including videoing of their classes, were implemented through iterations of use, reflection and modification of the resources and the associated pedagogy.

In the first year, the project involved two teachers in one primary school, who were co-teaching in a shared learning setting with 52 students. In the second year, the project included six teachers in four schools, including three new schools. This second year encompassed teachers and classes covering the junior, middle and senior levels of the primary school system (aged 7 to 12 years old). Design-based research principles and findings reflect the context and associated conditions in which they take place (Anderson & Shattuck, 2012), so providing space in the second year for new participants, in new conditions, recognised the importance of new contexts and participants emerging through the design process. Most of the data presented in this paper were related to the two teachers involved in both years.

As well as the purposive sample spanning the range of primary-school levels, there was also variance in the nature and organisation of the schools. Included in the participant schools are single classroom situations, innovative learning environments (double and larger), and a range of student and teacher-centred learning environments. The teachers also had varying levels of teaching experience and expertise with digital technology. All the schools comprised of a range of ethnicities. The research design was also aligned with teacher and researcher co-inquiry whereby the university researchers and practicing teachers work as co-researchers and co-learners (Hennessy, 2014). Allied to this was an emphasis on collaborative knowledge building. The research design was based on a transformational partnership arrangement that aims to generate new professional knowledge for both academic researchers and teachers (Groundwater-Smith et al., 2013).

The *ScratchMaths* resources identified by the teachers to use initially, were from module one and included: moving, turning and stamping a sprite, and creating circular rose patterns. The *ScratchMaths* resources and existing projects were used as starting points for the lessons, with the unplugged activities also incorporated into the sessions. Some of these class sessions and individual groups working on the tasks were video recorded. There were two iterations of the review and design process with videoing of classes each time, followed by co-researcher meetings to examine the classroom practice. One element of these meetings was the analysis of classroom video recordings. Discussions in the meetings were recorded, as were the teacher interviews. Analysis of the qualitative data from the interviews and observations was through thematic analysis with the research team identifying the initial themes, and the nodes for the NVivo analysis drawn from these themes. The data from the interviews and observations went through six phases of thematic analysis adapted from Braun and Clarke (2006).

The research question related to this paper is:

> *In what ways might the use of coding embedded within a mathematics curriculum context, influence teacher practice and children's coding and mathematics thinking?*

This research question was addressed through teachers and their students engaging with the *ScratchMaths* and modified resources, followed by teacher reflection, and re-modification of the materials and pedagogical approaches by the full research team. Episodes from the video-recorded observational data were analysed by the teacher researchers and the resources reviewed through this and their in-class experiences. Several new materials and pedagogical approaches were developed through this process.

The research project gained approval from the University of Waikato ethics committee. This approval included having all participants being invited to participate, giving informed consent (and participant assent for the student participants), confidentiality (e.g., transcriber confidentiality agreements), anonymity (e.g., use of pseudonyms), mitigation of the potential influence of power differentials, and participants' right to withdraw. Validity was enhanced through: the design of the project matching the purpose of the research questions; using a range of methods to generate the data; the design of the analysis plan; the range of contexts and participants (given the place of context in design-based research); the frequency of design iterations, the collaborative teacher/researcher research team; and ongoing peer-review of the formative findings through the research team and their colleagues.

Results and Discussion

This paper reports on teachers' perceptions of how using *ScratchMaths* facilitated the learning process in three key areas: problem solving when debugging script, working collaboratively, and using unplugged activities.

Debugging Script

The teachers consistently commented on how using *ScratchMaths* fostered a problem-solving approach as the students collaboratively debugged their codes to make them work. The process of debugging code was a particular aspect that many students became immersed in. This is a part of computational thinking that involves reviewing the code through trialling and when it didn't produce the desired output, problem-solving for possible solutions. It might also involve the output unexpectedly stopping or going into continuous loops. While the aspect of debugging was highlighted by the teachers, usually students were self-motivated with this process through wanting the script to be consistent with their expectations of the output. Marama commented on the student debugging process:

> There would not be many things that would have them that focused on what they're doing so intensely. They would be doing debugging the whole time.

The students found solutions to unfamiliar problems in mathematical contexts, through a variety of approaches. For example:

> Annie: The children were problem solving, risk taking and learning from failure

> Marama: For some activities there are no instructions for how to get them from there to there, they just had to work it out.

The students use of *ScratchMaths* within the problem-solving process at times led to enhanced engagement. The teachers identified that the students not only appeared more cognitively engaged but that the process facilitated enjoyment.

> Marama: Even though its heavy-duty problem solving, they're having fun, they're smiling and enjoying working with each other too. It's talking about what they are doing and its excited talk.

Some of the problem solving involved particular mathematical thinking. The teachers also indicated that the mathematical thinking related to both concepts and processes arose more naturally within the *ScratchMaths* activities. For instance:

> Annie: I think because maybe the opportunities with this program and what it's actually focused on with the angles and the measurement side and the negative numbers … that's probably been more cemented than what it could have been if we had been teaching it in isolation.

While the teachers made the mathematical thinking explicit to the students by referring directly to the mathematics and using mathematical language, some of the mathematics thinking emerged through attempting to solve and accomplish the tasks, and the collaboration on the coding aspects. In this way, some of the learning was initiated when the need arose, and not confined to the expected curriculum level for that age group.

> Annie: It was just-in-time learning around the maths concepts. The use of angles was very in-depth. They used negative numbers, degree turns and always mathematical language.

Negative numbers are not part of the curriculum for this particular age group. In a later discussion the teachers identified some of the other mathematical thinking that occurred: identifying patterns and relationships, exploring variations, precision with language, methodical thinking, and strategies for problem solving. Their spatial awareness, understanding of angles, and positioning sense through the use of coordinates, were all engaged to varying degrees. There was also evidence of relational thinking as students made links between their input, the actions that occurred on screen, and the effect of specific variations of size in coding procedures. They discussed how the students negotiated meaning, came to conclusions and gave explanations of what they had done. They predominantly did this through collaboration.

Collaboration

The students interacted with each other as they investigated the problems and explored potential solutions. They then collaborated on the debugging to make their codes more efficient. As they worked to design the scripts and subsequently make the codes more economical, they shared ideas and potential solutions using language that included coding terminology or was related to the mathematical or coding processes that they were discussing. The teachers noted this in the interviews. For instance, Annie indicated how the collaboration fostered their shared understanding of language, and hence from her perspective, their mathematical and computational thinking:

> Annie: So, it gives a context for social interaction to happen where they're learning to code and learning maths.

> Annie: So, then we can look at different ways of how children create a script to get to an end product and look at just simplifying the script.

Marama identified instances when students found efficient ways to code that were valued by other students, enhancing their mana (respect) within the class. Sometimes this was the students who were not usually perceived as being more capable in mathematics, so it readjusted those perceptions.

> Marama: There are kids that are capable but then someone quietly just comes up with this really simple code to do something that someone else has taken a long time to do and they think they're good so it's kind of just levelled everyone out.

This also indicated how using *ScratchMaths* facilitated collaboration. Collaborative learning can be perceived as going beyond the sharing of ideas and task coordination to the ongoing negotiation of perspectives and meanings (Mercer & Littleton, 2007). Collaborative learning in line with this definition was identified:

> Annie: It supported students' learning through communicating with friends, problem solving, increasing their mathematical knowledge and mathematical and coding language, bringing that all into the norm of how we can talk about coding.

> Marama: They're all in different roles all the time, sometimes they're teachers, sometimes they're learners.

While the ongoing negotiation and evolving perspectives are evidenced here, this also indicates that the students' roles were flexible and contingent on personal, and group understandings. Observational data also suggested that there was contestation of ideas during the collaborative work. Not only did the students interact through the ongoing dialogue as they problem solved to find solutions, but students also did at times became leaders of learning.

> Marama: One of the girls solved this thing that really no-one else was managing to do and she managed to crack it. Well, the whole class was whoosh over there, so that's fantastic that she's having to explain it and off they go all excited.

Within this collaboration, students often drew insights from the unplugged activities.

Unplugged Activities

The unplugged activities were valuable in terms of developing instructions or codes that designated actions, including movements. Some of these were repetitions, such as a series of dance moves, and some were a single task. The children wrote code that another student would enact. Once they began to trial their script it often became clearer where the debugging was needed. A teacher from the second year of the project commented.

> Katarina: When we introduced the repeat, I tried to do something unplugged with it so we did the dance. They created the five-step dance on the grid and then they had to repeat it, and they had to work out how many times they needed to repeat it for one person to complete the grid or for two people etc. And so, it's quite good for making sure that the instructions were accurate, so that everybody got the right steps at the same time to do it and to get there (the end of the grid). And then, if it didn't work ask why it didn't work, and then that introduced the debugging.

Another teacher indicated that the unplugged activities consolidated the coding moves needed and hence assisted with the coding process.

> Annie: We wouldn't have thought to use the unplugged if it wasn't in the resource but what we have found is that using the unplugged really helps to consolidate and cement in the children's minds how to create an object or whatever it is they've been asked to create. It's like the first step and then they can go and create that object on a device.

Annie also indicated the value of the unplugged activities, as students oscillated between the coding in the app and a physical activity. This helped them with developing the code and with debugging it. She identified that the unplugged activity moved their thinking.

> Annie: Also, in the teaching (of coding) when children are struggling, it's good to go back to that process so they physically do it using the unplugged... I think what I'd do more is using unplugged more.

A fourth aspect reported here is the teachers' pedagogical approach, which varied from their usual approach when teaching mathematics.

> Marama: I don't know that I need to know everything. Most of the time it's the kids that are the ones that solve things. They are learning off each other a lot more, they're going to each other a lot more, they're talking a lot more.
>
> Annie: The classroom approach is to explore, but the mathematics and coding objectives are explicit. At times (we) start with *ScratchMaths* for say, angles. There is a purposeful context for the learning.
>
> Marama: The teachers' role is facilitating learning – actively scaffolding processes and content.

The teachers were consistent in their belief that positive student learning had occurred including through student collaboration when problem solving. They were also consistent that their personal learning of coding processes had developed markedly, while acknowledging that their role in the classroom had evolved.

Conclusions

Although findings are presented separately, in practice, they were mutually influential elements that the teachers perceived had contributed to student engagement and learning. The teachers' perceptions of students' learning suggested that learning through *ScratchMaths*, including the unplugged activities, facilitated collaborative problem solving, enhancing student engagement and learning. The teachers' perceptions of their teaching approach when working with *ScratchMaths* indicated that the process influenced their practice, moving them towards a more faciliatory approach. The students' thinking and learning in coding were tied to their solving of both mathematical and coding problems, while the explicit language of both seems to have contributed to the communication of processes, concepts and solutions. These processes appeared to facilitate thinking. Students at times became leaders of the learning as well.

There was also conceptual understanding and thinking related to the Geometry and Measurement strand of the NZ curriculum; in particular, angles and spatial perception. However, the process the participants undertook, in the perceptions of the teachers, facilitated thinking through the collaborative problem-solving it evoked, and the development of logic and reasoning as they debugged code, negotiated understanding, and responded to various forms of feedback.

While the findings were limited by the size of the project, and the particular contexts in which they were enacted, they nevertheless give insights into the ways learning in coding, including unplugged activities, might be enhanced through the *ScratchMaths* resources. The research is ongoing, with the teachers in the project now leading this learning in their schools. Further research into the assessment and analysis of students' computational thinking could be undertaken which might give more comprehensive insights.

Acknowledgement. This research was supported by the University of Waikato Research Trust Contestable Fund.

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Understanding the Relationship Between Cognitive Activation and Academic Emotions: A Comparison Between Students with Different Mathematics Achievements

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Previous studies have identified the relationship between cognitive activation and academic emotions. However, little is known about the underlying process behind this relationship. Considering that cognitive activation strategies may have different effects on students of different abilities, latent multi-group structural equation modelling was used to compare the mechanism mentioned above. Findings showed that (1) academic self-concept and value significantly mediated the link between cognitive activation and academic emotions among students with different mathematics achievements; (2) the mediating effects of academic self-concept and value are more significant in the below-average students than in the above-average students.

Quality of teaching, a core element of classroom environment, has been theoretically and empirically suggested to be a critical factor in impacting students' emotions and learning-related beliefs and values (e.g., Lazarides & Buchholz, 2019; Pekrun, 2006; Zhang et al., 2021). Evidence during the past decade showed that teaching characteristics, such as clear and activating teaching, well-organised classrooms, or supportive and cooperative teaching, could induce students' positive learning experiences and lower students' negative emotions (Goetz et al., 2020; King et al., 2012; Zhang et al., 2021). As an essential component of teaching quality, cognitive activation integrates the critical characteristics of mathematics teaching, including activation of prior knowledge, challenging tasks, and participation in practice related to the teaching content (Klieme et al., 2009). What remains scarcer is the knowledge about how cognitive activation is associated with emotions in the learning situation. As implied by the control-value theory (Pekrun, 2006), cognitive activation may, directly and indirectly, affect students' emotions. Specifically, teaching may influence students' emotions via two cognitive appraisals (i.e., academic control and value appraisals). However, such a mediation assumption of the control-value theory has been rarely tested. Therefore, the study reported in this paper examined the underlying process behind the relationship between cognitive activation and academic emotions. In line with the definition of Pekrun (2006), academic emotions are defined as emotions closely related to students' academic success or failure.

Another purpose of this study was to compare the link mentioned above between cognitive activation and academic emotions among students with different prior mathematical knowledge. Prior research work revealed that students with high mathematical performance reported more positive emotions (e.g., pride and enjoyment) but less negative emotions (e.g., anxiety and boredom; Holm et al., 2017; Wu et al., 2014). However, few studies have examined the effect of activating teaching on students' emotions, focusing on their different mathematics achievements. Klieme et al. (2009) have described that cognitive activation may increase students' negative emotions, particularly for students with mathematical difficulties. Reflection on this topic would shed more light on how an activating teaching approach could be employed in mathematics, considering students' different prior knowledge.

Theoretical Framework

The control-value theory provides a sound framework in examining the link between cognitive activation and academic emotions (Pekrun, 2006). One central idea of the control-

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 106–113. Launceston: MERGA.

value theory is that the classroom environment is a distant influencing factor of students' emotions, with proximal antecedents, such as academic control and value, having an indirect influence on emotions. Control appraisal is often conceptualised as academic self-concept or self-efficacy in existing research. It refers to students' competence beliefs in achieving academic success or completing learning-related tasks well (Pekrun, 2006). Academic self-concept is used in this study because students' confidence in mathematics learning is emphasised rather than their capability to solve specific mathematics problems successfully. Value appraisal denotes students' perceived significance or importance of learning mathematics (Pekrun, 2006).

In education, researchers classified academic emotions in three dimensions: valence (i.e., positive or negative emotions), activation level (i.e., high- and low-activating emotions), and object focus (i.e., activity- and outcome-focused emotions; Pekrun, 2006). For example, enjoyment is a positive and high-activating emotion focusing on the activity itself, while anxiety is a negative and high-activating emotion focusing on the outcome. The detrimental effect of negative emotions on students' engagement and outcomes has attracted more research attention, such as anxiety and boredom (Passolunghi et al., 2019; Preckel et al., 2010; Yi & Na, 2020). Good learning experiences, however, accompany positive emotions, contributing to the enhancement of students' engagement and interest and, in turn, their academic performance (Zhang et al., 2021). In the past decade, enjoyment received increasing attention with positive psychology. Existing evidence enhanced our understanding of the promoting role of enjoyment in mathematics achievement (Tze et al., 2021), classroom engagement (Liu et al., 2019), and homework effort (Luo et al., 2016). Given that enjoyment, anxiety, and boredom are frequently reported by students (see Pekrun et al., 2002), these three emotions will be measured in the study, as will the fact that these three emotions cover the three dimensions of academic emotions.

Cognitive activation refers to a wide range of teaching practices that enable students to participate in higher-level and abstract thinking, promote conceptual understanding by connecting facts and ideas, and provide opportunities for students to reflect on, evaluate, and openly discuss their perceptions (Förtsch et al., 2017). In a recent longitudinal study, Lazarides and Buchholz (2019) observed that cognitive activation was favorably related with enjoyment but negatively connected with boredom; besides, the influence of cognitive activation on anxiety was not significant. Contrary to this conclusion, Kunter et al. (2013) only found the relationship of cognitive activation with mathematics achievement but not with enjoyment. Concerning the relationship between cognitive activation, cognitive appraisals, and academic emotions, although currently there is no empirical evidence, such a relationship might exist. More recent research provided limited evidence on the relationship between cognitive activation and students' self-concept (Kitsantas et al., 2021) that would further influence students' emotions according to the control-value theory. Lastly, given that students' prior mathematical knowledge is an essential factor in determining academic emotions (Pekrun et al., 2017), further exploring the relationship between variables in students with different mathematics achievements is also very necessary for the activating strategy use in the classroom.

Current Study

Using the control-value theory and the previously provided empirical information, this study explores the cognitive activation/academic emotions relation via control and value appraisals in two groups of students with high and poor mathematics achievements. Evidence showed that students' emotions begin to decrease as their age increases, especially during the transition from primary to secondary school (Vierhaus et al., 2016). Meanwhile, the development and modification of academic emotions start in the early stages of adolescence

(Pekrun et al., 2002). Furthermore, students' emotions in mathematics are more intense than in other disciplines (e.g., Science and English; Goetz et al., 2006). Combining with the domain-specificity of academic emotions (Goetz et al., 2007), junior high school students in mathematics classrooms were investigated in this study. The following two research questions framed the study:

(1) What are the relationships among cognitive activation, cognitive appraisals and academic emotions in above- and below-average students?

(2) Are there any differences in the relations among students with different mathematics achievement?

Method

Participants

This study comprised 470 middle-school students who participated in the survey that included questions about quality of teaching, cognitive appraisals, and academic emotions. These students were randomly selected from 16 mathematics classrooms in three schools in Jiangsu, China. Educational resources were abundant in Jiangsu and intense educational reforms had been implemented to improve students' academic performance and provide positive learning experiences. There were 220 boys and 247 girls who participated in the investigation, and three other students who failed to report their gender. Furthermore, 203 students were in Grade 7, 233 in Grade 8, and 32 in Grade 9, with two students failing to report their grade. Ethical approval was obtained from the Human Research Ethics Committee of the author's university before data were collected.

Instrument

In addition to demographic information (e.g., gender and grade), the current study contains six self-reported variables (i.e., cognitive activation, control and value appraisals, enjoyment, anxiety, and boredom). All variables were assessed using a 5-point Likert scale that ranged from 1 (strongly disagree) to 5 (strongly agree).

Eight items from the PISA 2012 Student Questionnaire (Organisation For Economic Co-operation and Development, 2012) were used to assess cognitive activation. Respondents were asked to evaluate how often a set of facts came up in class with their mathematics teacher. The sample item is "The teacher provides problems in various situations so that students may assess their understanding of the concepts." Because of the growing criticism leveled against Cronbach's alpha, which may exaggerate the reliability of results, McDonald's omega coefficient was employed in the current study (Dunn et al., 2013; Flora, 2020). The omega coefficient for cognitive activation was 0.918, implying that data had good reliability.

Academic self-concept was examined with four items using the Motivated Strategies for Learning Questionnaire (MSLQ; see Pintrich, 1999). The sample item is "I am confident I can understand the basic concepts taught in mathematics." Similarly, three items from the MSLQ were used to measure academic value. The sample item is "I am very interested in the content area of mathematics." The omega coefficients for self-concept and value were 0.856 and 0.766, respectively.

Academic emotions were measured with 12 items, which were selected and modified from the Achievement Emotions Questionnaire-Mathematics (AEQ-M; Pekrun et al., 2005). The following are some examples of items. "I am motivated to attend to mathematics class because it is fascinating" was used to assess enjoyment, "I am concerned that others will comprehend more than me" was used to assess anxiety, and "When I feel bored, my mind begins to wander" was used to assess boredom. The omega coefficients for enjoyment, anxiety and boredom were 0.890, 0.770 and 0.919, respectively.

Students' mathematics achievement was reported by themselves. In Jiangsu, students are not given specific scores after taking the examination but grades (i.e., A-plus, A, B-plus, B, C, and D). A denotes excellence, B represents the average, C denotes below-average, and D indicates failure. In this study, students who obtained A-plus, A, and B-plus were classified into the above-average group, while the other students were classified into the below-average group.

Data Analysis

Missing data should be considered before examining the mediation assumption. The missingness of each measurement item is no more than 0.4%. Results of Little's MCAR test showed that the missingness was not entirely at random (χ^2 (278) = 382.518, p < .001). Therefore, it is not recommended to remove missing data directly using the Listwise delete method (Little & Rubin, 2019). Instead, the default function of full-information maximum likelihood (FIML) in the Mplus was used (Muthén & Muthén, 1998–2015).

Then, the confirmatory factor analysis (CFA) was used to test the fit indices of the measurement model. Several fit indices in the study of Hu and Bentler (1999) were reported in this study, such as *TLI/CFI* (≥ 0.95), *RMSEA* (≤ 0.06), and *SRMR* (≤ 9.08). Because measurement invariance (*MI*) is an unavoidable issue or prerequisite in multi-group analysis, configural, metric, and scalar invariance will be examined to see whether the mediation assumption between students with different prior mathematical knowledge is comparable (Shahzad et al., 2020). A series of fitting indices of *MI* were proposed. Byrne (2001) argued that a significant $\Delta\chi 2(\Delta df)$ means a significant change in the model fit. As a result, the differences in model fit of the three nested models were assessed using changes in the chi-square and the degree of freedom.

Results

Preliminary Results

Table 1 shows the descriptive and correlation analysis of the measured variables. In addition to gender and cognitive activation/academic emotions, other variables were correlated. Furthermore, results of the measurement model suggested a good model fit, with several fit indices showing that χ^2 (193) = 425.820, *TLI* = 0.961, *CFI* = 0.953, *RMSEA* = 0.052 and *SRMR* = 0.043. The factor loading of the measurement items ranged from 0.556 to 0.907. As Hair et al. (2006) described, a good convergent validity necessitates significant standardised factor loading higher than 0.5.

Table 1
Descriptive and Correlation Analysis Among Main Variables

	M (*SD*)	1	2	3	4	5	6
1. Gender	-	-					
2. CA	4.357 (.656)	-.079	-				
3. Self-concept	3.469 (.857)	-.226**	.326**	-			
4. Value	4.202 (.617)	-.125**	.434**	.475**	-		
5. Enjoyment	4.199 (.643)	-.066	.433**	.487**	.515**	-	
6. Anxiety	3.071 (1.075)	.070	-.112*	-.363**	-.186**	-.251**	-
7. Boredom	1.680 (.763)	.065	-.504**	-.391**	-.476**	-.541**	.268**

Note. CA means cognitive activation. **p < 0.01. *p < 0.05.

Table 2 shows the results of the measurement invariance test, and findings supported the configural and metric model ($\Delta\chi 2$ = 19.287, Δdf = 16, $p > 0.05$) but not the scalar model ($\Delta\chi 2$ = 48.335, Δdf = 32, $p < 0.05$) in the above-average and below-average students. According to Cheung and Rensvold (1999), a multi-group model is acceptable when the weak invariance (i.e., metric invariance) is supported.

Table 2
Results of the Measurement Invariance Test

Model	χ^2	*df*	*CFI*	*TLI*	*RMSEA*	*SRMR*	$\Delta\chi^2$	Δdf	*p*-value
1.Configural	780.821	388	.926	.912	.070	.062			
2. Metric	800.108	404	.926	.915	.069	.066	19.287	16	$p > 0.05$
3. Scalar	829.156	420	.923	.916	.069	.071	48.335	32	$p < 0.05$

Results of the Multi-group Analysis

The latent multiple-group structural equation model of all samples indicated a good model fit (χ^2 (418) = 838.200, *CFI* = 0.921, *TLI* = 0.913, *RMSEA* = 0.070, *SRMR* = 0.086). Figure 1 presents the results of the multi-group analysis between students with different mathematics achievements. Results suggested the multiple mediating roles of academic self-concept and value in the relationship between cognitive activation and academic emotions. There was, however, no significant relationship between value and anxiety (or self-concept and boredom) among below-average students. Furthermore, as shown in Table 3, the relationship between cognitive activation and academic emotions was stronger among below-average students than above-average students. Lastly, by comparing the model fit between an unconstrained and a path-constrained model (Byrne, 2001), only the path from value to anxiety showed significant differences between the two groups.

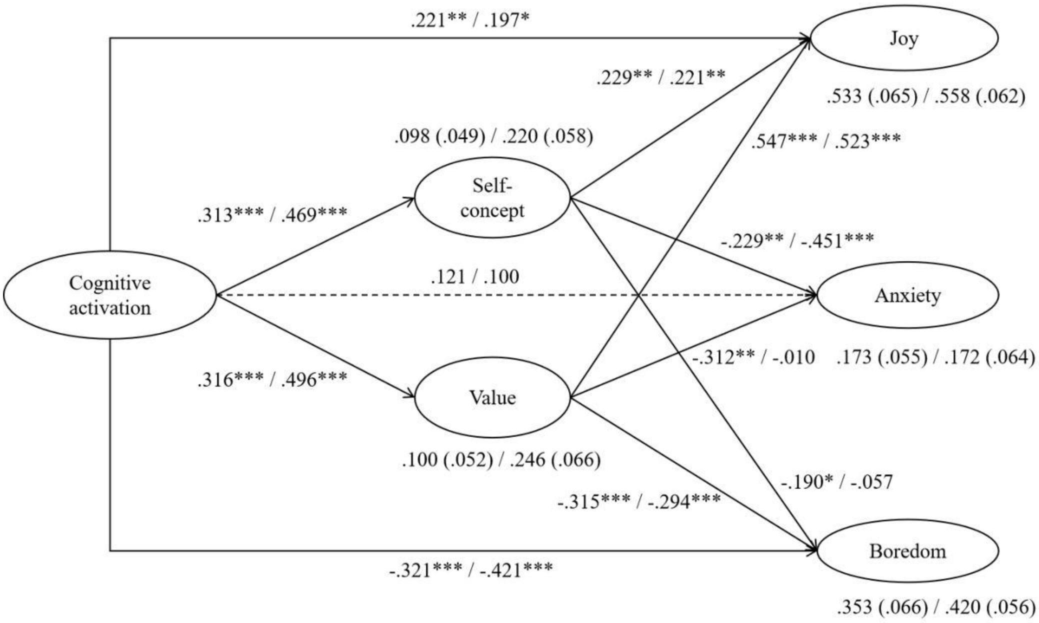

Figure 1. The mediating effects model of above-average/below-average students in mathematics.

Table 3
Indirect Effects of the Multi-group Structural Equation Model

Indirect paths	Above-average students		Below-average students	
	Standardised β	95% CI	Standardised β	95% CI
CA→Self→Joy	.072*	[.009, .135]	.104**	[.041, .178]
CA→Value→Joy	.173**	[.072, .274]	.259***	[.142, .377]
CA→Self→Anxiety	-.094*	[-.176, -.012]	-.211***	[-.318, -.105]
CA→Value→Anxiety	-.099*	[-.183, -.014]	-.083	[-.097, .087]
CA→Self→Boredom	-.060	[-.122, .033]	-.027	[-.096, .043]
CA→Value→Boredom	-.100*	[-.178, -.022]	-.146**	[-.232, .061]

Note. ***$p < 0.01$. **$p < 0.01$. *$p < 0.05$

Conclusion

The predictive power of cognitive activation on academic emotions was examined in students with different mathematics achievements, focusing on middle-school mathematics classrooms. In line with the control-value theory (Pekrun, 2006), findings showed the multiple mediators in the cognitive activation-emotions relationship. For example, for students of different abilities, cognitive activation enhanced their positive learning experience by enhancing their competence beliefs and academic values. Findings regarding enjoyment mean that the joy or fun students get from mathematics learning was related to their control and value perception, supporting previous findings that value and control are two essential antecedents of enjoyment (Buff, 2014; Goetz et al., 2010). However, such a mediation assumption does not always exist among below-average students. Specifically, academic self-concept mediated the teaching-anxiety relation, while academic value mediated the teaching-boredom relation among the below-average students. That finding means, for below-average students, their anxiety was more rooted in a lack of confidence while their boredom was strongly related to the less value perception in mathematics. Activating teaching approaches could deepen students' conceptual understanding from a higher cognitive level and contribute to their value perception, resulting in less boredom during mathematics learning (Pekrun et al., 2014). Notably, Chinese mathematics teachers emphasise basic concepts in their instruction, and learners of varying ability levels are considered, whether in variation teaching, classroom interaction, or training practice (Fan et al., 2015). In daily classroom teaching, teachers usually aim to enable each student to grasp the most basic knowledge and concepts, rather than focusing only on high-achieving students' cognitive needs and development. In this sense, cognitive activation may decrease below-average students' anxiety via increasing their self-concept.

Based on the result of multiple-group latent variable analysis, this study found that the mediating effects of academic self-concept and value were significantly higher in the below-average students than above-average students. That requires further explanation, combined with consideration of the Chinese mathematics education context. Cognitive activation requires reflective or high-level thinking, which challenges students to relate new lesson content to previous knowledge or everyday life experience (Baumert et al., 2010). In Chinese mathematics classrooms, teachers often encourage students to solve the same problem in different ways; meanwhile, when a student gives an answer, the teacher usually discusses it in front of the class and asks the student to explain the answer (Fan et al., 2015; Gu et al., 2004). However, constrained by the set teaching time and a significant amount of teaching content, teachers cannot provide all students with sufficient time to think and understand what is being taught. Students with ordinary or low ability levels sometimes require more time comprehending the information and learning the meaning of various strategies. Therefore, the influence of cognitive activation on below-average students' emotions is more considerable.

The findings reported in this paper make a vital contribution to the existing literature by demonstrating different path coefficients from cognitive activation to academic emotions via cognitive appraisals. Meanwhile, there are several practical implications. When activating teaching approaches are used, teachers should give students more time to think, reflect, and understand. Besides, in daily teaching, teachers should cultivate students' confidence and transfer more mathematical value during teaching.

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Reflection Model to Facilitate Teachers' Adoption of the Constructivist Learning Design

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As teachers begin to construct an understanding of 'inquiry', and incorporate IBL into their classroom practice, they are challenged to be sensitive to the 'constructivist' nature of the CLD. This paper presents a reflection model structured to trigger thinking about beliefs in teaching and learning in order for teachers to re-examine their practice and adopt new pedagogies. The reflections by two secondary school mathematics teachers are presented as they experiment with inquiry-based learning in the CLD. The teachers showed awareness, monitoring and regulation of their teaching practices including new and existing beliefs.

The Singapore mathematics curriculum (Ministry of Education [MOE], 2012) encourages learning through inquiry where teachers "led students to explore, investigate and find answers on their own" (p. 23). However, the use of the inquiry approach was not visible. In fact, a study of the Enacted School Mathematics Curriculum involving 30 experienced and competent teachers in Singapore secondary schools shows that "direct instruction appears to be the dominant model of instruction that teachers draw on" (Kaur et al., 2019, p. 18). For teachers to engage in more inquiry-based pedagogies, literature suggests it may arise from teachers' beliefs (McLeod, 1992; Mosvold & Fauskanger, 2014; Richardson, 1996). Indeed, research studies have shown that experienced teachers' attempts to learn to teach in new ways are highly influenced by what they already know and believe about teaching, learning, and learners (Borko & Putnam, 1996). "Teachers' beliefs strongly influence their classroom practices which in turn influence their students' beliefs about mathematics and their ability to learn it" (Marshman & Goos, 2018, p. 519). "Teacher mathematics-related beliefs such as beliefs about nature of mathematics, mathematics teaching, and mathematics learning also become variables that play a role in guiding that knowledge to create meaningful mathematics learning (Purnomo, 2017)" (Siswono et al., 2019, p. 493). Literature reports that many teachers hold behaviourist beliefs, which has strong implications for the success of constructivist-oriented curricular reforms (Handal & Herrington, 2003). Modification of beliefs is possible for teachers when they engage in the process of reflective thinking (Mewborn, 2002). Reflective practices have been reported to support teachers' adoption of new pedagogies (Davis, 2006; Tan, 2016; Yaowiwat et al., 2019). However, limited information is provided to teachers on how to conduct the process of reflection, which may hamper its use (Marcos et al., 2011). This paper provides an example of a reflection tool to scaffold reflective practice. This paper also presents an account of teachers' use of the reflection tool to engage in deep reflection of their personal beliefs against their practice during their participation in the Constructivist Learning Design Project.

Reflective Practices

Reflection is a key strategy in many professional development programmes (Watanabe, 2016) to enable teachers to be aware of their own problems, for knowledge construction in

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 114–121. Launceston: MERGA.

teaching, and to promote self-regulation in teachers (Boud, 2007). Teachers are reported to become more engaged and more inclined to participate in professional development when they are given the opportunity to reflect on their practices (Yu, 2018). Reflective practice has also been reported to be a useful vehicle to mediate pedagogical innovation (East, 2014). "Reflection on beliefs appears as a crucial element and a good start toward changing beliefs and practices" (Girardet, 2018, p. 13). In professional development programmes, if teachers are led to question their prior beliefs before and during the study of alternative practices, they will be more likely to be open to considering the studied practices (Girardet, 2018). Most research papers on teacher reflection looked at phases, levels or stages of reflection (Rodgers, 2002). In the discourse analysis of 122 articles on teacher education, Marcos et al. (2011) reported the need for reflective practice to be articulated more systematically "by offering teachers more evidence-based or research validated information on what works in reflective practice" (p. 34). Clarà (2015) also suggested that the process of reflection is not well understood. The practical procedures and methods on how to reflect is not sufficiently specified for teachers to conduct the process of reflection (Marcos et al., 2011). There is a need for structure to link reflection activities explicitly with the intended change by providing reflection prompts (Girvan et al., 2016). In recent years, there are studies that reported the use of a framework or tool for teachers' reflection. Yu (2018) reports the study of the reflective practice using the critical incident framework and prompts (see Table 1) in a professional development workshop to guide the teaches' process of reflection. The prompts did not focus on teachers' beliefs. The findings show that participants did not reflect in the same way and some reflected without referring to the framework. The Teaching Learning Instrument (TLI) was used as a tool to support teacher reflection. The teachers used the prescriptive codes (see Table 1) in TLI to look out for specific instructional moves instead of teacher beliefs in their lessons that help them make specific inferences about their teaching to improve their instruction (Sabelski et al., 2019). Joseph and Atweh (2020) reported four teachers' reflection on the use of Productive Pedagogies as a tool to reform their mathematics classroom practice. However, the paper concentrated on the views of the teachers on the benefits of the Productive Pedagogies framework during classroom instructions instead of the reflection process. Girvan et al. (2016) reported on teachers' experiences of a new pedagogical approach as learners themselves in a professional development, before implementing the new approach in their own classrooms. The paper did not illuminate how the reflection prompts facilitates reflection although the use of reflection is reported to be a key feature to the PD model that supports the teachers' implementation of the new pedagogy approach in their own classrooms.

Table 1
Samples of Reflection Prompts from Studies

Critical incident framework (Yu, 2018)	Protocol/scaffolding Codes for TL1 (Sabelski et al., 2019)
Do you have an unforgettable educational experience? What happened? How did you feel about it? Why do you think it happened this way?	P1 Focus attention P2 Explain task SR Promote self-regulation WR Maintain warmth & responsiveness

A-Cube Change 2-dimensional Reflection Model for Teaching and Learning

The A^3 Change 2-dimensional reflection model provides mathematics teachers a structure to reflect their teaching by addressing their belief for better alignment of practice, knowledge, and belief (Lee et al., 2019). The model encapsulates how two dimensions of reflection interact. The first dimension comprises of the three *types of reflection,* namely (i) emotive reflection—

teacher's awareness with regard to his/her gut feeling of how successful a lesson is after completing the lesson (ii) critical reflection—teacher's detailed analysis of his/her lesson (iii) creative reflection—invitation for the teacher to create a new lesson episode based on his/her reflection in emotive reflection and critical reflection stages. The second dimension considers the teacher's *depth of reflection* (i) articulative—articulating and describing new/reinforced understanding (ii) assimilate—examining articulated new/reinforced understanding against their personal belief to assimilate and own the assimilated new schema of understanding (iii) appraisive—appraise the new schema for effective regulated use of new/reinforced understanding. Question prompts (in Table 2) were developed based on A^3 Change 2-dimensional reflection model to engage teachers in deep reflection.

Table 2

Sample of Reflective Question Prompts from A-Cube Change 2-dimensional Reflection Model

Depth/stages of reflection	Examples of reflective question prompts
Articulative—Emotive Reflection	How do I rate my lesson(s) based on a scale from 1 to 5 (1 being the worst and 5 being the best)?
Articulative—Critical Reflection	What worked? What did not work?
Articulative—Creative Reflection	What is my reinforced/new understanding?
Assimilative	Were my beliefs reinforced?
Appraisive	Any limitations to my new belief?

The CLD as an Inquiry-based Learning Approach

This study adopted the CLD, an inquiry-based learning (IBL) approach to teaching mathematics. Teachers who participated in the CLD project were provided specific teaching and learning packages involving IBL approach in the secondary mathematics classrooms. The IBL approach in this paper is akin to Teaching Through Problem Solving Approach (Cai, 2010), as reported in Cheng et al. (2021). The crux of the learning design rests on the building of students' ideas in the instruction of a mathematics concept—an embodiment of constructivist principles—hence the name Constructivist Learning Design (CLD). Unlike direct instruction with instruction followed by problem solving, the two phases in CLD starts with *problem solving phases* followed by the *consolidation phase*. In the CLD *problem-solving phase*, students first solve a complex problem that targets a concept that students were not formally taught. Students also explore and generate as many representations and solution methods (RSMs) (Kapur & Bielacyz, 2012) as they could to the problem. The teacher provides the necessary facilitation and affective support for students without revealing the solution in this phase. In the CLD *consolidation phase***,** the teacher consolidates, builds, compares and contrasts students' solutions, and discusses students' solution in accordance to the critical features of the solutions. In addition, the teacher makes "concise summaries and lead students to understand key aspects of the concept based on the problem and its multiple solutions" (Cai, 2010, p. 225) and helps to negotiate the meaning of the targeted concept. This paper addresses the following research questions: What does teacher reflection look like when using the A^3 Change 2-Dimensional reflection model during participation in the CLD project?

Methods

We use an instrumental, collective case study approach to frame the qualitative data collection and analysis in this study (Stake, 2000). Two teachers who participated in the CLD project were purposefully selected for us to gain in-depth analysis of what the teachers were reflecting using the A^3 Change 2-dimensional reflection model. The two teachers, Teacher A and B, taught in two different secondary schools in Singapore and implemented two different

CLD units, Angle Properties of Circles and Gradients of Linear Graphs, respectively. After the CLD lessons, the two teachers were invited to an individual reflection session which lasted about 30 to 40 minutes, where they were asked to reflect on their experiences in using the CLD and on their pedagogical beliefs and practices using question prompts in Table 2. The third author transcribed sections of the reflection sessions only if he identified them as articulative-the motive reflection, articulative-the critical reflection, articulative-the creative reflection, assimilative or appraisal. We met several times and used the transcripts to discuss our analysis. When we did not have agreement on the analysis, we went back to the audio recordings of the reflection, discussed the codes, and realigned our analysis against the codes until we reached agreement.

Results and Discussion

Articulative—Emotive Reflection

When asked to rate their CLD lessons on a scale of 1 to 5, with 1 being the worst and 5 being the best at the start of the reflection session, the two teachers rated their lessons to be between 3 to 4 out of 5. The average rating scores suggest that the teachers were satisfied with certain aspects of their lessons and they were aware of room for improvements in their teaching.

Articulative—Critical Reflection

In this stage of reflection, Teacher A said her problem-solving phase, was student-centred because students were highly engaged and displayed a “curiosity to learn” during the group work activity. She also uncovered several students’ thinking processes and misconceptions from the RSMs that students generated for the problem task. Teacher B was pleased that the students were able to reflect on their RSMs and identify the gaps or limitations to their methods or ideas. Both the teachers pointed out that their lesson objectives were achieved. Both teachers did not give themselves a higher rating because they identified some areas for improvement for their lessons. They were unsatisfied with the way they facilitated the discussion in the problem-solving phase. Teacher A said, “I had this perception that I am not supposed to tell them [the students] so many things, they are supposed to find out on their own”. Teacher B said, “… the more difficult part was facilitating the starting of the ideas that I thought was a bit of a challenge for myself. Because [students] are very wary of giving the wrong answers … I think this project also requires a lot of the facilitation skills from the teacher themselves.” In the consolidation phase, Teacher A said,

> Teacher A: For me I find that the second lesson was a bit dry because it is back to teacher-centred teaching, so I am not very sure if I have done it correctly. All I did was showing the slides and telling them what I see from their findings et cetera, so I don’t really feel very comfortable … So, when I ask them questions they don’t seem to respond, I feel very insecure.

Teacher A’s verbatim suggested that she constantly monitored the atmosphere of the class and related it to her enactment of the new pedagogy. However, she expressed her discomfort during the *consolidation phase.* She felt that her PowerPoint presentation on the RSMs in the *consolidation phase* made it difficult for her to engage students in rich discussion of the mathematical concept because the students “were just listening to a lecture.” Compared to the *problem-solving phase,* her *consolidation phase* was less student-centred, and she wondered how she could make her *consolidation phase* more student-centred and engaging. She suggested that guiding questions to be used during lesson enactment could help her facilitate the discussion. Teacher B felt that students who “are fundamentally weaker” in her mathematics class had difficulties making connections among the RSMs during the *consolidation phase.*

The teachers' reflections show that *emotive reflection* triggers teachers' *critical reflection*. Notably, unlike *emotive reflection*, which is based on the teachers' intuition and immediate emotional responses, *critical reflection* reported here illustrates teachers' objective evaluation of their lesson episodes by drawing on what worked and what did not work from their enacted lessons. *Critical reflection* not only bring to the teachers' awareness challenges they faced during enactment of the new pedagogy, it also provides teachers the opportunities to articulate their strengths, areas for improvement and identify professional development needs grounded with specific examples from their enacted lessons when using the new pedagogy. This finding supports the importance of teachers looking back to the classroom events, teacher actions and student reactions to learn, realise what they observe or to improve their own teaching practice (Hatton & Smith, 1995). Our findings also align with Korthagen and Kessels (1999) pedagogical framework that supports the view that teachers' perspectives be 'situation specific and relate to the context in which they meet a problem and develop a need' (p. 7) to help them find a course of action. The data also shows that Teacher A's perception and belief about her role in the *problem-solving phase* influence her facilitation of classroom discussion.

Articulative—Creative Reflection

When asked what they would have done differently if they were to conduct the CLD lessons again, Teacher A suggested using a gallery walk. Students could pin their solutions at different parts of the classroom for their classmates to discuss the multiple solutions generated in the *problem-solving phase* instead of her presenting the solutions using PowerPoint. She felt that this strategy can make her *consolidation phase* more student-centred. Students would then be able to analyse their peers' solutions, and this would encourage more student-student discourse and teacher-student discussion. Teacher B suggested a similar idea. She also suggested that she could have given her students the opportunity to analyse and critique the RSMs themselves during the *consolidation phase*. She said, "the process, I feel, could have been better thought through on my part, in the sense, in making connections for them or allowing them to make the connections themselves …." She also said, "I did a lot of the brainstorming myself, but it didn't occur to me what it would had been like if students have tried it". For "students whose thinking may not be as matured yet" Teacher B suggested more teacher involvement and more scaffolding for students.

From the teachers' suggestions, we observe that teachers' *creative reflection* was motivated by their *critical reflection*. Specifically, teachers' *critical reflection* on shortcomings of their lessons prompted them to think of creative ways to overcome the perceived limitations. Their suggestions (e.g., gallery walk) also indicated their willingness to explore alternative methods to enhance their teaching practice and students' learning experience. The findings here illustrate teachers' engagement in the highest level of reflection, (Larrivee, 2008; Manouchehri, 2002; Muir & Beswick, 2007) where teachers try to examine the events in the class and finds alternative ways to improve their teaching or solve those situations when reflection is viewed as a continuum.

Assimilative Reflection (Awareness of New or Existing Beliefs)

The reflection prompts to elicit teachers' beliefs need to be made explicit for both teachers before they reflected on whether the experience of implementing the CLD had reinforced or changed any of their pedagogical beliefs. Both teachers reported that the CLD reinforced or resonated with their existing beliefs. Teacher A believes that students should construct their own knowledge but was unsure how to realise it prior to the project. The experience of using the CLD has supported and reinforced her constructivist beliefs in some ways. Teacher A also shared her *change in belief* about students' capacity to try an exploratory task on a new mathematics topic that has not been introduced in her mathematics lessons. She also shared that she has been using a more teacher-directed form of teaching, and she was apprehensive about conducting student-centred activity in the CLD package at first. However, the student responses changed her views. Teacher A also reconsidered her beliefs on the effectiveness of collaborative work in mathematics learning from the group work activity (see Table 3). Teacher A's case shows that reflection helps her question her own assumptions about her teaching practice and the data from her enactment of the new pedagogy made her consider alternatives practices which aligns with Cochran-Smith and Lytle (1993) report on reflection.

Table 3
Teacher's Change in Belief on Student and Teaching Practices by Teacher A

Student Capacity	Collaborative Learning
From "… for me I never tried such lessons before whereby students were given a task and they were supposed to explore. I have been using the more teacher-centred teaching … So when I was teaching this lesson … initially I was quite apprehensive … I am worried that there won't be any responses … I am also worried that the lesson will be just a waste of time that they wouldn't be able to find anything." To "… when I did the lesson, I was actually surprised that the students were able to see certain properties by themselves, although they didn't phrase it in the best way …"	From "Previously, collaborative work for mathematics was not my cup of tea because I find that it doesn't work." To "But then after doing this I see that actually there is value in doing collaborative work whereby the students get to reason with each other, they get to work towards a task together … maybe getting them to acquire the knowledge in this manner through interaction, through discussion, through discourse, it probably gets them to retain knowledge better."

The shift in beliefs seems to have an impact on Teacher A's teaching practices. In her subsequent lesson on "angles in alternate segment", which is not part of the CLD research, Teacher A revealed that she had tried something constructivist in nature. While she would "tell" her students the proof of the theorem in her usual practice, Teacher A had instead asked students to try and prove it themselves. She provided her students with scaffolds and hints and asked them to share their solutions in class. Teacher A's description of her "angles in alternate segment" indicates a deviation from her teacher-centred teaching, and this deviation in practice aligns with her reported change in beliefs. This finding aligns with the literature on teachers' beliefs (Mewborn, 2002), which suggests that teachers' pedagogical beliefs have an impact on their teaching practices. It also shows that reflective practice is a useful vehicle for mediating pedagogical innovation (Tan, 2016; Yaowiwat et al., 2019). Prior to the project, Teacher B said she had tried something similar before but at a smaller scale. She believes that students should "learn [math] by doing" and that it is important "to hear students what they have to say". Since the CLD builds on students' responses, the pedagogy is aligned with her beliefs.

Appraisal Reflection (Awareness of Gaps in Understanding)

Both teachers noted that they are willing to try out the pedagogy for other topics or other classes, or to conduct something similar but of a smaller scale (e.g., problem that requires shorter exploration time followed by immediate consolidation within the same lesson). However, despite their belief in the effectiveness of the CLD in students learning, Teacher A and B highlighted that factors like time constraints might limit their use of the learning design. Both teachers also questioned if such a pedagogy would be suitable for all topics. This awareness would impact when and how the teachers are going to employ constructivist pedagogy in the future.

Conclusions and Implications

In this paper, we described the teachers' process of reflection by organising our findings using the reflective question prompts as headings. By doing so, we hope we have articulated the teachers' reflective practice more systematically and that more teachers can benefit from their reflective practice. In summary, this study shows how the reflection tool and its accompanying reflective question prompts provided teachers a structure on how to reflect on their teaching practice. The reflection model surfaced teachers' awareness of their practices, beliefs and change in practices and beliefs. The teachers became more aware of their teaching actions and practices as the reflection model "facilitated the teacher reflection process in ways that helped to make visible (clear or coherent) what is often invisible (or incoherent) in teaching" (Sableski et al., 2019, p. 325). The question prompts in the reflection model not only elicit teachers' awareness, encourage teachers to suggest creative ways to improve but also has a structure that takes into consolidation teachers' emotions, deep-seated beliefs and practices in order to impact practice. In this sense, the reflection model empowers teachers "to understand specific teaching situations in more depth to inform future actions" (Sableski et al., 2019, p. 326). This study also suggests that the way question prompts are sequenced in the reflection model are very helpful for them to relook into teaching practices and beliefs as they try to implement CLD. By doing so, the teachers are engaged in "a process of bringing coherence to an initial situation" (Sableski et al., 2019, p. 326), a pivotal first step in adopting CLD in their mathematics classrooms. This study has practical significance on the use of a structured reflection tool to promote self-regulation in teachers (Boud, 2007) and for teachers to adopt new pedagogies in teaching and learning mathematics. The critical and appraisive characteristics of the reflection tool can provide valuable insights for professional developers to plan for PD programmes that support the needs of teachers as they adopt new pedagogies in their classrooms. In conclusion, the reflection model reported in this paper has a great potential to aid teacher growth and adoption of new pedagogies in the light of 21st century—the age of digital technologies and knowledge—where innovation is vital.

Acknowledgement. This project is funded by the Singapore Ministry of Education (MOE) under the Education Research Funding Programme (DEV 04/17 LNH) and administered by the National Institute of Education (NIE), Nanyang Technological University, Singapore. The claims and opinions presented herein are those of the author(s) and do not necessarily reflect the views of Singapore MOE and NIE.

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Considerations for Teaching with Multiple Methods: A Case Study of Missing-value Problems in Proportionality

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In this paper, we present a case study of a secondary mathematics teacher, Isaac (pseudonym), and his considerations for teaching with multiple methods for solving missing-value problems. While his students preferred methods that drew more closely on their intuitive understanding of proportionality, Isaac emphasised the algorithmic cross-multiplication method. Analysis of Isaac's introduction and use of the cross-multiplication method suggest his key considerations were linked demonstrating the efficiency of the cross-multiplication method, while also helping students to make making meaning from the cross-multiplication method.

Teaching with multiple methods is widely recognised as beneficial for developing students' conceptual understanding and procedural fluency (Durkin et al., 2017). In the context of proportionality, a topic which is fundamental in mathematics, demonstrating multiple methods helps to develop students' proportional reasoning (Cramer & Post, 1993). This requires teachers to facilitate instruction that draws on the connections across methods and to explicitly discuss the multiplicative relationships between quantities (Fernandez et al., 2010). While some methods are more intuitive for students to understand, such as using the scalar factor between two quantities to solve for a missing value, the use of the cross-multiplication method is still prevalent despite its lack of meaning to students. In this study, we examine the case of a teacher who taught with three methods for solving missing-value problems: using the multiplicative factor between ratios, using the multiplicative factor within ratios, and the cross-multiplication method, of which he spent significant lesson time explaining and demonstrating the cross-multiplication method. Hence, the research question that guided this study was: *What considerations did this teacher have for teaching with multiple methods in proportional problems?*

Background Literature

Teaching with Multiple Methods

Research has reported that teachers perceive several advantages in teaching with multiple methods. In a study conducted by Lynch and Star (2014) with middle- and high-school mathematics teachers (n = 92), the most cited advantage was that demonstrating multiple methods could cater to students' individual differences. As students have different learning styles and come with different background knowledge, these teachers wanted to ensure their students could see methods that were understandable to them and that resembled their own thinking during lessons. Another consideration teachers had was that they felt multiple methods helped to deepen students' understanding through developing their problem solving and reasoning skills. However, as Durkin et al. (2017) noted, demonstrating multiple methods is unproductive unless teachers compare and discuss them with students. This includes not only making sense of alternative methods and their similarities and differences, but also helping students to recognise the affordances of certain strategies over others relative to the context

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 122–129. Launceston: MERGA.

and making connections with other concepts. They concluded that doing so successfully requires careful selection of strategies and problems for comparison.

One way this can be achieved is through the design of instructional materials, a common practice for Singapore mathematics teachers. Through the design of instructional materials, such as worksheets, teachers can embed their instructional goals into the selection, modification, and design of items (e.g., worked examples, helpful hints, practice questions) that will be implemented in their lessons. This was achieved by an experienced and competent teacher in a case study reported by Toh et al. (2021), who wanted to help students make connections across different representations and strategies for quadratic equations. She implemented tasks that presented multiple methods for solving a single quadratic equation and required students to choose a method to adopt and to explain why. This was followed by another activity where she provided three quadratic equations and called on students to demonstrate their preferred method for each one along with justification of their choice of method. As noted by Cramer and Post (1993), helping students to see and explain multiple methods is also encouraged in the teaching of proportionality to develop students' proportional reasoning and ability to see multiplicative relationships.

Teaching Proportionality

Proportionality is a fundamental topic in mathematics that is highly connected to several concepts (Lamon, 2007). Notably, the recent emphasis on *Big Ideas* in the 2020 secondary mathematics syllabus in Singapore listed eight clusters of ideas, of which proportionality is one. In his review of the literature on research and the teaching of proportionality, Yeo (2019) defined proportionality to mean the existence of a constant ratio between two quantities, which includes both direct and indirect proportionality. He clarified that two key ideas in proportionality were the equality of two ratios ($\frac{y_2}{y_1} = \frac{x_2}{x_1}$ for direct proportion) and the equality of two rates ($\frac{y_2}{x_2} = \frac{y_1}{x_1}$ for direct proportion). In the 2020 secondary mathematics syllabus, this is described more functionally as the "relationship between two quantities that allows one quantity to be computed from the other based on multiplicative reasoning" (Ministry of Education, 2019, p. 8).

Within the topic of proportionality, Lamon (2007) determined two main types of problems: missing-value problems (e.g., $1:4 \text{ is equivalent to } 5:x. \text{Find } x$) and comparison problems (e.g., Drink A is mixed with 3 parts juice and 8 parts water, Drink B is mixed with 4 parts juice and 9 parts water. Which drink is sweeter?). For missing-value problems, several researchers have documented the different methods that students and teachers adopt. Artut and Pelen (2015) reported that using the *factor of change* (i.e., determining the multiplicative factor represented by the equality of ratios and rates) was the most common method adopted by sixth-grader students. In comparison, Cramer and Post (1993) note that the *cross-multiplication algorithm* is the most commonly taught method in textbooks, but that the *unitary-rate method* was the most commonly used amongst seventh-grade students, followed by the factor of change which were both perceived by students as being more intuitive. In their study of the strategies that preservice middle- and high-school teachers used to solve proportion problems, Arican (2018) found that the preservice teachers actively avoided using the cross-multiplication method, when it came to more complex problems with multiple proportions and when the multiplicative relationships were not clear.

Across all of these studies, a common issue discussed was that while the cross-multiplication method was efficient, it has no meaning for students (Cramer & Post, 1993), and hence students and teachers have been found to apply it incorrectly or inappropriately with non-proportional contexts. The common consensus is that the cross-multiplication method is often taught in such a way that it doesn't require students to reflect on the number structure (i.e., the proportionality). Hence, it is suggested that while using multiple methods to teach proportionality is recommended, methods such as the unitary-rate and factors of change should take precedence to ensure students engage in proportional reasoning.

In this study, we explore the case of a secondary mathematics teacher, Isaac (pseudonym), who taught missing-value problems with multiple strategies but appeared to emphasise the importance of the cross-multiplication method. Given the existing literature on the disadvantages of teaching the cross-multiplication method, as students would likely apply it as an algorithm with no meaning or overuse it in inappropriate situations, this led us to wonder, why was the cross-multiplication method important to teach along with other methods? Despite alternative methods that have been shown to be more intuitive for students for engaging in proportional reasoning, what affordances did Isaac perceive with teaching the cross-multiplication algorithm that warranted its special feature across the two lessons?

Methods

The data presented in this paper is part of a larger study about mathematics teachers' design of instructional materials. Four teachers from two secondary schools in Singapore engaged in professional learning community (PLC) discussions at their respective schools with their colleagues, individual design of a set of worksheets on the topics of Ratio and Rates, and one-on-one semi-structured interviews with the first author about their worksheet design (e.g., selection, modification, creation of tasks). Then, they implemented their worksheets with their students and engaged in post-lesson one-on-one semi-structured interviews to reflect on their lessons. All worksheet drafts and teacher notes were collected, and all PLC discussions, interviews, and lessons were video-recorded and selected episodes were transcribed.

In this paper, we discuss the data of one teacher, Isaac (pseudonym), and the implementation of his first worksheet on ratios over two lessons. Of the four teachers, Isaac frequently demonstrated the use of multiple methods in his lessons, with special attention given to the cross-multiplication method, while the other teachers tended to adopt more advantageous and canonical methods. Our aim was to determine Isaac's considerations for teaching with multiple methods, which we analysed from his implementation on two levels of grain-size. At the item-level, we evaluated the mathematical content (e.g., relevant concepts, processes, possible solution methods) in each item in Isaac's worksheets and their potential affordances. At the set-level, we examined the connections of the content and affordances between items, and Isaac's overarching instructional goals. Adopting these two grain-sizes highlighted the unique role of individual tasks in developing students' understanding of proportionality, while also capturing how Isaac used these tasks collectively to deepen students' understanding of proportionality.

Results and Discussion

In this section, we begin by presenting a description of two tasks from Isaac's implementation and the questions that emerged from our item-level analysis about his considerations for teaching with multiple methods. As the cross-multiplication method emerged as a key method, we analyse a particular task that Isaac spent a significant portion of time discussing in the lesson at the item-level and set-level that will shed light on why Isaac emphasised this method.

Establishing Three Methods for Identifying Equivalent Ratios

While most of the items in Isaac's Ratio worksheet resembled "typical problems" found in textbooks, one item near the beginning of the worksheet caught our attention—a table of "Three methods for identifying equivalent ratios" (**Error! Reference source not found.**), which Isaac h ad designed himself. This table was intended to be a summary of methods that students could adopt to solve the problems in his worksheet and to discuss the different multiplicative relationships when dealing with equivalent ratios. As students were likely to have encountered Method 1 (M1, multiply by both side) and Method 2 (M2, multiplicative nature) in primary school, they responded confidently when Isaac asked them to determine the missing values in the examples using the multiplicative relationship between (M1) and within (M2) the equivalent ratios. These two methods use the *factor of change* (Lamon, 2007) to determine the missing value, which has been found to be a common strategy used by primary-level and early secondary students (Artut & Pelen, 2015). As students could clearly articulate that the missing values would be "five times" and "four times" another quantity, Isaac spent little time elaborating on these and swiftly moved on to introduce the third method.

When Isaac introduced Method 3 (M3, cross-multiply), he had difficulty communicating to students the meaning behind the procedure. Prior to the lesson, he anticipated that students would not immediately understand why cross-multiplying the ratios could be used to identify missing values. Isaac asked them if they realised that the products would be equal when they cross-multiplied the quantities, $1 \times 8 = 2 \times 4$. When he asked the class to think about why, the students said it was because "the quantities are multiplied by 2 on both sides" and "the second quantity is a quarter of the first". These were not the reasons that Isaac hoped students would use. He attempted to resolve this by demonstrating M3 on the same missing-value problem in M2, to show that when they cross-multiplied the values for the ratios $1: 4$ and $x: A$, they would arrive at the same solution as M2. Isaac's demonstration appeared to have little effect on the students; they remained silent and appeared to be unsure of why it was equal still. After the lesson he recounted that in the moment, "it's even difficult for me as the teacher to answer as well, I have to admit that". Isaac acknowledged that he only demonstrated the procedure and hadn't provided a proper rationale; hence, he was not surprised that students were unconvinced with M3. For the next lesson, he planned to revisit all three methods, to demonstrate their use on practice questions, and to explain M3 again.

THREE METHODS TO IDENTIFY EQUIVALENT RATIOS

Methods	Example	Come up with your own example!
Multiply by both side	×5 1 : 4 2×5 5 : …20…	
Multiplicative nature	1 : 4 ×4 x : 4x ×4	
'Cross-multiply' method	1 : 4 2 : 8 1 : 4 x : 4x. 1×A = 4×x A = 4x.	

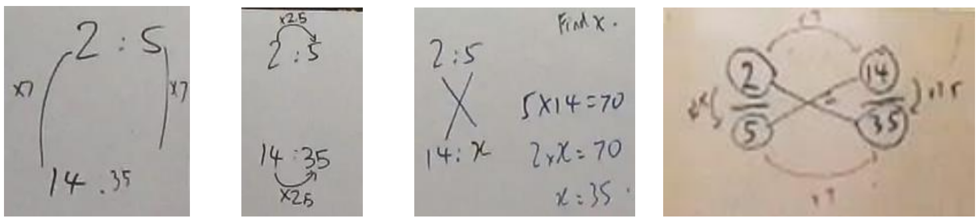
Figure 1. "Three Methods to Identify Equivalent Ratios" from Isaac's lesson.

Isaac's implementation of the table, especially his teaching of M3, raised several questions about his considerations for teaching with multiple methods. Why did he want to introduce M3 when he was having difficulty communicating the underlying reasoning and suspected that students would not be able to intuitively make sense of it? Notably, his students' explanations for M3 drew on the factor of change methods, M1 and M2, which Cramer and Post (1993) claimed are more naturally intuitive and meaningful for students than the cross-multiply method (M3). Although M1, M2, and the unitary method that is taught in primary school (Yeo, 2019) suffice to answer missing-value problems, what affordances did Isaac observe in M3? Further analysis of his implementation will shed light on his considerations.

Practicing Three Methods and Revisiting Method 3

After introducing the three methods, Isaac encouraged students to see how they could be applied across several problems in his worksheet. For all the questions in his Ratio worksheet, he invited students to write their solutions on the whiteboard or he would write the common methods that he observed amongst the class and asked the students to explain. When only one method was predominantly used by students—usually M1—Isaac typically called on students to describe additional methods and wrote these on the board. He frequently connected the three methods by pointing out that they produced the same solution due to the "proportionality" and "multiplicative relations" between the quantities in the ratios. However, he had not yet addressed the rationale for M3 or provided a reason for why students should try to understand and adopt it.

When Isaac attempted to address the rationale for M3 in the following lesson, he again encountered issues communicating the reasoning to students. At the beginning of the second lesson, Isaac generated a new missing-value problem, $2:5$ and $14:x$, and asked for three students to come to the board to demonstrate the three methods to find x. After each student came to the board to write their solutions (*Figure 2*, A-C), Isaac connected the three methods by explaining that "in all three methods, we claim that this ratio $(2:5)$ is *equivalent* to this ratio $(14:35)$ because the first quantity is *proportional* to the second quantity. In other words, I have *assumed* that the ratios are equivalent". Although the class could observe how to apply the three methods, only three students said M3 "made sense". Isaac explained that as the ratios could be expressed as equivalent fractions (*Figure 2*, D), by cross-multiplying not only would the products be the same but the multiplicative relationships between the quantities for M1 and M2 could be seen also. Isaac's students were struggling to make sense of M3, and even Isaac was having difficulty convincing students about the rationale.



(A) (B) (C) (D)

Figure 2. Isaac's task for revising the Three Methods in Lesson 2 (A, B, C by students, D by Isaac).

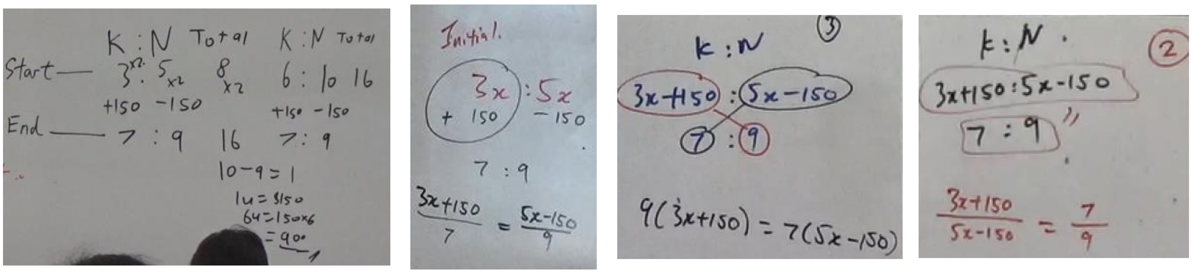
Isaac's continued attempts to demonstrate and explain M3 suggest that he thought it was an important method for students to understand—but *why*? While teaching with multiple strategies and helping students to see the relationships between quantities is beneficial for developing conceptual understanding about proportionality (Cramer & Post, 1993; Fernandez et al., 2010), Isaac's students can already use and explain at least three methods (M1, M2, unitary method) to solve ratio problems. It remains unclear what additional affordances Isaac saw in M3. What were Isaac's considerations for revisiting and *emphasising* M3, when alternative methods were sufficient, more intuitive, and more popular amongst his students?

As Isaac progressed through the problems he selected for his worksheet, the value of M3 became more apparent in one task near the end.

The Importance of Method 3: Cross-multiplication Method

In comparison to the other problems that were discussed in his lessons, Isaac's implementation of this problem about the exchange of money between two friends (Figure 3) began to shed light on his considerations for emphasising M3. When it initially came up near the end of Lesson 1, it was correctly solved by most of the students in the class using a unitary-rate approach (*Figure 3*, A) that reflected the corresponding worked-examples provided in the textbook from which Isaac sourced this problem. Although students had found the correct solution, Isaac's teacher notes showed that he wanted to discuss solutions that more closely resembled the three methods and involved expressing the ratios as fractions. One student in the class solved the problem by forming a fraction (*Figure 3*, B), so Isaac wrote it on the whiteboard. Noticing how his students were puzzled by the emergence of the equation $\frac{3x+150}{7}=\frac{5x-150}{9}$, when the end-of-lesson bell rang shortly after, he decided that he would revisit the problem the next day. Isaac acknowledged that using the unitary-rate method was easy for students to understand, but it was "super specialised"—suggesting there were some contexts where it might be inappropriate or inconvenient. In comparison, the other method (*Figure 2*, B) "is so incredibly difficult, at least for students at their level right now", but he wanted them to make sense of it as it would "help them to understand that ratios can be expressed as fractions, as equations, which in the future as they go to upper secondary in mensuration, they will need this."

Kate and Nora each have a sum of money. The ratio of the amount of money Kate has to that of Nora is $3:5$. *After Nora gives $150 to Kate, the ratio of the amount of money Kate has to that of Nora becomes* $7:9$. *Find the sum of money Kate had initially.*



(A) (B) (C) (D)

Figure 3. Four methods for solving a problem (A and B by students; C and D by Isaac).

On his second attempt to explain the problem the next day, Isaac was successful in communicating to students the efficiency of M3 by making connections with alternative methods. For solution B, he added that as the ratio $3x + 150 : 5x - 150$ was equivalent to $7:9$, an equation could be formed to represent "1 part" by dividing $3x + 150$ and $5x - 150$ by 7 and 9 respectively. Then to solve for x, it naturally followed to cross-multiply to form the equation $9(3x + 150) = 7(5x - 150)$. Finally, the students accepted this explanation and Isaac moved on to demonstrate the direct use of M3 (*Figure 3*, C), showing that it would produce the same equation. Immediately after, Isaac explained how the two ratios could be directly expressed as equal fractions (*Figure 3*, D) and again mentioned how the next step would be to cross-multiply, resembling the same equation in M3. The students accepted that across these three solutions, they all required cross-multiplication and eventually all arrived at the same algebraic equation $9(3x + 150) = 7(5x - 150)$ to solve for the total sum of money. In this task, Isaac had convinced students that M3 would be more efficient, while simultaneously making connections across other methods.

Our analysis of Isaac's implementation of this task surfaced two key considerations that Isaac considered for teaching with multiple methods and emphasising M3. At the item-level, this task demonstrated the *efficiency and generalisability* of M3. As the factors of change between ($\frac{7}{3x+150}$ or $\frac{9}{5x-150}$) and within ($\frac{5x-150}{3x+150}$ or $\frac{9}{7}$) the ratios are not immediately intuitive, applying M3 directly forms an algebraic equation which can be used to solve the problem. Instead of simply telling students to apply M3, Isaac made connections across the methods by discussing the general need for cross-multiplication, which would help students to see M3 as more than just a shortcut and to develop a more meaningful understanding of M3. However, research has reported on how the cross-multiplication method is often taught in textbooks and used by teachers as merely an algorithm without meaning (Arican, 2018; Cramer & Post, 1993). As a result, students typically avoid using it or use it inappropriately in non-proportional contexts due to a lack of understanding (Fernandez et al., 2010). Through forming different fractions and demonstrating how the equation $9(3x + 150) = 7(5x - 150)$ can be derived, Isaac imparted both efficiency and meaning to the method, and hence lessened the likelihood that students would simply apply M3 without being able to connect it to other methods.

In our set-level analysis, we determined that Isaac wanted to *build students' understanding* of M3 from their proficiency with M1 and M2. For the previous problems students encountered, students had no need to understand or use M3 because they had at least three methods they could use (M1, M2, unitary method). However, once students came to this problem, M3 became more than just another method for students to see; it became a key method for students to adopt both efficiently and meaningfully by building on their existing understanding of M1 and M2. As Isaac taught with multiple methods across his Ratio worksheet, including frequently inviting students to the board to write their solutions that included M1 and M2, his revisiting and re-explaining of M3 suggests he was deliberately preparing students to see the connections from one method to another. Although solving proportional problems using the cross-multiply method does not necessarily imply proportional reasoning has occurred (Arican, 2018; Lamon, 2007), as a collection of tasks where the Kate and Nora problem plays a unique role, the cross-multiply method became relevant to proportional reasoning with other tasks and through other methods.

Much like the teachers in Lynch and Star's (2014) study of teachers' considerations for teaching with multiple methods, Isaac demonstrated how he attempted to use multiple methods to deepen students' understanding of proportionality. He frequently made comparisons between the methods and selected a specific task that would help students to see the efficiency and to build meaning for the cross-multiplication method. Isaac's teaching of multiple methods reflected the suggested practices proposed by Durkin et al. (2017), which turned an otherwise meaningless algorithm for students to simply apply into a method that had its particular affordance for solving problems where the factor of change is not immediately obvious, as well as being connected to others.

Conclusion

Isaac's teaching of ratio revealed he had two key considerations for teaching with multiple methods: (i) to demonstrate the efficiency of methods in specific situations, and (ii) to help students make meaning of methods through connections. A cursory analysis of Isaac's teaching with multiple methods may raise several questions about his instructional goals; it might even suggest that he was an ineffective teacher who favoured the use of algorithms over meaningful strategies, such as the unitary method and factors of change. However, upon closer examination at the item- and set-level, it emerged that Isaac's efforts to emphasise and substantiate the cross-multiplication method were not misguided but instead directed towards preparing students to see its efficiency meaningfully as they approached a problem he had selected. Although the findings in this study are limited to a single teacher and in the context of missing-value problems, we argue that the case of Isaac is valuable because it uncovers that there is more than meets the eye when observing how teachers teach with multiple methods. It demonstrates the underlying complexity of teachers' considerations for teaching with multiple methods, even when the instruction appears to be ill-judged and unproductive at times. Future research should look to document more detailed cases of how teachers teach proportionality with multiple methods. Cases of teachers involving comparison problems and teaching of algorithms would contribute to greater understanding for teacher development programs. Finally, adopting both an item- and set-level analysis could help to reveal these underlying considerations.

Acknowledgements: The data reported in this paper is part of a larger research project known as "Big Ideas in School Mathematics Curriculum" (Grant number: OER 31/19 BK) funded by the Office of Educational Research, National Institute of Education, Nanyang Technological University, Singapore. We would also like to acknowledge the funding support from Nanyang Technological University: Research Scholarships.

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Procedural Flowcharts Can Enhance Senior Secondary Mathematics

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The senior secondary Mathematical Methods subject was introduced in Queensland in 2019 and parents were concerned with the low grades attained. Students in the subject need to develop extensive procedural knowledge and fluency to participate and engage successfully. This mixed methods study informed by constructivism investigated teachers' perceptions on how procedural flowcharts can enhance procedural fluency in the subject. Data were collected through a survey and semi structured interviews with the subject teachers. The study found that procedural flowcharts can foster a classroom environment that stimulates procedural fluency and student-centred teaching and learning of mathematics. The study suggests the potential for using procedural flowcharts in other mathematics subjects at different levels of study.

When the senior secondary Mathematical Methods subject was introduced by the Queensland Curriculum and Assessment Authority (QCAA) in 2019, parents raised the issue that students who used to obtain 'As' at junior level were now getting lower grades at senior level (Bennett, 2019). In Queensland, trends have shown student decline in participation and achievement in calculus-based senior secondary mathematics options (Chinofunga et al., 2022). Internationally, trends have shown a decline in participation in advanced mathematics subjects at senior secondary level in most countries (Hodgen et al., 2010). Researchers have pointed to pedagogy and classroom practices that are disengaging (Tytler et al., 2011) as one of the causes of the decline in participation and achievement in the advanced mathematics subjects. Additionally, students' limited procedural fluency has been highlighted as one of the causes that limits their understanding of mathematics ideas and solving mathematics problems (Kilpatrick et al., 2001), hence affecting participation and achievement.

Procedural Fluency

Procedural knowledge is a part of procedural fluency in mathematics education. Procedural knowledge is defined as knowledge of procedures and steps to a solution (Braithwaite & Sprague, 2021). Procedural fluency, on the other hand, is more than just being able to perform a procedure as it involves conception of the problem, choosing the appropriate method and adaptability in applying the chosen method (Bay-Williams, 2020). Moreover, procedural fluency involves "using procedures efficiently, flexibly, and accurately" (Bay-Williams et al., 2022, p. 178). Bay-Williams and SanGiovanni (2021) define "efficiency" as selecting the best method and using it to solve a mathematics problem within a set time and "accurately" as using a procedure correctly. While "flexibility" is conceptualised as knowing more than one procedure and being able to modify procedures when solving a mathematics problem (Star, 2005). "To support flexibility, teaching standards in numerous countries recommend that students be introduced to multiple procedures early in instruction and be encouraged to compare the procedures" (Rittle-Johnson et al., 2012, p. 437). Importantly, students demonstrate procedural fluency when they exhibit flexibility in using a skill, obtain the correct solution and are able to effectively communicate the method used (McClure, 2014). Therefore, procedural knowledge is part of procedural fluency and teachers are expected to help students build procedural fluency using different strategies and teaching styles.

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Teachers in Queensland use explicit teaching approaches to help students execute procedures accurately and to select the optimal method to solve a given problem while further practice brings flexibility and efficiency. In this approach, teachers demonstrate the skill, then guide students' practice and finally provide the opportunity for unprompted practice (Archer & Hughes, 2010). Thus, after the students have been explicitly taught a method to solve a mathematics problem, they must be given an opportunity to practice when and how to use the method (Bay-Williams et al., 2022). When students can identify the context where the procedure can be suitably applied, it provides the opportunity for procedure modification (National Council of Teachers of Mathematics [NCTM], 2014), resulting in deeper knowledge. Similarly, "procedural fluency, is a comprehensive way of navigating mathematical procedures; it includes mastery of algorithms and strategies, but it also includes knowing when to use them" (Bay-Williams & SanGiovanni, 2021, p. 25). However, procedural knowledge is perfected through "practice, and thus is tied to particular problem types" (Rittle-Johnson et al., 2015, p. 119), as mastery of procedures is key to developing this knowledge. As a result, repeated practice and guidance is one critical part of building procedural knowledge (Rittle-Johnson, 2015). Hence, procedural knowledge development is characterised by firstly being guided, mimicking and then through experience, adapting procedures to other complex familiar problems as part of procedural fluency.

Students who operate at high levels of procedural fluency are more likely to integrate and modify familiar procedures to solve complex unfamiliar problems (Blöte et al., 2001). However, in Queensland, simple familiar problems constitute 60% of examination questions and require use of procedures identified in the questions (QCAA, 2018). In this case, students have to identify the most appropriate procedure, apply it correctly and efficiently to pass the examination. Therefore, procedural fluency plays a key role in being successful in mathematics. This study focuses on how procedural flowcharts can enhance students' procedural fluency in the Mathematical Methods subject.

Procedural Flowcharts

A flowchart is the most efficient and concrete method to illustrate a procedure or multiple procedures to solve a problem (Toyib et al., 2017). The importance of flowcharts in developing procedural knowledge is supported by the definition of procedure established by Rittle-Johnson et al. (2015), that it is "a series of steps, or actions, done to accomplish a goal" (p. 588). In addition, they are effective in a class with students operating at different levels of prior knowledge; being more advantageous to those at the very low level as they help in decision making and provide problem solving skills (Hooshyar et al., 2015). Importantly, flowcharts play a significant role in promoting independent learning as students can refer to them after encountering a familiar mathematics problem (Marzano, 2017). Apart from showing contradictions and contrasting procedures, they promote representations of steps and procedures from different perspectives (Andrej, 2018). In procedural knowledge, relationships are sequential that is, steps follow each other (Hiebert & Leferve, 1986). Consequently, flowcharts are an important tool that a mathematics teacher can use to teach procedural knowledge as they guide students through the process allowing learning to be student centred and accommodative of different students' understanding.

It is a common experience for mathematics teachers to witness students applying procedures where they are not suitable just because they have memorised them (DeCaro, 2016). Minimising this common problem will improve students' participation and achievement. When procedures are taught using flowcharts, decisions are taken at every step. This is because, "flowcharts represent a sequence of decision making and information processing" (Marzano, 2017, p. 57). They are an "aid to thought" that help in analysing a problem and planning the solution (Ensmenger, 2016, p. 328). Consistent use of flowcharts helps students to develop the ability to identify suitable methods for solving mathematical problems, as well as helping them to gain sophistication in approaching complex problems (Newton et al., 2020). Superficial procedural knowledge might be limited to accurate and efficient use of one procedure, but deep procedural knowledge involves several approaches and knowing when to apply a particular strategy (Bay-Williams, 2020). As students apply a flowchart, decisions are made depending on how the solution, the processes, and steps being followed align with the flowchart. Due to the checks and balances provided by the flowchart, students will be able to determine relevant procedures that must be applied to solve a particular problem.

Applying tools that promote multi-solution strategies enhance students' capacity to solve a variety of problems. Teachers can use flowcharts to represent multiple ways or choices to a solution (Marzano, 2017), thus promoting procedural flexibility. Marzano noted flowcharts guide students through processes, steps and decision making which is critical for procedural fluency. "When students achieve procedural fluency, they carry out procedures flexibly, accurately and efficiently" (QCAA, 2018 p. 1). Thus, procedural flowcharts are a visual representation of available procedures and corresponding steps, showing all stages of evaluation and alternative paths of a solution to a desired result. This study explored teachers' perceptions on the use of procedural flowcharts. The research question was: What are teachers' perceptions on how flowcharts impact teaching and learning of procedural fluency in the Mathematical Methods subject?

Method

The mixed methods study informed by constructivism focused on teachers' perceptions on how procedural flowcharts can enhance the Mathematical Methods subject. Quantitative and qualitative data were collected and analysed to gain further insights into participating teachers' perspectives (Creswell, 2015). Ethical approvals were obtained from the Department of Education, Queensland: Reference number: 550/27/2383 and James Cook University Human Research Ethics Committee: Approval number: H8201. Sixteen senior secondary mathematical methods teachers participated in the study. The participants took part in a 10-minute video presentation based on procedural flowchart tools developed from a section on Functions in Unit 1 of the Mathematical Methods syllabus. Figure 1 below is an example of one of the procedural flowcharts developed from the Content Sequencing framework and used in the presentation.

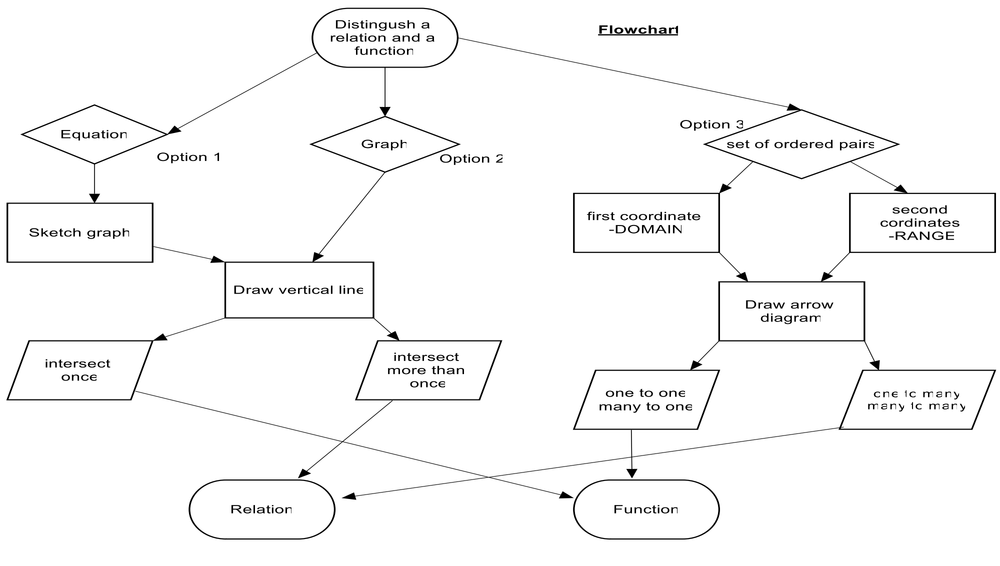

Figure 1: Procedural flowchart on distinguishing functions and relations.

Using Figure 1, students are asked to determine if a polynomial, graph or set of ordered pairs is a function or a relation. The decision is reached after applying the mathematical procedures. As the mathematical procedures are being implemented, they provide room for justification of choices given in the flowchart. This allows students to work through independently and be reminded of the steps and procedures that are critical in solving the problem. Students are also expected to learn about features of quadratic functions.

A procedural flowchart on features of quadratic functions that shows procedures of determining different features that students are expected to learn at Year 11 is also presented to students. Effective teaching of quadratic functions promotes students' comprehension of how different forms of algebraic representations relate to how features of the functions are determined, which is also included in the flowchart on quadratic functions. Equally, the relationships between these features also shown in the flowchart help students to build and broaden their understanding of the concept. The students are also reminded of key mathematical steps and procedures to solve problems related to the concept. Thus, procedural flowcharts are key in highlighting vocabulary expected in a concept such as intercepts, turning points and discriminant which is important in developing mathematical fluency.

Data Collection and Analysis

Participants were given a school term to embed the procedural flowcharts in teaching and learning of mathematics. They were then asked to complete a survey comprised of five-point Likert scale items and open-ended questions to gain a deeper understanding of participants' insight into the use of procedural flowcharts in the teaching and learning of mathematical methods. The 20-minute semi structured interviews were conducted with eight participants who completed the surveys. The thematic analysis and coding of survey open ended questions and semi structured interviews was done independently among the authors (Creswell, 2015).

Results

The survey data collected using the five-point Likert scale were analysed (see Table 1).

Table 1
Responses to Likert Scale Questions: Teacher Perceptions of Using Procedural Flowcharts

Questions	Strongly Agree	Agree	Not Sure	Disagree	Strongly disagree
1. Visual representation of mathematical knowledge enhances teaching and learning of mathematics.	94%	6%	0%	0%	0%
2. Procedural flowcharts (showing steps and procedures) plays an important role in developing students' mathematical skills.	56%	31%	6%	6%	0%
3. Procedural flowcharts promote fluency and recall.	69%	13%	19%	0%	0%
4. Procedural flowcharts can be used to highlight critical vocabulary	69%	19%	13%	0%	0%
5. Procedural flowcharts are a reference resources that can also be used for revision.	81%	19%	0%	0%	0%
6. Procedural flowcharts focus on students learning.	69%	19%	13%	0%	0%
7. Procedural flowcharts promote independent or collaborative learning.	69%	13%	19%	0%	0%
8. Procedural flowcharts can help evaluate or give feedback to students on their understanding and correct use of a procedure.	63%	31%	6%	0%	0%

The following themes were agreed upon after the independent thematic analysis, collaborative reviewing and revision: (1) procedural flowcharts can foster a classroom environment that stimulates procedural fluency when learning mathematics, and (2) procedural flowcharts can enhance student centred teaching and learning of mathematics procedures.

Theme 1: Procedural Flowcharts Can Foster a Classroom Environment That Stimulates Procedural Fluency When Learning Mathematics

The participants agreed that procedural flowcharts stimulate procedural fluency. The open-ended survey questions showed that participants supported the use of procedural flowcharts in enhancing procedural fluency. These included: (i) teacher created procedural flowcharts for students to use during explicit teaching phases or targeting students who have not achieved fluency or for students with identified learning needs, (ii) class generated procedural flowcharts during collaborative teaching phases to show the processes that were applied, and (iii) student generated procedural flowcharts to show common mistakes or misconceptions. These results demonstrate the flexibility of procedural flowcharts in enhancing fluency.

Feedback from semi structured interviews gave greater detail on how procedural flowcharts create a wide range of opportunities for developing procedural fluency in mathematics. Participants' perceptions after applying them as a teaching and learning resource provided some insight into how this resource can help develop students' procedural knowledge and skills. Participants noted that students are comfortable with visual representations more than just worded steps. In fact, they appreciated that most students are visual learners who respond well to diagrammatical representations than written steps. For example, Participant 8 said, "*Because it's a diagrammatic representation, students look at it favourably because it's easier to process and like I said most students are visual learners."* Participant 7 went further to give an advantage of a procedural flowchart by saying, "*it is steps in diagrammatic form which is easy to process and easy to understand*." Thus, students can follow easily and use the steps to answer problems with minimum help. Participant 2 noted that, "*if you had steps just written down in the book, it's hard to flip back through and find the information you're looking for, whereas if it's a diagram, it's easy to find."* Participant 2 went further and said, "*they enhance students' memory*". Therefore, flowcharts that are easy to navigate and use provide a better opportunity to recall and accurately apply information, which enhances procedural fluency. This will help in solving most problems in mathematical methods examinations as indicated by Participant 8 when he said, *"It is very handy for simple familiar questions which are mostly recall and fluency questions, but which are the majority in mathematics examinations."*

As students follow the steps on the procedural flowchart, they enhance their procedural knowledge and fluency. Participant 2 made this point when she said, *"really good how it organizes the steps and explains where you need to go if you're at a certain part in a procedure."* In addition, Participant 7 mentioned, "*the cycle approach, the feeding back in, feeding back out, that type of stuff, that's when we are starting to teach students how to think.*" Likewise, Participant 8 observed that, "*Complex procedural flowcharts like the one you provided guide students in making key decisions as they work through solutions which is key to critical thinking and judgement and these two are very important in maths.*" Procedural flowcharts enhance students' efficiency and flexibility in solving problems, deepening their understanding through reasoning and justification, which are mathematics proficiencies.

Procedural flowcharts provide teachers with the opportunity to determine students' procedural competences and misconceptions. Participant 8 emphasised that, "*I went further to ask my students to create their own procedural flowcharts … so that I can evaluate if they understand and represent their fluency on the chart."* Participant 1 went further to include procedural misconceptions, "*I use it to identify the potential students' misconceptions and I'll use it to identify student's competences."* Therefore, enhancing procedural fluency.

Theme 2: Procedural Flowcharts Can Enhance Student Centred Teaching and Learning of Mathematics Procedures

The participants agreed that the use of procedural flowcharts encourages and facilitates independent and student-centred learning. Open-ended survey responses highlighted use of: (i) students generated procedural flowcharts after explicit teaching, and (ii) student generated procedural flowcharts at the end of the lesson as part of lesson consolidation.

The use of procedural flowcharts by participants in the teaching and learning of mathematical methods made them realise that they promoted independent and student-centred learning. The response from Participant 8 was, "*they promote individual learning and learning which is student centred*." Moreover Participant 6 further alluded that a capable student, "*can teach themselves without even a teacher.*" Importantly, independent learning of students can provide a teacher with the opportunity to help struggling students. This view is supported by Participant 5 when he said, "*it gave me the opportunity to work with slower kids as they promote individual learning*."

Participant responses also indicated that procedural flowcharts enhanced students' engagement. When asked about how procedural flowcharts enhance students' development of procedural knowledge, Participant 8 said, "*I have witnessed more students engaging more in the YOU DO (student centred) phase*." The participant went further to say, "*I was so impressed because students engaged more with the task.*" A similar but more detailed observation was also witnessed by Participant 7 who said, *"mathematics goes from being very dry and dusty to being something which is actually creative and interesting and evolving, starting to get kids actually engaging and having to back themselves, and having to be less passive and more active as learners."*

Moreover, participants noted they help students to understand the importance of understanding procedures if they want to engage effectively with mathematics. Participant 3 shared her observation that procedural flow charts *"allow the students to move in both directions and it makes them see that the actual responses that they have to give are minimized rather than seeing every question as separate."* This is very important especially for questions which require procedural steps not in their most usual form or representation. The participants agreed that procedural flowcharts play an important role in enhancing procedural fluency and engagement in mathematics.

Discussion

One interpretation of these findings is that participants noted procedural flowcharts can enhance development of procedural knowledge and fluency. As highlighted previously, procedural knowledge is knowledge of steps and procedures to a solution (Braithwaite & Sprague, 2021). Thus, procedural flowcharts represent a series of steps and procedures that may include several approaches to reach a desired solution to a particular type of mathematics problem. Results show at least 80% of participants agreed or strongly agreed that procedural flowcharts enhance development of mathematics skills and promote fluency and recall. Fluency includes an understanding of vocabulary and 87% of participants acknowledged procedural flowcharts highlight critical vocabulary. Participants concurred that they do not only provide steps to be followed but facilitate decision making through reasoning as students evaluate the correct procedures to follow. Kilpatrick (2001) posited that procedural skills are central to students' learning of mathematics. Thus, practice in solving problems using sequenced steps and procedures promotes accuracy. Additionally, information processing and decision-making help in evaluating how the path to the solution aligns with the available procedures, thus enhancing "efficiency." This study highlighted that multi-solution procedural flowcharts provided an option for students to know more than one solution, thus enhancing "flexibility." Using mathematics steps accurately, effectively and efficiently develops fluency (Bay-Williams et al., 2022). Therefore, efficient use of procedural flowcharts helps students develop procedural knowledge and enhances procedural fluency. As a resource, it can also support explicit teaching which is one of the main pedagogies in Queensland.

Results show participants appreciated that developing the procedural flowchart during any of the stages of explicit teaching is beneficial. Firstly, teachers can develop the charts during I DO (teacher centred stage) by teaching students how to organise the steps, processes and loops for decision making. Secondly, the charts can be developed as a class during WE DO (collaborative stage) and lastly students can develop them during YOU DO (student centred stage). Participants perceptions show that having students develop their procedural flowcharts can be an efficient way of checking students' procedural understanding, misconceptions and evaluating learning. These results are consistent with Raiyn (2016), whose work concluded that visual representations require less time and are easier to process. Furthermore, presenting information in different forms such as verbal, numerical and diagrammatical helps students comprehend the phenomenon (Murphy, 2011). When students are given the opportunity to create their own procedural flowcharts, they represent their procedural knowledge and fluency diagrammatically. Moreover, procedural flowcharts are a tool that can also be used to promote engagement and student-centred learning.

Quantitative data analysis indicated that at least 81% of participants agreed or strongly agreed that procedural flowcharts enhance independent and student-centred learning. Qualitative data highlighted the importance of procedural flowcharts during YOU DO stage when using the explicit teaching approach. This is the stage where students are expected to interact and solve familiar problems to what they were taught and practiced as a class in the I DO and WE DO phases. This is because "routine practice is an extremely powerful instructional tool that not only helps students learn and retain basic skills and facts in a fluent fashion, but has positive outcomes when students attempt higher-order strategies" (Archer & Hughes, 2010, p. 21). Importantly consistent use of flowcharts helps develop mastery as they are an aid to thinking (Ensmenger, 2016). The participants perceptions are consistent with Marzano's (2017) work that concluded that when students come across familiar problems, they can refer to procedural flowcharts as they independently solve the problem. Likewise, in student centred learning, students develop knowledge and experiences they have acquired by further exploring using tools and resources as scaffolds (Marzano, 2017). When answering open ended and interview questions, participants emphasised that students could use procedural flowcharts during YOU DO stage. This provides students with an opportunity to engage with learning using the procedural flowchart as a scaffolding resource and minimum teacher assistance.

Conclusion

The study highlighted that teachers view the use of procedural flowcharts as a resource that can help to develop students' procedural fluency and participation in mathematics. The study suggests that this approach can be extended to other mathematics subjects at different levels of study. The present research, therefore, contributes to a growing body of evidence suggesting that representation of knowledge and processes in non-linguistic forms such as diagrams enhances participation and achievement. However, the main limitation of this study is the small number of mathematical methods teachers that was used. In terms of future research, we hope this study has provided a basis for further research in use of procedural flowcharts in mathematics teaching and learning.

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RESEARCH PAPER

Making Visible a Teacher's Pedagogical Reasoning and Actions Through the Use of Pedagogical Documentation

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Mathematics education research has focused on developing teachers' knowledge or other visible aspects of the teaching practice. This paper contributes to conversations around making a teacher's thinking visible and enhancing a teacher's pedagogical reasoning by exploring the use of pedagogical documentation. In this paper, we describe how a teacher's pedagogical reasoning was made visible and highlight aspects of his thinking in relation to his instructional decisions during a series of lessons on division. Implications for professional learning are discussed.

Teaching students for *relational understanding* (Skemp, 1978) in mathematics is challenging. Doing this requires teachers to choose, adapt, or design tasks that promote reasoning (Sullivan et al., 2014); orchestrate interactions and discussions amongst students around these tasks to focus on the key mathematical ideas while catering to students with different levels of readiness and interest (Lampert, 1985); and making in-the-moment instructional decisions by considering students' responses to these learning tasks (Jacobs et al., 2011). Much of the current research in mathematics education has focused on developing teachers' mathematical knowledge for teaching or what Kennedy (2015) described as the more "visible behaviours of teaching" (p. 6) to do the ambitious work of teaching. As argued by Kavanagh et al. (2020), it is critical that teacher education aimed at ambitious teaching "must involve unfolding the invisible professional thinking behind discrete elements of teaching practice" (p. 3). This paper contributes a new perspective to this ongoing conversation about enhancing teachers' pedagogical reasoning (Kavanagh et al., 2020; Loughran et al., 2016; Pella, 2015), an invisible aspect of ambitious teaching, by considering how a primary school mathematics teacher's pedagogical reasoning and action can be made visible in the context of day-to-day teaching activities. We explored how the idea of pedagogical documentation (Alcock, 2000; de Sousa, 2019; Lee-Hammond & Bjervås, 2021) can be used to make visible a teacher's pedagogical reasoning and actions about the teaching of division. We frame our discussion in this paper around the following research question: What insights can we gain into a teacher's pedagogical reasoning and action when his thinking during a series of lessons on division is made visible?

Theoretical Considerations

Our study was motivated by our interest in supporting teachers to do this ambitious teaching by providing meaningful opportunities for teachers to reflect and learn from their teaching practices. However, our premise differs from current perspectives about practice-based professional development (Chapman, 2014; Timperley et al., 2007) that also embed learning into teachers' work (e.g., lesson studies) in two ways. First, we want to go beyond the confines of professional development activities and explore how individual teachers can learn from their own teaching as well as their colleagues' teaching experiences. Next, we see a need to maximise learning opportunities for teachers by empowering them to learn from these opportunities as part of their teaching activities. Hence, our vision is to see professional learning as part of a teacher's daily teaching activities beyond a teacher's participation in

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 138–145. Launceston: MERGA.

professional learning communities. To introduce our ideas, we will revisit three aspects of this *invisible work* of thinking about one's practice (Kavanagh et al., 2020).

We begin by elaborating on the ideas of pedagogical reasoning and action as proposed by Shulman (1987), who perceived teaching as an iterative "act of reason" and an ongoing "process of reasoning" that culminates in a series of pedagogical actions (p. 13). This process involves taking what one understands about content and "making it ready for effective instruction" (Shulman, 1987, p. 14), through a cycle of activities involving comprehension, transformation, instruction, evaluation, and reflection, leading to new comprehension. Briefly, teaching first involves *comprehending* the content and its purposes in different ways and relate them to other ideas within and beyond the subject to be taught. The level of comprehension then influences how a teacher *transforms* his or her content knowledge into "forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (Shulman, 1987, p. 15). Transforming this knowledge involves preparation, representation, instructional selections, adaptations of these representations and tailoring the representations to specific students' profiles. While we may argue that comprehension and transformation can occur at any time during teaching, Shulman (1987, p. 18) sees these two processes as "prospective", occurring before *instruction*—an "enactive" performance in the classrooms. Shulman highlights *evaluation* and *reflection* as retrospective processes by which a teacher looks back at students' responses to instruction to learn from his or her experiences. This learning is encapsulated in the notion of *new comprehension* where teachers have a better understanding of teaching and learning. Shulman and Shulman (2004) position this kind of learning from experience through reflection as the capacity for enacting purposeful change. But *reflection does not necessarily lead to learning from experience*.

Second, we explain why some teachers learn from their teaching experiences, while others do not, by referring to Schoenfeld's (2011) conceptualisation of teaching as an goal-directed activity, which rests on a set of resources—a teacher's knowledge—and is driven by a teacher's orientations—one's beliefs and preferences. A teacher's existing knowledge provides the basis for an initial comprehension of content. The teacher then transforms the initial comprehended ideas into a form suitable for teaching the students, subjected to the teacher's cluster of goals and orientations. The orientations aspect is important because beliefs have high inertia (Schoenfeld, 2011), and this explains why it is sometimes difficult to change teachers' practices even after professional development activities. The iterative and cyclical processes of transformation, actual instruction, and evaluation feed forward to the reflection of the teacher (to different extents for different teachers). This process leads to some new comprehension, which may lead to an *expanded* set of resources, orientations, and goals (ROGs), corresponding to the idea of learning from experience, and the cycle repeats.

But what enables this learning from experience for an expansion of resources, orientations, and goals to take place? Here, we refer to Choy's (2016) ideas of productive noticing and argue that learning from experience occurs when a teacher notices critical aspects about the content, aspects of student learning, and appropriateness of teaching actions. Simply stating, a teacher's productive noticing is likely to raise an awareness of new possibilities (new comprehension), leading to an expansion of one's current cluster of ROGs (Choy, 2016). This expanded cluster of ROGs thereby becomes the base from which the teacher makes sense of instruction. Moreover, as Choy (2016) highlighted, productive noticing can take place during planning, instruction, and reviewing of lessons. Consequently, we argue that new comprehension leading to learning from experience can occur during any of the activities of Shulman's model. Putting these three ideas together, we conceptualise the invisible work of thinking about one's practice in terms of an adaptation of Shulman's model of pedagogical reasoning and action. Referring to Figure 1, we see how a teacher's prior set of ROGs may influence the initial comprehension of the content to be taught. This initial comprehension leads to what we perceive as an iterative cycle of transformation, instruction, and assessment. Note that we replace the term "evaluation" with "assessment" to denote a more comprehensive notion of assessing for learning and assessment of learning. By reflecting on the cycle of transformation, instruction, and assessment, a teacher may gain new comprehension, which may lead to an expanded set of ROGs, resulting in learning from one's experiences. The key to learning from the processes of reflection on experiences during planning, teaching, and reviewing of a lesson is to productively notice aspects of teaching and learning that promote student understanding of mathematics (Choy, 2016).

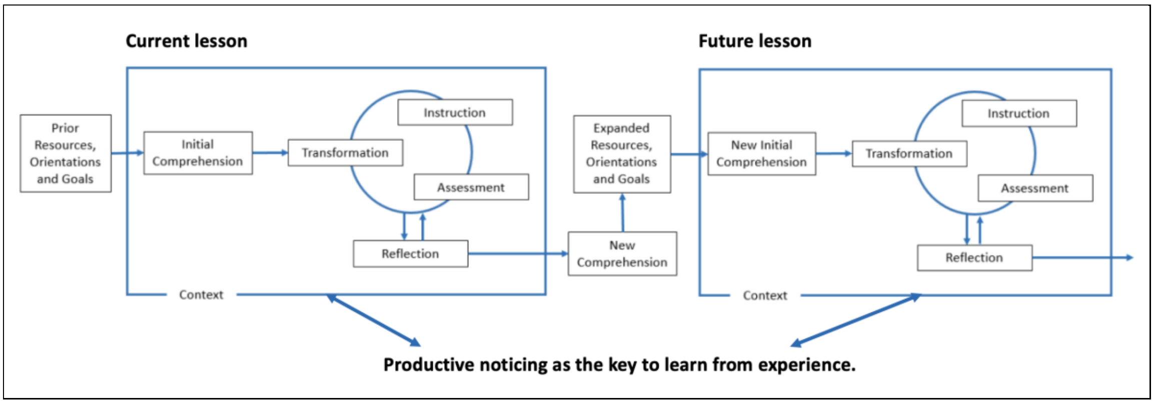

Figure 1. Adapted model of pedagogical reasoning and action.

Methods

The data presented in this paper were collected as part of a larger project, involving a total of 39 mathematics teachers from three different primary schools, which aimed to develop the proof of concept for a new professional learning model. Drawing on current theoretical perspectives of teacher noticing (Fernandez & Choy, 2019), we conceptualised professional learning sessions where teachers would have opportunities, in the context of a community of inquiry, to work and co-learn with us by engaging in activities related to our adapted model of pedagogical reasoning and action (see Figure 1). For this paper, we focus on a series of sessions and activities we had conducted with teachers from Sandy Shore Primary School (pseudonym).

First, we engaged all the teachers in two online professional learning sessions conducted over the Zoom platform. The sessions were conducted online because of prevailing COVID-19 restrictions at that time, which prevented us from holding in-person meetings. During the sessions, which were facilitated by one of the teachers, we took on the role of a knowledgeable other to share new ideas for teaching or critique existing ideas. The teachers from Sandy Shore Primary School decided to work on teaching of division for Primary Three pupils. Although we provided teachers access to relevant research and practice-based articles when requested, we did not insist that the teachers must adopt our ideas. Instead, we left all the instructional decisions to them because we wanted to investigate the thinking behind their instructional decisions. After the second session, we followed Shawn (pseudonym), one of the teachers in the discussion, as our focus teacher and observed his teaching of the unit on division. An accountant by training, Shawn had worked in the finance sector and had some part-time teaching experiences before he went on to be trained as a teacher in 2018. At the time of this study, Shawn had over three years of teaching experience at Sandy Shore Primary School. After Shawn's last lesson on division, we conducted the third and final professional learning session to collectively reflect on our learning. Data collected include voice and video recordings of the discussion during the sessions, photographs of lesson artifacts such as lesson plans, discussion notes, and when available, samples of students' work.

To make Shawn's pedagogical reasoning more visible, we turned to the idea of pedagogical documentation (Alcock, 2000; Lee-Hammond & Bjervås, 2021), which is widely practised in early childhood education settings. The practice of pedagogical documentation involves teachers in collecting written notes, audio and video recordings, photographs, or students' learning artifacts for describing what and how students learn, which then serve as a basis for reflection and making instructional decisions (Lee-Hammond & Bjervås, 2021). In this way, the documentation serves as both a product and a process to support teachers in their professional learning. For this study, we used Padlet (https://padlet.com/), a digital notice board, as a platform for Shawn to curate his pedagogical documentation. We did not impose any number for the reflections—instead, we asked him to post his reflections, photos, videos, or documents related to any incident that he had found interesting on the Padlet—and we left all instructional decisions to Shawn. Our role was to observe what he had learned from our sessions, his selection of tasks and the instructional decisions made during his lessons.

Findings were developed through analyses of Shawn's Padlet entries, his teaching materials, and his teaching videos. We identified parts of his reflections, artifacts, and actions that were related to aspects of division concepts (Brown et al., 2011; Takker & Subramaniam, 2018), students' confusion about division concepts and algorithm (Holland, 1942), as well as Shawn's knowledge and instructional decisions about the teaching of division (Simon, 1993; Takker & Subramaniam, 2018). By relating these selected artifacts to our adaptation of Shulman's (1987) model, we made inferences about Shawn's comprehension of division, his transformation of understanding into lesson materials, his instruction, assessment, and reflections. We also identified *critical incidents* (Goodell, 2006), which are "everyday" events encountered by teachers that made them question their instructional decisions, thus providing "an entry to improving teaching" (p. 224), that occurred during Shawn's lesson and reflection. More specifically, we wanted to examine if Shawn had *noticed* critical aspects of content, student thinking, and teaching approaches from these incidents (Choy, 2016).

Findings

Here, we present snapshots of Shawn's pedagogical reasoning as seen from his pedagogical documentation in the Padlet, supported by evidence from the lesson materials he used, relevant snippets from his teaching, and points of discussion raised during our professional learning sessions. It is important to note that we are *not criticising* his way of teaching nor evaluating his practices. Rather, we are *making visible* his pedagogical reasoning to gain insights into what he noticed about the content, student thinking, and the teaching approach adopted.

Shawn's Comprehension of Division: Division as Equal Sharing?

In our first two professional learning sessions, we elicited the teachers' prior understanding of division by getting them to articulate ideas about division and how they would explain $48 \div 4$ and $48 \div 3$ using different ways. From his Padlet entries, we see that Shawn was familiar with the idea of division as equal sharing or partitive division (Simon, 1993), or the idea of finding the number of items in each equal group. He wrote "For division, the idea is on equal sharing. For chapter 4, students need to understand that there is an algorithm to learn to solve 2 or 3-digit division." In Figure 2, we see his "demonstration" of the two division problems. He demonstrated his understanding of partitive division, his emphasis on place values (Holland, 1942), and his coordination between the pictorial representation and the symbolic representation of the long division algorithm. For instance, in $48 \div 4$, he "shared" the 4 tens into the four equal groups ("1 ten in each group") and the 8 ones into four equal groups ("2 ones in each group") and showed how he would explain the algorithm by referring to the place values of the digits (e.g., the digit 4 is "4 tens", which came from 1 ten × 4).

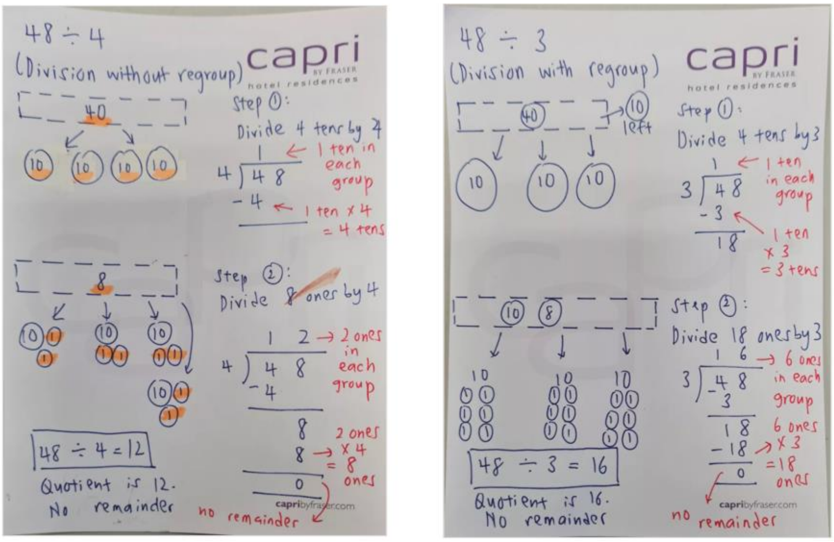

Figure 2. Shawn's demonstration of the two division problems.

Shawn's Transformation, Instruction, and Assessment: Emphasis on Equal Sharing and the "DMSCB" Algorithm

At this point, it appears that Shawn seemed to see division only as equal sharing. However, from the slides he used, it was clear that he was aware of division as equal grouping (see Figure 3, left). Nevertheless, an analysis of his explanations from the video recordings and his use of the lesson materials suggests a stronger emphasis on division as equal sharing, as evidenced from his transformation made visible by his lesson materials (see Figure 3, right).

We noted three interesting insights about his pedagogical reasoning from his materials and instruction. Firstly, we see that Shawn was aware of division as equal grouping or quotitive division (Simon, 1993) even though he did not mention the idea during the professional learning sessions. Besides the slide on equal grouping, he also selected the storybook "Divide and Ride" by Stuart Murphy, which depicted equal grouping scenarios (such as "11 divided by 2 = 5 full seats with 1 friend left over."). Next, Shawn incorporated a mnemonic "GET" to support his students in remembering the relationship between the number of equal groups (G), the number of items in each equal group (E), and the total number of items (T). As seen from Figure 3 (left), he used the equation "Total ÷ Each = Group" to remind students about the idea of equal grouping. Likewise, he had a similar equation for equal sharing. His thinking behind this move was captured in one of his reflections on Padlet, dated 30 March, as follows:

> Since P2, they have learnt that G × E = T, Group × Each = Total. In P3, they have to interpret this equation and understand that Each = Total ÷ Group and Group = Total ÷ Each.

Although we may question the use of such mnemonic in promoting relational understanding, it is clear that Shawn wanted to build on what students had already been taught in Primary Two to bring across the relationship between multiplication and division, as well as the two different notions of division—equal grouping and equal sharing.

Thirdly, despite his awareness of equal grouping, his transformation and instruction strongly suggested a preference towards the use of equal sharing. In Figure 3 (right), we see how Shawn used the idea of equal sharing to distribute the 3 hundreds, 6 tens, and 9 ones into three groups and coordinated the use of the division algorithm or what he called "DMSCB" algorithm ("Divide, Multiply, Subtract, Check, and Bring Down") with the different representations (number discs and symbolic).

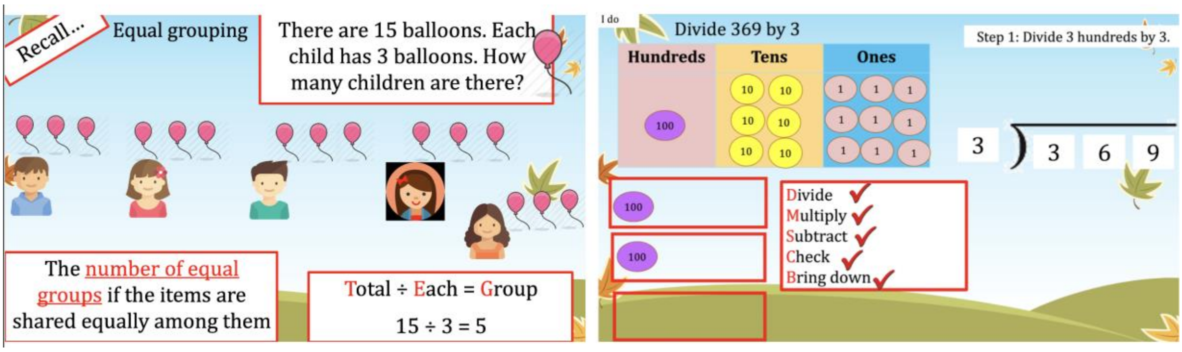

Figure 3. Snapshots from Shawn's teaching materials.

His instruction and assessment clearly reflected a strong emphasis on teaching the "DMSCB" algorithm. For example, he noted, in his reflection on Padlet, that "students had issue placing the wrong number at the wrong place value" and he "reinforced" the idea that students "have to clear the 'particular lane' (referring to place value) by doing the DMSCB first, before they proceed to the next lane" (Shawn's reflections, 5 April 2021). Again, he reflected that while "some students could follow the 5-step algorithm to derive the answer" to 682 ÷ 2, a "handful of them did not apply the 5 steps" (Shawn's reflections, 6 April 2021). He noticed that students might not have applied the 5-step algorithm because "the numbers 6, 8, and 2 are in the 2 times table and got the answer immediately by dividing". Interestingly, he acknowledged that he "should not dismiss that the method is wrong" even though he had actually insisted his students to use the full DMSCB algorithm during the lesson. While Shawn was cognisant of the idea that the long division algorithm "is a way of keeping track of the numbers", he seemed to prefer his students to follow the DMSCB method. This can also be seen from his proposed use of questions such as "672 ÷ 4" so that his students "have to rely on the long division method to keep track of the numbers" (Shawn's reflections, 6 April 2021).

Shawn's Reflection of a "Critical Incident"

This dominant use of the DMSCB algorithm was questioned by Shawn himself after the lesson on "long division with regrouping" when he realised that "students were initially confused by multiplication and division" (Shawn's reflections, 7 April 2021). In that reflection, he also wrote: "It suddenly occurred to me that there might be a simpler way of long division without going through the whole motion of DMSCB". We had also pointed out to Shawn that he "might have relied too heavily on the DMSCB method" (Shawn's reflections, 8 April 2021). Consequently, it appeared that he considered another approach that resembled a chunking strategy (see Figure 4), or what was commonly referred to as the partial quotient strategy (Takker & Subramaniam, 2018). In his reflection (dated 8 April), he wrote:

> In the question 72 ÷ 4, I solved it with the use of number disc, where I can share 1 group of 10 each equally into the 4 groups, then students recall that they can use 32 ÷ 4 = 8, all they have to do is to use 10 + 8 = 18 to get the answer. However, one drawback about this method is that it requires the use of number disc to solve the question.

As seen in Figure 4 (left), Shawn continued in his use of "equal sharing" to explain the chunking strategy in his reflection. It was interesting to note that Shawn did not think that this method could be an alternative to the DMSCB algorithm. He also highlighted that the method "requires the use of number discs to solve the question" and resisted teaching this method to his students even though he opined that his students might understand this method better. In addition, Shawn did not consider how an equal grouping notion of division might offer an alternative explanation for the division algorithm (Simon, 1993), and hence the chunking strategy. It appeared that Shawn did not consider the use of the chunking strategy because he did not notice the connection between the chunking strategy and the standard algorithm (Choy, 2016), as highlighted by us in Figure 4 (right, parts (a) to (c)).

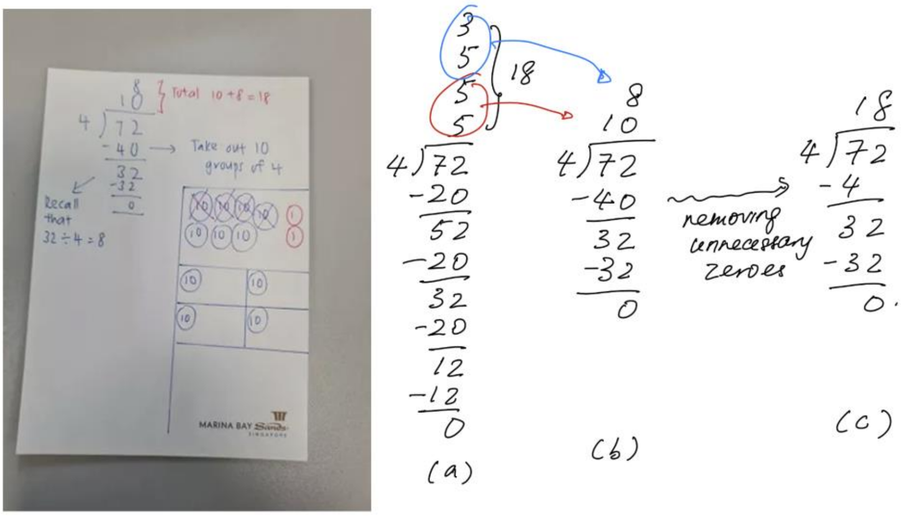

Figure 4. Shawn’s idea of chunking (left); relationship between chunking and standard algorithm (right).

Concluding Remarks

Our analysis of Shawn’s pedagogical reasoning and action made visible by his pedagogical documentation during his series of lessons on teaching division reveals *unseen* aspects of his visible practice (e.g., his understanding of division as equal sharing and equal grouping, the use of “GET” mnemonic and his preference for the DMSCB algorithm). More importantly, we uncover the complexity of learning from practice by highlighting that while Shawn might have learned some new ideas (chunking strategy), he might not have *productively* noticed (Choy, 2016) about the connections between this strategy and the standard algorithm to impact his practice. This paper also illustrates the affordances of pedagogical documentation to make visible a teacher’s thinking, beyond its usual use to reveal students’ understanding (Alcock, 2000; de Sousa, 2019). By making one’s pedagogical reasoning visible, we can make connections between a teacher’s visible actions and the corresponding invisible aspects of practice. This is critical for supporting teachers’ efforts to develop their adaptive expertise in teaching in three ways. Firstly, as a form of reflection, the teacher’s pedagogical reasoning made visible by the use of pedagogical documentation serves as a means for teachers to learn from their day-to-day teaching experiences (Shulman & Shulman, 2004). Secondly, snapshots of how Shawn learned from his practice reaffirms that learning takes place over a sequence of lessons. Hence, we need to follow a teacher’s pedagogical reasoning through several lessons to uncover the invisible aspects of practice. Thirdly, by making visible a teacher’s thinking about instruction, we can better pinpoint the components of pedagogical reasoning that needs attention and the support that teachers need. Doing this can potentially transform the way we approach professional learning and development. However, documentation is hard work even though it can unpack the invisible aspects of one’s practice. How this can be done sustainably in the context of a teacher’s busy schedule will be an important area for future research.

Acknowledgement. This study was funded by Singapore Ministry of Education (MOE) under the MOE Academy Fund Programme (AFD 06/17 CBH) and administered by National Institute of Education (NIE), Nanyang Technological University, Singapore. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the Singapore MOE and NIE.

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Perceptions of the Role of Primary Mathematics Leaders

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Primary mathematics leadership has become a focus for improving the teaching of mathematics within Australian primary schools. Much of the training and support for those in the role have concentrated on content knowledge, rather than leadership training. There are currently no guidelines or standards in place to support the complex and multi-faceted roles and responsibilities of primary mathematics leaders. This paper reports on the initial stage of a research project examining how primary mathematics leadership is conceptualised and experienced. It reports on survey findings regarding teachers and leaders' understanding of primary mathematics leadership.

Primary Mathematics Leadership

Australian students' mathematics and numeracy performance in national and international testing have generally remained unchanged or declined in the last 15–20 years (Australian Curriculum, Assessment and Reporting Authority, 2017; Thomson, De Bortoli, & Underwood, 2017; Thomson, Wernert et al., 2017). This has prompted a re-evaluation of the approaches and support for mathematics education (Education and Training Committee, 2006; State of Victoria, 2017). Improving teacher capacity and confidence in teaching mathematics in primary schools has become a prominent focus nationally and in particular, within the state of Victoria. One initiative designed to improve teacher competency and confidence in mathematics has been to appoint mathematics leaders as part of the leadership team in schools. These mathematics leaders/specialists are more experienced and trained to support generalist primary teachers. This supports the Victorian Department of Education *Literacy and Numeracy Strategy* and focuses directly on the utilisation of middle level leaders in schools. These middle level leaders are expected to be instructional leaders with deep content, assessment and pedagogical content knowledge and focus on instructional coaching (State of Victoria, 2017). This places mathematics leadership at the centre of improving mathematics teaching and learning within schools.

Literature Review

Teacher leadership is a central component of leading mathematics in primary schools. Teachers that undertake mathematics leadership roles in schools are often categorised as "middle level leaders". Middle leadership is a relatively new term used by schools and replaces the idea of "middle management". This reflects the shift in roles from manager to leader and differentiates between administrative aspects of the role to a more strategic leadership focus (De Nobile, 2017). There are varying definitions of middle leadership, with most focused on the secondary school sector. Definitions recognise that the work of middle leaders is comprised of a formal and/or significant responsibility for a particular area of the curriculum, initiatives, or processes (Bennet et al., 2007; De Nobile & Ridden, 2014; Gurr & Drysdale, 2013). Within schools, the name of this middle leader role in primary mathematics has varied, such as "numeracy coordinator" (Cheeseman & Clarke, 2005), "mathematics education leader" (Eacott & Holmes, 2010), "school mathematics leader" (Sexton & Downton, 2014), and "primary school mathematics leader" (Driscoll, 2017). For the purpose of this study, previous definitions of middle leaders have been adapted and the term "primary mathematics leaders" has been defined as:

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 146–153. Launceston: MERGA.

> Teachers who have a formal and significant responsibility for improving student learning through mathematics education leadership of teaching teams, curriculum, resourcing, planning, instruction and assessment processes within the school.

The work of middle level leaders can be both complex and ambiguous (Gurr & Drysdale, 2013). A key factor in the work of middle level subject leaders is the middle leader’s expertise as a teacher and in the subject matter that they are teaching (Harris, 2009). Their pedagogical content knowledge and assessment knowledge, along with their ability to analyse data has a critical impact on improving teacher impact and student learning (Bennett et al., 2007; Dinham et al., 2011). In a review of secondary curriculum leadership, Leithwood (2016) found a strong correlation between student performance and work conducted by the curriculum heads because, as an extension of the school’s administration, the leaders have direct contact with teachers and students daily. Some mathematics leaders in primary schools also have a teaching aspect to their role, which can have a significant impact on classroom practices and educational outcomes due to their leadership in and between classrooms (Grootenboer, Edwards-Groves, & Rönnerman, 2015). It is this role of the middle leaders that has been found to be key to the successful implementation of improved practices in mathematics; they are critical in connecting the vision of the school to the enacted curriculum at the classroom level (Jorgensen, 2016).

A middle leader’s role is varied and complex, containing many different aspects or practices that can be implemented in the undertaking of the leadership position. Leithwood et al. (2008) grouped these practices into four categories, similar to role categories developed by De Nobile (2017), who grouped them into six categories. There were also similarities found by Gurr and Drysdale (2013), where successful curriculum leaders had certain traits in common (see Table 1).

Table 1
Summary of Models of Middle Level Leadership Roles

Leithwood et al. (2008)	Gurr and Drysdale (2013)	De Nobile (2017)
Building the vision and direction	Focus on student learning	Student focus
Understanding and developing people	Interpersonal skills	Administration
Organisational change	Allocate resources	Organisation
Overseeing teaching and learning	Promotion and advocacy of area	Supervision
	Planning and organisation	Staff development
	Shared vision and purpose	Strategy
	Teacher learning	
	High expectations	

There are many influences that support or hinder the successfulness of leading and improving mathematics such as: the pedagogical content and assessment knowledge of the leader; high expectations and support for all students; the relationships the leader has with staff and the principal; the culture of the school; and the leader’s ability to drive vision (Balka et al., 2010; De Nobile, 2017; Leithwood et al., 2008). Driscoll (2017) found that leaders self-reported challenges and influences on their role included: time, leader confidence, leader’s expertise, teacher knowledge and funding. The alignment or acceptance of reform by the teaching body, and peer support for change have also been identified by mathematics teachers as challenging to successful leadership (Kitchen et al., 1997).

Traditionally in primary schools, the focus for mathematics curriculum leaders has been management of the curriculum, programs, and resources. Leaders must go beyond management, beyond making sure that things get done, a leader must enact change (Balka et al., 2010). Leadership is about recognising what the goal of the organisation or group is and ensuring that the goal is met. This is influenced through the actions and behaviours of the leader of the group (Balka et al., 2010).

The challenge therefore for practising primary mathematics leaders is to understand their own role and to develop their leadership skills in a context where there is often little guidance. This study examines the perceptions that teachers have of their roles as primary mathematics leaders and the challenges that they currently identify as barriers to successful leadership. This paper reports on a section from an initial survey that forms part of a PhD research study exploring how primary mathematics leadership is conceptualised and experienced by primary mathematics leaders.

Methodology

This study adopts a representative case study model where individuals from a range of relevant experiences are interviewed to provide insight, which may be generalisable when understanding how primary mathematics leadership is conceptualised. Yin (2017) defined case study research as a qualitative study that investigates a phenomenon (in this case, primary mathematics leadership), which is embedded within a real-life context. The context for this study is situated within a hierarchy of organisations, the school level, is the most basic level and dependent on localised factors. However, it is situated within the context of Victorian education policy and regulations, which are in turn informed by national policy and regulations. Using case study as a method for exploring roles within systems is also supported by Cohen et al. (2018).

The aim of this research is to gain a better understanding of how primary mathematics leadership is conceptualised by primary mathematics leaders, teachers, and other school staff, as well as those not in schools, who either inform policy and regulations or work with primary mathematics leaders, within the context of Victorian schools. In this research, data were collected from those in the role and those in the wider systems to build an understanding of the role of a primary mathematics leader and determine what can be learnt from these cases (Cohen et al., 2018).

Participants

Participants in the survey represented a wide range of roles in primary mathematics education, including teachers and leaders. Respondents also identified the postcode of the schools in which they worked, which enabled a range of perspectives from differing locations, according to the models classifying a geographical location based on the level of remoteness and population size (Modified Monash 2019 and ASGS 2016—Australian Geography Statistical Standard). There are more respondents from metropolitan areas then regional areas, but no respondents from rural areas.

Respondents were required to identify their role title from a drop-down menu. For those respondents who self-classified as a teacher, there were two distinct groups. The first group were classroom teachers who had responsibility for teaching mathematics in their classroom, or leading planning for mathematics within their team/year level group. The second group were almost all highly accomplished teachers or learning specialists, and one graduate teacher. This group had a much wider role and could perhaps be considered numeracy leaders, although they had not self-identified as mathematics leaders. They were, however, able to identify their leadership responsibility (Driscoll, 2017). Many of these teachers worked part time in the classroom and part time as a mathematics specialist or numeracy coordinator. They listed duties such as coaching and planning across the school, organising and running staff professional development, and supporting teachers.

Within the group who had formally identified as mathematics leaders, most respondents were again classified as highly accomplished teachers or learning specialists. There was also one leading teacher and one teacher at the proficient classification. This designated position of mathematics leadership also included some with a current teaching role (Grootenboer et al., 2015). Four respondents in this group also stated that they still had a classroom teaching allocation. This may be the case for more of this group, however, this was not explicitly stated in their responses. The duties that they listed included similar aspects to the second group above, and additionally included comments referring to use of data, coaching, mentoring, modelling, and collaboration.

It is interesting that the teachers themselves were distinguishing whether they thought they were leaders or not. Perhaps the first group saw themselves as more of a manager, using the title co-ordinator (Cheeseman & Clarke, 2005) instead of leader, even though their role descriptions included aspects of leadership. It is important to see leadership as a developing process or a continuum, and perhaps these respondents could be considered beginning middle leaders. Driscoll (2017) noted that that some mathematics teachers lack confidence in their role as leaders and that leadership expertise develops over time. The key difference in the roles as defined by the two different groups appeared to be coaching, mentoring, collaboration, leading professional learning communities (PLCs) and using data.

Method

This paper focuses on the initial stage of the research project, a survey undertaken to map the field. Teachers and mathematics leaders were surveyed online, via a Qualtrics survey, on their conceptions of primary mathematics leadership. There were over 60 respondents. The survey included a mix of demographic questions, Likert scale questions, short answer questions, and open-ended questions, some of which enabled longer answers. For the purposes of this paper, the focus is on one of the questions from the survey, where respondents were asked “What do you think mathematics leadership is?”

Qualitative analysis of the data was undertaken using an inductive approach to determine emerging themes and to define codes. The data were analysed and interpreted to obtain common themes and concepts (Thomas, 2006). Themes and categories were created, and then summarised, refined and modified based on subsequent analyses (Cohen et al., 2018). In the coding of the data, meaning was sought through close examination of the data and categorised according to relevance (Thomas, 2006). These themes were then considered in light of middle leader role definitions used by Leithwood et al. (2008), De Nobile (2017), and Gurr and Drysdale (2013).

Results

Responses to the question “What do you think mathematics leadership is?” included a range of answers from single words to short sentences to longer paragraphs. Initial analysis created a large number of codes; however there appeared to be some overlap in areas. Some names of codes did not accurately reflect the group either, such as “ongoing professional learning”. This could appear to be focused on helping others with ongoing learning but was intended to reflect the ongoing learning of the leader. Subsequently some new codes were developed or renamed, to reflect the overlap and to reflect the five emerging themes more clearly. These can be seen categorised below in Table 2 with examples of responses.

Table 2
Results of Analysis

Themes	Sub-categories	Example responses
Culture	Buy in	Being able to bring teachers along in the journey
	Positive relationship	…being a cheerleader, acknowledging teachers' achievements and celebrating their success
	Valuing mathematics	…promote and lead mathematics in the school community
	Whole school approach	Leading the entire Mathematics culture of the school
Knowledge expertise	Content Knowledge	Mathematics leadership involves a deep knowledge of mathematics content and pedagogy.
	Curriculum knowledge	Having an excellent understanding of how to teach the breadth and width of the curriculum to all year levels
	Pedagogical content knowledge	Being flexible but also knowledgeable about pedagogies
	Expertise development	… leadership requires being aware of both current teaching and learning practices and new approaches that are trialled and reported.
Administration/ management	Resourcing	Providing teachers with quality resources/ professional readings about current best practice
Teacher development	Staff development	Being able to understand your staff so you can upskill them in areas they need support.
	Use of PLCs	Developing strategies for improving teacher knowledge of curriculum and pedagogy with teacher through reflection in the context of a responsive and supportive learning community.
Student learning and assessment	Data and Assessment	To work with teams to look at their data and allow that to really drive what needs to come next for each student.
	Implementation and classroom practice	Leaders need to be able to support teachers implementing best practice
	Planning	Knowledgeable guidance and assistance in planning, implementing and analysing
	Student learning	Quality leadership in mathematics education is fundamentally about improving … student outcomes.

Discussion

Culture

Participants' responses coded for the theme of "Culture" appear to resonate with ensuring that there is a whole school approach to mathematics at the school. This validated the idea of the role of a middle leader to develop strategy and build a shared vision and purpose (De Nobile, 2017; Gurr & Drysdale, 2013; Leithwood et al., 2008). Other responses within this theme mention encouraging staff, getting them on side and working positively with them, especially through change. This connects with the role of managing organisational change (Leithwood et al., 2008). Positive relationships appear to be an important factor, which aligns with a leader's need for effective interpersonal skills (Gurr & Drysdale, 2013). Promoting and valuing mathematics in the school and wider community were also included in this theme, supporting the importance of promotion and advocacy of the area (Gurr & Drysdale, 2013).

Knowledge Expertise

Knowledge expertise had many items within the themes. Although this did not seem to be a focus of the roles in the models discussed, subject expertise was considered important by the respondents. De Nobile (2017) suggested that knowledge of curriculum, pedagogy and assessment is a factor that influences the effectiveness of the middle leaders but does not list expertise as a role of middle leaders. In the specific case of mathematics leadership, there has been an assumption that leaders are experts (Jorgensen, 2016) and that they will pass on their knowledge and share expertise with teachers within their schools (Driscoll, 2017). Respondents concurred that expertise of the leaders in content, curriculum and pedagogy was central to primary mathematics leadership, which Harris (2009) suggested is necessary for curriculum leaders and stated that knowledge should be evidence based, and continually developing. Seeking feedback from other teachers and leaders about their own classroom practice was part of critical reflection and inquiry to improve their own practice. Respondents stressed the importance of knowing and understanding not only the content but also knowing and understanding how children learn that content and the best ways to teach it. Leaders were seen to exemplify best practice.

Administration/Management

The Administration/Management theme had the least number of items from respondents; however, it has been included due to responses in other survey items, such as, "Briefly describe the duties of your role", "What do you think are important attributes for a good mathematics leader?" and "What skills do you think are needed for a good mathematics leader?" These questions have helped to build a broader picture, but for the purpose of this paper, have not been included. Administration/Management is an important aspect of the role of a middle leader. Responses focused on organisation of classroom resources, tools and professional learning resources, aligning with De Nobile's (2017) identification of the roles of administration and organisation, and Gurr and Drysdale's (2013) classification of taking responsibility for area planning and organising and the allocation of resources.

Teacher Development

This theme included the largest number of responses. Responses varied from ensuring that teachers receive the professional development that they need as well as ensuring that all teachers are focused on school priorities. Participant responses included providing professional support to colleagues to strengthen their mathematics teaching pedagogy and supporting teachers to seek, analyse, and act on feedback on their practice. De Nobile (2017) referred to the role of supervision for middle leaders. In this context, supervision is focused on mentoring and reflective practice, rather than regulation. Middle leaders adopt more collegial approaches, and are seen taking on a supportive role, rather than a disciplinary one. Teachers surveyed spoke of leading and collaborating with staff in building teaching practice to improve learning outcomes. Responses also included the responsibility for mentoring and/or coaching teachers and guiding professional learning. Respondents had discussions with teachers about professional development available to help them to address their performance and development plan goals. Teacher learning, along with high expectations of staff (Gurr & Drysdale, 2013), developing people and overseeing teaching and learning (Leithwood et al., 2008) support the theme of teacher development.

Student Learning and Assessment

Initially "student learning" and "planning and assessment" were separate themes. However, there was considerable overlap between the themes and the two were combined. Leithwood (2008) also combined teaching and learning together with a middle leader taking responsibility for the oversight of this area, while Gurr and Drysdale (2013) and De Nobile (2017) separated the role aspect to focus on student learning and did not tie it to planning. In a primary context, the monitoring of student cohort data was considered an important aspect of teaching practice. As stated by respondents, there was a real need for a mathematics leader to be able to analyse data, interpret that data and then target planning to identify needs at their school. This really focuses on overseeing teaching and learning through assessment. A leader's knowledge and skills should be used to ensure all students are learning and continuing to improve in their learning, and assessment and data supports the process of identification of student need.

Conclusion

Analysis of the survey data in response to the question "What do you think mathematics leadership is?" has seen five themes emerge that reflect the perceptions of the respondents. These themes included middle level primary mathematics leaders viewing their role as complex and multi-faceted. Themes included: culture, knowledge expertise, administration/ management, teacher development and student learning and assessment. These themes show alignment with previous research on middle level leadership (De Nobile (2017; Gurr & Drysdale, 2013; Leithwood et al., 2008) along with a clear need for expertise in primary mathematics content and pedagogical content knowledge. The survey will be followed by semi-structured interviews and document analysis (including school-based, policy, and procedural documents) to further investigate the conceptions and experiences of primary mathematics leaders and those who work with them.

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Resource Materials as Structured Guidance in Practice Change

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This study reports on an early career Year 2 teacher's reflections of enacting specifically structured resource materials and her practice change when participating in a mathematics professional learning project. A qualitative case study was designed to examine the reflections of a teacher's pre-service and in-service teaching experiences. Data collection instruments included a timeline graphing tool and a semi-structured interview to capture and communicate her reflections. Findings reveal that providing structured resource materials within a professional learning project supported changes in pedagogical practice.

Professional learning (PL) is a critical component in enhancing quality teaching and learning of mathematics, which impacts student learning outcomes (Guskey, 2002). Further, curriculum resources are frequently used as instruments to support or drive professional learning and teachers' change in practice (Rezat et al., 2021). Educational research on the impact of mathematical resource materials typically focuses on textbook use and student outcomes rather than changes in teachers' practice (Pepin, 2018). However, in the last decade, a focus on how and why teachers used an array of resources to support change in practice has become of increasing interest to researchers (Remillard, 2018). Utilising a range of resources may be challenging for early career teachers.

Curriculum resource materials may be defined in many ways. Typically, they are referred to as, "a package of resources assembled by developers for the purpose of guiding instruction and student learning" (Rezat et al., 2021, p. 1189). Educative curriculum resources are those "that are intended to promote teachers' learning" (Davis & Krajcik, 2005, p. 3) in addition to supporting student outcomes. In the context of this study, the term resource materials will be used to describe the educative curriculum materials the PL project team provided to the project teachers.

We report a case study of an Australian early-career Year 2 teacher's reflections of enacting practice change and her use of specifically structured resource materials. The teacher was participating in a two-year mathematics professional learning project titled *Exploring Mathematical Sequences of Connected, Cumulative and Challenging Tasks* (EMC^3). The project explored "ways to support both teacher and student learning [through]… an approach to resource development and teacher professional learning that uses the notion of relentless consistency to encourage innovative practices" (Sullivan et al., 2020, p. 11). These practices include sequences of cognitively demanding tasks, and lesson structures that support problem-solving and reasoning. (Sullivan et al., 2020).

The research question we sought to answer was:

> *How do resource materials support the experiences of an early career Year 2 teacher to change her practice when participating in mathematics professional learning?*

Literature Review

This section provides an overview of the literature pertinent to this study, including professional learning and resource materials, concluding with a framework for analysing teacher change.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 154–161. Launceston: MERGA.

Professional Learning and Teacher Change

PL evolved from professional development with an aim to enhance student learning through improving teacher practice. The shift to PL recognises the continual, reflective, and critical thinking characteristics of effective teacher learning (Bobis & Tregoning, 2019). Teacher change is an outcome of participation in PL. "[C]hange is identified with learning, and it is regarded as a natural and expected component of the professional activity of teachers and schools" (Clarke & Hollingsworth, 2002, p. 948). While change is open to many interpretations, Darling-Hammond et al. (2017) purport that "teacher knowledge, practices, and improvements in learning outcomes" (p. 2) are frequently the subject of teacher change. In our study, we focussed on self-reported practice change but acknowledge that the three are interrelated.

In her small-scale qualitative study of Year 7 and Year 8 teachers, Lee (2001) identified four factors that influenced teacher change. These four interrelated factors were time, professional development opportunities, support from policy-makers and administrative decision-makers, and availability of resources. Similarly, Forrest et al. (2019) described six factors that influenced teachers' change in practice: teacher knowledge, collaboration, reflection, ownership, time, and pressure. While these combined nine factors are all interconnected, in this study we focused on the impact of specific resource materials from the EMC^3 project and a case study of one teacher.

Curriculum Resource Materials

A resurgence of research on curriculum resource materials was evident in the 2000s, which led to the emergence of educative curriculum materials to support school reform (Remillard, 2018). Curriculum materials were often constructed to support grade or age-level student learning on a particular topic of goal, whereas educative curriculum materials were designed to enhance teachers' pedagogical approaches and promote student learning (Davis & Krajcik 2005; Rezat et al., 2021). According to these authors, educative resource materials should help to enhance teachers' knowledge for teaching mathematics and assist the development of general knowledge, which could be applied in new situations. Choppin (2011) termed this phenomenon as *learned adaptations* describing "knowledge-based adaptations designed with respect to what teachers have learned from prior enactments" (p. 335). Learned adaptations suggest teachers' understanding of the intentions of the educative curriculum resource and an ability to apply observations from previous experiences when enacting the material. Lloyd et al. (2008) stated that:

> …teachers are central players in the process of transforming curriculum ideals, captured in the form of mathematical tasks, lesson plans and pedagogical recommendations, into real classroom events [and] what they do with curriculum resources matters. (p. 3)

In other words, understanding what teachers do with resource materials and how these resources influence teachers' practice change is of importance. Both Choppin (2011) and Lloyd et al. (2008) called for further research to investigate how such materials support teacher learning and the materials influence classroom activities and future learning.

Resource materials are intended to deliver messages about teaching mathematics. The extent to which teachers adopt these messages is dependent on their beliefs, knowledge of the curriculum and pedagogical approaches (Choppin, 2011; Stein & Kim, 2008). Remillard's (2018) study in the United States of a Year 4 teacher's participation in the *Maths in Focus* project captured this complex relationship between resource materials and teacher interpretations and enactments. Findings showed that a proficient teacher was more likely to interpret and enact resource materials aligned to the designers' intent. A study in the United States by Stein and Kim's (2008) of curriculum resources concurred with the aforementioned author that educative curriculum materials are more likely to be enacted to the intentions of the designer than resources that provide no support.

> When curriculum materials are not transparent, teachers can have difficulty redirecting students who fall off the expected learning route; in such cases, teachers' on-the-spot decisions about how to guide them back to the path can be hampered by limited understanding of the underlying purpose of the lesson. (Stein & Kim, 2008, p. 51)

Clarke and Hollingsworth's (2002) framework offers a lens to analyse and investigate the complexity of teacher change in practice. *The Interconnected Model* of professional growth encompasses four interconnected domains that represent key change factors. The External Domain represents elements that sit outside of the teacher's personal world. It may involve stimulus from professional learning, such as the one described in this study. The Personal Domain recognises individual teachers' personal knowledge, beliefs, and attitudes. The Domain of Practice represents teachers trying new things in professional experimentation. Lastly, the Domain of Consequence describes the salient outcomes observed by teachers (Chan et al., 2019). The domains are linked via enactment or reflection (Clarke & Hollingsworth, 2002). Where enactment is the process of putting new ideas into action, reflection requires "active, persistent and careful consideration" (p. 948). The model depicts the complex nature of the teacher change process and is a suitable framework for this study when analysing the impact of resource materials on an early career teacher.

In summary, teacher change is an expected outcome of participation in professional learning. Educative curriculum resources are designed to support teacher learning and their interpretations. The Interconnected Model (Clarke & Hollingsworth, 2002) offers a way to analyse complex change processes and the impact of EMC^3 resource materials on an early career teacher's practice change.

Method

This case study employed a qualitative methodology to investigate an early career teacher's reflection of her experiences when using resource materials as part of the EMC^3 project. The context and participant along with the instruments, data collection and analysis are described here.

Context and Participant

The larger EMC^3 project provided PL to early years teachers over two years in the form of workshops, a mentor to support implementation, and project resource materials (14 learning sequences). Each sequence focused on a different mathematical concept, providing sequences of illustrative challenging lessons and supported differentiation, consolidation, and student agency (Sullivan et al., 2019; Sullivan et al., 2020). The explanatory statement for each task and planning documents assisted teachers to interpret and implement innovative pedagogical approaches when teaching. While the lesson sequences played an important role in teachers' PL, the goal was for teachers to adopt the pedagogies into their practice. This may include adapting the EMC^3 resource materials or incorporating the pedagogical features into other resources (Sullivan et al., 2020). Case study participant Andy (pseudonym) was selected from seven Victorian Catholic primary schools that participated in 2020-2021 PL. Andy, an early career teacher, was selected to reflect on her teaching experiences including practice change when implementing EMC^3 resources.

Instruments, Collection, and Analysis

Two instruments were used to collect data related to the teacher's experiences of using the resource materials. The instruments were a timeline graphing tool (Bobis et al., 2021) and a semi-structured interview (Galletta, 2013). The purpose of the timeline graph was to elicit Andy's pre-service and in-service experiences of teaching mathematics, including her participation in the project. Data were collected via an online interview (due to COVID-19), 18 months after the commencement of the project. Prior to the interview Andy was required to complete the first graphing tool (due to time constraints); the second was completed during the 45-minute semi-structured interview and video recorded for data analysis. The recorded file and Andy's timeline graphs were uploaded to NVivo and coded using a thematic analysis (Braun & Clark, 2006). Following a process of becoming familiar with the data, initial codes were developed. Initial codes included: resource materials, collegiate support, leadership support, team planning, team cohesion, and personal attitudes. Codes were grouped under themes of External, Personal, and Salient Domains, and the Domain of Consequence (Clark & Hollingsworth, 2002). For this research paper, we focussed on Andy's reflections and report the impact of the EMC^3 resource materials, on her change in practice.

Results and Discussion

Following a brief overview of Andy's background, data from her semi-structured interview and time-line graph tools will be reported, discussed, and analysed.

Summary of Andy's Background

Prior to participating in the project, Andy's mathematical confidence was low; "Growing up, I knew that I wasn't very good at maths. So, I've always had a negative perception." At the commencement of the project, Andy was in her second year of teaching Year 2 after having previously completed one year as a casual relief teacher (CRT).

Summary of Andy's Experiences

Andy was quite daunted to join the EMC^3 project. First, as an early career teacher, she was nervous about the intensity of simultaneously commencing her career and participating in a PL project. The initial negative reaction was expressed in her timeline graph (see Figure 1), as "doubtful of expectations." Andy elaborated on this further in the interview. "Coming in with having pre-service and CRT [experiences], and then expecting to learn and follow what our school implements for maths and coming on to the project. I thought it was all a bit blue."

The reaction stemmed from Andy's perception the project would add rigidity to her teaching. She thought her involvement would lead to a lack of flexibility in her practice. Second, at the time of the PL, the COVID-19 pandemic occurred across the world leading to an unstable environment in education. With possible impending government restrictions to manage the health crisis, Andy was concerned her team planning arrangements and teaching being may be impacted.

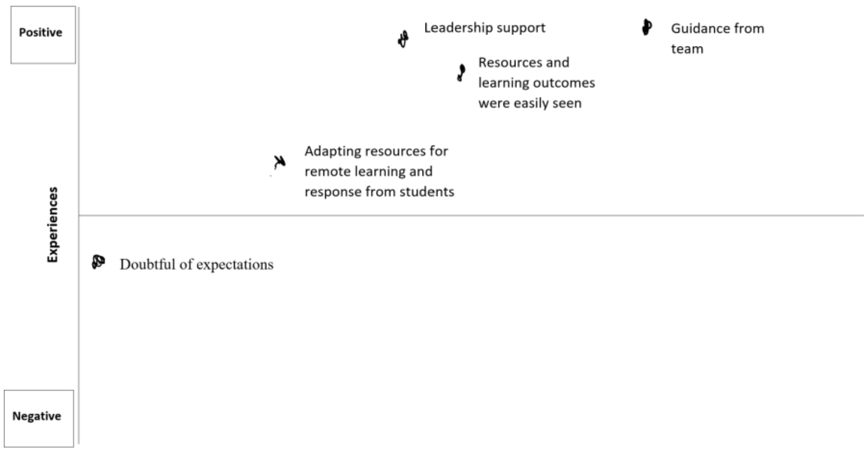

Figure 1. Andy’s timeline of EMC^3 project experiences.

However, after commencing the project, Andy’s hesitation changed. In particular, she found the resource materials helpful for implementing the pedagogical approach for teaching with challenging tasks. Her positive reflection and shift in her practice can be seen in her timeline graph with the comment “adapting resources for remote learning and response from students.” In the interview she elaborated further with:

> I think having resources and examples are really important. As a graduate teacher, I kind of second guess what I'm doing and I’m always asking, ‘Am I doing it right?’… So having the examples from the resource book is great.

Resource materials provided Andy with structured guidance in the first instance to assist her to implement the EMC^3 pedagogical practices. The resource materials provided a structure to be guide her teaching so that she wouldn’t “second guess herself”.

> It was great having all these tasks and ideas to implement. It was easy to follow, for us to make into seesaw tasks [remote learning tasks]. It was already there, what was expected, and the curriculum was there for us to follow… So, it [the resource materials] was quite good.

The resources provided her with practical exemplars of sequences of lessons, consisting of challenging tasks, curriculum content, and pedagogical considerations. As Andy became more familiar with teaching and planning with the resource materials, she reflected that they had a positive influence on her practice change. She reflected that the “resources and learning outcomes were easily seen” (see Figure 1) making it easier for her to interpret the pedagogy and the curriculum.

Andy noticed that when using EMC^3 resource materials, her students were experiencing mathematical success. She reflected that launching the lesson before instruction enabled students to make mathematical connections and apply their skills during the lesson. Such experiences provide students with positive learning experiences where they share their mathematical thinking and strategies. This approach is a key EMC^3 pedagogical practice. This was evident in Andy’s reflection of the “Making Things Equal” number sequence (Sullivan et al., 2019) elaborated next.

> One of my favourites would have been the tasks Making Things Equal, and how this particular quiet student finds maths challenging … she actually got the concept before anybody else. She was saying, “hold on, it doesn't say that I can't take away and I can't add, so I can keep adding.” I said, “Yes. What do you mean about keep adding?” [Student responded with] “Because I can do 100?” And I said, “Yep, exactly! Well done. Show me what [strategies] you can do.”

Witnessing students’ positive learning experiences helped Andy gain confidence and this, in turn, supported her practice change. With continued guidance from the resource materials, she reported building confidence in her mathematics pedagogy by adapting the tasks to meet her students' needs. An example of how Andy adapted a task was through her experimentation with differentiation. She implemented enabling (helping students who are struggling) and extending prompts (for those who finish the main task). Andy's reflection of using prompts was, "You can actually differentiate them [the tasks] to modify them for the enablers or those who are extending." The prompts made it easier for Andy to implement this approach.

> I would plan to teach it one way. Then I would go back into the [resource] book and say, "Okay, I'm on the right track." Or I would see that I could do something else instead and adapt my plan.

Once Andy developed her pedagogical approach, the resource materials became a source of confirmation rather than structured guidance. Instead of following the tasks as outlined in the learning sequence, Andy demonstrated learned adaptations (Choppin, 2011). She would continue to use the resources and apply her knowledge of teaching and students, and the EMC^3 pedagogies to develop a teaching plan.

Summary of Andy's Change Process

The experiences above are summarised next to describe Andy's change process. Figure 2 shows these changes mapped to an adapted Clarke and Hollingsworth (2002) model. The following numbered bullet points describe our interpretation of Andy's change process.

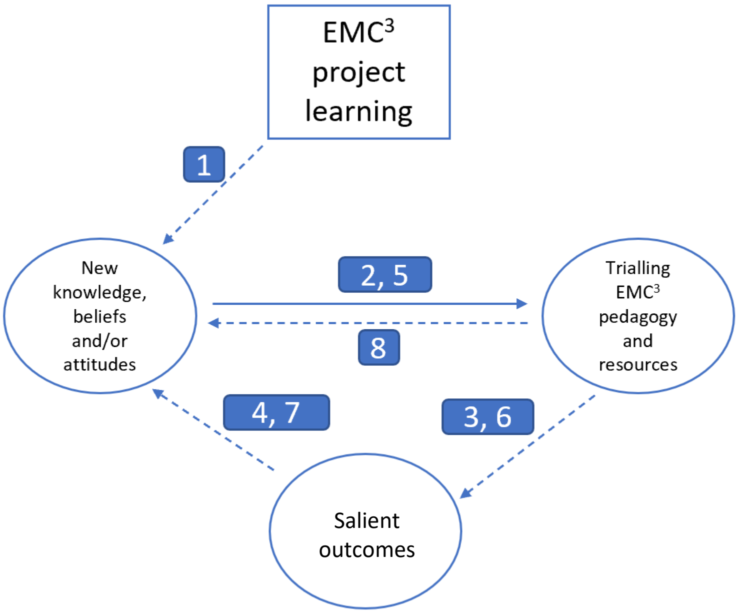

Figure 2. Andy's change sequence as interpreted by the researcher.

1. *Reflecting* on the introduction to the project during the COVID-19 pandemic, and adapting to online teaching, was overwhelming.
2. *Enacting* new knowledge of teaching sequences of challenging tasks.
3. *Reflecting* on students' positive learning outcomes supported confidence when implementing the pedagogical approach.
4. *Reflecting* on adapting the resource materials to meet students' needs, supported confidence to teach mathematics.
5. *Enacting* new knowledge of differentiation.
6. *Reflecting* on adaptation and implementation of resource materials supported student learning.

7 & 8. *Reflecting* on new pedagogical knowledge supported confidence in teaching mathematics with EMC^3 pedagogies. Now describing learned adaptations of the resource materials as confirmation rather than structured guidance.

In summary, there are four results. 1) When participating in professional learning, resource materials were one factor that supported an individual's practice change. 2) The combination of PL and resource materials provided opportunities to enact a new approach. 3) Witnessing her students' success, the teacher's confidence, and motivation to continue increased. 4) Once the teacher was confident with the approach, she adapted the resource materials to meet her students' needs while maintaining the EMC^3 pedagogical intentions.

Conclusion

The intention of this research was to investigate the experiences of an early career Year 2 teacher and the influence EMC^3 resource materials had on her practice change. Results suggest that the EMC^3 resource materials supported Andy's increased confidence to teach mathematics and her adoption of pedagogical practice change. The main finding from these results is that the provision of structured resource materials combined with the professional learning increased the teacher's confidence to implement an innovative pedagogical approach. A subsidiary finding was the benefit of using the timeline graphing tool to capture a teacher's reflections over time.

The main finding supports Davis and Krajcik's (2005) research on educative curriculum materials. They suggested that these materials "should help to increase teachers' knowledge in specific instances of instructional decision making but also help them develop more general knowledge that they can apply flexibly in new situations" (p. 3). In addition, our finding strengthens Stein and Kim's (2008) conjecture that educative curriculum materials are "more likely to lead to successful enactments in the classroom than materials that do not provided these [pedagogical approach] supports" (p. 44). However, we acknowledge that focusing on one teacher's reflections of her experiences as captured in a short interview, was a limitation.

As researchers we found inclusion of the timeline graphing tool facilitated questioning to generate rich dialogue and valuable reflections. As Bobis et al. (2021) posited, "combining a graphing tool with a semi-structured interview "encourages participants to provide rich descriptions of past experiences" (p. 137). Further exploration of this tool with experienced teachers will be of interest and extend this current study. Other research opportunities could include investigating the effects of combining PL with resource materials with larger cohorts of teachers.

Acknowledgment. We wish to acknowledge the contributions to the research design and this paper by Dr Sharyn Livy (Monash University).

The research that formed the basis of this article was funded by the Australian Research Council, Catholic Education Diocese of Parramatta and Melbourne Archdiocese Catholic Schools (LP 180100611).

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Designing an Early Number Sequence for Teaching

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This paper discusses the design modifications of a sequence of specified learning goals for teaching early number, which characterise an instructional resource developed in a design research collaboration with teachers in Mexican preschools. Highlighted are the types of research involvement that appear to be necessary in designing resources for teaching meaningful mathematics at scale. We argue that, and illustrate how, such involvements need to venture beyond addressing the problems of learning, into the territory of problems of teaching.

In this paper, and in our recent work, we grapple with the relation between researcher-produced instructional design resources and mathematics teaching. We take the perspective that teachers ultimately shape how mathematical activities play out (Pepin, 2018), and what forms of mathematical reasoning become characteristic of conversations in their classrooms. We then see the job of design researchers to be that of re-sourcing teachers (Pepin et al., 2013) for the demanding, intellectual task at their hand.

From a perspective of insiders to a design research tradition in mathematics education (Cobb et al., 2003), we draw out a distinction between material-and-conceptual-resources that are a product of classroom design experiments (CDEs) focused on student mathematical learning (e.g., Cobb, Gravemeijer, et al., 1997) and those that could be viable for re-sourcing teaching at scale. We seek to address the research question of what we need to understand when designing resources for teachers' use.

To open this question, we share our failures and successes in designing for teaching in a context of early number in Mexican preschools. We exemplify our learning that resulted from taking problems of teaching seriously across cycles of design. Specifically, we discuss iterations of the goals for student learning we shared with teachers, and how the teachers' interpretations of and responses to those goals led to their subsequent, more viable redesign.

Our design research included (a) an adaptation of an instructional theory in early number (Cobb, Gravemeijer, et al., 1997) for the learning of younger children; (b) an intensive collaboration with a single kindergarten teacher Jesica, the third author, conducting a CDE (Peña, 2018; Peña et al., 2018); (c) a collaboration, orchestrated by Jesica, with a group of teachers; and (d) the facilitation of intensive online workshops for groups of preschool teachers, and of an online community that formed around the designed resource.

Designing for Teaching

When conducting CDEs, researchers design, test, and modify the means of supporting the learning they strive to study, because to be able to study such learning, they must be able to first generate it in the classroom (Cobb et al., 2003). For instance, guided by an adaptation of the theory of Realistic Mathematics Education (Cobb et al., 2008), Cobb and colleagues pursued questions about collective mathematical developments that *can be generated* in classroom communities (even if such developments currently do not take place in any classrooms), and what forms of reasoning need to be sequentially supported in a classroom for such developments to become a reality. It may appear that both the formulated learning goals

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 162–169. Launceston: MERGA.

and the means by which these were achieved in CDEs should be shareable with mathematics teachers rather seamlessly. However, this does not appear to be the case.

Teachers have been shown to be highly capable of developing the complex and demanding teaching practices that require that they analyse, adapt, test and refine, in their own classrooms, the resources that were developed and proved effective in CDE classrooms (e.g., Visnovska & Cobb, 2019). However, the structural and institutional support conditions that are available to design research teams are rarely in place for teachers. As a result, while teachers may face problems in their students' learning, they face additional, immediate and pervasive *problems of teaching*, related to affordances and constraints of a mandated curriculum, assessments, or how their teaching is locally organised (Lampert, 2001). The resources produced in CDEs with a focus on student learning (e.g., Cobb, Gravemeijer, et al., 1997) are typically not designed to address these additional problems. In this paper, we discuss cycles of design research through which we strove to address the problems of teaching via resource design to make the use of novel resources viable for teachers.

It is important to clarify that when discussing a designed resource, we do not refer to a collection of classroom tasks that are given to the teachers to implement. The resource could instead be better imagined as a system of assumptions and forms of mathematical and pedagogical reasoning that allow teachers to independently pursue, in their classroom, a sequence of specified learning goals, while assessing the completion of these goals and deciding when to adapt/design additional classroom activities and when to *move on* to the next goal. The paramount goal of the designed resource is that teachers experience agency over the collective mathematical learning and education that takes place in their classroom and perceive their job as manageable.

Background and Method

The collaboration with Jesica started when she enrolled in a Masters degree program, with the first author as her advisor. Jesica taught in a public preschool attended by children from very-low-income families. In Mexico, *preschools* serve children in three year levels, prior to their transition to Year 1 of a primary school. At the start of preschool, the youngest children are 2 years and 8 months old. During her six years of teaching, Jesica was mostly in charge of Year 3 preschool classrooms, comparable to a Foundation Year in Australia.

For her Masters degree, Jesica conducted a CDE on early number with a classroom of 22 Year 3 preschool children, findings of which she reported in Peña (2018). Her study was a part of a dual design experiment (Gravemeijer & van Eerde, 2009), in which the first two authors collected and analysed data on Jesica's learning and resource use. Jesica later took a teaching job in a larger preschool, where her teaching practice attracted attention of colleagues, and led, over time, to a broader exploration of the teaching resource, when additional teachers trialled it in their classrooms. At this time, Jesica acted as a broker between the communities of the researchers (the 4 authors) and of teachers in her school and school-cluster. Efforts at supporting the group of teachers led to an emergence of a sizeable online community, and a provision of intensive teacher professional development events.

The method that allows us to make claims about teachers' uses of the designed resource is an adaptation of the method used to study student learning in CDEs (e.g., Cobb & Whitenack, 1996). Researchers collect data to document (a) research conjectures on what teachers will do and why doing so would be reasonable within their context (e.g., how they would use a specific aspect of the resource and why), (b) what teachers actually did, and (c) how teachers explained and justified their teaching decisions. During the ongoing analysis, researchers aim to understand where their initial conjectures about teachers' resource use "went wrong", adjust their understanding of how teachers reasoned with the resource in the specific context of their schools, adjust conjectures about teachers' future activity and, if needed, redesign the resource to better support teachers' decision-making process. A retrospective analysis is conducted upon completion of a study cycle to interpret, with hindsight, which resource modifications made the difference to the teachers and how the resource could be further modified prior to the next cycle of trials.

Both during the dual design experiment, and in her role as a broker, Jesica was the key person responsible for the data collection—on her own and others' attempts at the resource use, and on the reasoning that underpinned these uses. Multiple corrective processes are in place during design research studies to correct for possible inaccuracies in data collection and interpretation, the most powerful of them being the reality check. Had the data not represented the resource uses with integrity, the design modifications developed on their basis would have had no chance of better supporting teachers' reasoning with, or use of, the resource. Indeed, even modifications based on reliable data often fail to succeed, as designing for a real difference requires considerable cumulative learning on the researchers' part. A modification that leads to a productive change in teacher resource use, retrospectively, verifies the viability of the model of teacher reasoning that gave rise to it. The researcher's (in this case Jesica's) results and failures in CDE teaching, through which the resource was initially developed or tested, provide an important interpretive tool in making sense of other teachers' classroom experiences.

The Cycles of Designing for Teaching

At the start of the project, the research team shared a concern for the limitations in early number teaching in Mexican preschools. The sources of these concerns were, however, different. The members who were trained researchers were aware of the general poor performance of Mexican students on national and international standardised assessments and of the findings indicating that underperformance in preschool is predictive of subsequent mathematical performance (e.g., Jordan et al., 2009).

Jesica's concerns came from her experience as a preschool teacher, where she had seen only very few of her Year 3 students ever meeting the learning goals specified in the Mexican curriculum. This was the case despite her commitment to her students' learning, her interest in mathematics education, her commitment to taking professional development courses in the subject, and the attention she paid to the recommendations published by the Mexican Ministry of Education on teaching mathematics in preschool, which she tried to faithfully apply.

The initial project involved supporting Jesica in conducting a CDE, in which she tested the viability of the *Patterns and Partitioning* (P&P; Cobb, Gravemeijer, et al., 1997) instructional sequence. The sequence was designed to support the collective development of early number ideas by providing instructional opportunities for students to reason about patterns and partitions of collections of up to ten items. For example, students are supported to develop familiarity with pairs of numbers that add up to five, by finding the different ways in which five monkeys could be in two trees (Cobb, Boufi et al., 1997).

Jesica's CDE project was justifiable as the P&P sequence had been tested with older, US students (Year 1), with a significantly different socio-economic background. Prior to engaging in the CDE, these students already had a command of the verbal number sequence up to 10, recognised written numerals up to 10, and had an understanding of cardinality when counting small collections of objects.

Cycle 1: Adapting the Instructional Sequence to Teach Younger Students

At the start of the CDE led and taught by Jesica, we assessed the pre-schoolers. The results indicated that most of them were not ready to productively engage in the initial activities of the P&P sequence, as they yet had to develop elementary number abilities. Some children were only successful with the word number sequence up to three and could correctly identify the names of only one or two single digit numerals. Activities like *Monkeys* (above) would not be within their reach. This led to the reformulation of the sequence (see Table 1).

Table 1
The P&P Instructional Sequence Implemented in Cycle 1

Phase	Overarching teaching goal	Specific learning goals
1	Support the development of the essential number understandings up to five	Master the number word sequence Enumerate with one-to-one correspondence Use fingers to represent numbers Identify the names of written numerals
2	Support students' reasoning about patterns and partitioning with numbers up to five	Reason about (and subitise) spatial patterns Reason about (and subitise) finger patterns Reason about number partitions in the 10-frame Subitise and reason about spatial patterns in the 10-frame Reason about arithmetic problems
3	Support the development of the essential number understandings up to ten	Master the number word sequence Enumerate with one-to-one correspondence Use fingers to represent numbers Identify the names of written numerals
4	Support students' reasoning about patterns and partitioning with numbers up to ten	Reason about (and subitise) finger patterns Subitise and reason about spatial patterns in the 10-frame Reason about number partitions in the arithmetic rack Reason about arithmetic problems

Prior to introducing the original P&P sequence activities, we included a phase, in which we intended to support the children to develop essential number understandings with numbers up to five (Phase 1). We also formulated a general strategy to first support the reasoning about P&P with numbers up to five, and only then proceed to working with larger numbers (we separated Phases 2 and 4 and added an explicit focus on essential understandings of larger numbers in Phase 3). This distinction was not key in the initial sequence, given the more advanced starting point of the US students.

The CDE consisted of 21 instructional sessions that were implemented over a 5-month period. Our analysis indicated that the reformulated instructional sequence was viable. Peña (2018) reported that the learning goals of Phase 1 were met after five sessions, in which the teacher supported collective engagement in activities of repeated counting with words and symbols. The activities included singing number songs, playing number-word games and board games. To encourage the repeated counting of small collections, the students were asked to help the teacher's friend, who owned a candy factory, to find out how she could sell the candies without having to count them one by one. Using a large supply of Unifix cubes, students created rods of *candy packs* of the same size (e.g., 4 cubes).

The learning goals of Phase 2 were met after six more instructional sessions, where each of the five learning goals were accomplished (Peña, 2018). To illustrate, when supporting students to reason about number partitions up to five in the 10-frame, the teacher used a narrative involving a watermelon stall with two decks (see Figure 1, left).

The narrative involved a teacher's friend, Doña Esperanza, who sold watermelons at a market. Once students had understood the situation, the teacher asked them to advise Doña Esperanza on arranging a certain number of watermelons on her stall. As students proposed different ways, the teacher kept a record of their suggestions on the board, specifying how many watermelons would be in the top and in the bottom deck (see Figure 1, right).

Figure 1. A watermelon stall and the capture of the record the teacher kept.

On Jesica's suggestion, the learning goals of Phase 3 were pursued in tandem with those of Phase 2. She used the final 20 minutes of sessions 8 to 11 to support her students' development of essential number understandings up to ten, using similar activities to those in Phase 1, but with number words, collections, and written numerals up to 10.

The learning goals of Phase 4 were met over the final 10 sessions of the CDE, and the students came to reason about number patterns and partitions when solving relatively complex arithmetic problems with numbers up to ten (Peña, 2018). In Session 21, the teacher presented the class with a problem about passengers getting on and off a *Tour Bus*. The bus left a park with 4 tourists, made a stop at a museum, and arrived at the final destination with 10 tourists onboard. Students were asked to explain what happened at the museum.

When the teacher first asked the whole class, it was considered obvious that more tourists had boarded the bus. Lupe, asked by the teacher, explained: "Six got on because six are missing for ten". Hernan, when asked whether he understood what Lupe said, responded: "Yes! Six are missing because there were four, and six are ten." Both Lupe and Hernan were amongst the children who, at the beginning of the CDE, showed the least understanding of early number. Their responses to the problem illustrate how, by the end of the CDE, the great majority of the students did not only solve rather complex additive problems correctly, but how they did so by reasoning about number patterns (i.e., the amount 4 is included in the amount 10, the complement of 4 to make 10 is 6), not by counting by ones.

Cycle 2: Modifying the Instructional Sequence for Broader Teacher Use

After finishing her degree, Jesica returned to teaching in a new school and used the instructional sequence she researched, to teach. Initially, she was questioned by her principal and colleagues, as she focussed on small numbers and used whole class activities, instead of "maintaining the challenge" by teaching more complex tasks from the Mexican curriculum that involved larger numbers and working with small groups of students, as was usual in her school. She defended her teaching decisions by referring to her students' assessments (e.g., lack of familiarity with larger numbers), the results of the CDE, and to the research literature.

Her teaching soon became of interest to her principal and supervisor because of what she was achieving with her students. The students were much more eager to participate in mathematics than what was typical in the school, and the parents were very pleased with her teaching. She was asked to give short workshops during the staff meetings, both at the school and the school-cluster levels. Several of her colleagues started to approach her for advice.

At that point, we decided to create a website that contained the materials developed during the CDE, to make these readily available to the interested teachers. It was then that we noticed (based on download data and teachers' questions) how Jesica's colleagues were much more interested in the classroom activities aimed at supporting "essential number understandings" (Table 1, Phases 1, 3), than in the rest of the resource. The teachers readily recognised the importance of their students developing basic counting skills. In contrast, we conjectured, the notion of *patterns and partitions* meant little to them, as did the purpose of related activities.

From their perspective, there were few clear incentives for investing time and effort in learning to pursue the goals of Phases 2 and 4 of the sequence (see Table 1).

We were aware that if teachers were to focus solely on supporting the development of "essential number understandings", their students would not come to solve the *Tour Bus* problems with flexibility when larger numbers were involved, as the only number patterns at their disposal would be those of sequential order (i.e., counting up, and perhaps down, by ones). However, we also knew that in the prevailing teacher culture of Mexican preschool, "problem solving" was a key focus, as it was a goal of Mexican curricula over past decades. Jesica shared that in relation to this goal, there was much frustration amongst her colleagues, because only very few of their students ever became proficient in solving problems.

We thus conjectured that the goals of Phases 2 and 4 of the sequence (see Table 1) would seem much more appealing if preschool teachers recognised them as a means of supporting children to become problem solvers. We introduced modifications aimed at *connecting the learning goals to the problem of teaching* the preschool teachers already faced (see Table 2).

Table 2
The P&P Instructional Sequence Implemented in Cycle 2

Phase	Overarching teaching goal	Specific learning goals
1	Support the development of the *basic number skills* up to five	Master the number word sequence Enumerate with one-to-one correspondence Use fingers to represent numbers Identify the names of written numerals
2	Support the development of *advanced number skills* up to five (problem solving)	Reason about (and subitise) spatial patterns Reason about (and subitise) finger patterns Reason about number partitions in the 10-frame Subitise and reason about spatial patterns in the 10-frame Reason about arithmetic problems
3	Support the development of *advanced number skills* up to ten (problem solving)	*Use basic number skills up to ten (transition)* Reason about (and subitise) finger patterns Subitise and reason about spatial patterns in the 10-frame Reason about number partitions in the arithmetic rack Reason about arithmetic problems

At the crux of these modifications was our intent to convey to preschool teachers that (a) by supporting their students to reach the learning goals of the P&P instructional sequence, they would be providing students with valuable means for problem solving, and (b) while the essential number understandings were necessary for becoming a proficient problem solver, they were not sufficient. The first modification involved renaming "number understandings" to "number skills" to align with the language, in which the teachers made connections to their practice. The second one involved renaming the teaching goals as addressing "basic" vs. "advanced" number skills. This allowed us to both remove the non-transparent "P&P" language and facilitate the image of the advanced number skills as a continuation of the basic skills, which enhanced students' problem solving beyond the activity of counting.

The third modification involved downgrading the Phase 3 (see Table 1) into a transition stage for the following Phase (see Table 2). This were to further support teachers in recognising the P&P goals (now *advanced number skills*) as the key ones in student learning.

By and large, the modifications appeared to be helpful. Jesica reported that the teachers at her school became quite interested in *advanced number skills* and hopeful that these would lead to better problem solving (field journal). Similarly, the two members of the research team who facilitated two 4-hour professional development workshops in Jesica's school, noted that the participating teachers were highly interested in the modified version of the instructional sequence, and that the advanced number skills seemed meaningful to them.

Cycle 3: Highlighting Enjoyment as an Independent Learning Goal in the Sequence

In facilitating the teacher workshops, we became aware that the “fun” activities of Phase 1 were not common in preschool mathematics teaching. We realised then that during the CDE, particularly at the beginning, we had worked to engage the students by making sure that they experienced joy, success, and belonging, regardless of how competent they were. We based the activities on stories and games, and Jesica always focused on conveying to all of her students that they were good at what she was asking them to do.

We thus became aware that this aspect of the teaching during the CDE, which we already took for granted, would not be immediately obvious to, or valued by, the teachers who were interested in the instructional sequence. This led to a further modification, the inclusion of a dedicated Phase 0 (see Table 3), with the main goal of supporting children’s willingness to engage in early number activities, and enjoyment of doing so. We developed a set of instructional activities and teaching routines (Lampert et al., 2010) that aimed at supporting teachers in pursuing this initial teaching goal.

Table 3
Addition to the P&P Instructional Sequence Implemented in Cycle 3

Phase	Overarching teaching goal	Specific learning goals
0	Support the development of an *interest in* and a *taste for* counting and numbers	Become interested and show joy when engaging in activities that involve counting or working with numbers

The final version of the instructional sequence has caught the interest of an unexpected number of teachers. In 2020, after Mexican schools closed for COVID-19, we started to collaborate with several teaching organisations and offered intensive online workshops, organised in 2-hour increments over three consecutive days. Although we do not know yet how the attending teachers incorporated the designed resource to their teaching, they valued the experience positively, to a surprising degree. One of the workshops had an attendance of 850 teachers who were present during all three days, even though no external incentives were provided to participate. The Facebook community that we created to keep in touch with the teachers interested in the resource reached 5000 members.

Discussion and Conclusions

In the case of P&P instructional sequence, the research findings that were a product of the initial CDEs addressed the problem of classroom learning. They established that it was to the benefit of learners when the number relationships beyond counting sequence were explored in the classrooms by the means that leveraged quantitative meanings and where students’ activity focused on comparing and manipulating quantities that numbers represented. These issues are not addressed by tools such as rainbow numbers, where rainbow—not quantitative pattern—is used to help students memorise numbers with their complements of ten.

Additional design work was essential in making the instructional sequence produced in the CDEs viable for re-sourcing teachers at scale. For us, this included understanding the problems of teaching presently experienced by the teachers, such as the lack of meaning or relevance of some of the design features within their context; the policy-mandated need to support problem solving; and the novelty of legitimacy of cultivation of students’ interest in numbers through mathematical activities. We do not claim that the changes in wording and the structure of the resource goals alone made the difference. It was, indeed, critical that the teachers saw, reasonably readily, an associated change in *their* students' engagement, problem solving, and mathematical reasoning. Yet the appropriate wording made the teachers' access to and interest in the ideas presented in the resource possible.

It is at this time not unusual that—like in the Mexican curriculum—the learning outcomes are specified in the form of end-goals (e.g., ability to solve additive word problems in preschool). In absence of resources to suggest otherwise, teachers are encouraged to work *on* the specified end-goals directly (e.g., train students to solve word problems with large numbers). The provision of reasoned classroom journeys towards the curricular end-goals, and the means by which teachers could support such journeys, are needed. It is important to remember that such provision does not seamlessly follow from research on student mathematical learning but requires research into the realities and problems of mathematics teaching.

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Teacher Agency and Professionalism in the Context of Online Mathematics Instructional Platforms

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Current trends in mathematics education emphasise student-centred learning; and internet-based pedagogies, such as online mathematics instructional platforms, go even further in reducing the role of the classroom teacher. In this paper I consider how teachers may exercise agency and professionalism in the context of online mathematics instruction. Whilst there is some evidence for the reduction of the teacher role, there is also considerable space for teachers to enact professional identities as teachers of mathematics. However, I suggest that care needs to be taken to ensure that there is a balance to the overall mathematics programme, including opportunity for teachers to make their own decisions and assessments regarding students' learning.

One might argue that the student-centred learning trend in mathematics education is now well established. This is evident in 'reform' mathematics (e.g., Ma & Singer-Gabella, 2011), inquiry learning (e.g., Hunter & Anthony, 2011), problem-solving focused mathematics (Ingram et al., 2020), and in many technologies enabled by the internet (Engelbrecht et al., 2020). Research in mathematics education certainly appears to push a student-centred agenda. However, I wish to trouble this orientation a little. Student-centred pedagogies are situated within a wider discourse of what Biesta calls "*learnification*" (Biesta, 2004, 2012); the "reduction of all that matters educationally to questions of learning" (Biesta, 2012, p. 36, italics in original). Learnification is problematic because it makes it difficult "to ask the crucial educational questions about *content, purpose* and *relationships*" (Biesta, 2012, p. 36, italics in original). Biesta argued, in contrast, that teaching matters and thus teachers should be allowed to teach. In this paper I follow Biesta in taking the position that teaching does indeed matter, as I look at the teachers' role in a student-centred mathematics pedagogy. Specifically, I consider the challenge to teacher professionalism and agency within the context of the increasing use of online mathematics instructional platforms (OMIPs) in primary schools.

Learnification, and the de-centreing of the teacher, has happened within a neoliberal educational context, in which education is a marketplace (O'Neill, 2011), and people are seen as being individually responsible for their own learning (Biesta, 2012). Aotearoa New Zealand, like many other Western countries, has adopted many aspects of neoliberal ideology into its education system (O'Neill, 2011); for example, the notion of school choice, professional development provision, and private operators being allowed to profit within public education (Thrupp et al., 2020). One example of the latter is the educational technology (Ed-tech) industry, whereby digital curriculum resources are sold to parents and schools (Wright & Peters, 2017). Particularly relevant to mathematics education, primary schools subscribe to dozens of different instructional platforms, or OMIPs (Darragh & Franke, 2021b), such as *Mathletics* (3P Learning, 2022), *Maths-whizz* (Whizz Education, 2022), or *Education Perfect* (Education Perfect, 2022). OMIPs promote a student-centred approach by emphasising personalised (or individualised) learning (Boninger et al., 2019). The idea is that every student has their own curriculum pathway designed for them (see also Engelbrecht et al., 2020) using algorithms embedded in the platform, in other words the "netflixing" of education (Roberts-Mahoney et al., 2016). It is certainly worth asking where the space for the teacher is in such platforms; rather than being a professional who makes decisions for their student, the teacher is relegated to a "coach" (Ideland, 2021) or "facilitator" (Biesta, 2012) of learning.

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Given this socio-political context, and particularly since distance learning during the COVID-19 pandemic has created perfect conditions for the rise of Ed-tech (Moore et al., 2021; Williamson et al., 2020), it is time to give attention to the teacher role when using technologies such as OMIPs.

Background

Digital technology in mathematics education is a massive and growing domain (Borba et al., 2017; Calder et al., 2018), and yet research predominantly addresses potential *benefits* to learning and teaching (Attard et al., 2020; Borba et al., 2017; Young, 2017) and is thus more concerned with measuring impact on learning (Reinhold et al., 2019; Robin & Kwak, 2018; Young, 2017) or how to encourage teacher uptake with technology (Bennison & Goos, 2010; Remillard, 2016; Utterberg et al., 2019). OMIPs, by contrast, have received less research attention, despite them being a growing phenomenon in Australasian schools (Darragh & Franke, 2021b; Day, 2014; Nicholas & Fletcher, 2017). OMIPs are subscription-based mathematics curricular platforms that are available for schools or parents to purchase. OMIPs operate globally yet adapt their curriculum to each local country context. Their features include games for learning (practising) mathematics, instructional videos, interactive digital objects, reward systems, embedded assessment, adaptive technology for individualized progression, and supplementary teacher resources. The majority of schools in Aotearoa New Zealand subscribe to at least one of more than a dozen different OMIPS (Darragh & Franke, 2021b), and many parents accessed them during distance learning during COVID-19 lockdowns (Darragh & Franke, 2021a). OMIPs support a neoliberal ideology due to their capitalist, competitive features (Darragh, 2021; Ideland, 2021; Macgilchrist, 2018) as well as their focus on the individual learner. OMIPs provide a good example of how the Ed-tech industry has considerably raised its marketing efforts since the onset of the COVID-19 pandemic (Moore et al., 2021; Williamson et al., 2020).

Within the field of media studies, critique of Ed-Tech centres on two main aspects: the personalisation of learning (Boninger et al., 2019; Knox et al., 2020; McRae, 2013; Roberts-Mahoney et al., 2016), and the use of data analytics (Knox et al., 2020; McRae, 2013). Along with personalisation of learning comes an implicit side-lining of the teacher. Some critique goes further to wonder whether Ed-tech may lead to the end of the teacher, at least as we know them today, and notes the potential "re-localization of power from teachers to ed-tech companies" (Ideland, 2021, p. 11). Others consider the reduction of the teacher role due to the invisible process in which a student's grade or next learning steps are assigned (Boninger et al., 2019). A concern of teachers is the lack of control they have over the learning assigned to their students by the computerised system (Utterberg Modén, 2021). Given this potential for teachers to become side-lined by the presence of the mathematics instructional platforms in their mathematics programme, the research question to frame this paper is:

> *How do teachers exercise agency and professionalism as mathematics teachers when using online mathematics instructional platforms?*

Methods

Conceptual Framework

To understand the data in this study, I use Holland and colleagues' (1998) concept of figured worlds. A figured world is "a socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et al., 1998, p. 52). In the case of this study, the figured world is the mathematics classroom; and teachers and students are the characters recognized in this world. A number of mathematics education researchers have used the concept of figured worlds (Esmonde & Langer-Osuna, 2013; Horn, 2008; Langer-Osuna & de Royston, 2017; Ma & Singer-Gabella, 2011), typically examining the figured world of the mathematics classroom and also the identities offered to students or teachers in these worlds.

Important to the concept of figured worlds is this notion of identity. "[I]dentities are hard-won standpoints that, however dependent upon social support and however vulnerable to change, make at least a modicum of self-direction possible. They are possibilities for mediating agency" (Holland et al., 1998, p. 4). It is the identity of the teacher character, in the context of the mathematics classroom figured world that I consider in this paper, together with attention to the agency enabled them when using OMIPs. I wish to understand the extent to which the teacher may take up the identity position of "professional" as a mathematics teacher in this OMIP context.

Clearly, it is also necessary to consider the meaning of "professional" here; this is a term likely to mean different things to different people, and certainly what it means to be a professional teacher has changed over time. Day and Smethem, (2009) advocate a "new professionalism in which teachers are entrusted to make wise, evidence informed and accountable judgements about their teaching and pupil progress" (p. 154). Key to this definition is that teachers are trusted to make teaching and assessment decisions and they experience agency in these teaching decisions (see also Adams, 2017). For the purpose of this paper, I operationalised the notion of professionalism as the teacher's perception of being able to make their own decisions regarding the teaching, learning, and assessment of mathematics.

Context

The research presented here is part of a wider study that examined OMIPs in Aotearoa New Zealand primary schools. Other data collected in the study included a survey of school leaders asking about their rationales for choosing to use OMIPs (Darragh & Franke, 2021) and a discursive analysis of the websites of the four most popular OMIPs (Darragh, 2021). In this paper I present data from a third data source – that of interviews with teachers in schools that use OMIPs.

Data Sources and Analysis

During 2019 and 2020, I interviewed 12 teachers from 5 different schools about their use of OMIPs. The interviews were semi-structured and the planned interview prompts included: "Please tell me about yourself as a mathematics teacher"; Tell me about teaching mathematics in your school; What sorts of resources are available to you to support your mathematics teaching? How do you use [the OMIP]? What are the benefits, drawbacks, uses, of the platform? How do you think using [the OMIP] limits or strengthens your mathematics teaching? Interviews typically lasted 40 to 60 minutes and were transcribed in full. These transcripts were sent back to the teachers for comment and/or correction. The participating teachers ranged in their years of experience, the school level they taught, and the particular OMIP they used in their classroom programme.

I followed Taylor and colleagues' (2015) approach to qualitative data analysis. Specifically, I begin by reading and re-reading the data. I created codes, using both inductive and deductive methods, and developed themes from these codes. Some of the broader (and overlapping) themes constructed during this analysis included: Teaching challenges and supports, Classroom organisation, Teaching beliefs/philosophies, Other digital resources, Assessment/surveillance, Engagement and motivation, Mathematics learning, Identity (generally as learner/teacher of mathematics), and sub-theme of Agency and professionalism. I wrote analytic memos about each theme and created propositions that I then checked via re-reading all the interview data again in order to nuance the theme using data coded in the other themes as well. In the findings section below, I present two propositions based on the theme of agency and professionalism.

Findings

The interview data supported two contrasting findings regarding the use of OMIPs. First, OMIPs reduced teachers' agency and professionalism as a mathematics teacher; and second, despite this reduction, teachers maintained a level of professionalism and demonstrated agency in a number of ways. Due to constraints of space, here I present data from three teachers from one school, who each provided a range of responses related to their agency and professionalism. Whilst the data analysis was informed by the responses of all 12 teachers, I suggest it is more useful to give a deeper and contextualised look at the use of one OMIP, in one school.

The school was an intermediate school in central Auckland. The teachers, Holly, Giselle, and Peter (all pseudonyms) were in their first, second, and third years of teaching respectively. They taught to Years 7 and 8 (students typically aged 11 to 13). The OMIP used at the school was *Maths-whizz* (Whizz Education, 2022). Despite the fact grouping based on attainment (ability grouping) is typical in Aotearoa New Zealand schools (Anthony & Hunter, 2017), at this school mathematics was not taught in that way. Under the guidance of the mathematics curriculum leader, and an outside "coach" who delivered regular professional development in mathematics, teachers at this school used a combination of whole class mathematics discussions, called "number talks"; small group teaching in flexibly arranged groups to "workshop" an area of identified learning need; some rich mathematical tasks in small collaborative problem-solving groups; and independent work on *Maths-whizz*. This variety of mathematics teaching demonstrated the emphasis on having a 'balanced mathematics programme' that was commonly described by school leaders throughout the country in the wider survey data (Darragh & Franke, 2021b). Each of the three teachers interviewed mentioned all these different mathematics teaching approaches during their interviews. In the rest of this section, I present responses from the three teachers to illustrate the two propositions regarding agency and professionalism when using OMIPs for mathematics teaching.

OMIPs Reduce Teacher Agency and Professionalism

Two broad ways that OMIPs reduce teacher agency and professionalism emerged from the data. The first was evident in the lack of control over the platform's features and the second was a subtle undermining of the teacher role. The lack of control impacted on teaching decisions; for example, 'identifying' a "maths age" for the students:

> [They get given a maths age] and the annoying thing is they immediately compare it to their actual age and they say it says here I'm a maths age of 8 but I'm 11 and then sometimes it puts the brakes on a little bit which can be frustrating (Peter).

Clearly the OMIP "decision" to assign students a "maths age," according to embedded assessment algorithms, was not a decision Peter would have himself made. Peter also expressed his frustration at the restrictions in deciding content:

> … because I would like to be able to have all the kids on probability while we are teaching probability, but the system doesn't actually allow it. … [Usually] I can force the system to teach a particular topic for a week or two I think it is. [But for probability, only] some kids have it available. (Peter)

In this instance Peter was unable to align the content of the OMIP with what he was teaching in class at the time, because those students at a lower "maths age" were not permitted to learn probability, according to the OMIP. This limited Peter's ability to cohere different aspects of his programme together. Another complaint was that *Maths-whizz* taught in a way that did not match with his approach to teaching addition, for example:

> … in Maths-whizz there are little boxes under the answer and then those ones are where you put the carried number. But it is not clear and the kids get confused and they don't know how to use it and more often than not at the beginning of the year they come up quite low for their addition and subtraction and multiplication whereas they actually know it and that causes a problem. (Peter)

Here, evidently, not only was the teaching approach different, but it resulted in an assessment of the students' capability that would be different to Peter's assessment.

In contrast, Giselle did not find the platform limited her teaching beyond issues with logistics, such as when one student had log on difficulties. Holly, however, reflected more deeply about the impact of *Maths-whizz* on her professional teacher identity:

> … one of the effects it has made it sometimes maybe a bit out of touch with what the kids know that I know that is because I'm not making the most of it. So, I know you can go in and see exactly what each kid is doing and how they are doing with it. (Holly)

> Sometimes it makes me feel less competent almost as a maths teacher. For example, when the students make a reflection, they talk about what they learned from Maths-whizz … but there is nothing about all the stuff we did in class. (Holly)

From these quotes we can see that Holly felt undermined as a teacher by the *Maths-whizz* platform, both in the estimation of the students and in her own feeling of being 'out of touch' due to not completing the assessments with the students herself. It is worth noting two differences between Holly and her colleagues. First, the fact she had only been teaching for six months at the time of interview could have led to feelings of inadequacy. Second, Holly was the only person who arrived at the interview with notes for her interview responses (I sent an indication of the sorts of questions I would be asking prior to the interviews) and thus her responses may indicate a greater depth of reflection.

To summarise, there was some evidence of the reduction of the teacher role as suggested by the literature; including the lack of control over aspects of the platform (Utterberg Modén, 2021), taking assessment out of the teachers' hands (Boninger et al., 2019) and the sense of de-professionalism (Ideland, 2021). However, these impacts were minor incisions into these teachers' overall sense of agency and professionalism as mathematics teachers, as to be demonstrated next.

Teachers Maintain a Level of Professionalism and Demonstrate Agency

One reason the teachers were able to maintain their sense of professionalism is because the OMIP did not feature too prominently in their overall mathematics programmes. All three teachers expressed how flexible their mathematics programmes were, as can be seen in Giselle's response:

> [We were talking] as a class and from there I said right I am going to do a workshop on factors, I know it was a bit random but it linked in with yesterday's lesson because I noticed a few students going huh when I was doing factor stuff. So [I thought] okay we will do a workshop on it tomorrow and so the kids come down to the mat with me and the rest of them were working independently. (Giselle)

The other two teachers similarly referred to moments when they saw a learning need and were able to act on it by adjusting the planned programme to include a "workshop" lesson for those who opted in to it. In fact, *Maths-whizz* helped to create the space for this flexibility as it was easy for the teachers to assign *Maths-whizz* to the other students whenever this kind of change in plans occurred.

Another way that *Maths-whizz* afforded different kinds of learning and teaching was expressed by Holly: "… what I actually kind of like about Maths-whizz is I said here I feel freer to focus on specific aspects of maths and delve deeper into it." *Maths-whizz* allowed Holly to feel that the curriculum was "covered" and thus enabled space for explorative problem solving. Holly also mentioned that *Maths-whizz* gave her confidence that both the high and lower attaining students were catered to:

> Yeah, I guess Maths-whizz in a sense because it is catering a lot more to their levels especially the extremely like one at the very bottom kind of thing and 3 at the very top. It is kind of hard to do a workshop for [just one or two students]. (Holly)

Giselle similarly expressed that *Maths-whizz* was "really helpful for their individual learning path."

To summarise, and in contrast to the first proposition, here we see evidence that these teachers maintained a sense of professionalism and agency when using the OMIP, and at times the OMIP allowed them space to make agentic teaching decisions that they might otherwise be unable to execute.

Discussion and Conclusions

Returning to the definition of professionalism in a teacher of mathematics, we may reconsider the findings to evaluate the extent to which teachers were able to exercise agency and professional identities. We might ask if these teachers were "entrusted to make wise, evidence informed and accountable judgements about their teaching and [student] progress" (Day & Smethem, 2009, p. 154). There appeared a mixture of freedom and constraint for the teachers regarding their mathematics teaching decision-making. The OMIP allowed them to feel confident that their high and low "ability" students were catered to, despite their concerns for this in the context of mixed-ability grouping required by the school. This confidence in turn enabled the teachers more freedom to explore other, perhaps richer, pedagogies. Yet there was a little less agency for the teachers regarding issues of assessment. *Maths-whizz* assigned a "maths age", which Peter felt constraining. Additionally, the assessment algorithm was hidden and prevented Holly from being "in touch" with her students' learning progress—a criticism of Ed-tech noted in the literature (Boninger et al., 2019). Key to these teachers maintaining professionalism as mathematics teachers was their particular school context, in which, for the most part, they were entrusted to run their own mathematics programmes while the OMIP was simply a small part of it.

Biesta's (2004) notion of learnification is evident in the teachers' responses. Their concern for individual student progress reflects Ed-tech industry priorities (Darragh, 2021; Wright & Peters, 2017) and neoliberal ideology (Ideland, 2021; O'Neill, 2011). Biesta's educational questions about content, purpose, and relationships might also be raised. It appears the OMIPs sometimes limited teachers' choice of *content*, exercised a dubious *purpose* in the assigning of "maths age", and could at times be undermining of the teacher-student *relationship*, as noted by Holly. Consequently, the OMIPs were not without risk to teacher agency and professionalism, despite these three teachers' mainly agentic responses.

There are a couple of limitations I would like to note here. First, the interviews were conducted prior to the COVID-19 pandemic and thus do not consider the impact that distance learning may have had on the use of OMIPs for these teachers—both during and after lockdowns. Second, the three teachers were volunteer participants who all worked at a school with considerable mathematics support and freedom. Some other interviewees had less support and less individual freedom to make decisions about their mathematics teaching in general, without even considering the impact of the OMIPs on this freedom. It is also fair to assume all volunteers for such interviews would have a certain level of professionalism in their mathematics teaching, those without are less likely to volunteer.

These limitations notwithstanding, it is worth celebrating the way in which teachers were able to exercise agency and professionalism as mathematics teachers within a neoliberal educational context and when using OMIPs in their classroom. The experiences of these teachers also point to the importance of having a balanced mathematics programme that allows teachers freedom to experiment with a range of different pedagogies for the teaching of mathematics.

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RESEARCH PAPER

Primary School Mathematics Leaders' Actions that Facilitate Effective Mathematics Planning and Support Teachers' Professional Learning

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All teachers of mathematics aim to provide productive learning experiences that cater for students in their care. The planning of effective and engaging mathematics lessons is complex and requires expertise. In a larger study survey data, observations and interviews were used to investigate the ways in which School Mathematics Leaders supported teachers to learn. This paper reports results from case study research and focuses on the actions of one School Mathematics Leader during planning meetings. Findings highlight a range of supportive actions, which included developing constructive working relationships with teachers, fostering knowledge of mathematical content and curriculum and facilitating collaborative team planning.

Prior to teaching mathematics, teachers need to have given some thought to, and ideally planned in detail, the lessons they are about to teach. Planning decisions made by teachers, whether working in isolation or in teams, involve deciding what it is they are hoping to achieve with careful thought given to the curriculum, suitable tasks, the possible pedagogy, differentiation and the use of assessment data (Davidson, 2017). Earlier studies have suggested the importance of a School Mathematics Leader in supporting teachers to plan (Clarke et al., 2012a; Davidson, 2017, 2019; Sexton & Lamb, 2017). This paper aims to document the actions of School Mathematics Leaders as they support teachers' mathematics planning.

For the purposes of this paper, one case study has been chosen from a larger study of mathematics leadership (Driscoll, 2021), where primary School Mathematics Leaders were observed working with groups of teachers in their school settings. According to Vale et al. (2021), "few studies have reported what leaders of mathematics in schools actually do" (p. 401). The evidence provided here will address a gap in the research. The research question addressed in this paper is: *How do primary School Mathematics Leaders support teachers' mathematics planning?*

Literature Review

It is an expectation in Australian schools that teachers of mathematics will plan lessons based on the curriculum, before they actually teach (Davidson, 2019; Sullivan et al., 2012). Planning plays a critical role in teacher practice. According to the Australian Professional Standards for Teachers (Australian Institute of Teaching and School Leadership [AITSL], 2017) Standard 2.3 Curriculum, assessment and reporting, teachers are expected to "use curriculum, assessment and reporting knowledge to design learning sequences and lesson plans" (p. 12). It is difficult to imagine how teachers can teach mathematics effectively without substantial planning (Roche et al., 2014). However, while curriculum is important, it is also critical that teachers consider experiences and approaches that are student-centred (Davidson, 2019; Vale et al., 2010) and that take into account appropriate challenge, student commitment, confidence, and understanding (Hattie, 2012).

Previous studies (e. g., Davidson, 2017, 2019; Roche et al., 2014) have focused on teachers' planning of mathematics lessons. According to Clarke et al. (2012a, 2012b), when planning mathematics lessons, teachers will draw on: their own knowledge and experiences, resources and influences that include curriculum documents, commercial publications, web-based resources, information about students based on assessments, and experienced colleagues. It is

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 178–185. Launceston: MERGA.

also reasonable to assume that teachers' mathematical knowledge for teaching is important when it comes to the decisions teachers make when planning (Davidson, 2017, 2019). Teachers need to "know their mathematics and know how to teach it" (Gaffney, Clarke et al., 2014, p. 4). Teachers also need to understand the most important mathematical ideas on which to focus (Clarke et al., 2012b).

Interestingly, according to Roche and colleagues (2014), it is the processes that teachers use as they plan mathematics lessons that need further investigation. Sullivan et al. (2012) found that planning processes could be improved and suggested that any related professional learning from teacher educators should begin with processes used by teachers as they plan. Given the fact that teacher leaders have the potential to significantly affect the quality of teaching and learning in schools (Calderone et al., 2018), it could be assumed that necessary support would be provided through the actions of teacher leaders, or in this case School Mathematics Leaders, when they attend planning meetings.

School Mathematics Leaders are teachers with the responsibility of leading improvement in the teaching and learning of mathematics in schools and play an important role in improving mathematics education (Driscoll, 2021). These leaders "are knowledgeable, on-site teachers who support their colleagues' efforts to interact about all facets of mathematics teaching" (Campbell & Malkus, 2013, p. 198). School Mathematics Leaders are considered by some as an exemplary source of in-school support, and "the most immediate source of professional development in mathematics for the primary teacher" (Millet & Johnson, 2007, p. 19). As instructional and curriculum leaders, School Mathematics Leaders are key to providing professional learning related to content and pedagogy and in-class support for teachers to develop quality practices (Jorgensen, 2016). It is through their actions and interactions with teachers that School Mathematics Leaders "make a difference to the lives and learning of others" (Gaffney, Bezzina et al., 2014, p. 68), including their colleagues and students.

While it is common in Australian schools for teachers to plan in collaborative teams (Davidson, 2016) and to discuss evidence of student learning, achieving maximum impact "depends on teams of teachers working together with excellent leaders" (Hattie, 2012, p. 35). Cobb and Jackson (2015) also noted that teacher collaboration provided significant learning opportunities. However, the extent to which this collaboration supported teacher learning depended on the quality of leadership and the inclusion of already accomplished teachers (Cobb & Jackson, 2015). Accomplished teachers in the form of School Mathematics Leaders, while working together in professional learning communities, share their "wisdom of practice" (Clarke, 1994) and expertise as they review student work, and discuss curriculum and pedagogy with colleagues.

Furthermore, Fullan and Quinn (2016), proposed that teachers need deeper collaborative experiences built on teacher input and choice, connected to their daily work of designing and assessing tasks that have the power to influence student learning. While Davidson (2019) reiterated this view, she also contended that schools aiming to promote productive collaborative planning practices may benefit from providing support in the form of an experienced team leader or School Mathematics Leader.

Theoretical Framework

The purpose of the larger study was to identify the ways in which School Mathematics Leaders supported teachers' professional learning about effective teaching of mathematics. Teacher learning was the theoretical lens used to frame and guide the research design and data analysis. In addition, a leadership framework (Fullan, 2001) was used for describing and analysing the practice of leaders of mathematics. Therefore, two fields of research literature were interrelated throughout the research—how teachers learn and leadership. Specifically, Lave and Wenger's (1991) work on social theory and Fullan's (2001) Framework for

Leadership (Figure 1) were used to guide the study. As part of this framework for leadership, Fullan (2001) described five components which included: moral purpose, understanding change, relationship building, knowledge creation and sharing and coherence making. Leaders who displayed all five components with the personal characteristics of energy, enthusiasm, and hope were found to be most effective (Fullan, 2001). These attributes of effective leadership practice were used to frame this study and the interpretation of the results.

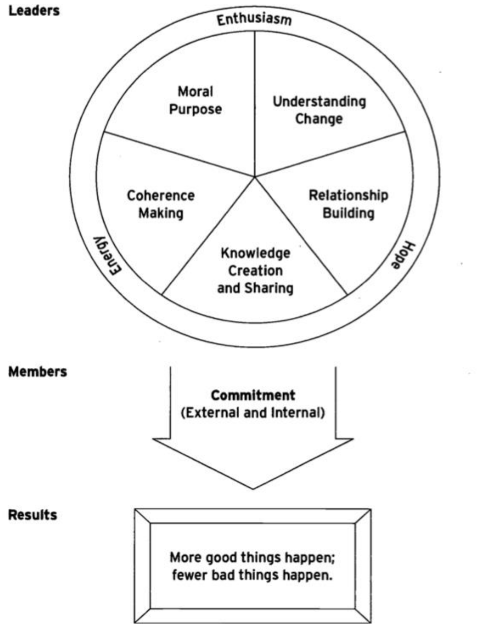

Figure 1. A Framework for Leadership (Fullan, 2001, p. 4).

Methodology

Qualitative research was chosen for the larger study because it enabled exploration of the ways in which School Mathematics Leaders supported teachers' professional learning. The qualitative research design was a combination of two types, case study together with open coding procedures, which were used for data analysis (Yin, 2016). This research was comprised of two phases. Phase 1 included collecting and examining data from a survey of leadership practices. Phase 2 comprised of observations and interviews that contributed towards the reporting of case studies. Phase 2 also included a series of prompted written reflections from the School Mathematics Leaders as well as field notes and other documents. Prompted written reflections were completed by the School Mathematics Leaders based on events that they believed were significant in their interactions with teachers. To allow for triangulation of data all sources of evidence were used when examining the case studies.

Initially, a state-wide survey (n = 56) was designed and implemented to collect data about the nature of the role of primary School Mathematics Leaders, and aspects that influenced the effectiveness of the role. The survey also documented ways in which School Mathematics Leaders supported the professional learning of their colleagues, including the support given to teachers during the planning of mathematics lessons. Finally, information gathered from the data collected about individual School Mathematics Leaders was used to select participants for the case studies.

As a means of understanding the ways in which support was provided with mathematics planning, it was necessary to observe the actions and interactions that occurred between School

Mathematics Leaders and teachers as they worked in their specific school settings. This paper reports on one case study participant, who was selected because of her experience as a School Mathematics Leader. Jane (pseudonym), the School Mathematics Leader, was observed and interviewed in her primary school workplace on four occasions over a ten-month period. During two of these observations, Jane supported teachers in the Foundation team during two-hour long planning meetings. Jane was also interviewed following the observations.

Data Collection and Analysis

Interviews and observations occurred during the same day of each visit. The planning meetings were video-recorded, and the interviews were audio-recorded. The most effective method for recording evidence of the meetings and capturing the actions and interactions of participants was to use a video-camera. Video recording allowed for detailed analysis through repeated viewing and reviewing of the footage. Events of interest were transcribed. The events chosen were based on themes that were emerging and were related to the research literature that framed the study. One of the advantages of using image-based methods, is that it is possible to capture a scene far more discreetly and effectively than you can with recorded notes (Thomas, 2016). Comparisons can also be made between the direct observation of School Mathematics Leaders' actions in meetings, and their statements recorded in the interviews, which leads to "very rich, high-quality data" (Punch, 2014, p. 155).

Data were collected and transcribed then added to NVivo. Printouts were also viewed many times. The coding process began by reducing the data into categories based on the two fields of research that informed the study, leadership and teacher learning. As the coding process continued, codes moved to higher conceptual levels and themes and concepts began to be identified in the data. Analysis for this research aligned with a grounded theory approach and was based on the five-phased cycle proposed by Yin (2016): compiling, disassembling, reassembling, interpreting and concluding. These processes supported the purpose of this study and enabled analysis to proceed in a methodical manner.

Results and Discussion

Working with Teachers During Planning

Evidence was gathered related to the ways in which School Mathematics Leaders supported teachers' professional learning during planning with the intention of analysing and interpreting the ways this occurred. The evidence highlighted the important role that the School Mathematics Leaders played in supporting teachers as they offered guidance and advice in planning meetings. While in many schools, School Mathematics Leaders do not have the time or opportunity to support teachers in the planning of mathematics lessons, and in some cases do not possess the pedagogical content knowledge to do so (Driscoll, 2021), this paper reports the actions of one School Mathematics Leader who was fortunate to be involved in planning.

In each of the two observations, the year-level team leaders facilitated the meeting, supported by Jane as the School Mathematics Leader. Jane's perceptions of aspects of the meeting outlined at the follow-up interview provided further evidence of the situation and were triangulated with the survey data and Jane's written reflections. Emerging themes informed by the findings and the literature became apparent. These themes identify actions that could be considered to be part of School Mathematics Leaders' practice in supporting teacher planning.

The Story of Jane

Jane worked in a modern, government primary school located in an outer suburb south-west of Melbourne in the state of Victoria, Australia, and had worked for a number of years as a School Mathematics Leader. When working with teachers to support the planning of mathematics lessons, Jane demonstrated some specific key actions. These actions included:

Developing constructive working relationships with teachers. As Jane worked with the Foundation team, it was apparent through her actions and interactions that she was aware of the importance of developing constructive relationships with teachers in her school and made attempts to enhance this. Relationships are complicated, but at the same time are crucial and make a difference to the success of an organisation (Fullan, 2001). Jane explained that the teachers at her school had built a close professional relationship with her as she supported them during mathematics planning sessions, and with weekly analysis of student assessment data. For example, as the team discussed possible lesson ideas and teaching strategies, Jane spent time encouraging the teachers to contribute using open-ended questions, which served as prompts to focus the discussion. For example, "What do you want them to know?" and "Where are you starting?" occasionally directing questions at certain teachers by name. Questioning encouraged teachers to feel included in the discussion, showing that their opinions and ideas were valued, and contributed to building a constructive working relationship. Seeking teacher input is likely to build "trust, respect and commitment" (Goleman, 2000, p. 10) and contribute to building positive relationships.

However, "relationships are not an end in themselves" (Fullan, 2001, p. 65) and teachers often express doubts and differences of opinion. During one planning meeting a particular teacher in one of the teams demonstrated signs of resistance through her limited participation in the discussion and her demeanour. As Jane mentioned in one of the interviews, "there's a lot of work going on there," referring to open-to-learning conversations as a strategy to improve the relationship. While the relationship was definitely strained, an effective leader understands teacher differences of opinion and appreciates resistance (Fullan, 2001) and can learn from this. It was obvious from Jane's comments that the focus was on improving this relationship with the aim of making the mathematics planning sessions more productive.

Fostering knowledge of mathematical content and curriculum. The planning of mathematics lessons often brings many challenges related to curriculum knowledge, mathematics content knowledge and pedagogical content knowledge. At Jane's school, building teachers' mathematical knowledge for teaching was important. Jane explained as a reasonably new school it had taken individual teachers a long time to establish pedagogy and build mathematics content knowledge amongst the teachers. During planning meetings, Jane supported teachers by offering guidance and advice as they planned sequences of mathematics lessons. Jane suggested resources, lesson ideas, professional reading options, and posed questions as she challenged teacher thinking.

Early in her role, Jane also provided a mathematics teaching and learning curriculum document to support teachers with developing their mathematics knowledge. This document was used by teachers as they planned and included links to the curriculum, big ideas and understandings, games, equipment, misunderstandings, key resources, and teaching sequences. Jane created this document with teacher input to support teacher planning, as according to Jane, it was one way to ensure teachers were covering the expected curriculum and it built their mathematical content knowledge. The fostering of knowledge building and sharing in an organisation is an important element of effective leadership (Fullan, 2020).

Prompting and pressing teachers to contribute ideas. As Jane enacted her leadership role during planning meetings, the actions of prompting and pressing teachers for further ideas were obvious. School Mathematics Leaders can influence how planning meetings progress by prompting and pressing teachers on key issues (Cobb & Jackson, 2015). Jane attempted to provide a balance between pressure and support based on individual teacher needs. While Jane encouraged teachers to construct and develop their mathematical knowledge, she prompted teachers to choose the most appropriate tasks based on the effective teaching and learning of mathematics. However, the challenge experienced by Jane was to know when to prompt and press for ideas to add to the planning, as opposed to “telling” teachers what to do, which intentionally leads to the next action.

Attempting to hold back from telling. As a means of encouraging teachers to make a decision and build their confidence in planning, on several occasions, Jane purposefully asked a question and held back from sharing her opinion. For example, in one observation the team leader asked, “So what is the learning focus? The learning focus is about multiplicative thinking” and Jane replied, “I don’t know. Is it?” On this occasion Jane sat quietly without adding any more comments letting the teachers take responsibility.

During another observation, it was apparent that Jane was waiting for teachers to respond as she sat with her chin on her hand and did not participate in a discussion that was occurring between members of the Foundation team. After some time, and some initial holding back, Jane finally stepped in and shared her thoughts on teaching problem-solving in a context, not just as a topic. Jane explained this point further in an interview with this comment, “Sometimes you can let them go, but other times you have to go, ‘No, I don’t want you to do a unit on problem solving. I want problem solving in your practice.’”

During an interview, Jane commented that although “I was really conscious at the start in my role that I was always telling … I was really aware of that,” there were times when “I still [couldn’t] help myself from blurting out, ‘I think you should do this.’” Jane went on further to say, “I am getting much better at waiting. But sometimes, they just don’t know what they don’t know.” Jane believed that more professional reading by teachers would help, and went on further to say, “At the end of the day, they’ve just got to get their planning done” and sometimes “there are some gaps in their knowledge, where I have to go, ‘Hey, this is the sequence, you’ve got to get this in.’” For Jane knowing when to “hold back from telling” (Roche & Clarke, 2014) was not always obvious, and in the end, it was about finding the balance between allowing teachers to make their own decisions and voicing her opinion based on her experience. Although Jane admitted it became easier as she became more experienced working with teachers and understood more about how teachers learn.

Encouraging teachers to reflect on and evaluate possible lesson ideas. Significant learning opportunities occur when teachers and School Mathematics Leaders are engaged in actions that have the potential to improve teacher knowledge and practice. Such actions include, encouraging teachers to reflect on ways to facilitate student learning, and planning lessons by anticipating students’ responses and solutions. Actions also include, supporting teachers to reflect on previous lessons in relation to student engagement and possible misunderstandings, and encouraging them to ask themselves if a task has the potential to engage students in exploring and understanding mathematical concepts. Reflective practices can maximise student engagement and learning, and potentially affect the actions of School Mathematics Leaders like Jane, as they work with teachers to plan mathematics lessons.

There were many occasions when teachers were observed evaluating lesson ideas. At one stage as the teachers discussed possible lessons, one teacher explained how she had implemented the “Cookie Count” lesson in a previous year, then the teachers in the team discussed the possible sequencing of lessons that would work best based on the students’ previous experience. Teachers often make decisions based on reflective practice, and decide on the direction to take, and where to go to next based on their knowledge of curriculum and student needs (Hattie, 2012).

Facilitating regular collaborative team planning. Many teachers plan in collaborative teams and are supported by a year-level team leader, and in some cases School Mathematics Leaders. The value of collaboration cannot be underestimated, as teachers working together discuss, and influence others, as they draw on a variety of assessment data and curriculum documents, then select appropriate learning activities. In this study, working in a collaborative team to plan helped build a sense of community (Gaffney, Faragher et al., 2014), and provided an opportunity for sharing of ideas, shared decision making, and shared responsibility for student learning (Du Four et al., 2010). Jane encouraged teacher collaboration as a means of developing a common understanding of the mathematics content and ways to evaluate the impact of planning on student learning. Teacher learning is enhanced when teachers participate in shared experiences, and in Jane's case, being an active participant in a community of practice (Lave & Wenger, 1991) led teachers to plan activities that had the potential to cater for their students' needs.

In summary, although the mathematics planning in the Foundation team was not always what Jane considered "the best," and at times she believed that she needed to step in and give the team more direction, Jane continued to challenge her team while supporting them to build their mathematical knowledge for teaching. Through implementation of these identifiable actions, Jane supported teacher learning during mathematics planning meetings in multiple ways.

Conclusions

Findings from this study support the notion that successful mathematics planning promotes teacher professional learning. Teacher learning is ongoing and more effective when it is embedded into the regular practices of a school community. Engaging in discussions of curriculum and pedagogy, analysing student data, evaluating previous lessons and collaborating to plan possible learning tasks provide significant learning opportunities for teachers. In terms of the theoretical lens used to frame this research, through her actions, Jane incorporated elements of Fullan's (2001, 2020) leadership framework as she developed relationships with teachers, fostered mathematics content and curriculum knowledge, and attempted to seek coherence. These key actions, which also included: stimulating discussion amongst teachers as they planned, probing and questioning teachers' contributions, developing data literacy, encouraging mathematics related professional reading, evaluating mathematics planning, and suggesting possible lesson sequences, potentially led to improved planning practices. The implication here is that for School Mathematics Leaders to implement actions that contribute to supporting teachers to build their mathematical knowledge for teaching during planning, these leaders of mathematics require opportunities to be involved in the meetings, to bring knowledge and expertise to encourage ongoing professional learning and to make a difference.

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School Mathematics Leaders' Support of Primary Teachers' Professional Learning in Meetings

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School Mathematics Leaders seek to improve the mathematical learning outcomes of the students in their schools. Recognising that improved learning is dependent on high quality teaching, leaders are often keen to support teachers' knowledge of mathematics content and pedagogy. This paper reports ways in which School Mathematics Leaders support and lead the professional development of teachers in team meetings. Results from survey data and case study research are reported in this paper to describe and highlight the supportive actions undertaken by School Mathematics Leaders as they work to provide professional learning opportunities for teachers.

Research findings related to the work of primary school leaders of mathematics are scant. Understanding the nature of the work and how to maximize its positive impact is an emerging field. This paper is based on a recent study conducted in Victoria, Australia (Driscoll, 2021). The purpose of the study was to investigate ways in which School Mathematics Leaders in primary schools supported the professional learning of the teachers in their teams.

Like in other parts of the world, there are various titles and responsibilities given to leaders of mathematics in Australia (Clarke et al., 2013; Driscoll, 2017). Here we use the name *School Mathematics Leader* to mean a teacher working in a primary school who has responsibilities for leading other teachers in that school to improve the teaching and learning of mathematics. The School Mathematics Leader often acts as an *agent of change* (Fullan, 1993) with fellow teachers who need support and encouragement to improve the mathematical outcomes of their students. Little has been written about the nature of the support leaders offer teachers to encourage professional learning. The research question that this paper addressed was:

> *How do School Mathematics Leaders support primary teachers' professional learning in the context of meetings?*

Background

The importance of effective leadership of mathematics in schools has been noted (Cheeseman & Clarke, 2005, 2006; Sexton & Downton, 2014; Sexton & Lamb, 2017). Through their supportive actions School Mathematics Leaders make a difference to the learning of others, including teachers and their students, as they share ideas and insights about effective teaching of mathematics (Faragher & Clarke, 2014; Gaffney et al., 2014). As critical educators in improving mathematics teaching and learning School Mathematics Leaders provide a link between the principal and classroom teachers and possibly "have the greatest impact on teacher learning and development" (Grootenboer et al., 2015, p. 278).

Teacher Learning

Researchers such as Clarke and Hollingsworth (2002) have described models of professional development initiators. Similarly, Goldsmith, Doerr and Lewis (2014) provided important information in relation to the need for teachers to continue to develop their knowledge and skills in teaching. The ways in which practicing teachers continue to learn and develop the knowledge that enables them to teach well are complex. While it could be assumed that "teachers who know more teach better" (Cochran-Smith & Lytle, 1999, p. 249), knowing

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 186–193. Launceston: MERGA.

exactly what it is that teachers need to know and how they will learn this knowledge has been the subject of much research. A focus by researchers and educators at all levels is to try and understand the best ways for teachers to "learn to develop and refine their practice" (Hollingsworth & Clarke, 2017, p. 458). According to Kim et al. (2019) developing the ability to analyse, interpret and understand students' mathematical thinking through various means supports teachers to acquire necessary pedagogical content knowledge.

While a number of researchers have emphasised "active learning [that] requires opportunities to link previous knowledge with new understandings" (Cochran-Smith & Lytle, 1999, p. 258) through a process of change (Bransford et al., 2000; Clarke & Hollingsworth, 2002). Learners bring prior knowledge and experience to learning situations and create new concepts by constructing links to their existing knowledge (National Academies of Sciences, Engineering, and Medicine, 2018), rather than being told information by others (Cochran-Smith & Lytle, 1999). Research also suggests that learning "takes place over time rather than in isolated moments" (Cochran-Smith & Lytle, 1999, p. 258) and needs to be situated in meaningful and relevant contexts (Bransford et al., 2000); which are likely to be school-based, collaborative and continuous, and aimed at student learning (e.g., Hiebert et al., 2002).

Learning Communities

Developing communities of practice creates opportunities for teacher collaboration where teachers participate in shared experiences and discourse around student data and learning (Bransford et al., 2000). According to Lave and Wenger (1991), learning occurs through participation in a community of practice, where newcomers are transformed into old-timers, whose changing knowledge and skills became part of a developing identity, and in turn they became a member of a community of practice. The literature emphasises the benefits of working together collectively in professional learning communities and the impact it can have on student and teacher learning (DuFour et al., 2010). According to Cobb and Jackson (2015), teacher collaboration provides significant learning opportunities for professional learning. Teams of teachers gather evidence of student learning, discuss teaching strategies, then implement these ideas and analyse their effectiveness (Darling-Hammond & Ball, 1998). Fullan and Hargreaves (2016) believe that success in schools is achieved through the establishment of a culture where teachers work collaboratively and grow and learn on a daily basis through feedback and joint work, by engaging in pedagogy, and developing mutual trust.

Theoretical Framework

The theoretical lens used to frame the research incorporated leadership (Fullan, 2001, 2020) and teacher learning (Lave & Wenger, 1991). In particular, Fullan's *Knowledge Creation and Sharing*, *Relationship Building*, and *Coherence Making* components of his leadership framework applied to the study of School Mathematics Leaders, and the ways they supported teachers to learn. Components of the leadership framework were used to guide the data analysis and discussion of the findings. Lave and Wenger's (1991) idea of *communities of practice* as a context for learning, also framed observations in schools. Investigating School Mathematics Leaders' creation of learning opportunities allowed these theoretical constructs to be compared to evidence from practice. The broad theoretical underpinning in this research is a socio-constructivist view of learning which holds that meaning is made by the learner building new knowledge on existing knowledge in a social setting (von Glasersfeld, 1987).

Methods

The research reported here is part of a larger study (Driscoll, 2021). The data examined in this paper are three observations of each of four School Mathematics Leaders as they worked with teachers during planning or Year level team meetings (12 meetings in total). In addition, following each meeting an interview was conducted in the workplace with each School Mathematics Leader (12 interviews). Video recordings were made of the observed team mathematics meetings, audio recordings were made of the researcher's interviews with each leader, and leaders' written reflections of events they considered significant were collected. In these ways the events were documented, and participants' views of the events were recorded. The data were collected and analysed by the first author whose perspective as a researcher and as an experienced School Mathematics Leader enabled a subtle interpretation of the evidence. Data were compiled, disassembled, reassembled, and interpreted, and conclusions were drawn to address the research question (Yin, 2016). Individual case studies were assembled, and a cross-case analysis was conducted to find similarities and differences in the ways that each School Mathematics Leader supported teachers' professional learning in their schools. The findings here describe how School Mathematics Leaders support teachers' professional learning in meetings.

Results and Discussion

Meetings took two different forms in the four schools reported here. Planning meetings and professional learning team meetings often comprised teams of teachers from different year levels attending, depending on the context. Planning meetings provided the opportunity for a team of teachers to meet and discuss, decide on, and record, a sequence of mathematical learning experiences teachers planned to teach the following week. Whereas, Professional Learning Team meetings were focused more on "big picture" data, where teachers engaged in the analysis of student work samples, discussed data and planned and evaluated assessment tasks. Despite the differences in aims and organisation of these meetings, the leaders' supportive actions had characteristics in common. The data in Table 1 is listed to describe School Mathematics Leaders' actions without making a formal distinction between the meeting types. The term collegial team meetings will be used to encompass both meeting types.

Table 1
Team Meetings: School Mathematics Leaders' Actions (n = 4)

School Mathematics Leaders' Actions	*S	J	A	R
Built mathematical pedagogical content knowledge	✓	✓	✓	✓
Focused discussion on students' mathematics learning	✓	✓	✓	✓
Developed mathematics knowledge and understanding of the curriculum	✓	✓	✓	✓
Stimulated teachers to select high quality tasks, representations, and materials	✓	✓	✓	✓
Encouraged teachers to contribute ideas to planning	✓	✓	✓	
Guided teachers with suggestions of possible lesson sequences	✓	✓	✓	
Highlighted important ideas and connections between concepts	✓	✓	✓	✓
Encouraged teachers to reflect on, and evaluate, possible lesson ideas	✓	✓	✓	
Provided teaching and learning documents and teacher reference books		✓	✓	
Challenged teachers' ideas while supporting them to learn		✓		
Analysed and discussed assessment tasks during moderation	✓	✓	✓	✓
Suggested mathematics professional reading	✓	✓	✓	✓
Created and refined a range of whole school 'rich' assessment tasks				✓
Attempted to hold back from telling teachers	✓	✓		✓

* S = Susan, J = Jane, A = Amy, R = Robyn (all pseudonyms)

Table 1 shows that the School Mathematics Leaders (n = 4) supported teachers by sharing aspects of their mathematical pedagogical content knowledge with members of the team; encouraged teacher discourse related to student learning of mathematics; and initiated opportunities for teachers to develop their content and curriculum knowledge during collegial meetings. The School Mathematics Leaders encouraged teachers to select possible tasks and suitable representations and materials as they designed activities. Three School Mathematics Leaders encouraged teachers to contribute their ideas to mathematics planning and guided them to make decisions about sequencing learning steps during the implementation of tasks. The results also revealed that the during these meetings all four School Mathematics Leaders highlighted important mathematical ideas and helped teachers to make connections between mathematical concepts. All four School Mathematics Leaders also spent time supporting teachers in their analysis of salient mathematical content knowledge during moderation of assessment tasks and suggested mathematics professional readings. The actions described in Table 1 were intended to support teachers to build their mathematics knowledge for teaching.

Although many of the actions exhibited by the School Mathematics Leaders in this study were specific to their school context, it became clear that there were commonalities across the cases between the ways in which these leaders supported teachers to learn. Limitations of this paper permit discussion of the first five categories only. An example of the nature of support provided by the School Mathematics Leaders will be included, along with a final finding common to all four School Mathematics Leaders.

Built Mathematical Pedagogical Content Knowledge

It was clear that the majority of supportive actions displayed by the School Mathematics Leaders in this study during meetings focused on developing teachers' mathematical knowledge for teaching. Each of the School Mathematics Leaders at some stage shared elements of their mathematical pedagogical content knowledge with teachers in these collegial meetings. For example, during a team meeting as teachers discussed teaching a lesson on the topic of length, one School Mathematics Leader pointed out the advantages of using an open-ended measurement task and said, "I think you should open the lesson up. You've probably got kids who could use a ruler," which indicated that students needed to be provided with more flexibility and challenge in tasks. School Mathematics Leaders who share the depth and breadth of their mathematical knowledge for teaching with others have the potential to support teachers to learn and improve their mathematics teaching (Ma, 1999). In fact, the same School Mathematics Leader when discussing how she promoted the use of challenging tasks at her school, during an interview, pointed out that, "I sort of [felt] like that was my thing to do." During an observation of a meeting, it was obvious that this School Mathematics Leader believed this was part of her mathematics leadership role, as she encouraged teachers to use more challenging problems structured by a "Launch, Explore, Summarise" model and as a result she provided resources to support this approach.

Focused Discussion on Students' Mathematics Learning

During collegial meetings all four School Mathematics Leaders encouraged teacher discourse related to student learning as they reviewed student assessment and data. The teachers were supported to develop data literacy in the context of formal assessment as they discussed student work samples and responses. There were occasions, when teams of teachers worked with the School Mathematics Leaders to moderate assessment tasks and reflect on elements of their teaching, which "is critical for professional development" (Kim et al., 2019). For example, one Year 3/4 team used an assessment task called Packing Pots from *Scaffolding Numeracy in the Middle Years* (DET, 2018), to discuss student responses and strategies for solving multiplicative situations. Discourse around student learning was encouraged as teachers worked in "a collaborative and collective effort" (DuFour et al., 2010, p. 14) to inform their professional practice. In another school, a team of teachers led by the School Mathematics Leader, created, evaluated, and refined a range of whole school 'rich' assessment tasks. This team of teachers discussed and created the assessment tasks based on the effective teaching and learning of mathematics, then decided on the direction to take with planning, and the suitability of possible follow-up lessons based on their knowledge of curriculum and student needs (Du Four et al., 2010, Timperley, 2008). Structured opportunities that encouraged discourse during meetings, allowed teachers "to share and reflect on each other's practice [which] are all facets of the change environment that act to afford or constrain teacher growth" (Clarke & Hollingsworth, 2002, p. 955). Such opportunities cannot be underestimated.

Developed Mathematics Knowledge and Understanding of the Curriculum

While planning mathematics lessons can be challenging, particularly as the level of teachers' mathematical content knowledge for teaching (Ball et al., 2008) varies, planning in collaborative teams offers many advantages, as it allows School Mathematics Leaders to have more impact across groups of teachers. Supportive actions by the School Mathematics Leaders included encouraging discussion of content and curriculum as teachers planned and evaluated lessons and discussed elements of their practice. One School Mathematics Leader supported teachers at her school by providing them with a detailed curriculum document, that teachers used to inform their planning. This document included a scope and sequence chart linked to curriculum documents, central concepts, common misconceptions, and valuable resources for teaching. There were also regular professional learning community meetings at this school where teachers met and discussed readings related to mathematics content and curriculum. In one case, led by the School Mathematics Leader, teachers debated a reading that emphasised the teaching of mathematics content and its connection to the four proficiencies. While two other School Mathematics Leaders developed documents with teams of teachers at their schools to support teacher content knowledge that outlined the "essential understandings", or the priority areas of the mathematics curriculum. The teachers collectively agreed on the priority areas that needed to be taught, which in the long term supported the development of teachers' knowledge of mathematical content and the intended curriculum.

Stimulated Teachers to Select High Quality Tasks, Representations, and Materials

Results indicated making decisions in relation to the most suitable tasks, representations, materials, and possible lesson sequences to include when planning mathematics, was a constant dilemma for some teachers. Judging from the observations, this was a particular area where the actions of the School Mathematics Leaders influenced teacher learning. For example, teachers in one Foundation team meeting debated for a considerable amount of time the possible tasks and tools to use as they planned a sequence of lessons on measuring length. As the School Mathematics Leader attempted to guide teachers with their choice of task, she questioned the team, and encouraged them to reflect on their prior experiences when teaching the topic. In the end, it was necessary for this School Mathematics Leader to step in and support the teachers to extend their knowledge and thinking. This point was evident in the following comments made by the School Mathematics Leader during a planning meeting:

> Well, the problem is that they have got to be measuring something or comparing different things to say which is longer. It's just that some of them [children] need to able to just hold them [pencils], and go that one, and others you want them to be justifying it [and] measuring it.

> The other thing is you can't say matchsticks because some aren't ready. So, if you think about that. What's your core task so the kids who are ready can do it that way, but the kids who aren't can be doing direct comparisons and going this is longer than this. [She demonstrates using a pen and phone]

There was a constant struggle within the team as they tried to decide on and select the most appropriate tasks and materials, as well as the most effective lesson sequence. Deep discussions created opportunities for teachers to make informed decisions, by reflecting, noticing, anticipating, and negotiating changes, allowing teachers to expand their knowledge related to the complexity of teaching (Kim et al., 2019). As part of their work in collegial meetings, all four School Mathematics Leaders also supported teachers to understand the important mathematical ideas and to make connections between underpinning concepts.

Encouraged Teachers to Contribute Ideas to Planning

There was also evidence that three of the four School Mathematics Leaders prompted and pressed teachers to contribute their ideas to the documentation of teaching plans. While in some cases, limited mathematics content knowledge caused a degree of reluctance by the teachers to contribute to planning, it could also have been that teachers possibly lacked confidence or were worried "about admitting they [did not] know or understand for fear of colleagues' reactions" (Bransford et al., 2000, p. 195). Alternatively, teachers may also have felt that they did not need to contribute when the area team leader and the School Mathematics Leader dominated the discussion, which occurred in one school. There were occasions when two of the School Mathematics Leaders described their struggle between knowing when to prompt teachers, in contrast to telling teachers what to do. One School Mathematics Leader expressed her dilemma in this way:

> At the start, when I worked with them, I'd let them go more, and I'd say, what about this, I'd throw in a suggestion. I wouldn't shoot things down straight away; I'd let them go with things that I probably wouldn't have normally gone for. And then pose that question the week after, how did it go? Okay, but what about you try this? I've been very wary. I've worked with leaders in the past and they would just say, "No, you're not doing it!" and I don't think anyone learns from that. I think that they do need to learn if something's not going to go well, that's fine, because that's reality, and let them go with that, and then maybe after that provide the solution. But then the flipside of that is that some teams will be a little bit too reliant, they will just say, what do you think, or what can we do? They're not willing to go out on that limb. So, it's finding that balance.

Support from a more knowledgeable experienced other, such as a School Mathematics Leader, during these meetings provided a potentially powerful opportunity to improve teacher learning, but as this quote demonstrates, it is about finding the balance prompting and telling.

School Mathematics Leaders Actions in Fostering Opportunities for Team Collaboration and Collegial Support

Examination of the practices and actions displayed by the School Mathematics Leaders during collegial meetings led to a further finding of significance. While the School Mathematics Leaders contributed towards improving teacher practice as they designed, facilitated, attended and advocated for collegial meetings, each School Mathematics Leader established processes and protocols for working with teams of teachers. It was also clear that all four school mathematics leaders took steps to develop constructive working relationships with teachers in their schools as they built relational trust, respect and commitment (Goleman, 2000). Developing constructive relationships and relational trust with colleagues is critical to leading mathematics successfully (Fullan, 2001).

Finally, the aim of the study was to understand in detail the ways in which School Mathematics Leaders supported teachers to learn in the context of meetings. While the structure, frequency and effectiveness of each meeting differed according to school context, it was possible to examine and gain insight into the types of actions and interactions that fostered teacher learning, and that potentially led to improved teacher practice. Although each of the categories discussed were treated separately, for the purposes of this paper, it was obvious that all categories were intertwined and connected.

Interestingly, Vale et al. (2021) found that the most frequent activity School Mathematics Leaders participated in was mathematics team planning. Despite all four School Mathematics Leaders advocating for team planning to occur, time and structures were not in place in two of the schools that participated in the study to allow teams to meet for planning, even though Professional Learning Team meetings were mandatory. This situation is reflected in Table 1 where fewer actions were displayed by two of the School Mathematics Leaders. However, all meetings observed were focused on collective responsibility, linked directly to student learning, and created meaningful opportunities for teachers to learn, while working towards *coherence making* (Fullan, 2001, 2020) in a *community of practice* (Lave & Wenger, 1991).

Conclusion

Implications for school wide improvement in mathematics arose from the study. School Mathematics Leaders who have the opportunity to meet, plan and discuss learning in collegial teams are able to: work with teachers to link decisions to their core purpose of improving mathematics; develop teachers' mathematical knowledge for teaching; and create opportunities for relationship building in professional learning communities. The supportive actions undertaken by the School Mathematics Leaders as they worked to provide professional learning opportunities for teachers in focused team meetings encouraged teachers to build their pedagogical content knowledge, fostered discourse related to student learning, and developed a shared understanding of effective mathematics teaching practice. The support provided by School Mathematics Leaders for teachers' ongoing professional learning is critical, as it has the potential to improve the mathematical outcomes of students.

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RESEARCH PAPER

Methodological Choices Made When Using Design Based Research to Explore Mathematics Education: An Updated Analysis

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Design based research (DBR) has become a popular methodology for exploring various aspects of mathematics education due to its focus on theoretical development and practical implementation. Drawing on a recent meta-study into trends in mathematically focused DBR studies, this paper explores how this method is being used in comparison to its original goals. Findings indicate that most studies presenting as DBR are essentially isolated case studies exploring individual teaching interventions lacking the iterative development needed to meet the intentions of DBR. Reasons for the current state of research are suggested along with a potential research architecture for mathematics education.

Design based research (DBR) is a methodological genre with similarities to many overlapping research methodologies such as design experiments, educational design research and design-based implementation research. When exploring education issues, these approaches attempt to iteratively build on theoretical understanding of learning and teaching whilst also establishing effective practical implementations. This richer and more contextual view of education has led to increased adoption of DBR throughout the education research community—particularly in investigations relating to mathematics—but the complexity of its ideals has led to greatly varying interpretations. As such it is important to examine the current practical application of DBR in order to identify whether the original conception of DBR is aligned with its actual implementation.

In arguably the first significant review of DBR to be published, Anderson and Shattuck (2012) where still quite tentative as to whether the methodology was meeting its promised benefits in the decade after it first came to prominence (e.g., Barab & Squire, 2004; Sandoval & Bell, 2004). A particular concern raised by Anderson and Shattuck was that DBR was not supporting the widespread adoption or scaling of tested innovations. Their analysis demonstrated the early adoption of the methodology was only resulting in "small improvements to the design, introduction, and testing of sustaining technologies and practices in classroom or distance education contexts" (Anderson & Shattuck, 2012, p. 24) but not in sustained or scalable change.

The purpose of this paper—and another we have published with a focus on how DBR tends to be used for different kinds of *theoretical* development in different parts of the world (Fowler et al., 2022)—is to report how the methodology has matured after another decade of use. To do so, we present the results of a meta-study making use of tools emerging from the "digital" social sciences to visualise trends in the literature with respect to the *methodological* choices made in DBR in mathematics education research. The questions guiding this meta-study were:

- *How is DBR being used to improve outcomes in mathematics education?*
- *How has the use of DBR in mathematics education research changed over time?*
- *To what extent are DBR studies in mathematics education meeting the goals of the early advocates of DBR?*

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 194–201. Launceston: MERGA.

The key finding we report here is that Anderson and Shattuck's (2012) conclusion of "cautious optimism" is still appropriate. In many ways the research being undertaken under the DBR banner has grown stronger, but there remains only scant evidence that the method is supporting real-world change any more than traditional research methodologies. Our findings suggest, though, that we may need to look not at the conduct of individual projects but rather at our research infrastructure if DBR is to fulfill its early promise.

Method of Review

This paper reports on the maturing nature of DBR methodology and adds an additional 56 papers (15% increase) to our original meta-study (Fowler et al., 2022). This is notable as these more recent papers do show maturation in the methodology. Similar in some ways to a scoping review, meta-studies are a largely qualitative research methodology that can help generate new understandings by treating the research literature itself as the source of data—becoming essentially "research of research" (Paterson et al., 2001, p. 5). This study investigated the methodological choices of the researchers using DBR. It is not a quantitative meta-analysis and does not seek to aggregate the results of the corpus of studies included.

Inclusion Criteria

Papers demonstrating a clear recognition of using DBR to investigate mathematical education issues were sourced from ERIC and Scopus. To ensure quality studies were investigated, only peer-reviewed papers demonstrating well thought out and practically implemented projects were included. As such, conceptual papers, book chapters and reports did not meet these criteria and were excluded. Conference papers were also rejected as they tended to lack methodological detail for coding in this meta-study.

The search period included papers published from after the Anderson and Shattuck (2012) review until February 2022, when the search was conducted. The full search terms are reported in Fowler et al. (2022). The following PRIMSA flow chart (Moher et al., 2009) in Figure 1 details the search method.

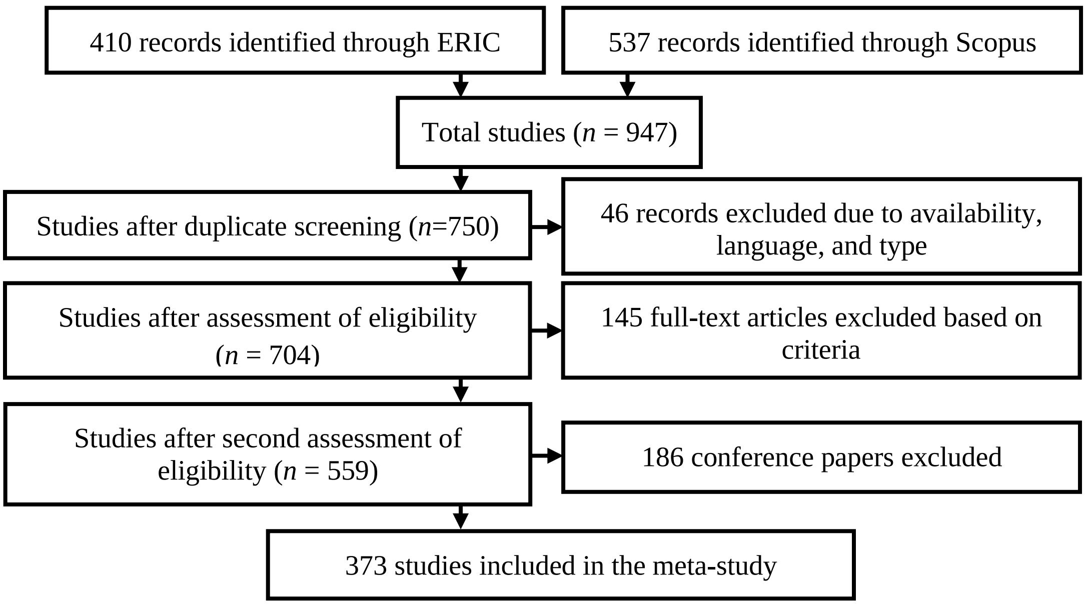

Figure 1. PRIMSA flow diagram of search method.

The qualitative coding system, also explained in full in Fowler et al. (2022), explored locus of refinement (Pedagogy, Educational tools, Understanding of student thinking, Theoretical foci, Systemic environment and Epistemology), methodological choice (Meta-analysis, Survey/Interview, Empirical observation, Quasi-experimental, Randomised control trials and Case Studies with triangulation) and the number of iterations reported. Intercoder reliability between two coders was determined with a 15% sample of the papers. Codes relating to iterations, focus of the intervention and method were compared. Cohen's κ was run to determine if there was agreement between the two coders. There was moderate agreement between the two coders on iterations ($\kappa = 0.477$ $p < 0.001$) and the focus of the study ($\kappa = 0.411$, $p < 0.001$). Reasons for coder error are discussed in the results. There was substantial agreement about method ($\kappa = 0.647$, $p < 0.001$) (McHugh, 2012).

Results

Locus of Refinement of Papers

To illustrate the types of educational phenomena DBR used to investigate, coding emphasis was placed on the focus or "locus of refinement" of the papers (see Figure 2).

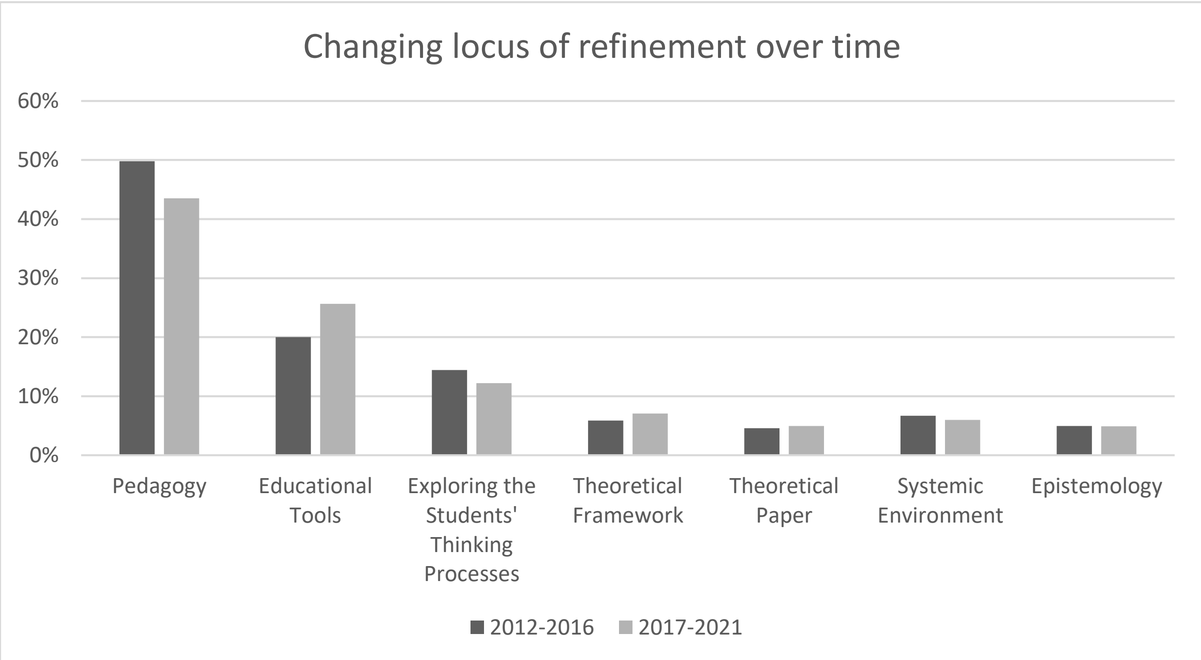

Figure 2. Changing locus of refinement over time.

Seven codes were formulated for this analysis. By far the two most coded items were *Pedagogy* and *Educational Tools*. *Pedagogy* was coded for studies that explored how teachers developed their students' learning but also, in the case of some papers, how this could be improved. *Educational Tools* on the other hand, indicated studies aimed at improving digital or analogue tools (e.g., textbooks, sets of problems) for use by schools. The third most important code related to the confluence of the two former codes in the form of *Exploring the Students' Thinking Processes*. These three codes understandably had significant overlap and focus in DBR studies of mathematics, which may have caused the lower inter-rater reliability scores as each paper was given only one focus code.

Theoretical Papers were coded for studies exploring issues relating to mathematics and DBR that did not use research methods to collect and analyse data. Studies were coded *Theoretical Framework* when studies used research data to assess the practical implementation of a framework to enable greater refinement of the theoretical models. The *Systemic Environment* code referred to studies that explored how policy was formed and enacted within the larger educational system.

Most categories tended to remain quite stable throughout the study period apart from a general decline in the still dominant pedagogical focus and an increasing interest in educational tools; however, this decline was not statistically significant. It should be noted, however, that pedagogies have been studied in more complex ways moving beyond quasi experiments using classroom testing as evidence and adopting a greater use of multiple data sources.

Method Choices

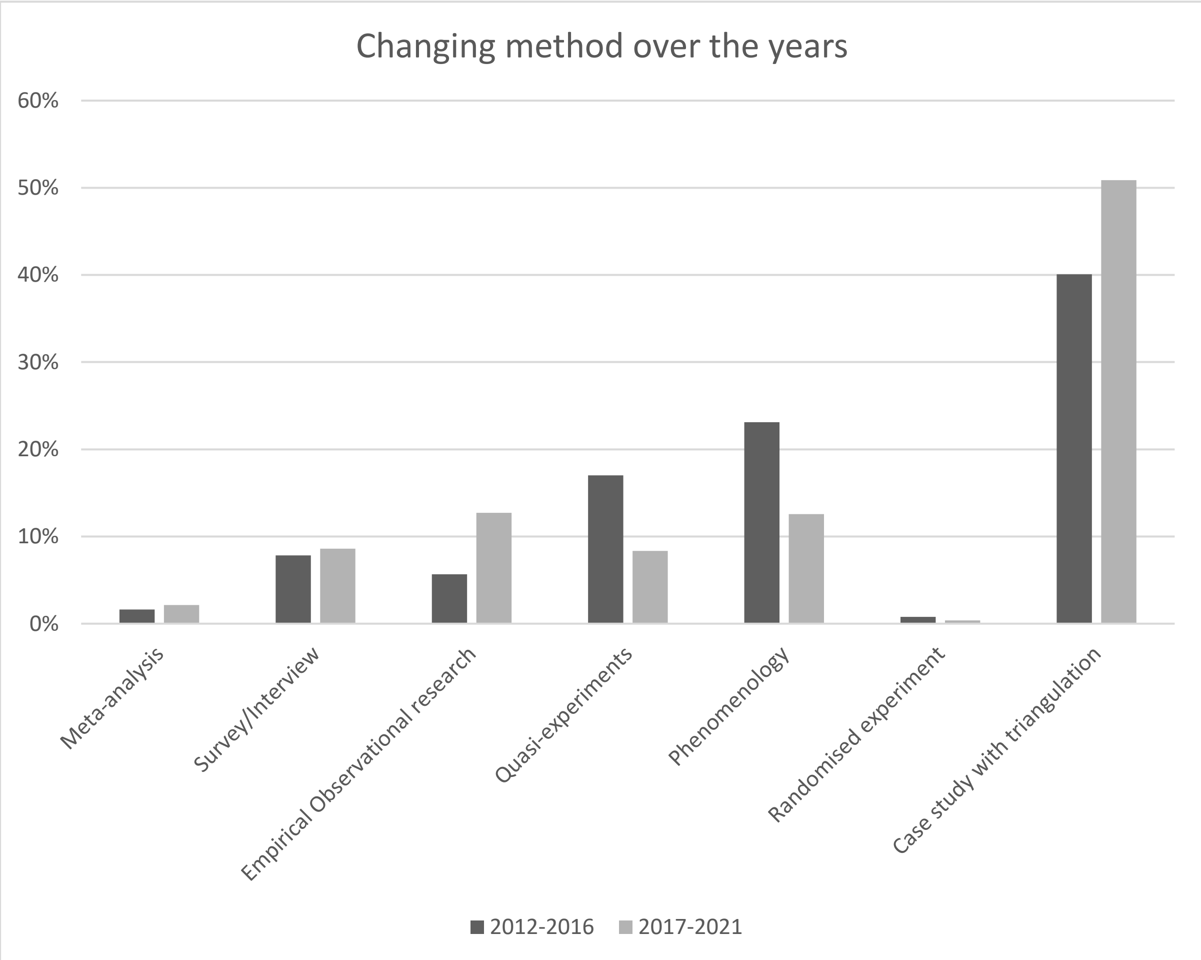

Figure 3. Changing method choice over time.

Much in the spirit of the original design experiments that predated DBR (e.g., Brown, 1992), a case study approach seeking increased credibility through the use of multiple sources of both quantitative and qualitative data were the predominant choice of methods reported in the papers analysed (see Figure 3). In recent years the greater use of technology has facilitated more accessible and effective ways to collect and analyse data, leading to deeper descriptions of educational phenomena. Common data sources used for triangulation in these studies were interview data, work samples and video of participants.

Quasi experiments and phenomenology were a more common methodology in earlier studies, and in non-OECD countries (e.g., Indonesia), but their use has declined over time as researchers have tended to source more data for triangulation. Phenomenology was most prevalent in studies of pedagogy (21% of papers with this foci) and epistemology (22% of papers with this foci) where deeper descriptions of experience were more important than more quantitative measures.

Surveys, interviews and forums have remained important, but they are now often supplemented with additional data. Technological advancements (e.g., improved coding software, online surveys) and accumulating knowledge about effective educational research have most likely been influencing factors in the greater collection and more thorough analysis of educational data in DBR studies. Empirical observational research, which seems to ignore the interventionist nature of DBR (McKenney, 2018), was generally conducted using coded video, work samples or academic tests. Empirical methods were more strongly favoured in studies of educational tools (12% of papers with this foci) and understanding of student thinking (17% of papers with this foci).

Iterations

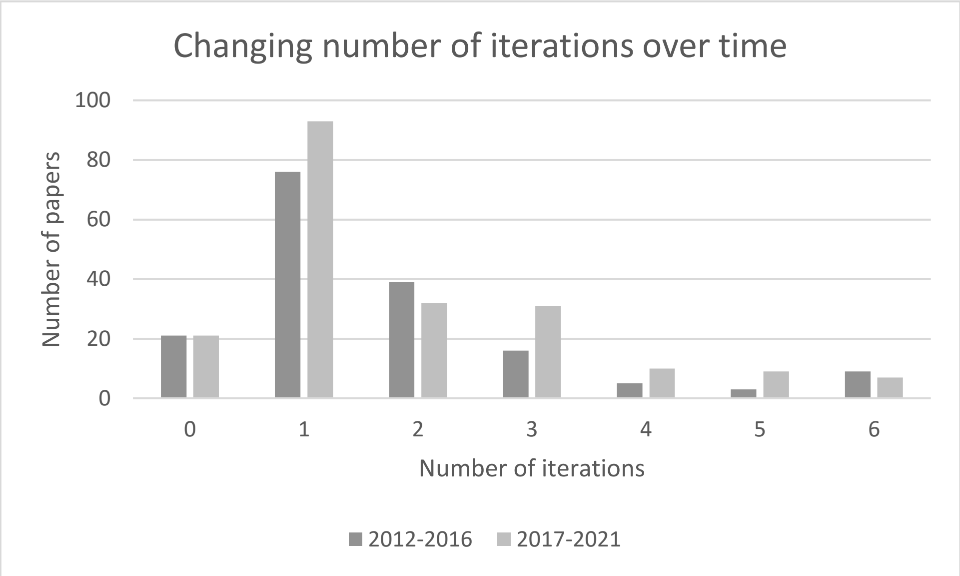

Figure 4. Changing number of iterations over time.

Figure 4 displays the number of iterations. Whilst there was little trend evident over time the distinction of iteration by the papers was quite varied making the coding of this aspect difficult. Theoretical papers involving no practical implementation were coded as 0 and those that applied an intervention but did not clearly explain iterative development were coded as 1. Some of these single (non)iterative studies treated different phases or stages within a study as iterations but they supplied little evidence that data from the former phases effected the development of the next phase. Instead they often referenced phases such as “analysis”, “design” and “evaluation”, derived from influential proponents such as McKenney and Reeves (2012), whilst ignoring or failing to report on larger macro-cycles (Gravemeijer & Cobb, 2006). In effect, many papers represented literature review or hypothesising as “iterating” when no data were being used to affect change. Frequently the goals did not change, and the parts of the studies were pre-determined, suggesting they might be thought of as “phases” rather than “iterations” (Easterday et al., 2018).

Another definition of iterative development often presented was the teaching and tinkering of multiple lessons. Whilst this showed improvement of method (usually pedagogy) through repetition, the qualitative and quantitative data collected in the study were rarely used to inform decisions or change. Instead, they were used to assess the overall span of lessons. As such, the types of teaching activity described in papers such as Avcu and Çetinkaya (2021) showed tinkering and the collection of a range of data, but the data did not influence the change and hence could only be identified as a single iteration.

Discussion

Summarising the intent of the methodology, Anderson and Shattuck (2012) identified DBR as interventionist, iterative, mixed methods research conducted collaboratively between educational researchers and practitioners in real educational contexts to identify design principles for future practice (pp. 16–17). Many of the studies investigated were true to parts of this definition. Most seemed to recognise the importance of context and referred to the important symbiotic relationship between the researchers and educators, even when little input from the practitioner was noted. Also, many of the projects reported little development through iteration. While some interventions showed adaptation throughout the process, this was often through teacher “tinkering”, highlighting that “iteration” may sometimes be ambiguously defined.

Addressing this lack of genuine iteration, which ought to be the primary driver of theoretical development in the DBR methodology, will be critical if DBR is to support sustainable and scalable change. After almost two decades of use—more if we count the early design experiment movement of the 1990s—however, it is not likely that this problem will be addressed simply though upskilling researchers. Rather, we would argue that to fully realise the potential of DBR seen by Anderson and Shattuck (2012), as well as in our own studies, that our field must give consideration to some parts of our research practice and infrastucture.

A clear reason for a lack of iteration, for example, is that DBR is a resource intensive research methodology. This was a key finding in our initial study (Fowler et al., 2022), which showed that studies invovling multiple iterations almost exclusively eminated from the richer OECD countries, and even then were primarily the result of a PhD program with at least the candidate devoted to the project full time for 3–4 years. This challenge has been exacerbated by the trend identified in this paper towards greater use of mixed method case-study over and above simple quasi-experimental approaches based on a single measure. The time-frames required for this work, particularly when one considers the efforts required for realtionship building, funding and so on, are not consisent with our standard “publish or perish” publishing cycles, which instead encourage the rapid publication of early results and lead to a literature littered with reports on early work that did not proceed when sufficient resourcing could not be found.

As the need for near continuous publication is not likely to recede any time soon, thought must be given then to the modes of “serialising” DBR research that will be acceptable within our research community. In their oft-cited DBR methodology handbook, McKenney and Reeves (2014) suggest the option of collated studies such as that recently provided by Prediger et al. (2021) in summation of their 15-year DBR KOSIMA textbook project. Such an approach can allow for greater elaboration of the macro-cycles that house the micro-cycles of DBR research (Gravemeijer & Cobb, 2006), although they are hardly timely, which would detract from an approach for maximising the impact of DBR on scalable change.

A second concern emanating from the data is the lack of true collaboration between educators and researchers. A key definer of DBR has been the interventionist nature of the methodology and its potential for not only producing high quality research but increased epistemic alignment to reduce the theory/practice divide (Dunn et al., 2019; Fowler & Leonard, 2021). Whilst making reference to the researcher being in context, many DBR studies in mathematics education are still placing the researcher beyond the "fourth wall" (Kavanagh et al., 2022), where they can supposedly remain impartial in their observations. When used, this is seen to describe the study more empirically, but it lowers the sustainability of interventions and ignores the rich contextual understandings of the educators who can more fully identify subtle changes. There are signs of progress in this meta-study that show that this important aspect of DBR is improving, but mostly in well developed, and well-funded, research communities.

A way forward may be the development of "grey literature" DBR project sites that support groups with common goals to report and iteratively build on each other's work whilst providing opportunities for educational sites to indicate interest in projects. To some extent, this is an approach that the authors of this paper have essentially chosen to develop "in house" within our own research centre. We have been favoured with a large team within our research centre who make use of DBR to explore problems in mathematics education. We have also been fortunate enough to develop research practice partnerships with multiple educational stakeholders, allowing us to adopt an "accretion" model to our DBR work which draws together findings from multiple sites of research. These collaborations also encourage our research to address issues relevant to our educational partners resulting in clearer frameworks of understanding essential to high quality investigations and implementations. The pathway to regularly publishing the meta-research that emanates from this context, though, is not clear.

A glimpse of the first steps to overcoming this reporting issue, we suspect, can be found in pre-print peer review sites such as *Academic Karma* or the American Association for the Advancement of Science's *PRE-val*. What we are suggesting may simply be a development on this kind of platform. An "open" research repository such as ResearchGate may even serve the purpose. Whatever platform is chosen, though, it will be essential that contributing to this kind of activity is seen as valid use of researcher time within performance management systems of the universities, and so it will be essential that common and accepted approaches to scaling DBR are championed through organisations such as the Mathematics Education Research of Australasia (MERGA). Greater clarity of the macro-cycles and improved collaborative research practices through research practice partnerships will progress this popular methodological genre towards not only matching, but exceeding, the original goals of DBR.

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RESEARCH PAPER

One Teacher's Pedagogical Actions in Eliciting and Developing Mathematical Reasoning Through a Contextually Relevant Task

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In this paper, we report on the pedagogical actions of one teacher in eliciting and developing students' mathematical reasoning during one mathematics lesson. The findings illustrate that through the careful design and planning of a contextually relevant task (the construction of a manu tukutuku), and the implementation of specific teacher actions, a group of marginalised students were provided access to exploring the concept of equivalence.

In this paper, we aim to illustrate pedagogical actions teachers may take to engage marginalised students in learning important mathematical concepts. To support students in learning mathematics, teachers need to enact specific instructional actions to ensure all students can access the content. One of these actions is to provide multiple opportunities for engagement in collaborative mathematical discussions characterised by students asking questions, making conjectures, justifying, and generalising, not only with their own thinking, but also with the thinking of their peers (Selling, 2016). Some teachers, however, encounter challenges in creating the kinds of collaborative learning environments required for students to participate in productive discourse. One barrier could be the teachers' strong beliefs that mathematics is best learned through students reproducing teacher directed procedures with solution focused outcomes. Another challenge is that some teachers hold deficit views about certain students being more or less capable of learning mathematics than others. One way to mitigate these challenges is for teachers to provide students with many opportunities to engage in learning mathematics collaboratively through contextually relevant tasks.

In this paper, we report on findings from a small inquiry, where one teacher used one contextually relevant task to engage marginalised students to reason with mathematical concepts. The findings also highlight that when the teacher enacted several certain instructional actions, the students were supported to reason collectively and engage in mathematical discourse. The specific research question explored in this paper is:

How can a contextually relevant task engage marginalised students to reason mathematically?

Literature Review

One teacher action that supports productive engagement in mathematical reasoning is providing students with an appropriately challenging task. These kinds of tasks need to be open-ended, high-level, focused on important mathematical concepts, with various entry and exit points allowing for student creativity and multiple solution pathways. When teachers design tasks in these ways, they are providing opportunities for students to think, reason, and problem solve in cognitively demanding ways (Smith & Stein, 1998). Solving routine tasks using memorised procedures, without connections to mathematical concepts does not allow for new learning to occur, rather, completing the task is simply practicing a learned procedure or formula. When these kinds of classroom practices take place, there is little opportunity for students to explore or deepen their understanding of important mathematical concepts. Askew (2012) suggested that mathematics tasks should prompt students to engage in mathematical reasoning as opposed to being led by direct-teacher instruction. Such a pedagogical approach allows students to become active participants in the learning process, co-constructing new

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 202–209. Launceston: MERGA.

knowledge through improvising solutions, rather than relying on teacher direction. Furthermore, both Selling (2016) and Mueller et al. (2014) agreed that challenging tasks provide teachers with affordances to facilitate meaningful discussions with students, that elicit their thinking and ideas. Marginalised students can benefit from learning mathematics in this way as when they experience success in finding solutions to problems in multiple-accepted ways, and are not always being shown what to do, they can begin to see themselves as knowers and doers of mathematics (Sullivan et al., 2020).

When first presented with a high-level challenging task, without any direct teacher direction on which strategy or solution pathway to use, students may initially feel overwhelmed or confused, anxious, or resist attempting the task altogether (Selling, 2016). While high-level cognitive demanding tasks can initially appear daunting to some students, Smith and Stein (1998) emphasised that low level tasks do not provide pathways to high-level mathematical reasoning. Askew (2012) described how the gap between mathematical content and individual understanding can be bridged using a context that is meaningful to children. He suggested that students may not generate solutions to a problem if they do not see the relevance of the problem to real-life experiences. Bills and Hunter (2015) found that the use of relevant cultural contexts can provide marginalised students with support for the development of conceptual understanding. Sullivan et al. (2020) emphasised the benefit for teachers to design a range of tasks sequencing mathematical concepts that can be explored and consolidated. Therefore, it is imperative that teachers focus on specific mathematical concepts and learning goals their students need to explore. High-level tasks should also provide opportunities for students to make conjectures, reason about, and justify their mathematical thinking (Hunter, 2014). For students to engage in mathematical discourse characterised by conjectures, explanations, and justification, they need to be explicitly supported to do so.

Students can be supported to engage in productive mathematical discourse by specific teacher action. One pedagogical approach is the use of *talk moves* (Chapin et al., 2003). Talk moves are teacher-initiated requests for students to add on, repeat, or revoice ideas presented by their peers. Talk moves also support the development of mathematical argumentation as students can be asked to agree or disagree with the mathematical content of their peers. Threaded through these moves is the intentional use of wait time, an action which provides students with time to construct their responses, or to clarify and reason with ideas being presented by their peers. Another instructional action teachers can utilise to establish purposeful discourse is to select and sequence group explanations after students have worked collectively on mathematical activity. Selecting and sequencing explanations offers affordances for mathematical ideas and concepts to be explored in greater depth (Stein et al., 2008; Sullivan et al., 2020). Furthermore, when mathematical explanations are purposefully sequenced, students are able to contribute to the construction of mathematical understanding in ways that draw on their strengths. Finally, it is essential that the teacher connects students' responses and ideas to important mathematical concepts. Selling (2016) described this as a *reprisal* move-an action where the teacher goes beyond simply accepting the explanations groups of students have presented. Rather, the teacher makes explicitly draws a connection to the mathematical concepts inherent in the groups' solution strategies and explanations. This action ensures access to deeper mathematical understanding for all students.

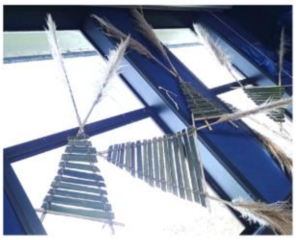
Research Methods

The present qualitative inquiry was grounded in a sociocultural perspective and was undertaken in one mathematics class in a regional primary school in Aotearoa, New Zealand. The students were aged 11–13 years old. The academic achievements in mathematics for these students highlighted that many of them were not yet achieving at the expected level (Level Four) of the New Zealand Curriculum, as evidenced by performance in standardised school assessments. The teacher was a participant in a nation-wide professional learning and development project focused on equity and inclusivity in mathematics education.

Qualitative data were collected from field notes, classroom artifacts, videorecording, and transcripts of student and teacher dialogue. Analyses comprised of multiple reviews of the recorded footage and transcribed dialogue, and thematic identification. Classroom episodes, where opportunities had been provided for students to engage in mathematical reasoning generated, which were then used for coding.

The task designed for this lesson formed part of a larger unit on Fractions. The students had some prior experience ordering fractions and their relative decimals and percentages. The big mathematical idea in this task was equivalence. The task provided opportunities for students to explore how fractions with different denominators could be subtracted from a whole. Prior to the lesson commencing, the teacher stated that she was aware that while some students could use a learned procedure to convert to equivalent fraction, being able to reason with why the procedure worked was the intended focus of the lesson.

The context of the task had been written by the teacher after noticing how the students had worked together to create manu tukutuku (traditional Māori kites) during Matariki (Māori New Year) celebrations. She had reflected on conversations among students about how they could get the most pieces out of their lengths of harakeke (flax). This task provided connections to both a lived experience of the students, and a cultural connection for Māori students in the class, thus serving as an entry point for all students (Bills & Hunter, 2015; Sullivan et al., 2020).



Dion and Jayden were cutting harakeke for their manu tukutuku. They discovered that if they cut carefully, they could cut one long piece of harakeke into 4 different sized pieces, that fit on their manu tukutuku.

Piece one is ¼ of the harakeke, Piece 2 is 2/5 of the harakeke, piece 3 is 3/10 of the harakeke.

What fraction of the harakeke is piece 4?

Figure 1. Culturally responsive task used in lesson.

While planning this task, the teacher had anticipated several possible strategies the students might attempt as they solved the task. This included drawing four different rectangles or circles and comparing each sized piece and trying to work out how much was left, or, drawing one rectangle or circle and trying to show each fraction on it, or being able to convert fifths into tenths and then possibly twentieths, or the use of decimals and percentages to convert all four fractions to determine how much of the harakeke was left for piece four.

The teacher launched the task by initially inviting the students to discuss what they had experienced while cutting their harakeke into the various sized pieces. The students shared how they had measured the pieces and cut carefully so that no harakeke was wasted, and how they could get as many long and short pieces out of one length. The teacher then prompted them to describe how they had to work together to build their manu tukutuku. These ways were then connected back to the social norm of working together to solve the task. These actions have been identified by Hunter and Civil (2021) to show the students the connections and value of prior skills to the current mathematical activity.

Findings and Discussion

The findings and discussion are presented chronologically. The specific instructional actions the teacher used are emphasised. Key themes appear as headings for each section. The explicit outcomes and opportunities for deepening mathematical reasoning are illustrated through the students' responses and dialogue.

Teacher Noticing and Responding to Students' Mathematical Reasoning

As the groups worked on solving the task, the teacher monitored the groups carefully by observing closely and taking care to notice the different strategies groups were using to solve the problem. This specific instructional action has been highlighted by several researchers (e.g., Jacobs & Empson, 2016; Smith & Stein, 1998) as an effective way of responding in the moment to students mathematical reasoning.

Initially, the teacher noticed that most groups explored the problem with representations of the fractional pieces of the harakeke. One group began by drawing three circles and attempted to divide the circles into different fractional regions. For example, dividing one circle into fifths, and another into tenths. After heading into difficulty to divide the circles accurately into the fractional sections, the students altered their approach. One student reminded the group members that the task concerned the length of harakeke (flax) and suggested it might be easier to use a rectangle to represent the harakeke. The teacher noticed that another group represented their reasoning by stacking several rectangles to illustrate all fractional pieces at the same time.

Whilst monitoring the small group work, the teacher listened carefully to students' explanations and questioning. This was an important pedagogical action, as listening to the developing mathematical discourse meant the teacher was able to begin selecting which groups would share their thinking, and the mathematical reasoning could be purposefully sequenced to develop student understanding.

Selecting and Sequencing Students' Solutions

All teacher anticipated responses were evident across the groups. Careful attention was also paid to claims students were making about the fractions such as "one-quarter is the biggest piece" and "one-quarter is two and a half one-tenths". Thought then went into how these statements could be shared with the larger group as points to argue and how the representations students had drawn could prove or disprove these claims. The sequencing of the group explanation aimed deliberately to support the explanations building on one another in increasing sophistication. This specific instructional action has been documented to support students developing conceptual understanding of important mathematics ideas (Stein et al. 2008).

The first group to share was chosen as they had not found the remaining fraction, however, they had made a mathematical claim about the need to convert all the fractions to equivalent fractions to be able to subtract them. Further, they stated that they did not think that tenths could be converted to quarters. An extract from this discussion can be seen below:

> Student 1: We decided to start with three-tenths, and we have tried to find the equivalent of one and one-tenth
> Teacher: Because?
> Student 2: Because they have to be the same fraction and one-quarter is bigger than one-tenth because this piece is bigger than that piece (points to diagram of two rectangles drawn on the board) and you cannot split it
> Teacher: What do you mean by 'you cannot split it'?
> Student 1: You cannot evenly split tenths into quarters
> Teacher: So, you are making a claim that you cannot evenly divide tenths into quarters?
> Student 2: Yes, because half of 10 is five, and then it is not even

The teacher used sustained questioning and then a reprisal move (Selling, 2016) to clarify thinking and elicit a deeper explanation from the wider group. These moves acknowledge the small group explaining as valid contributors to the developing mathematical thinking, in spite of not yet having solved the task. The students continued building on their explanation. The excerpt below illustrates the deepening dialogue:

> Teacher: Can you show on your diagram what this looks like that shows what you mean by its not even?
>
> The group draw one rectangle divided into tenths and dissect the rectangle showing half, labelling each half as one-fifth
>
> Student 1: See, you cannot halve five evenly

The group stood silently waiting. The teacher then extended an invitation to all students asking if anyone would like to add to what had been said.

> Student 3 (On mat): Well, it is still even because half of 5 is 2.5

The teacher asked the small group to consider how this claim could be represented or proved. Together, they draw lines on the rectangle to show quarters. Once the group had completed their drawing, she asked them think about a mathematical statement they could make about the number of tenths that were equivalent to one-quarter. Two students offered the following reasons:

> Student 1: Oh wait! It is two and a half
>
> Student 4 (On mat): Its it is two and a half one-tenths. And the other bit is three-tenths so that is five and a half one-tenths altogether.

In this episode, the teacher had initiated a group discussion where all ideas were considered. Specifically, she utilised wait-time and adding on, both of which are important pedagogical actions supporting the development of productive discourse, as highlighted in the work of Chapin et al. (2003).

The teacher continued developing the collaborative discussion by asking the students if they agreed with the statement that two and a half one-tenths was the same size as one-quarter. Several students agreed while others looked confused. The teacher waited and then prompted the students to turn and talk with each other to make sense of what had been stated. Some students used both explanations and written representations (diagrams) to communicate their reasoning with other members of their groups. The teacher then asked two students who had initially appeared confused to share their explanations with the larger group. She asked two other students to add on to the previous claim that they had cut five and a half one-tenths by adding one-quarter and three-tenths. The students were able to do this by pointing to the diagram the group had drawn on the board. Now satisfied that there was an improved understanding of the relationship between one-quarter and two and a half tenths, the teacher offered a reprisal move (Selling, 2016) to highlight the mathematical reasoning that has just emerged.

> Teacher: What you have just explained and shown the group is that by dividing the harakeke into tenths you can show where the cuts were made to get the pieces that were one quarter and three-tenths long.

The teacher drew attention to the clear explanation the group had made. Continuing, she asked the second group to build on the explanation and explain how they had converted two-fifths to four-tenths. This group of students drew two stacked rectangles, one divided into fifths, and the other into tenths. Some students became confused as to why there were now what appeared to be two separate pieces of harakeke. The teacher then prompted one of the students to ask the group explaining why they had represented their thinking in that way:

> Student 1 (On mat): Why have you got two pieces of harakeke? There was only one
>
> Student 2 (From the group explaining): There is only one, this is just to show how tenths and fifths are

the same

Teacher: Why do you think it is important to know how many tenths are the same as one-fifth?

Student 3 (From the group explaining): Because then we can work out how big each piece was that we cut

Teacher: What are we trying to find out in this task?

Student 4 (On mat): Oh, how big the left-over piece was

Teacher: So, can you now show on one rectangle where the cuts are, if we know that one-quarter is the same as two and a half tenths, two-fifths is equivalent to four-tenths, and we also have a piece that is three-tenths?

The group explaining then drew a new rectangle and divided it into tenths. They shaded each unit and were left with what they described as half of one-tenth. The teacher then asked the students to think about what fraction the left-over piece was, asking whether half of one-tenth was the best way to describe it. One student responded stating that half of one-tenth was not a real fraction. Another group of students explained that they had recognized that quarters, fifths, and tenths could all evenly divide into twentieths as seen in the following extract:

> One-quarter is the same as five-twentieths, and three-tenths is six-twentieths, and two-fifths is the same as eight-twentieths.
>
> When we add these together, we get nineteen-twentieths, with one-twentieth remaining

Satisfied with the development of students' reasoning, the teacher proceeded with connecting all the students' ideas to important mathematical concepts inherent in the task.

Connecting Students' Reasoning

To connect these ideas the teacher drew another rectangle on the board. This time she divided it up into twentieths. She asked the students to think about where halfway was. One student explained it was "at 10 because 10 is half of 20". She then asked where one-quarter would be, and another student explained that one-quarter would "be at five because 20 divided by four is five." The teacher then counted the twentieths out in groups of five to illustrate this. She then turned to the final groups' explanation and asked the students to think about where the fifths were. They explained that 20 divided by five was four, so, fifths would be the same as four twentieths. The teacher drew lines to highlight where the fifths were. Finally, she asked where the tenths were. At this point, the teacher was now certain that the students understood why twentieths could be used to solve the problem. She then asked all the students to shade in the pieces of harakeke using twentieths. The teacher actions in connecting the different groups ideas supported students to understand that there was now only one-twentieth remaining. The teacher then extended student reasoning to generalizing with how many twentieths were equivalent to four-fifth, three-quarters, and seven-tenths-which many of the students were able to do.

Conclusions

The findings of this investigation have illustrated how using a contextually relevant task, and specific instructional actions can support underachieving students to learn important mathematical concepts. Underpinning these actions was evidence of the teacher utilising an asset-based approach to teaching and learning mathematics. Teaching mathematics from a strength-based approach involved the teacher holding high expectations that all students were capable of learning mathematics, and countering deficit theorising about others.

Analyses of the data highlighted that when all students, particularly underachieving students were supported to learn mathematics though collaborative engagement in challenging tasks, they were provided affordances to explore important mathematics concepts. These affordances included opportunities to explain their mathematical reasoning, reason with their peers' explanations, ask questions for clarification, and justify thinking. The findings from this inquiry also illustrated that one specific way students could be supported to participate effectively in collaborative mathematical activity was through purposeful and consistent development of social norms. Evidence suggested that when effective ways of working together had been established, students developed confidence to discuss their ideas publicly, even when solutions were incomplete or showed obvious errors or misconceptions. Parallel to developing effective collaborative norms, was the importance of planning tasks that were meaningful to students.

In this investigation, effective planning involved thoughtful consideration of the cultural identities of the students and drawing on these to connect to important mathematical concepts in contextually relevant ways-in this instance, the fractional reasoning required for making manu tukutuku. Planning actions also included teacher anticipation of probable student responses and the identification of possible misconceptions or common errors. Planning mathematical tasks in this way ensured the teacher had clear reference points for monitoring student collaboration. Monitoring is an instructional action characterised by the teacher noticing and in-the-moment responding to students working together on mathematical activity. In-the-moment noticing and responding in this inquiry resulted in students clarifying mathematical explanations, asking questions, and representing their reasoning in different ways for conceptual understanding. Wider student discussions were also made possible when the teacher extended invitations for all students to contribute to the mathematical reasoning.

Group solution pathways were also sequenced in a way that afforded deeper mathematical thinking through shared discussion about the different mathematical ideas. This action meant that students' collective reasoning could be built on in increasing sophistication. The findings presented in this paper highlighted that when clear and explicit connections were drawn between students' co-constructed mathematical reasoning, the students had access to many ways to think about the mathematical concept of equivalence within a contextually relevant task. The implications of these findings demonstrate the benefits for all students when teachers adopt pedagogical approaches to teaching mathematics that encompass equity and inclusion. Teaching and learning mathematics with equity is particularly important in New Zealand, as for many students, mathematics is an unyielding gatekeeper where access is often only provided for those who exhibit speed and accuracy. When teachers purposefully plan mathematical activity that connects authentically to their students' experiences and utilise specific instructional actions that provide opportunities for creative exploration and reasoning, conceptual mathematics learning becomes accessible to all.

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RESEARCH PAPER

How Big is a Leaf? Using Cognitive Tuning to Explore a Teacher's Communication Processes to Elicit Children's Emerging Ideas About Data

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The intangible concept of data, as part of statistical literacy, can be complex for young children to grasp. Inquiry as a pedagogy has potential for supporting student development of statistical literacy as the investigation process is driven by the inquiry question. The aim of this paper is to gain insight into how a teacher's communication processes with her students supported their emerging understandings about the abstract concept of data. In this exploratory case study, we present data from a Year 4 classroom (age 9) in a guided mathematical inquiry within the STEM context of agricultural science. The inquiry question the students addressed was, "How big is a leaf?" The inquiry focused on linking data to the real-life context the data represented.

The research question guiding our research was,

> *How can a teacher's communication processes support students' emerging mathematical understandings about measuring area and the abstract concept of data?*

Area as an attribute to measure was a new concept for student exploration. Students' emerging mathematical understandings about data included the abstract idea that numbers used in a statistical investigation represent, data which in this instance, depicted the area of leaves and ultimately, the leaves they investigated.

The study took place in a Year 4 classroom ($n = 25$) in a metropolitan school in Queensland. Student ideas about data were analysed using the theoretical framework of cognitive tuning to illustrate how one class built common understandings about the concepts of area and data. The focus was on how the teacher supported the processes of *normalisation*, *conformity* and *innovation* (Wit, 2019). In this paper, we look closely at the teacher's communicative processes around the *normalisation* of the problem task, and *conformity* of common understandings about area and data. Teacher dialogue will be used to explore how the processes of cognitive tuning made the connections more visible between the teacher's support and the students' emerging ideas.

Background

In classroom data modelling, data and its attributes are defined and negotiated to suit a purpose or to address the context in which the problem is set (Leavy & Hourigan, 2018). For example, the mathematical concepts of area (as an attribute to measure) and data (for investigation and interpretation) in a biological crop model, are very closely linked. Similar to Meyer and Land's (2005) description of a threshold concept, in a biological model the concepts of area and data are tied together (integrative) and understanding the first would change the way a student looked at the latter (transformative). Threshold concepts would potentially be troublesome for students but once a relationship between the mathematical concepts was established, it would be difficult to unlearn.

Mathematical challenge and problem solving in the classroom can present students with uncertainty and even anxiety (Buckley & Sullivan, 2021). However, the *Australian*

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 210–217. Launceston: MERGA.

Curriculum: Mathematics emphasised the proficiency of problem solving and encouraged teachers helping students through "active participation in challenging and engaging experiences" as part of the rationale for the curriculum (Australian Curriculum, Assessment and Reporting Authority. [ACARA], 2014). A socio-cultural perspective is often acknowledged as underpinning inquiry pedagogy and socio-mathematical norms of an inquiry classroom, which differs to the traditional classroom sense of individual success and classroom teacher as "sage", and include the embrace and normalisation of intellectual challenge (Goos, 2004; Hunter & Hunter, 2018; Makar & Fielding-Wells, 2018). An inquiry classroom is characterised by collaborative groupwork and a knowledge-building culture. This in turn, can foster positive mindsets and strengthen intellectual engagement (Australian Government Department of Education, Skills and Employment, 2022; Confrey, 1995; Fielding-Wells et al., 2017; Goos, 2004).

Key in group problem solving situations is the communication between its members. Collaboration is part of the problem-solving approach used in inquiry and centres on communication between students to develop thinking together (Allmond et al., 2010). Research articulates the argumentation processes students encounter in inquiry as they defend their mathematical findings with evidence (Fielding-Wells, 2013). The teacher's role in inquiry settings, to orchestrate classroom conversations, is critical to facilitate problem solving without giving away the solution (Fry & Hillman, 2018).

Theoretical Framework: Cognitive Tuning

Cognitive tuning was used by Wit (2019) to explain the communicative processes involved in group settings when reaching a common understanding, or when making progress towards a socially anchored representation of a task. Although Wit applied the framework to an adult setting, cognitive tuning encompasses modalities, which seem to align with teacher goals more generally of pursuing commonality in their students' thinking when teaching mathematical concepts. The basic modalities of *normalisation*, *conformity* and *innovation*, are offered to build a commonly shared frame of reference in group settings and will be used to gain insight into the ways one teacher scaffolded student thinking in one classroom, to address the research question,

> *How can a teacher's communication processes support students' emerging understandings about measuring area and the abstract concept of data?*

Normalisation describes the moment when the participants (problem solvers in this instance) do not yet have a shared interpretation of the task and provides time for a group to build a normative frame of reference in which to work together, for creating solutions *to* the problem. Wit's (2019) framework acknowledges the influence individual prior experience presents to a group setting and the cognitive conflict participants may encounter. A common understanding of a task can lead to a common response. *Conformity* describes the communicative processes when a less-experienced group member lacks confidence and faces conformity pressures from a majority, or a common belief. In a classroom context, this can be likened to a teacher's efforts to reach conformity about mathematical ideas or learning goals, and the pressures a teacher can exert to guide and scaffold student learning. *Innovation* is when a naïve individual resists pressure from the majority and challenges the group norm. Although consistency of thinking is often a classroom goal for teachers, an individual's line of reasoning (which may be in the minority) may also have innovative impact by introducing a new frame of reference. To have impact, the new frame of reference needs to be publicly supported to build less-threatening environments that encourage innovation.

In this exploratory study, the authors are aware of the importance of building a shared frame of reference about the statistical concept of data specifically, related closely in this instance to the concept of area. In this paper we use cognitive tuning to look closely at the ways students were supported by the teacher, to develop their thinking about the concepts of area and data, framed specifically by the processes of *normalisation* and *conformity*. The cognitive tuning framework may also provide insights into innovations in problem solving, developed by students, that are shared.

Research Design

Setting and Participants

There were 25 students in this classroom, situated in a metropolitan public school in Australia. With slightly more boys than girls, all students had their own learning needs, special needs and achievement levels—typical of a Year 4 class in this setting. The inquiry took place over nine lessons (1–1.5 hours each), across two weeks. This was the first mathematical inquiry the students had encountered although they were familiar with solving problems posed by open questions. The first author, also the joint classroom teacher working in a shared partnership at the time, worked with her classroom teaching partner to plan the inquiry based on conversations with their STEM professional; an agricultural scientist from Commonwealth Scientific and Industrial Research Organisation (CSIRO). The agricultural scientist and the teachers had met three times (once face-to-face and twice over the phone) to discuss the key mathematical concepts involved in the modelling agricultural scientists do to predict crop size.

Context

The mathematical problem of "How big is a leaf?" arose when the classroom teacher and the teacher-researcher (classroom teacher partners) worked with an agricultural scientist to learn how mathematical modelling could be used to predict crop production. The relationship between the length and/or width of a leaf and its area was an important first step in determining how much energy a plant produces, using sunlight. A passionfruit vine on the school grounds provided the model of a biological system to explore (passionfruit) and this presented an authentic STEM context in which Year 4 students could informally calculate the area of leaves and irregular shapes. However, the classroom teacher and teacher-researcher partner felt that the relationship between the length and/or width of a leaf and its area was important and difficult for the students to consider. Surface area was a topic that had not been explored in the classroom prior to the inquiry and it was not initially clear to the students, which part of the leaf might absorb sunlight for photosynthesis. Student strategies for solving the problem were important as the focus for learning, as outlined in the curriculum, compare the areas of regular and irregular shapes by informal means (ACARA, 2014). The mathematical focus also included making comparisons between objects involving familiar metric units. Once area was established to determine size, this information would constitute data for investigating the kinds of relationships (between data points) that would be useful to an agricultural scientist in the field.

The strand devoted to learning statistics in the *Australian Curriculum: Mathematics* (ACARA, 2014), brings focus to data representation and interpretation from an early age (Foundation Year) with students answering yes/no questions to collect information. Understanding that data are information is the basis for considering data, and (part of) a Data Science K–10 Big Idea to Inform Teaching Practice (Bargagliotti et al., 2020; YouCubed, 2022). Little is reported on how primary-aged children perceive the concept of data and it is a wonder that children can accept the answer "information" in response.

Data Collection and Analysis

All nine classroom sessions were video recorded by the teacher-researcher and field notes were taken to record events. Powell et al. (2003) present a model for analysing video data, used by researchers to inquire into students' mathematical activity. The approach supported the process of analysis of the classroom videos in this study, offering a glimpse into the non-linear learning progression of students, anticipated in inquiry settings. First the authors viewed the video sessions to familiarise themselves with the lessons and to briefly describe the content when the term "data" was used in discussion between the teacher and students, and between students. Time-coded descriptions allowed the researchers to review and identify critical events—where discourse demonstrated change in understanding about data. This was crucial to consider in terms of Wit's (2019) cognitive tuning processes of normalisation and conformity. These critical events were transcribed for closer analysis and annotated for themes and patterns, enabling the researchers to make sense of the data and to construct a storyline. The narrative below is composed of the storyline revealed through analysis of the classroom data.

Results

Building Shared Interpretations of the Measurement Attribute of Area

Prior to Year 4, Australian students' experiences with measuring area involve making direct physical comparisons. There were two mathematical aims of this inquiry; (1) to support students to compare areas of regular and irregular shapes by informal means (ACARA, 2014); and (2) students needed to look for, identify and describe patterns in surface area measurements in order to make conjectures or hypotheses about ways to predict leaf size, as part of the mathematical modelling used by agricultural scientists. When asked in the first session to consider how much sunlight a leaf 'gets' (photosynthesis), the students appeared unsure about how to approach this. The teacher needed her students to have a shared interpretation of the task which required knowing what part of the leaf constituted the surface area. She guided her students to consider this through their initial explorations. Students brainstormed ways to measure how *big* their leaves were, and the teacher recorded suggestions on the board: length, width, mass (weight), thickness (depth), how much it grows, how much light it reflects and how much sunlight it takes in. Based on this, the students devised ways to measure their leaves taking out rulers to do so. However, curved edges made this difficult.

> Liam: They're curved.
>
> Kyrie: Well that makes it really hard
>
> Liam: Wait! (he picks up a 30cm ruler)
>
> Kyrie: Have you got something curved in your desk? A piece of paper or something?
>
> (They both shake their heads and rip a small corner from a page in their book. They measure the piece of paper and then line it up with the edge of their leaf.)
>
> Kyrie: One centimetre. One centimetre. One centimetre.

The teacher brought the class together in a Checkpoint, to share plans for measuring leaves but it was clear that no table groups had a plan yet. The students returned to this task.

> Isla: Measure the stem from here to here (She points to the bottom of the stem to the top of the leaf [furthest point]. Her neighbour explains the importance of measuring across the leaf also (the width)).
>
> Liam: Don't we have to measure the whole thing? (Moves his hand across the surface of his tri-lobed leaf.) I think we should measure that, and that, and that (Points from the centre of his leaf along each lobe/leaf section).

Isla rephrased what Liam said and pointed to the sections of her leaf to show her agreement.

> Liam: But how can we find around this bit? (Runs his finger along the edge)
>
> Isla: I don't know.
>
> Helena: I've got this idea if we just measure across the middle. We should measure it through the middle.
>
> Isla: Yeah but if you want to know there to there then you have to measure it that way (she points from the bottom of the stem along to the end of a leaf section/lobe). Because then it's not the actual leaf.

The students continued to explore this for a further twenty-five minutes (with a lunch break in between). One student decided to cover her leaf with a round container, whereas all other students struggled with rulers to measure different dimensions of their leaves. This student was the only person to try this approach and although she explained her idea to them, peers at her table group relied on their own approaches. This was an idea the classroom teacher wanted to encourage, and she focused attention on this as a successful approach.

The teacher asked if students in the class had ever tried to cover something to measure area and one student recalled a previous experience about a little cube – putting little cubes on something. Bringing focus to the covering method now validated the approach, and other students started to cover their own leaves in different objects to measure area. Marbles turned out not to be useful to cover a leaf as they did not tessellate, but shape tiles proved popular. Triangles had "pointy bits" like the leaves and combined with a rhombus shape, could cover most of a passionfruit vine leaf.

In the following lesson, the teacher reminded students of the importance of standard units of measurement. Two students had thought overnight about using grid paper to cover their leaves and the teacher built on this idea by introducing students to the 'square centimetre.' Using grid paper to measure surface area became the new focus of the inquiry.

The teacher continued to guide her students through repeated opportunities to measure the sizes of their leaves using grid paper displaying square centimetres. Two students introduced the idea of drawing a box around their leaf and this was pursued by all students in the class. By the fourth lesson, students were able to express the area of their leaf as a fraction depicting the relationship between the area of the leaf and the area of a box drawn around the leaf. Now that the students were aware of the mathematical focus on area and what part of the leaf this related to, the teacher turned the focus to the statistics notion that the measurements they had collected and recorded (on the board) constituted "data."

Building Shared Interpretations of the Statistical Concept of Data

It was important to compare leaves that were very different in size, proportionally. A number of data points were also required before making conjectures about how to accurately predict leaf size. Lesson six began with the first acknowledgment by the teacher, that the solution involved a focus on the statistical concept of data. In fact, their investigation by now had resulted in a "heap of data" to consider. The students were encouraged to look at the data and to share what they noticed (Figure 1).

> Teacher: When we look at data we try and make sense of what the data means. We look for patterns. We look for things they have in common or things that are really different, things that really stand out—things that are similar.

The students were encouraged to consider what "this" (Figure 1, the data) actually meant.

> Rafael: They're representing how big our leaves are.
>
> Teacher: (agrees) Precisely what is it representing?
>
> Akayla: How much of the box is taken up by the leaf.

One student described the pink fractions as easier to compare because they were "not as big" and the teacher prompted for further response (pictured in Figure 1).

Kyrie: Because the pink ones have the same denominator.

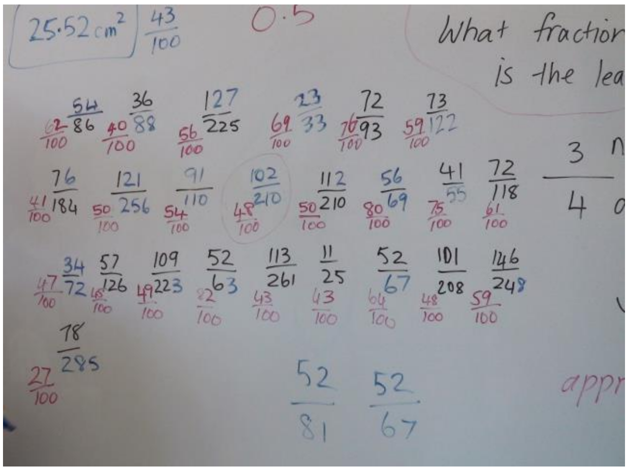

Figure 4. Classroom board depicting "*heaps* of data."

The teacher was pleased with this result as it was a mathematical response – the fractions written in pink (Figure 1) all included hundredths as the denominator and this made them easier to compare than their original fractions, such as 76/184 and 146/248 for example. Two leaves were represented by the fraction 43/100 and the owners of the leaves stood up to show their leaves to the class. Surprisingly, one leaf was very small in size and the other leaf was larger than the teacher's hand! The class continued to explore the data and the teacher made multiple connections between the fractions recorded on the board and the leaves they represented.

By the seventh session, the teacher was still dissatisfied with the messy data on the board and asked the students what they could do. Suggestions by students were recorded on the board: bin it, sort it out, arrange it, organise it, and group it. Students sorted the data in different ways but appeared unsure of how to make a statement about what the data showed them. They started the next lesson looking at one student's workbook—selected by the teacher—projected on the screen in front of the class. The student had made the statement, "This data shows us that a lot of people have their decimal fractions between 40 and 48 [hundredths]" and the class considered this. However, the teacher explained how the statement was not specific nor a comparison. Other students shared what they noticed about the data now it was sorted into columns. The teacher encouraged all students to predict what they would expect to find if they placed all the leaves together, prompting them to think what the leaves might have in common. What might students expect to see?

Sandy: A big fraction?

Teacher: No, I mean if I get all these leaves in a group, what do you think those leaves are going to look like?

Sandy: I think they will be big leaves.

The students were still unsure of what the teacher meant and so she took a leaf that matched a data point in the 40s (hundredths). She pointed to another data points and asked for the leaf it represented to physically compare the two leaves. The students made predictions about what they might see before the leaves were placed in groups that matched the sorted data.

Eventually, students were encouraged to connect the leaves with the data on the board through sorting to reflect ways the data were organised. Students recorded statements to demonstrate their ideas about the relationship between the area of the leaf to the area of the box around it. This is one example from Naomi:

> Most boxes are 40% to 60% taken up by the leaf. Then you can fill the box up 40% to 60% and that could be close to the area of the leaf. But with a mono leaf, it will take up between 70% and 80%.

Discussion

Inquiry presented the pedagogical approach in this classroom, to solving the open-ended and complex problem, "How big is a leaf?" *Normalisation* is the initial phase in cognitive tuning (Wit, 2019) and in this classroom, is used to describe the initial phase of problem-solving when the teacher supported her students to build a shared understanding of the task, or a normative frame of reference about the requirements of the problem: finding ways to determine the size of a leaf in relation to the amount of sunlight it received. This would support the mathematical focus on data and support students to develop understandings about area as an attribute to measure, as a threshold concept. Building a normative frame of reference about the problem being posed, assisted the class with moving forward as a whole group. *Normalisation* encompasses the problem-solving aspects of moving from not knowing to knowing and involves communication and mathematical reasoning between students and the teacher to reach the normative frame of reference, or shared interpretation of the task. The results depicting *Building shared interpretations of the measurement attribute of area* are lengthy and involve much student talk. However, teacher moves such as allowing students time to think and struggle with not knowing, validating particular approaches, introducing measurement conventions (square centimetres), and allowing multiple opportunities to measure, communicated the importance of the shared frame of reference about the mathematical concept of area.

Beliefs that are common to a majority in a group situation, can influence the beliefs of those with less confidence. These pressures constitute the idea of *conformity* (Wit (2019). Although inquiry places students at the centre of the problem-solving process, conformity pressures by the teacher were reflected in the valuing of specific student approaches that related to surface area. The struggle for students to figure out which part of the leaf to measure was important and only one student approached the problem with the idea to cover the surface of the leaf. In the results for *Building shared interpretations of the statistical concept of data*, attention turned to student data illustrating leaf measurements which students were required to interpret to identify patterns. Conformity in a group can be difficult to achieve and teachers are certainly aware of the difficulties faced in supporting all students to demonstrate particular learning goals. Here, the teacher made repeated connections between data points and the real-life objects they represented and emphasised students' particular ways of sorting data. Attention was drawn to patterns or clusters in the students' data that could be interpreted, and students could begin to perceive relationships between the size of a leaf to the size of the box around it.

Cognitive tuning places importance on providing opportunities for student discussion as well as listening to ideas shared. The openness of GMI supported *innovation:* one student showed innovation in their approach to measuring area by covering her leaf with a lunchbox container. This new way of thinking had innovative impact as it proved a way forward in problem solving. The mathematics classroom should be a place which supports innovation and fosters positive dispositions towards mathematics. This responsibility lies with the classroom teacher in establishing a safe environment to do so.

The focus of this paper was on how the classroom teacher supported her students to develop understandings about area and data. Cognitive tuning presented one way to describe the process of supporting students through stages of not knowing to knowing, recognising the role of the teacher to develop a knowledge building classroom culture. This process may prove useful for describing the processes of reaching shared interpretations of a problem-solving task. It acknowledges how knowledgeable others, in this instance the classroom teacher, can apply pressure within a safe learning environment, to conform or guide student solution processes, while allowing for innovation in student approaches.

Acknowledgments. The first author wishes to acknowledge the support received by Professor Lyn English, in her role as advisor during the Post-doctoral Fellowship, awarded by Queensland University of Technology. Professor English' support and insights into mathematical modelling in the primary classroom enabled the classroom research to gain important insights into the problem-solving processes young children use.

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Designing Specific Tools to Enhance the Numeracy of Adults with Intellectual Disabilities

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Design Research (DR) has been used to develop means of supporting mathematical learning for typically-developing students. This study investigated the use of DR to develop context specific tools to support adults with intellectual disabilities (ID) to improve their numeracy capabilities and engagement in daily tasks. Using observation and interview data, findings demonstrated increased engagement and participation in the numeracy demands of these tasks. Participants reported positive perceptions of improving competence and increases in independence. This study demonstrates the application of DR to the field of numeracy and adults with ID and the usefulness of context specific designed tools to support numeracy learning and independence.

Higher levels of numeracy lead to a better quality of life (Tout & Gal, 2015), however, adults with intellectual disability (ID) lack opportunity and expectation to engage with numeracy learning (Lambert & Tan, 2019). The term numeracy has evolved since first being coined by Crowther (Ministry of Education, 1959), and current conceptualisations of numeracy value more than just mathematical knowledge (Geiger et al., 2015). An ability to apply that knowledge in different contexts, a positive disposition towards using and applying mathematical knowledge, and a willingness to engage with and solve problems involving mathematics, are considered vital qualities of a numerate individual. Learners with ID need to have the opportunity to engage with numeracy learning at school and continue to have ongoing learning opportunities once they leave school.

Although research on inclusive school mathematics education for students with ID is ongoing (Bennison et al., 2020), research into numeracy learning opportunities for adults with ID is sparse (Prendergast et al., 2017). This study aimed to demonstrate one way of continuing to support numeracy learning for adults in work and social settings by investigating the way specifically designed tools could support numeracy learning and task engagement.

Background

What counts as numeracy has changed in an emerging technological environment (Bennison et al., 2020). Gaining a mastery of computations and fluency with numbers, previously seen as the foundation of school mathematics, has evolved into an understanding that being numerate requires the ability and the dispositions to use mathematics when solving problems in the context of home, community and work life (Geiger et al., 2015). Further, Faragher (2019) argued that mathematics for students with ID, should include the consideration that students now need to master the use of appropriate *tools*, such as calculators or smart phone apps, that may be used to support them to complete *basic mathematical skills* to develop more complex mathematical understandings. For example, students with ID may be able to learn to complete perimeter and area problems with their same age peers if they have access to calculators for the computation steps of the problem.

One model of numeracy that encompasses these conceptualisations is the 21st Century Model of Numeracy developed by Goos and her colleagues (Geiger et al., 2015; Goos et al., 2012). This model consists of five elements; *mathematical knowledge*, *tools*, *dispositions*, *context*; and *critical orientation*. Numerate individuals can use *mathematical knowledge* and select useful *tools* to solve problems and make sense of mathematical situations. In doing so, they demonstrate *positive dispositions* towards situations that involve mathematics.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 218–225. Launceston: MERGA.

Additionally, the *context* of the problem can dictate the required mathematical knowledge and the available tools to support problem solving (Geiger et al., 2015). Finally, considered essential by Goos et al. (2012), is a *critical orientation* to numeracy; the ability to challenge and critically evaluate a situation involving mathematics. The 21st Century Model of Numeracy has been used to frame this study of developing numeracy for adults with ID.

Opportunity to participate in mathematical learning in different contexts is essential to developing numerate individuals (Schreiber-Barsch et al., 2020); however, for learners with ID, that opportunity is limited (Lambert & Tan, 2019). More than 30 years ago, Mastropieri et al. (1991) identified differences in the research on mathematics and numeracy education for learners with ID and typically-developing learners. Learners with ID were mainly exposed to a narrow range of mathematics curriculum, and teaching approaches constrained by behaviorists theories of learning. Mastropieri et al. (1992) noted the focus on constructivist approaches in mathematics education research for typically-developing learners and identified the need for mathematics research for students with ID to broaden the range of mathematics topics and variety of approaches. More recently, these results were confirmed by Lambert and Tan (2019) with the authors calling for significant changes in mathematics education research for students with ID that pays attention to "participation in general education mathematics" (p. 28) and documents students with ID in the "dominant pedagogical orientations in mathematics education" (p. 28.). Research that focuses on skills can lead to the segregation of students with ID into "lower quality mathematics instruction and may lead to low expectations of mathematical competence" (Lambert & Tan, 2019, p. 5).

Post school, the lack of opportunity and expectations continues with Schreiber-Barsch et al. (2020) suggesting that there are limited opportunities for adults with ID to continue learning when they leave school. This lack of opportunity to learn contributes to the lack of opportunities in employment and an "ordinary life" (Lysaght & Cobigo, 2014). Children with Disability Australia (CDA) commissioned a report on post school transition of children with disabilities in 2015. They found that people with disability in Australia "are only half as likely to be employed as people without disability" (CDA, 2015, p. 19). Thus, investigating ways of supporting adults with ID to continue learning post school need to be investigated.

Prendergast et al. (2017) suggested that adults with ID wanted to learn numeracy that is meaningful and useful to them and Schreiber-Barsch et al. (2020) identified learning in context as an important aspect of adult education. Thus, this study aimed to investigate ways of further developing adults' numeracy that is meaningful and useful to them by working with adults with ID in their work or social contexts to answer the following research question:

> *In what ways can DR support the development of specifically designed tools to support numeracy learning and task engagement for adults with ID in social contexts?*

Method

Qualitative research approaches enable the collection of rich data and are best suited to situations where a deep understanding of social contexts and phenomenon is required (Merriam & Tisdale, 2016). In this study, qualitative approaches were chosen because a rich analysis of the context of numeracy in the actual experiences of adults with ID was required. Additionally, Design Research (DR) is used in mathematics education to study mathematical learning and the development of tools to support learning (Cobb et al., 2003). In this study, designing tools that specifically targeted the participants' learning in the context of their daily tasks was required, thus DR was adapted to the context of adult learning.

Using a qualitative research design, observations and interviews were conducted with four adults with ID in their work or social settings to determine participants' numeracy needs and design individual goals. Based on these goals, DR was then used to develop, trial and refine task specific tools to support numeracy learning. Participants were three male and one female adult with ID ranging in ages from 19 to 41 years. This paper draws on data from two participants competing in ten pin bowling, Ben, a 19-year-old male and David, a 41-year-old male (pseudonyms). The study comprised two phases.

Phase 1 of the study comprised 6–8 one-hour long audio recorded observations (Merriam & Tisdale, 2016) over 7–12 weeks to document numeracy demands of chosen tasks of participants. Interviews with participants and significant others, such as support workers, were conducted at the end of Phase 1 to clarify researcher interpretations of observation data and provide participants with a voice. These data were used to identify learning goals and design tools to support participants' numeracy development in Phase 2, a further 11–13 one-hour long audio recorded observations over 16 weeks. Ongoing analysis throughout Phase 2 supported the design and modification of the tools. Phase 2 observation data were analysed with further interview data collected at the end of Phase 2, to determine the effectiveness of the designed tools to support participation in the numeracy demands of the chosen tasks.

Results

In this section, using data from Ben and David at the bowling alley as evidence, the process of designing individualised tools to support learning is discussed. Original design of tools to support learning was based on observation and interview data from Phase 1 of the study, and then refined through iterations of the DR cycle, illustrated in Figure 1.

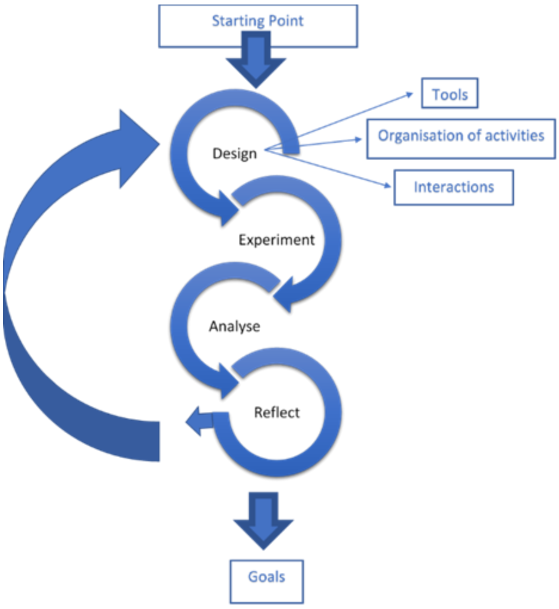


Figure 1. Design research cycle.

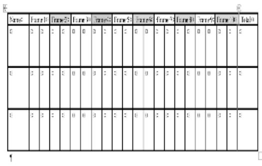
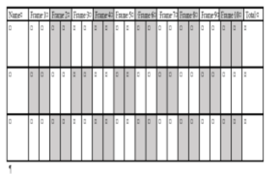
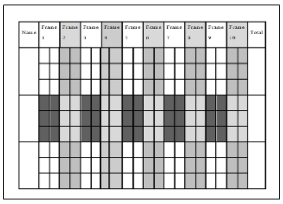
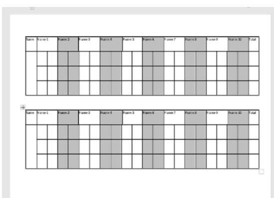
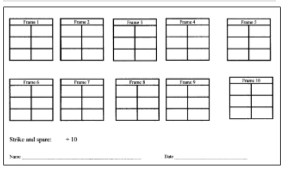
Using data from Phase 2 of this study in the context of the bowling alley, the development of viable tools using the DR cycle will be demonstrated. A *Scoresheet* was the tool designed to support Ben and David to reach their goal of being able to determine their current running total when strikes and spares were scored. This example was chosen for this paper as a number of iterations of design were required before the resulting tool supported these participants' progress towards their goal at the bowling alley.

Designing a Tool at the Bowling Alley: The Scoresheet

The starting point for this DR experiment was the identification of the goal. Identification of goals in this study were discussed in a previous publication (Gaunt et al., 2019). Based on the analysis of Phase 1 data, the identification of the goal for Ben and David was to be able to calculate their running total when a spare or strike was scored. The next step was to determine the mathematical knowledge to calculate the running total after scoring a spare or a strike (add 10 to the current score) and the current capabilities of the two participants. From Phase 1 data, Ben and David demonstrated strength in reading, comparing and understanding numbers (observed 85 times during Phase 1 with 100% accuracy from both bowlers). Both bowlers showed difficulties in determining their current score using mental calculations (out of 25 attempts during Phase 1 observations, 11 were correct and 14 were incorrect). Additionally, remembering information, such as how many points they scored for a strike, was difficult (Out of 16 attempts, only David answered correctly on one occasion). To support numeracy learning, a *Scoresheet* was designed to be used with the support of a *Calculator*.

Designing a visual *Scoresheet* similar in layout to the scoreboard would facilitate the bowlers' understanding and use of the tool. Design 1 was developed (see Table 1) and trialled by the researcher. After analysing and reflecting on the design during observations, two further iterations of the DR cycle resulted in Design 3, the first design trialled with participants.

Table 1
Scoresheet Design

Experiment: Design no	Design	Analyse: supports	Analyse: difficulties	Reflect: Considerations for next design
Design 1 Trailed by researcher		Can record scores exactly as on score board	Difficult to track across score sheet	Provide alternate shading of frames
Design 2 Trailed by researcher		Can track across scoresheet	If participant scores a strike, nowhere to record interim score	Include extra row to record score
Design 3 First design shown to participants		Frames identifiable with shading and space to record interim score after strike.	A significant amount of recording would be required by each bowler if they were to record all bowlers scores	Each bowler only records their score
Design 4: First design used by participants		Larger boxes and bowlers only recorded their own score	Boxes too small. Difficulty tracking across the scoresheet. Difficulty remembering strike and spare = +10.	Separate frames and make boxes larger. Add visual prompt
Design 5 Final design		Frames separated. Boxes larger. One game per side of page		

The Design 3 *Scoresheet* was modelled on the scoreboard and provided space to record the scores for the usual three bowlers for each game (only two of the bowlers were participants in the study). Each frame was distinguishable by the use of alternate shading, and within each frame, there were three rows instead of the two found on the scoreboard to allow the recording of the interim total if a spare or strike was scored.

During Observation 1 of Phase 2, the researcher demonstrated the *Scoresheet* to David and Ben. Both bowlers checked in after each turn and were shown how to record their scores, by demonstrating the use of the calculator to add 10, and where to record scores on the scoresheet. During this trial, it was conjectured that simplifying the amount of recording required for each person would further support the bowlers in completing their *Scoresheet* independently, allowing them to concentrate on only their score for recording purposes. The subsequent Design 4 *Scoresheet* (see Table 1) was thus intended for just one bowler to record the usual two games that were completed for each competition.

Design 4 was the first design where Ben and David completed their own scores. When the bowlers began using their *Scoresheets*, it became apparent that Design 4 did not support the activity well. Figures 2 and 3 show copies of a section of the recorded *Scoresheet* for Ben and David respectively, compared to the same section of the researcher's master *Scoresheet*.

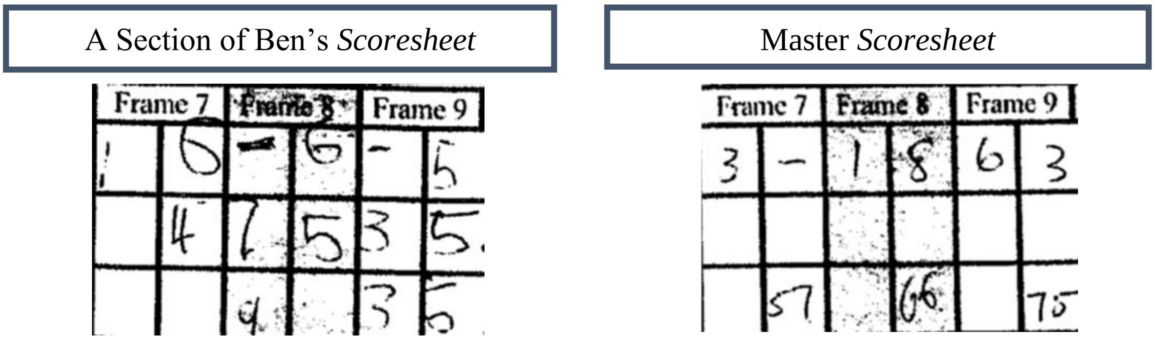


Figure 2. First trial of Design 4 Scoresheet for Ben.

As Figure 2 shows, Ben had difficulties keeping the numerals within each box, and distinguishing the different frames on the *Scoresheet*, even though they had been alternately shaded. His written numerals were quite large and even though the *Scoresheet* covered the top half of the A4 page in landscape (each box was 12 mm x 14 mm), Ben struggled to fit the numerals within the boxes. When offered help by pointing to the correct boxes on the *Scoresheet* for the next score, Ben often rushed ahead to write his scores in, without assistance. In doing so, he had difficulty following the table setup, and as a result, Ben's final *Scoresheet* carried no resemblance to the master score sheet for his game (see Figure 2).

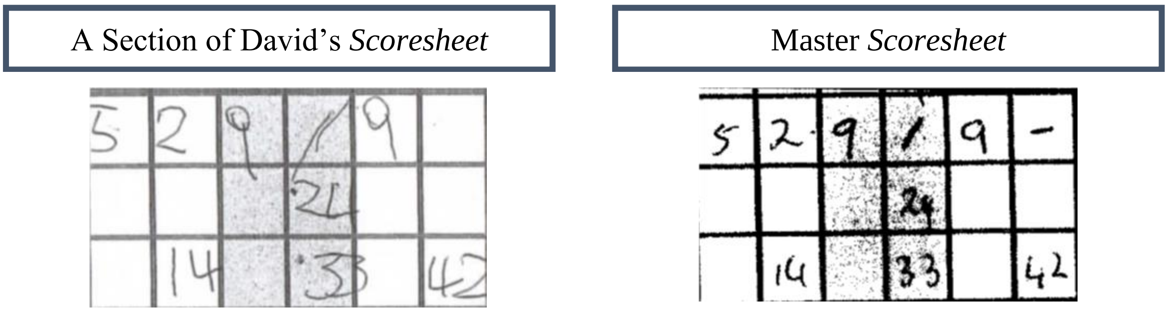


Figure 3. First trial of Design 4 scoresheet for David.

David was hesitant in writing his numerals. He would look at the scoreboard (or *Calculator*), and back at the *Scoresheet*, and then back to the scoreboard repeatedly, before writing the number down. However, when asked what his score was, he could answer immediately. David had difficulty finding where to write each score. It was not sufficient to point to the sheet and say, ‘write your score in here’ and then move away. The researcher had to hover the pen over the correct box and wait for David to check the scoreboard a few times before he wrote the number in the box. If the pen was moved away, David did not know where to write his score. As Figure 3 shows, a dot was put in the empty box to add a further visual support and indicate the correct place. However, this strategy was unsuccessful as it was even more difficult for David to fit his numbers in the square as he would not write the number over the dot. Despite these difficulties, Figure 3 shows that David’s score sheet accurately matched the master *Scoresheet* for this section of the game.

Additionally, Ben and David had difficulty remembering the number of points to add in order to calculate their running total (10). During this observation, David responded with “*seven points*” three times, 10 once and “*I don’t know*” once. When he scored a spare, Ben responded with the first bowl of his spare (observed twice out of two times a spare was scored). On the one occasion Ben scored a strike, he responded, “*I don’t know.*”

The ongoing analysis and reflection of participants’ activities with the designed tool facilitated the subsequent modifications within the DR cycle (Figure 1). The difficulties presented by the participants in using the *Scoresheet* informed specific adjustments that were subsequently trialled to accommodate the needs of these adult learners with ID. The next re-design of the *Scoresheet* (Design 5) included the enlargement and separation of each frame. Additionally, the *Scoresheet* was double sided so only one game was recorded on each side. This allowed for larger boxes to record scores (each box was 18 mm x 14 mm). It was conjectured that this would both support the participants in finding the appropriate box more easily and allow for writing larger numerals. Additionally, further scaffolding was added by including written instruction at the bottom of the *Scoresheet* to add 10 for spares and strikes.

Design 5 of the *Scoresheet* assisted both Ben and David with more accurate recording of the scores. The larger boxes made it easier to keep large numerals within the boxes. Separating each frame made it easier to track frames in the game. Ben required some assistance when recording scores for spares and strikes, but he could copy his scores from the scoreboard independently. The process of separating each frame, made it easier for Ben to independently follow the *Scoresheet*. Both participants stated that Design 5 was much easier to follow. This was the final design of the *Scoresheet* used during Phase 2 of the research.

The design of the scoresheet supported both participants in reaching their goal of being able to calculate their running total when they scored a spare or a strike. While both participants achieved their numeracy goal, the impact of that achievement went beyond the simple ability of knowing their current score in the game. An increase in participation and engagement was observed as the participants progressed in their skills.

During Phase 1, particularly if a number of spares or strikes had been scored in a row, estimating who was currently in the lead was difficult. In such situations, both bowlers were frustrated by not knowing the current score. For example,

1. *Ben*: [Bowls 8 and checks his score] *64, yes!*
2. Bowlers congratulate Ben. One person has bowled two strikes and scoreboard is inaccurate.
3. *Ben: Thanks, but I don’t know what your score is!*
4. The other bowler then has his turn and bowls a third strike.
5. *Ben: Well done buddy! A turkey. Turkey dinner tonight. We still don’t know what your score is, but I think you are winning!*

Transcript 1: Phase 1, Bowling, Observation 1

This excerpt shows the frustration that was evident (see Line 3), particularly when a number of spares or strikes were scored in a row (Line 5). Frustration with not knowing the score was observed 39 times during Phase 1. In those situations, the bowlers could not calculate, or estimate accurately, the current score. During the game discussed in transcript 1, the bowler in question scored four strikes in a row. For 50% of the game, the scoreboard showed incomplete information and bowlers were unsure of the score until the last frame.

Given the significant delay in the scoreboard displaying the score, and the complexity of updating the scores mentally, David and Ben were often unaware of their score and who was winning. Hence, while Ben and David made use of the scoreboard, it was not always sufficient for their purposes. The *scoresheet* was a tool designed to support them to calculate and record their scores, but knowing their scores influenced both interactions with each other and their engagement and participation in the game.

As the participants became independent in calculating their scores, the focus of their conversations changed from discussing the score and guessing who was winning to knowing their place in the game and discussing what was needed to maintain the lead or catch up.

1. *Ben*: [scored 8 (4 and 4)] *I got 4 and 4 and now I have 48* [wrote independently].
2. David scored a strike and the onlookers cheered.
3. *Researcher: Well done!* [To third bowler] *I wonder if David has caught up to you.*
4. *David: Yep, I reckon I have, I'll work it out* [wrote X on *Scoresheet*, got *Calculator*] ….
5. *David:* [puts "+10 =" in *Calculator*] *65! I've nearly caught up to* [bowler] *He is on 71.*
6. *Ben:* [Came to write in score] *I have 49, I am not too far behind.*
7. *David:* [Bowled 8 and wrote score in] *73, Now I am in front!*
8. *Ben: Yep, you are but I am not far behind.*
9. *David*: *49, you have some catching up to do.*
10. *Ben: I might need a strike then!*

Transcript 2: Phase 2, Bowling, Observation 10

Transcript 2 demonstrates the focus of conversations on scores with bowlers now discussing who they *knew* was winning (Lines 5–9) and what they needed to do in their own game to change that (Line 10). This focus is different from the earlier observations (Transcript 1) where conversations often focused on who the players *thought might* be winning.

Discussion

This research demonstrates the usefulness of DR to frame the design of tools to support the achievement of numeracy learning goals for adults with ID. In DR in mathematics education, the design research cycle focusses on the tools, activities and other means that would support students' progress from their current understandings towards a goal, usually predetermined by the curriculum (Cobb et al., 2003). In contrast, when using DR to support the individual numeracy learning of adults with ID, goals are designed for the context of the adult's activity (Gaunt et al., 2019), and the design of tools that supports adults to achieve numeracy goals leads to adults with ID demonstrating greater participation and engagement in the activity or task. This was seen in the changes in the conversations about scoring demonstrated in Transcript 2.

In designing tools to support the adult learner with ID, it is important to design and trial these tools within the context in which the learner requires those tools (Faragher, 2019; Prendergast et al., 2017). The design process (Figure 1) of trialling tools, with ongoing analysis and reflection of the impact and ease of use, is an important factor in the resulting successful development of the tool. While the mathematical aspects of the task are the focus of the initial design of the tool, consideration of the adult learner as a whole is a vital aspect in the design process for a successful outcome. This was seen in the enlargement and separation of the boxes for recording scores on the scoresheet. The design research cycle has much to offer learners with ID and has been shown to be valuable in supporting adults with ID in this study.

For adults with ID, continued learning post school supporting increased participation and engagement in a task or activity can lead to greater independence in, or enjoyment of, that task.

Cuskelly et al. (2021) identified that the benefits of continued learning for adults with ID go beyond simply achieving a learning goal, and include better outcomes in employment health and friendships. While the current study does not provide sufficient evidence to make definitive claims, the increases in engagement and participation indicated by Transcript 2, show a different aspect to the interactions between the bowlers. Their camaraderie, competition and friendship are on display in the transcripts. The increase in participation and engagement demonstrated in this study is an area that warrants further research.

This study has shown that designing specific tools to support numeracy learning in specific contexts may have benefits beyond the achievement of a learning goal. Although access to continued learning post school is still limited for adults with ID, the benefits shown by this research and others (c.f. Schreiber-Barsch et al., 2020) demonstrate the value in continuing to advocate and research more opportunities for adults with ID to promote post-school learning.

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RESEARCH PAPER

A Typology for Instructional Enablers of Mathematical Modelling

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Competency with mathematical modelling is increasingly important for career and informed and engaged participation in personal, civic and work life. In this paper we report on an aspect of a three-year longitudinal study that aimed to identify and describe enablers of mathematical modelling. Teacher interview data has been drawn upon to exemplify key features of a typology for instructional enablers of mathematical modelling. Findings highlight the importance of the didactical contract and socio-mathematical norms in promoting students' mathematical modelling competency, as well as teachers' anticipatory capabilities.

An inability to use mathematics limits an individual's career aspirations, social well-being, and financial security (Paulos, 2000). Competency with mathematical modelling, the use of mathematics to deal with real world problems, is increasingly important for career aspirations (e.g., STEM, economics) and informed and engaged participation in personal, civic, and work life (Geiger et al., 2018; Maass et al., 2019). Recognition of this competency is reflected in the inclusion of mathematical modelling in school mathematics curricula in a growing number of countries, including Australia (Geiger et al., 2021).

Research has provided insight into factors that influence the development of mathematical modelling competency, including: teachers' and students' mathematical and extra-mathematical knowledge (e.g., Blum, 2011); dispositions and beliefs (e.g., Jankvist & Niss, 2019); blockages between stage transitions (e.g., Galbraith & Stillman, 2006); use of digital technologies (e.g., Geiger, 2017); implemented anticipation (e.g., Niss, 2010); and the effective design and implementation of tasks (Geiger et al., 2022). Despite these and other studies, how to best develop students' modelling competency and teachers' instructional competency in relation to modelling remains an unresolved challenge in educational research and practice.

We report on an aspect of a national project that aimed to identify, apply and refine teaching approaches that support secondary students' competency development in mathematical modelling. In this paper, we address a dimension of this aim through a focus on mathematical, cognitive, social and environmental factors that *enable* students to implement the modelling process. Accordingly, we respond to the following research question:

What specific mathematical, cognitive, social and environmental aspects of instruction are conducive for assisting secondary students to implement the mathematical modelling process?

The theory/practice gap identified in this question involved collaboration with teachers and students. Our combined insights have resulted in the generation of a typology that consists of actions and conditions that foster productive activity when working on mathematical modelling tasks – both for learning and instruction. Limited space permits the presentation of findings related to instruction alone. To do so, we first present a concise synthesis of relevant research literature. Second, the methodological approach will be outlined. Third, the outcome of our analysis of data, in the form of an *instructional enablers typology,* will be described. Fourth, we will substantiate key aspects of the typology using illustrative comments from teachers. Finally, selected implications for practice and future research will be discussed.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 226–233. Launceston: MERGA.

The Nature of Mathematical Modelling

The goal of mathematical is to understand or make predictions about real world phenomena, typically to inform decision-making or action. While there are differing perspectives on mathematical modelling in the literature, there is general consensus that the key phases consist of: (1) identifying and specifying a real-world problem; (2) developing a mathematical representation; (3) generating a mathematical solution; (4) interpreting the mathematical solution within the context of the real-world problem; and (5) evaluating validity of the solution relevant to the original context. While each of these stages, is separately important, the goal of instruction is to develop holistic competency with the total process.

Mathematical modelling often requires iterations of this process to improve the model or refine solutions. Thus, it is typically depicted as cyclic in nature. While the representation is cyclic, associated diagrams are analytic reconstructions of the process and not depictions of the routes necessarily taken by actual modellers (Niss & Blum, 2020). In the representation below (Figure 1) (Galbraith, 2013), the heavy clockwise arrows (1 to 7) depict the flow of the modelling process (stages A to G). The double headed arrows indicate that intermediate transitioning/revisiting, within and between stages, are likely as metacognitive reflection both reviews and amends progress to date and anticipates moves yet to be enacted.

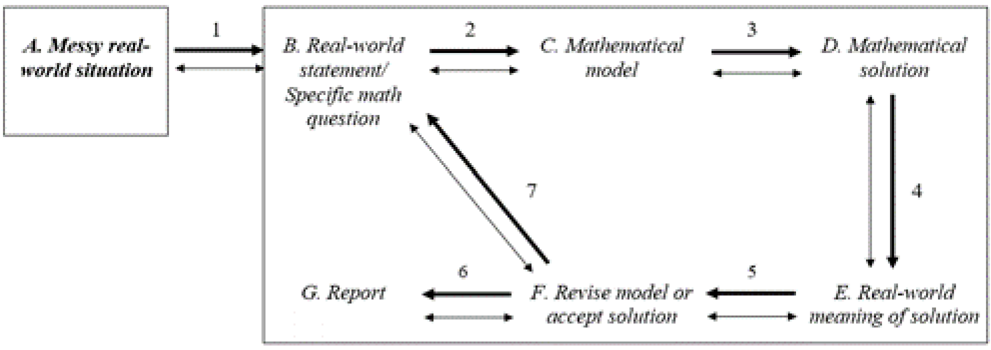

Figure 1. Representation of the modelling cycle (Galbraith, 2013).

Developing Mathematical Modelling Competency

Research into mathematics teacher competency has tended to explore cognitive aspects of performance (Blömeke et al., 2014). There is, however, increasing recognition of the situated nature of teaching, highlighting the affective dimensions of competence (Schoenfeld, 2011). Similarly, research into how teachers assist or inhibit learners' development of mathematical modelling competency has identified: a tendency to intervene and reduce cognitive challenge (de Oliveira & Barbosa 2010); disposition towards guiding students toward pre-determined solutions (Tan & Ang, 2016); ways to diagnose student difficulties as modelling competency develops (Jankvist & Niss, 2019); the importance of task authenticity (Galbraith, 2013); and advantages of openness to multiple solutions (Schukajlow et al., 2015).

Borromeo Ferri and Blum (2010) developed a model for mathematical competency by defining the cognitive demands of task creation, quality instruction and assessment of modelling activity. While providing a multi-dimensional perspective of modelling competency, this model is restricted to cognitive considerations alone. To include other factors that may influence the teachers' approaches to the development of mathematical modelling competency, we drew on Brousseau's (1984) notion of a didactical contract, which positions the actions students take to promote their own learning in the context of teacher expectations.

For example, what kind of activity will teachers expect students to complete and what is reasonable in terms of cognitive demand and time frame. Consistent with this perspective, we further considered the role of socio-mathematical norms (Yackel & Cobb, 1996), which define valued modes of reasoning and ways of working aimed at developing solutions to mathematical tasks. Socio-mathematical norms are seated within a didactical contract, which itself is shaped by these norms. Thus, if teachers are to change the established practices within a classroom, for example, introducing or placing greater emphasis on mathematical modelling, they must renegotiate the existing didactical contract as well as associated mathematical norms.

Further to these perspectives, we also drew on the notion of anticipation (e.g., Niss, 2010) in enabling modelling competency, as this requires the capacity to look forward and backwards, when evaluating progress, in order to make decisions about future action. Niss (2010) coined the term implemented anticipation and outlined three key processes (Table 1).

Table 1
Niss' (2010) Processes of Implemented Anticipation (adapted)

Structuring of an extra-mathematical situation, to prepare it for mathematization, must be focused on features that are anticipated as essential in addressing a problem situation.
Anticipation of mathematical representations that are suitable for capturing a situation must be familiar to the modeller and, ideally, the modeller would have had experience with their use in mathematizing simpler or similar situations.
Anticipation of how the mathematization and resulting model will provide a mathematical solution to the questions posed. Thus, that the outcomes of applying selected mathematical procedures and problem-solving strategies must also be anticipated after mathematization.

While these processes are vital to successful mathematical modelling, our study indicated that *anticipation* was also key for effective instruction as teachers must be capable of looking forward and backwards to evaluate the progress of students on a modelling task. This implies they must have a clear understanding of the task and how it may be solved, be capable of anticipating where students may encounter blockages, including the nature of these blockages, to their progress and have the capacity to make decisions about relevant advice *in situ*.

Research Design and Implementation

We adopted a design-based research approach (Cobb et al., 2003), a methodology suited to applied research that aims to develop contextualised theories of learning and teaching, The goal of such research is to address educational problems situated in a wide range of contexts.

Participants included six teachers and their intact Year 8–11 classes drawn from schools in Queensland (Australia). Teachers were selected purposively (Burns, 2000), volunteering because of their interest in promoting student modelling competency. Their experience with teaching modelling varied from novice to highly experienced. The curriculum context in which teachers practiced required the inclusion of modelling assessment tasks in Years 11 and 12, although relevant documents provided little advice on how to promote students' competency in this area, especially to inexperienced teachers in lower secondary.

The study utilised an iterative process of design-implement-reflect to facilitate researcher/teacher collaboration that informed the identification of instructional enablers, or dis-enablers, of mathematical modelling. This process was operationalised through three whole-day researcher/teacher meetings and two classroom observation visits per year over three years. Classroom visits took place between researcher/teacher meetings. Iterations of these activities (Figure 2) took place in each year of the project (see Figure 2). Researcher/teacher meetings focused on task development and planning for implementation as well as the identification of factors that enabled or dis-enabled mathematical modelling. This facilitated the trialling of tasks and pedagogies informed by identified enablers during classroom visits, leading to the refinement of tasks and elaboration of instructional enablers.

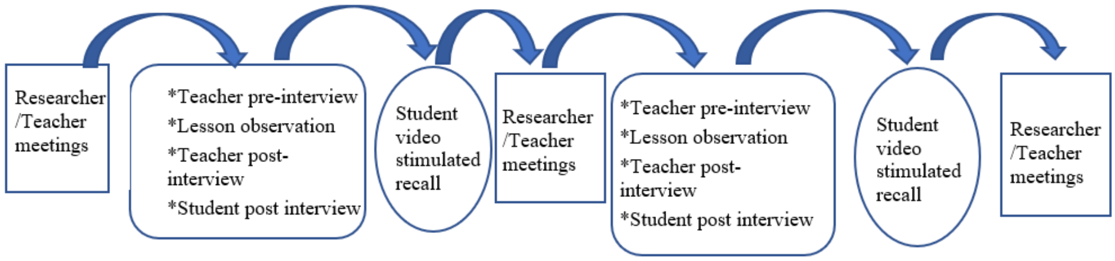

Figure 2. Yearly cycle of researcher/teacher meetings and classroom observation visits.

In this paper, we draw on data from teacher pre- and post-lesson interviews to substantiate elements of the instructional enablers of a mathematical modelling typology. Enablers identified in early stages of the project, were used as the basis for initial coding of data. This process led to the refinement of code definitions and the identification of additional enablers. Further iterations of identification and refinement were continued until definitions stabilised.

In the next section we describe a typology of instructional enablers and illustrate key features via reference to teacher commentary.

A Typology for Enablers of Mathematical Modelling Instruction

A typology for instructional enablers of mathematical modelling is presented in Table 2. Space limitations means that only selective aspects of the typology can be illustrated here.

Table 2
Typology for Enablers of Mathematical Modelling Instruction

Overarching enablers	Generic and specific enablers of mathematical modelling
Impact of learning goals – external/internal/ personal	*Generic enablers* Aligns teaching/learning with broader educational goals (e.g., promoting 21st century skills) *Specific enablers* Links task to formal curriculum/syllabus (e.g., addresses a specific content goal)
Classroom expectations/ways of working	*Generic enablers* Encourages diverse approaches and risk taking Encourages questioning Encourages student collaboration Provides opportunity for reporting findings *Specific enablers* Shapes the physical environment to support modelling Few restrictions on the use of digital technologies
Managing the learning process	*Generic enablers* Manages the learning process in an active manner *Specific enablers* Makes explicit reference to the modelling process Utilises/generates their own resources to support learning in modelling (e.g., representation of modelling process)

	Supports the development of a *mathematical question* Measured Responsiveness -Provides measured guidance in relation to aspects of the task -Provides measured guidance in relation to students' initial ideas -Provides measured in relation to students' proposed directions -Direct Responsiveness -Provides direct clarification of aspects the problem -Provides direct clarification of new terms and ideas
Teacher anticipation	*Generic enablers* Demonstrate knowledge of capabilities/needs/experiences Notice and respond to emerging issues *Specific enablers* Understands and makes explicit reference to the modelling process Identifies possible barriers to solving the problem through reference to the stages of the modelling process

The typology consists of four overarching enablers, each inclusive of both generic enablers relevant to mathematics instruction and enablers specific to mathematical modelling. The overarching enablers are: impact of learning goals–external/internal/ personal; classroom expectations/ways of working; managing the learning process; and teacher anticipation. These overarching enablers are associated with specific actions or expectations teachers employ, or classroom conditions they manage, to enable the development of students' mathematical modelling competency. We now elaborate on each of these over-arching enablers and describe actions/expectations/conditions that researchers/teachers identified as enablers of mathematical modelling activity. In addition, their absence was noted as a dis-enabler.

Findings

Impact of Learning Goals: External/Internal/Personal

Teachers indicated that external factors, for example, curriculum requirements or expectations of approaches employed for instruction in schools influenced the way they implemented modelling. Some teachers saw modelling as aligned with relevant curriculum documents or their own broader educational goals, for instance, 21st century skills.

> I like the modelling. It fits into the Scientific Method and Scientific Process and it's a big thing that we should actually be doing in 7, 8, 9, 10 … The kids are doing all the work. They're coming up with the pathway to solve it and everything … so it ticks all of the boxes for [local curriculum authority].

There were others, however, who were influenced by perceived restrictions related to curriculum requirements or school expectations about instruction, for example, the need to ensure each student acquired a high degree of fluency with set material within limited time.

> We've got a nine-week term plus all the disruptions ... kids are going off on this excursion … or whatever else, photo days. There're just constant disruptions. So, you don't have your kids there …

These are examples of conditions that can act as enablers or dis-enablers of mathematical modelling instruction which can shape the didactical contract and classroom mathematical norms (Brousseau, 1984; Yackel & Cobb, 1996).

Classroom Expectations/Ways of Working

This over-arching enabler is concerned with: what teachers expect students to do; how they work; and valued modes of reasoning and forms of student/student/teacher interaction. Thus, this enabler, consistent with the notions of the didactical contract and classroom mathematical norms (Brousseau, 1984; Yackel & Cobb, 1996), relates to the supportive classroom culture necessary for the development of modelling competency. Specific enablers that were supportive of such a culture included encouraging: diverse approaches to tasks (risk taking); questions/queries; student collaboration within and across groups; and reporting of findings to the whole class. There were instances where teachers supported modelling by shaping the physical environment to support modelling. Few restrictions were placed on the use of digital technologies. Illustrative examples of each of these actions/conditions follow.

In the following instance, a teacher noticed an approach that was different to what they had anticipated. The task had required students to determine the optimal approach to refuelling a car, given petrol stations were at different instances from their starting points and offered different prices. The teacher was comfortable with students solving the task in different ways.

> They were looking at having the most fuel in the car ... So, they were looking at a full tank is better than losing fuel when going to cheap one as it's further away. I didn't think they would come up with that but with these … problems there's a few points of view and that's a valid point to if you're looking at having the most or to have the tank full. So that's another avenue that you'd have to factor into your scheme.

In a different task related to construction, a question was welcomed by the teacher. Importantly, they linked their response to a key aspect of modelling—making assumptions.

> That was an excellent question, so you might need to decide what kind of concrete. And that's part of defining the problem … remember, once we make a decision, we need to include that in our assumptions.

A typical comment related to collaboration as an element of modelling can be seen in the following quote. The caution, however, is a salient reminder that not all collaborative work is productive, and teachers have a key role in guiding students modelling activity.

> Two-heads are better than one. Although you need to be clear what you're talking about. It's easy to get people on the same page when they're looking at the same information.

Students often reported their findings at the end of a lesson or wrote a report in order to receive feedback from the teacher. One teacher provided additional scaffolding to this end by way of a booklet in which they recorded their work in a structured fashion.

> And after we've finished the two lessons, we're going to write a little report ... So, the more you write in the booklet, the easier that report's going to be because you'll just go back to the start of the booklet and start typing what you've actually written … and the report is going to be our assessment.

Teachers also made use of the physical environment of the classroom or other resources. This was highlighted during a lesson in which desks designed to be written on were available. Students had no hesitation in writing notes, diagrams and other prompts that helped them share their ideas with members of their group. It was clear this was a regular occurrence.

> I think the set-up of the room, having particularly some shared information, is really valuable. So, I noticed the groups with the whiteboard tables are quite good because they can write, and they can see what each other are doing. And I think that enhances the collaboration straight away ....

The use of digital technologies was encouraged with few restrictions. Teachers saw such resources as supportive of students' thinking and in mediating collaboration.

> … they had graphs up and they were sharing them so that they could see them on their own computers ... So again, I think that real shared information is important, particularly when working in a group.

Managing the Learning Process

Learning was managed in an active fashion through preparatory and developmental phases. Key to preparatory phase was the identification of an extra-mathematical question that would lead to a mathematical question—a statement that encompassed the goals of students' activity expressed in a mathematical sense. The generation of this question was begun in groups but then opened-up to public scrutiny by peers. This question was the foundation for other aspects of the modelling process, such as the identification of assumptions.

> … so, I'm just going to get you to write them there when they come up in your group … feel free to chat amongst your group … What's the mathematical question we need to answer from this real-life situation?

During the developmental phase (body of a lesson), teachers employed a balance of direct and measured responsiveness to students' requests for assistance. In the case of new terms or ideas within a task, clarification was provided in a direct fashion. For example, one task was concerned with replacing the foundations of a building. An activity with which students were unfamiliar. In this case, teachers provided a direct explanation so that progress was not limited by factors unrelated to the use of mathematics to solve the problem.

By contrast, fostering students' modelling development often requires measured responsiveness (Geiger et al., 2022). This involves providing only enough information for students to make progress without directing them towards a pre-determined solution (Tan & Ang, 2016). Support in this circumstance was in the form of questions aimed at assisting students to clarify their thinking or by referring to stages of the modelling process.

> So, the question is, in a foundation, are there multiple blocks or one big block? So, what do you think? Have you seen foundations before? Is a foundation made of blocks? It's made of concrete. Is it made of concrete blocks or poured concrete? Well, you might need to decide before you do this activity.

While teachers understood the value of this approach, they also admitted that, at times, it was hard to let students lead the work.

> So, I knew once we got to that stage that they would be right. It's very hard to take that step back … I mean, I trust them, and I know how good they are and it's still hard to take that step back.

Teacher Anticipation

To provide effective instruction about mathematical modelling teachers needed to have a clear understanding of the modelling process themselves and ensured a task was worked through before implementation in the classroom. This preparation informed instructional anticipation—providing insight into where students may encounter difficulties, and why.

> … once we were looking for the prices of concrete. Not every student managed to take in all the information I gave them ... So, they didn't look on the OneNote where I provided a couple of links.

Knowledge of the modelling process also informed teachers' advice to students about possible ways forward when an unexpected difficulty or blockage occurred, taking into account students' previous experiences and individual students' current capabilities.

> The ones working with the cycle have a better grasp of where they are in terms of arriving at a reasonable solution. And so, you're seeing, "Here's an initial solution but, actually, I might need to think about these other assumptions that I haven't yet made. How is that going to affect the solution?"

Discussion and Conclusion

In this paper, we have provided insight into some of the important mathematical, cognitive, social and environmental aspects of instruction that enable students' mathematical modelling. Our analysis has identified generic and specific instructional enablers of modelling in the form of a typology. It is important to note that the absence of these enablers represent dis-enabling factors in modelling activity. The findings of this study confirm the results of previous research in relation to instructional factors that promote or constrain productive modelling activity, while providing, at the same time, insight into how teachers can manage the learning processes. In particular, the outcomes of the study indicate that attention to the didactical contract, socio-mathematical norms, and teacher anticipation are key to successful student outcomes in modelling and is thus an important area for future research in the field.

Acknowledgement: This paper is an outcome of DP170101555 (Geiger, V., Stillman, G., Brown, J., Galbraith, P., & Niss, M.) funded by the Australian Research Council

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George Preferred Learning Fraction Concepts with Physical Rather than Virtual Manipulatives

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This case study aims to describe the learning characteristics of a child and evaluate his preferences for using physical manipulatives (PM) and virtual manipulatives (VM) to solve fraction problems. The participant in this study was a fourth-grade child. The child was given similar problems to solve using PM and VM. Data sources were observations and interviews conducted with the child during and after the tasks were completed. The results showed that the child engaged and preferred solving fraction problems using PM more than VM. The child stated that PM helped him quickly understand the relationship between various representations of fractions and model them using manipulatives. He reported the VM did not help him solve the problems.

The use of manipulatives in teaching mathematics is explicitly encouraged in various studies to support students to understand mathematics concepts easily (e.g., Getenet & Callingham, 2021; Golafshani, 2013). These studies reported that students who use manipulatives in their mathematics classes outperform those who do not. They suggested that manipulatives could support students' mathematics learning through a wide range of visual representations, reducing anxiety, increasing engagement, and improving problem-solving skills. Recent studies by Donovan and Alibali (2021) and Basargekar and Lillard (2021), suggested that using perceptually rich manipulatives improved students' problem-solving skills and retention of information. As a result, teachers were encouraged to use manipulatives and technologies to help children understand complex mathematical concepts in the primary context (Reys et al., 2018). There seems to be a consensus that manipulatives help teachers enact effective pedagogy that makes abstract mathematical concepts concrete and relevant to children's lives. Naiser et al. (2003) found that the use of manipulatives is one way that teachers can make the lessons more engaging by creating a concrete experience and providing an effective way for children to represent their thinking. A study conducted in a New Zealand classroom by Getenet and Callingham (2021) showed that manipulatives helped a teacher transform her pedagogical practice by encouraging children to concretely demonstrate fraction concepts.

Technological innovations now allow teachers to use virtual manipulatives (VM) for teaching mathematics in place of physical manipulatives (PM). VM are computer-based versions of physical mathematics manipulatives. These digital substitutions have become popular over the past few years for supporting children's learning of mathematics concepts. There are various studies on VM and PM use in teaching mathematics in general (e.g., Day & Hurrell, 2017; Hunt et al., 2011; Wong, 2010). However, there are limited studies on children's preference between PM and VM to learn a specific mathematics concepts. The purpose of this study was to explore the preference of a year four child, George, in using various PM while engaging in activities such as paper folding, shading paper strips, and manipulating wooden area models or substituted VM for solving fraction problems. The study answers the research question, "How does a child engage with and prefer between VP and PM while learning fractions?" The author reports the observation and interview results from the fourth-year child in the Australian curriculum context. The importance of this paper is its implication for mathematics teachers on the use and preference of using various forms of manipulatives to

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engage and support children to learn fractions. Research on the use of various manipulatives will eventually help to discern the most effective uses of these manipulatives.

Literature Review

As highlighted above, mathematical manipulatives can build foundational knowledge for children to understand various mathematical concepts, which can support children in solving abstract mathematical concepts.

Teaching Fraction Concepts

Fractions are difficult to learn for children and create pedagogical challenges for mathematics teachers (e.g., Hackenberg & Lee, 2015; Siemon et al., 2015). These difficulties are observed across all year levels (e.g., Gupta & Wilkerson, 2015). Different reasons have been identified for these difficulties, particularly in the primary school context, including the complex nature of the concept itself and teachers' pedagogical approaches.

Hackenberg and Lee (2015) showed that limited understanding of particular aspects of the different meanings of fractions affected children's ability to generalise and work with fraction concepts. Similarly, Siemon et al. (2015) indicated that learning fraction concepts were difficult because they were commonly used to represent a relationship between numbers rather than an absolute quantity. Other studies (e.g., Blömeke et al., 2011; Getenet & Callingham, 2021), however, have shown that teachers' professional competencies, including pedagogical knowledge, are essential to the learning and teaching of mathematics concepts. For example, equivalent fractions are introduced in the fourth grade and the subsequent grades in the Australian curriculum (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2018). However, a majority of children do not thoroughly understand equivalent fractions due to ineffective teaching of equivalent fractions (e.g., Reys et al., 2018; Wong, 2010). Studies showed that children could develop the necessary conceptual understandings of fractions with teaching approaches that emphasised many representations—manipulatives, pictorial, real-world and symbolic—over more traditional didactic and procedural approaches (e.g., Bouck et al., 2020, Way, 2011; Wong, 2010). For example, Bouck et al.'s. (2020) study results showed that using virtual manipulatives to teach fractions to third grade increased significantly their test results and acquired knowledge of fraction concepts. This was because the children's understanding of fractions was influenced not only by how their knowledge was structured but also more profoundly, by how the concept was taught and structured by the classroom teachers, reported similarly by Getenet and Callingham (2021).

Using Manipulative for Teaching Mathematics

The use of manipulatives can be traced back to Piaget's (1952) suggestion that children cannot comprehend abstract mathematics through explanations and lectures; therefore, they need models and instruments to grasp the mathematical concepts. Piaget's ideas of using manipulatives are well received in today's mathematics classroom. Teachers are encouraged to start with manipulative materials to teach for understanding, then transfer to representational models like pictures or diagrams, leading and bridging learning to the abstract level of understanding symbols and operation signs (Reys et al., 2018). Getenet and Callingham (2021) suggested that children could be supported to understand the links between ratio and measurement concepts of fractions using manipulatives such as a thin strip of paper. Strip paper can be folded into halves, quarters and so on and later, children can use length partitioning to represent fractions as points on a number line. Jordan et al. (1999) compared the teaching of fraction concepts to fourth-grade children using PM and a traditional textbook approach. The finding showed that children who used manipulatives showed more significant gains in acquiring fraction concepts and skills than children receiving traditional instruction. Similarly, Strom (2009) reported that children who use manipulatives in their mathematics classes outperform those who do not. They suggested that manipulatives could support children's mathematics learning through a wide range of visual representations, reducing their mathematics anxiety and increasing their engagement.

The increased access to computers, software and internet access has brought VM into the majority of classrooms (e.g., Day & Hurrell, 2017; Dewi & Verawati, 2022; Hunt et al., 2011). It is assumed that VM can offer a visual image or a pictorial model, and they can be manipulated like a physical model. Furthermore, they can allow differentiation for the varied ability levels of the learners. Moyer et al. (2002) showed that VM supported children learning to work at their own pace. In addition, they argued that VM were great resources for classroom use because of their unique features to record and store user movements online and their potential for alterations, such as size and colouring.

Similar to their international peers, teachers in Australian schools are encouraged to use manipulatives such as fraction bars and pattern blocks or VM such as fraction circles. These resources can help children develop concepts about fractions (Wong, 2010). Some of the most effective materials, such as paper strips, fraction bars and counters, are readily available and used in most Australian school classrooms. Children can make concrete models (e.g., fraction bars) and then use them to find equivalent fractions. They can be further used to order fractions and connect the concrete device to the symbolic representation (Wong, 2010; Wu, 2013).

However, there are a few discussions and studies on students' preference for using either physical, virtual or both VM and PM and their effectiveness when teaching mathematics in general (e.g., Day & Hurrell, 2017; Hunt et al., 2011; Moyer et al., 2002). Day and Hurrell (2017) showed that VM do not necessarily provide the same experience as concrete materials. They can provide a bridge between the concrete materials and other representations. Additionally, Hunt et al. (201l) and Moyer et al. (2002) recommended VM to record and store users' movements, online and constant availability and their potential for flexible learning. However, a recent study by Dokić et al. (2022) showed that there was no difference in 4th grade (10–11 years old) students' 3D geometry achievement regardless of the learning support through either VM or PM. Few studies have explored children's preferences and experiences using the two forms of manipulatives to learn a specific mathematics concept, such as fractions.

Method

Purpose of the Study

The *Australian Curriculum: Mathematics* (ACARA, 2018) states that children in Year 4 should be confident to solve fraction problems using the concept and skills associated with equivalent fractions in various contexts (ACMNA077). For example, children are expected to explore the relationship between families of fractions (halves, quarters and eighths or thirds and sixths) by folding a series of paper strips. The purpose of this study, therefore, was to evaluate a child's preferences for using the two forms of manipulatives while learning fractions. In this case study, the author made the two forms of manipulatives available to a child, George, to solve fraction problems. The VM were from the National Library of Virtual Manipulatives (NLVM), and PM examples are shown in Figure 1.

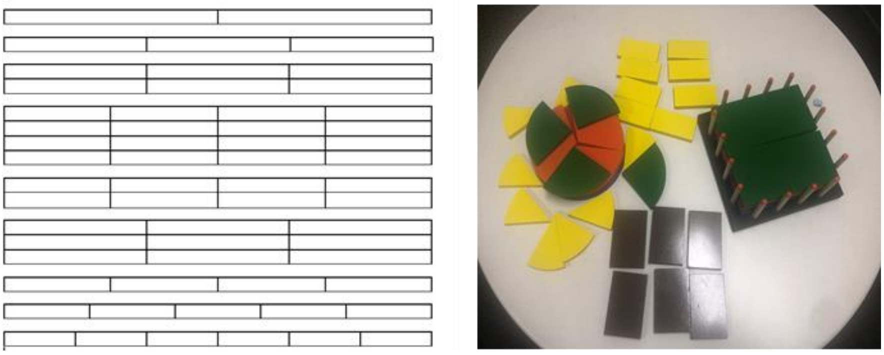

Figure 1: Examples of the physical manipulatives used by the child.

The Procedure of the Study

George was provided with two separate but similar fraction problems to solve using various manipulatives. First, George was asked to represent a list of fractions using various PM and later using a VM (see Figure 2).

Poblem 1. Model the following fractions by

1. Shading the provided paper strip
2. Folding the provided paper strip
3. Using the provided wooden resources
4. Using the virtual manipulative in your computer

The list of fractions

a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ b) $\frac{1}{3}, \frac{1}{5}, \frac{1}{6}$

Poblem 2. First, model the following fractions by

- Shading the provided paper strip
- Folding the provided paper strip
- Using the provided wooden resources
- Using the virtual manipulative in your computer

Second, group equivalent fractions togather

The list of fractions

a) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{4}, \frac{2}{6}, \frac{2}{8}, \frac{4}{8}, \frac{3}{9}, \frac{4}{16}, \frac{3}{8}$

Figure 2. Fraction problems provided to George.

Next, George was provided with a list of fractions and asked to model the fractions using VM. Screen captures of the front page of the VM the child used is shown in Figure 3. The problems further asked George to group the equivalent fractions. Using these VM allowed the child to compare fractions and determine if two fractions are equivalent. The VM George used were sourced from the NLVM website. In addition, George was provided with various forms of PM, such as paper strips and wooden manipulatives (Figure1) to solve the same problems.

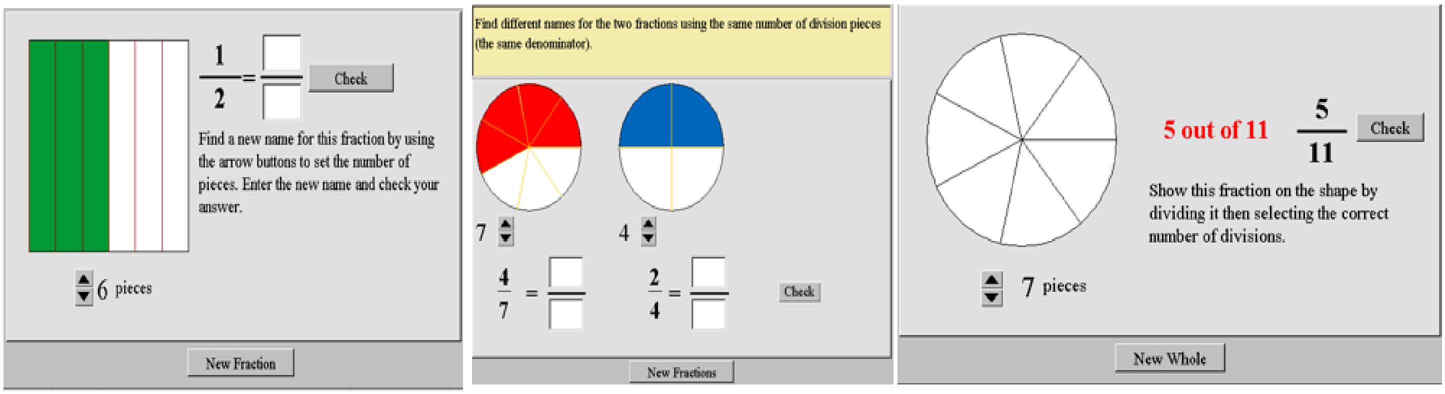

Figure 3. The screen capture of various sections of the NLVM apps.

Data Source and Analysis

The data were collected using interview and observation techniques. The author observed and video recorded while George solved the problems. At the conclusion of each activity, George was asked four questions to reflect on this process, which included, “Which resources do you think helped you to solve the problems” and “Which resources do you like or dislike and why?”

The author transcribed and analysed the interview and observation data (watching the recorded video, which showed George’s actions while using various manipulatives). Consistent with Barron and Engle’s (2007) advice, the analysis emphasised the characteristics of the child’s learning using the manipulatives, such as how the child interacted with the resources and how he worked to make sense of particularly equivalent fractions. The analysis involved viewing the recorded video and interview data by iteratively revising until the transcripts eventually provide a reliable record of what the researcher views as the most relevant aspects of the research question (Barron & Engle, 2007).

Results and Discussions

The results are presented in two sections - the first section describes the child’s experience of using the PM, and the second section presents the child’s experiences of using VM.

Using Physical Manipulatives

As described, George was asked to group sets of fractions using the provided PM—see Figure 1. He used the manipulatives for diverse purposes, as highlighted in Table 1.

Table 1
The PM Used to Solve the Fraction Problems

Manipulatives	Used in the challenge
Folding papers	To show different sizes of fractions
Shading	Compare fractions and group equivalent fractions
Wooden manipulatives	Identify different sizes of fraction and group equivalent fractions

George stated that he believed the PM helped him to be more successful when solving the problems. He used manipulatives to represent various fractions and later identified the equivalent fractions. This was demonstrated in his response to the interview question. “Which resources do you think helped you to solve the problems?”

“Blocks, you can tell equivalent fraction by putting on one on the top of the other.”

Figure 4 shows George demonstrating equivalent fractions putting one on the top of the other (e.g., two halves on the top of the whole).

Figure 4. George demonstrating equivalent fractions.

A similar study by Jordan et al. (1999) showed that manipulatives helped children represent fractions and build a foundational knowledge for various mathematical concepts, which can then lead to understanding abstract mathematical concepts. Before moving to operations with fractions, you must make sure that children have a clear understanding of equivalence (Reys et al., 2018). In addition, the child enjoyed working with folding papers. He said:

"Folding papers, you have to think ways to fold to the right fraction. It gives you more exercise to your brain."

The observation results and George's responses highlighted the importance of teaching and learning using manipulatives to increase student engagement when learning mathematics. This approach is supported by previous studies such as Hunt et al. (2011) and Strom (2009). Hunt et al. (2011) showed that manipulatives could help children learn mathematics through visual representations by increasing engagement. However, George mentioned the wooden area model manipulatives reduced his engagement in solving the problems. This was reflected in one of his responses to the interview questions:

"The fraction is already done for you. Your task is to take it out."

This finding resonates with Wong (2010), who showed that children could develop the necessary conceptual understandings of fractions with teaching approaches that use the right manipulative for each concept. In addition, the literature reports teachers' effective pedagogical approaches were strained in this area (e.g., Blömeke et al., 2011; Getenet & Callingham, 2021).

Using Virtual Manipulatives

George loved and enjoyed being around technologies. He spent holidays playing with computers, iPads and Nintendo switches. However, George inclined more towards using PM than the VM in solving the fraction problems. He had the opportunity to use the VM for various purposes, such as grouping equivalent fractions and shading fractions sizes as part of a whole. During the interview, George mentioned that he had no positive experiences in using the VM. He was not as engaged with VM compared to the PM to solve the problems. His responses from the interview and the author's observation supported this conclusion. He said:

"Using the computer was not fun, it is only answering questions, but folding the paper is more fun and helps me to think."

This result supports the argument made by Day and Hurrell (2017). They showed that VM do not necessarily provide the same experience as concrete materials but still bridge the concrete materials to other representations.

George identified a few advantages and disadvantages of using the two forms of manipulatives (Table 2). He mentioned that one advantage of VM was for checking answers. Similar studies by Hunt et al. (201l) and Moyer et al. (2002) showed that using VM helped teachers to record and store users' online responses and movements, which makes the resources suitable for flexible learning.

Table 2
Likes and Dislikes Reasons for the Two Forms of Manipulative

Manipulatives	Likes reasons	Dislikes reasons
Physical	It helps to solve the fraction problem through thinking. The blocks are great to compare fractions. The blocks are great to find the equivalent fractions. You can do this by putting one on the top of the other. Folding papers is more fun and helps to think.	The blocks do not help to think when identifying fraction sizes. The parts are already made for you.
Virtual	You can check your answer. Different responses to a question.	Some of the questions are confusing and were not fun.

Conclusion

Various studies and literature stated that using a variety of manipulatives in mathematics classrooms is beneficial for children to understand mathematics concepts (e.g., Day & Hurrell, 2017; Reys et al., 2018). However, there have been a few studies on whether children equally prefer VM and PM to learn mathematics (Day & Hurrell, 2017; Dokić et al., 2022). On the one hand, various studies (e.g., Hunt et al., 2011; Moyer et al., 2002) showed VM were innovative and useful ways to enhance mathematics teaching. On the other hand, there is a line of thought that states VM should follow, not precede, the use of concrete manipulatives (e.g., Wu, 2013). In this study, George preferred and was more engaged when solving the fraction problems using PM than when using the VM. This finding highlights the need for further studies on VM use in teaching mathematics. It is worthy determining VM impact on students' learning and understanding of mathematical concepts.

The results of this case study contribute to the body of literature in several ways. First, they replicate previous findings in the positive effect of using manipulative in learning mathematics concepts (e.g., Day & Hurrell, 2017; Hunt et al., 2011). Second, the present findings suggest teachers should be selective when using PM and VM in their classrooms. Teachers should consider children's preferences and the resources pedagogical advantage when using these forms of manipulatives. It appears, in a well-planned teaching setting, both physical and virtual manipulatives can encourage students to make their knowledge explicit and help to build concrete mathematical knowledge. Finally, the study provides insight into using PM over VM to teach fraction concepts and skills. Perhaps some concepts are supported better by one or the other form of the two manipulatives.

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The Role of Technologies to Enhance Pre-service Teachers' Engagement in an Online Mathematics Education Course

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This study reports part of a larger study that explores three technologies—Padlet, video-embedded quizzes and Google docs and their effectiveness for enhancing pre-service teachers' (PSTs) learning engagement in online mathematics education. The data reported in this study are a survey, learning analytics and observation data. We found that Padlet heightened PSTs' social and collaborative engagement, and these dimensions were further enhanced in Google Docs activities. PST's cognitive engagement was enhanced through adding quizzes based on lecture videos. This study contributes to selecting relevant technologies to enhance PSTs' engagement in online learning in general and in mathematics education more specifically.

As learning in higher education is moving more online, this presents a challenge to engage mathematics education students. Various studies showed that technology use in online teaching and learning can improve student engagement (e.g., Attard & Holmes, 2020; Redmond et al., 2018). Accordingly, various interactive technologies are used in online teaching to promote students' engagement and participation in mathematics education. These include online platforms, social media networks, and other digital technologies embedded in university learning management systems. Such technologies can increase students' digital skills, deepen their discipline knowledge and give diverse learners the flexibility to study at their preferred modes of engagement (Lee & Martin, 2020). The current study aimed to improve PSTs' engagement in an online mathematics education course using Padlet, Google docs (GD) and video embedded quizzes. The study used Redmond et al.'s (2018) Online Engagement Framework (OEF) to analyse PSTs engagement in their online learning. This study investigated how embedded technologies enhance PSTs' engagement in an online mathematics education course? Previous studies link technologies to student engagement in general terms; however, this study specifically investigates how PSTs' social, cognitive, behavioural, collaborative, and emotional engagement is enhanced using the selected technologies.

Literature Review

Online Learning, Technologies, and Engagement

Technologies have become central to higher education, affecting all student experience, including engagement. A range of mathematics-specific software are available to provide opportunities for active learning and enhanced student engagement (Attard & Holmes, 2020). As a result, a wide range of technologies have been used in online higher education courses such as Mentimeter, GD, Padlet, and Panapto quiz. In the Australian context, Attard and Holmes (2020) showed that technology used in mathematics education can improve student engagement and increase the number of students wishing to extend their mathematical knowledge. Using a multidimensional view of engagement and the Framework for Engagement with mathematics as a lens, Attard (2018) showed that technologies enhanced students' engagement in learning mathematics. Salvatierra Melgar et al.'s (2021) findings showed that the Mentimeter tool promoted PSTs' engagement in learning mathematics, including

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 242–249. Launceston: MERGA.

enhancing their experience and increasing their mathematical knowledge. Suwantarathip and Wichadee (2014) found students' engagement and scores significantly increased by using GD to collaborate on writing assignments compared to those not using GD. Similarly, Ellis (2015) used Padlet to make lessons more interesting by introducing student-generated content and reducing barriers to students contributing to discussions. Ellis's study found that lessons using Padlet were more engaging (83%), posts by other students enhanced students' experience (79%), and students were more likely to contribute to discussion via Padlet than verbally (42%) (Ellis, 2015). Studies have also shown that video-embedded quizzes may reduce online student dissatisfaction and assist with preparing for assessments (Prince, 2016). However, there are limitations to certain technologies. For example, in an exploration of student use of Padlet, Dianati et al. (2020) reported that students considered it easy to use but unwieldy when overpopulated with content. While most of these studies link the technologies to student engagement in general terms, few studies investigated in depth the specific dimensions of engagement that are enhanced and how this occurs.

Students Online Learning Engagement and Indicators

Students' engagement in the traditional learning mode may be limited to activities where they work independently or in small groups to enhance their cognitive engagement. Recent students' engagement has moved away from examining only students' cognitive processes to more aspects of engagement and how technology currently allows students to engage with learning (Redmond et al., 2018). Garrison's (2011) Community of Inquiry Framework for e-learning has three interrelated types of presence—social, cognitive, and teaching—enhancing students' educational experiences. When these three dimensions inform online course design, students and their educators share a community focused on collaborative learning and thinking (Garrison, 2018). Fredricks et al. (2004) define engagement as a multidimensional construct operating at behavioural, cognitive, and emotional levels for a deeper student relationship with mathematics. However, it has been challenging to measure online engagement, particularly in higher education.

There are a few frameworks that were used to measure student engagement. Bote-Lorenzo and Gomez-Sanchez (2017) identified 16 indicators to measure student engagement in an online course, including the percentage of totally watched lecture videos and assignments submitted. In mathematics education, Fredricks et al. (2004) measured engagement in relation to behavioural, cognitive, and emotional levels resulting in a deeper understanding of mathematics concepts. Redmond et al. (2018), which informed the current study, proposed the OEF for higher education comprising five dimensions (see Table 1). Redmond and colleagues identified several indicators representing each engagement dimension. The authors recommended the framework as an "audit tool or point of reference" (p. 196). This framework informs this study for two reasons: it describes each dimension, and the context of the framework is in higher education.

As shown in Table 1, PSTs create purposeful and trusting relationships with others in social engagement. Cognitive engagement involves "the active process of learning" (p. 191), and behavioural engagement involves "demonstrating positive learning behaviours and attitudes" (p. 193). A collaborative engagement included "the development of different relationships and networks that support learning, including collaboration with peers, instructors, industry, and the educational institution" (p. 194), and emotional engagement "related to feelings or attitudes towards learning" (p. 195).

Table 1
Online Learning Engagement Framework (Redmond et al., 2018, p. 190)

Engagement	Indicators (illustrative only)
Social	Building community, creating a sense of belonging, developing relationships, and establishing trust
Cognitive	Thinking critically, activating metacognition, integrating ideas, justifying decisions, developing deep discipline understandings, and distributing expertise
Behavioural	Developing academic skills, Identifying opportunities and challenges, developing multidisciplinary skills, developing agency, upholding online learning norms, supporting, and encouraging peers
Collaborative	Learning with peers, relating to faculty members, connecting to institutional opportunities, and developing professional networks
Emotional	Managing expectations, articulating assumptions, recognising motivations, and committing to learning

Methodology

This study is part of a larger study conducted at a School of Education and a Pathway College at a regional university in Australia across four courses. The School of Education prepares early childhood, primary and secondary school teachers. The Pathway College provides alternative entry options to enter university. This study focused on a primary program mathematics education course. The course has been designed to provide PSTs with various pedagogical and content knowledge understandings to teach mathematics in the primary school context. Zoom was used to share Padlet and GD in breakout rooms and share screens. Padlet activities were mostly reflections upon lectures and gathering PSTs' feedback and ideas anonymously about tutorial topics. GD was used to create problem solving activities using Google Sheets. The GD and Padlet were also made available asynchronously to involve PSTs who did not attend the live sessions. The live sessions were attended by 8 to 20 PSTs. Panopto quizzes were used in five lecture videos. One quiz in each video was located either at the beginning, middle or end of the lecture videos. The quizzes consisted of three or more questions formatted as multiple-choice or fill in the blank as part of the lecture viewing experience.

Data Sources and Participants

The data were collected from PSTs enrolled in the primary program mathematics education course. The data were collected using a survey, observation and web analytics. The survey comprised of a series of PSTs' experiences using Padlet, GD and video embedded quizzes. The survey was administered online through Google forms with 5-point Likert scale questions ranging from "5 = Strongly Agree" to "1 = Strongly Disagree." The questions were adapted from Redmond et al. (2018) OEF indicators except for the emotional dimension. For example, the question, "Padlet helped me think critically" was included to understand PSTs' level of agreement on their cognitive engagement while using Padlet (see Table 1). The authors believe that the emotional dimension is difficult to capture using a survey and focused on the observation data to capture this dimension. Furthermore, demographic information, including gender, age, and mode of study, was collected. The survey was distributed to 90 PSTs; however, this study reports on the preliminary data from 12 PSTs responded to the survey. All the survey participants were female (n =12), and most of the participants were studying off-campus (n = 7) and full-time (n = 8).

All live online video sessions were recorded while PSTs were using GD and Padlet. The course lecturer conducted the observations as part of normal teaching duties; however, an observation checklist was used to evaluate PSTs' engagement. The checklist was adapted from our survey questions (yes or no), followed by descriptive examples. A total of four video recordings with durations of 10 to 30 minutes while using these technologies were analysed. Each of the two recordings integrated either Padlet or GD. The technology, the PSTs who attended the live session (*n*), description of the topics, and activities are described in Table 2. Some of the live session participants could be different from survey participants.

Table 2
Technologies, Number of PSTs Involved in the Live Session (n) and Topics Taught

Video	Technology	PSTs(*n*)	Topic	Activity description
1	Padlet	20	Numeration system	Reflect on various numeration system and their experiences at schools
2	Padlet	12	Teachers' knowledge for teaching mathematics with technology	Reflect and comment on various forms of teachers' knowledge for teaching mathematics
3	GD	15	Learning mathematics with technology	Identify sum, mean, and generate graphs to identify the best technology for teaching a specific mathematics concept
4	GD	8	Problem based learning for effective mathematics teaching	Solve problems through posting pictures, generating graphs and calculations from the provided data

Panopto videos were used to embed the quizzes in the course lecture video and were available to all PSTs (n = 90). Panopto is a media tool with interactive features such as embedded quizzes and learning analytics. It allows one to see who has taken the quiz and their results and quiz scores. The PSTs who participated in the embedded quizzes and Padlet were identified from the Panopto and Padlet analytics.

Analysis

The data collected using the survey were summarised using descriptive statistics, including numerical data showing the strength of participants' responses to the survey items. The observation checklist rated PSTs' engagement by watching video-recorded lessons and making notes in the space provided in the observation checklist emphasising those parts of the lessons relevant to the research question. Consistent with the advice of Barron and Engle (2007), the analysis emphasised aspects of how the PSTs use the technologies to enhance their engagement. A deductive quantitative count was conducted during the analysis to describe the PSTs' engagement and calculate the frequencies of the occurrences of each engagement dimension.

Results and Discussion

Survey and Analytic Data

The PSTs' perception of the use of technology for teaching is presented in Table 3. Table 3 shows that PSTs tended to agree, for example, with being confident to use technology in learning (M = 4.25) and strongly agreed that learning with technology will influence how they teach with technology in the future (*M* = 4.58). However, they were not sure about the importance of learning with technology (*M* = 3.00).

Table 3
Pre-service Teacher Perception of Technology Use in Teaching

Items	Mean (*n* = 12)
I am confident using technology for learning at the University	4.25
I am confident teaching with technology	3.75
Learning with technology is important to me	3.00
Learning with technology will influence how I teach with technology in the future	4.58
The university courses offer good opportunities for learning with relevant technologies	4.00

Strongly agree = 5, Agree = 4 Neutral = 3, Disagree = 2, Strongly disagree = 1

PSTs have shown various levels of agreement on using the technologies to facilitate the various domains of engagement in studying the mathematics education course (see Table 4).

Table 4
Pre-service Teachers Mean Agreement on Technologies Engagement (n = 12)

Elements	Indicators	GD	Padlet	Panopto quiz
Cognitive	Think critically	3.25	3.50	3.50
	Develop deep discipline understandings	3.25	3.50	3.58
	Use expertise gained from other courses	3.25	3.50	3.42
Behavorial	Develop academic skills	3.00	3.50	3.67
	Develop agency	3.50	3.50	3.50
	Understand online learning norms	3.42	3.67	3.50
Collaborative	Engage with lecturers or tutors	3.50	3.67	3.33
	Connect to opportunities at the university	3.33	3.33	3.33
	Develop professional networks	3.00	3.25	3.25
Social	Create sense of belonging	3.25	3.25	3.25
	Develop relationship with others	3.33	3.50	3.25
	Develop a sense of community among others	3.33	3.42	3.33

Strongly agree = 5, Agree = 4 Neutral = 3, Disagree = 2, Strongly disagree = 1

The PSTs were unsure that GD enhanced their cognitive engagement (e.g., thinking critically). However, they agreed on the importance of Padlet and Panopto quizzes increasing their cognitive engagement, such as developing deep mathematics content understandings which echoed Salvatierra Melgar et al.'s (2021) suggestions of using technologies to enhance PSTs' mathematical knowledge. The PSTs agreed that Padlet and Panopto quizzes facilitated their behavioural engagement more than GD. However, PSTs equally valued the importance of GD, Padlet and Panopto quizzes to enhance their collaborative engagement. Similar to the findings of Ellis (2015), Padlet (e.g., developing a relationship with others) and GD (e.g., developing a sense of community) facilitated PSTs' social engagement.

The results presented in the following section are derived from the Panopto and Padlet analytics. Table 5 reports PSTs' engagement report from the Panopto quizzes. There was a noticeable difference between the number of PSTs accessing quizzes in relation to their placement within the video. The low percentage of PSTs accessing quizzes was shown either in the middle or end (12.2–22.2%). The PSTs tended to attempt the quizzes when they were located at either the beginning or middle of a video. The PSTs were less likely to attempt the quizzes when located at the end of a video.

Table 5
Primary Course Panopto Quizzes Engagement Pattern (n = 90)

Video	Location of quiz in video	Type of quiz	# of questions	Video length	# PSTs accessed
1	Beginning	Mixed	4	4:00	38 (42.22%)
2	Middle	Multiple	3	3:00	20 (22.22%)
3	Middle	Multiple	5	3:00	19 (21.11%)
4	End	Multiple	7	3:00	9 (10.00%)
5	End	Mixed	6	2:00	11 (12.22%)

Padlet provides limited information on its analytics system; however, the number of posts, comments and contributors were accessible. There were 123 posts and 20 comments from 50 PSTs contributors, which might indicate high involvement of the PSTs.

Observation Data

The recorded sessions, which integrated with Padlet and GD, were analysed, and the results are reported in Table 6.

Table 6
Pre-service Teachers Engagement in Padlet and GD on Observed Frequencies

Elements	Indicators	GD (*n*)	Padlet (*n*)	Descriptive example
Cognitive	Think critically	11	5	*GD:* PSTs discussed critical ideas while interpreting the data provided. *Padlet:* To craft responses to questions
	Develop deep discipline understandings	13	6	*GD*: Raised and answered questions from data *Padlet*: Represent a number using a different number system
	Use expertise gained from other courses	7	3	*GD and Padlet*: Drew upon previous mathematics courses to answer questions
Behavorial	Develop academic skills	12	4	*GD*: Learned calculating mean, Standard Deviations etc. in Google Sheets
	Develop agency	4	5	*GD:* Supported each other while using formulas in Excel Padlet: Supported the other PSTs on how to embed video in Padlet
	Understand online learning norms	7	5	*GD and Padlet*: Tracked while reflecting after the live sessions
Collaborative	Engage with lecturers or tutors	10	8	*Padlet*: Answered questions for the lecturer in text form
	Connect to opportunities at the university	0	0	
	Develop professional networks	7	8	*GD and Padlet:* PSTs shared links

Social	Create sense of belonging	4	7	*GD and Padlet:* PSTs managed activities while working in groups
	Develop relationship with others	8	4	*GD*: worked in pairs to answer questions *Padlet:* Commented on other PSTs responses
	Develop sense of community	8	6	*GD and Padlet*: worked in groups and created a community for further discussion
	Managing expectations	2	3	*GD and Padlet*: Commented on the expectations to the activities
	Articulating assumptions	2	9	*GD and Padlet:* supported each other to explain assumptions
Emotional	Recognising motivations	1	5	*GD and Padlet:* PSTs appeared motivated to calculate on Google Sheets and comment in the Padlet
	Committing to learning	4	7	*GD and Padlet*: Carefully finished the activities on time

The observational data showed that both Padlet and GD engaged the PSTs across all five dimensions of Redmond et al.'s (2018) framework to different degrees. When PSTs were using Padlet there were instances of engagement across all dimensions but mainly on the emotional dimension (n = 23), such as PSTs articulating their assumptions about what they were learning (n = 9) and the motivation about why they were learning those topics (n = 5). GD better supported PSTs to engage cognitively (n = 31) and behaviourally (n = 23). Similar to the findings of other studies (e.g., Lee & Martin, 2020; Salvatierra Melgar et al., 2021), the cognitive engagement included developing deep discipline understandings (n = 13) and thinking critically (n = 11). Examples of social engagement promoted by GD included developing a sense of community among others (n = 8) and developing relationships with others (n = 8). However, the emotional dimension was least observed while using GD (n = 9).

Conclusion

The research question guiding this study was:

> *How do embedded technologies enhance PSTs' engagement in an online mathematics education course?*

The study used Redmond et al's. (2018) OEF to analyse the data. The PSTs were engaged in various domains of engagement, particularly the dimensions beyond cognitive and behavioural, which are additional dimensions to the previous studies (e.g., Fredricks et al., 2004; Garrison, 2011). The technologies showed minimal support for PSTs' emotional engagement; however, they enhanced their cognitive, social, and collaborative engagement. As evidenced by the survey and observation results, the role of each technology on PSTs engagement is shown in Table 7.

Padlet was beneficial to PSTs who wished to contribute anonymously to the live sessions. The observation results further showed that GD, combined with Zoom breakout rooms, works well for focused group activities. The quizzes embedded within lectures supported seamless formative feedback and influenced PSTs' cognitive and behavioural engagement. There was a strong inference that Padlet technology encourages emotional engagement. Its flexible layout allows PSTs to participate via text, audio, and images.

Table 7
Padlet, GD and Video Quiz and Summary of their Contribution to Engagement

Element	Padlet		Video quiz		GD	
	Survey	Observation	Survey	Observation	Survey	Observation
Cognitive	Agree	31	Agree	N/A	Neutral	14
Behavioural	Agree	23	Agree	N/A	Neutral	14
Collaborative	Neutral	17	Neutral	N/A	Neutral	16
Social	Neutral	20	Neutral	N/A	Neutral	17
Emotional	N/A	9	N/A	N/A	N/A	24

N/A – observation or survey was not conducted; numbers indicate frequencies observed

The PSTs valued the importance of GD for cognitive engagement, including thinking critically, developing deep mathematics content understanding and developing academic skills. In addition, using GD provided strong support for social and collaborative engagement. However, the emotional dimension was the least observed while using GD due to the nature of the activities, which focused PSTs on collaborative problem-solving and discussion. The results showed that using the technologies in teaching can enhance PSTs engagement and improve their use of various technologies for their future profession. In addition, it showed the importance of teacher educators understanding and identifying the types of technologies most suitable to enhance each engagement dimension. The small sample and the few selected technologies, however, limit the generalisability of the study. In addition, the specific topic being taught might have influenced the applicability of the findings. Future research could benefit from more robust samples and studying specific mathematics contents.

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RESEARCH PAPER

Developing Proficiency with Teaching Algebra in Teacher Working Groups: Understanding the Needs

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This study reports on a professional learning (PL) initiative aimed at establishing a community of practice, through teacher working groups in which teachers can explore and develop their algebraic pedagogical content knowledge (PCK). Here we report on teachers' solutions to three differently represented algebraic problems and explore what the nature of their solutions tells us about their algebraic reasoning and their PCK. The findings showed that most participants favoured only one solution and provided useful insights for the value of teacher working groups in PL activities to develop teachers' algebraic reasoning, understanding, and extend their range of problem-solving strategies.

The present study was developed from a previous professional learning (PL) initiative led by the first author, with a cross-disciplinary team of tertiary academics from mathematics, science, and mathematics and science education. The team established a Peer Learning Circle (PLC), funded by the *University of Tasmania Community of Practice Initiative Program*, focusing on using self-generated external representations in the teaching of mathematics and science. The success of that PLC community of practice suggested a collegial and theoretically grounded means of exploring mathematical concepts needed in teaching and highlighted the potential of PLCs for PL in both schools and higher education (Hatisaru et al., 2020). The current study expands the focus of the mentioned PLC towards developing secondary school teachers' proficiency with algebra teaching (Years 7 to 10) within a community of practice. The algebra focus stems from a research agenda to study the teaching and learning of algebra, in respect of teachers' pedagogical content knowledge (PCK; see, e.g., Ball et al., 2008; Chick & Beswick, 2018).

Algebra learning plays an important role for students in college level studies (e.g., McCallum et al., 2010). Students' algebra learning outcomes are, nevertheless, sometimes poor in both national and international assessments. For instance, in Australia, only 15% of Year 9 Victorian students gave the correct answer to what is regarded as an appropriate-level question: $2 \times (2x - 3) + 2 + ? = 7x - 4$ (Sullivan, 2011). Research studies show that students' algebra learning outcomes can be enhanced through effective forms of instruction that attend to algebraic proficiency, but also suggest that teachers need to be supported in developing such effective instructional practices (e.g., Star et al., 2015).

We aimed to establish a teacher working group in which participant teachers could solve and discuss algebraic problems with an emphasis on student thinking, develop a deeper understanding of algebraic processes and solution strategies, and allow us to examine the effectiveness of teacher working groups as a PL approach. We envisaged regular virtual meetings in which group members would be sent an algebra problem to solve themselves first, and then asked to anticipate how students might solve it, with the solutions used to guide the substance and direction of the following discussions. A workshop held at the *2021 Annual Conference of the Mathematical Association of Tasmania* (*MAT*) (hereafter referred to as the workshop) provided an opportunity to introduce the teacher working group study and start to build a comprehensive understanding of the needs of teachers in algebra. Here, we analyse and

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 250–257. Launceston: MERGA.

report the workshop participants' solutions to three algebraic problems and provide some reflections on the possible needs of teachers in teaching algebra.

Types of Algebraic Activity and the Workshop Problems

Effective teaching practices can inspire and develop mathematics learning. Kilpatrick et al. (2001) identify five intertwined strands to achieve what they propose constitutes mathematical proficiency: conceptual understanding (comprehension of mathematical concepts and operations); procedural fluency (skill in carrying out procedures accurately and efficiently); strategic competence (ability to formulate and solve mathematical problems); adaptive reasoning (capacity for logical thought, reflection, and justification); and productive dispositions (seeing mathematics as sensible, useful, and worthwhile). Corresponding to the first four of these elements, the *Australian Curriculum: Mathematics* (*AC: Mathematics*) targets four desirable proficiencies for students as outcomes of studying mathematics: understanding, fluency, problem solving, and reasoning (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2019). The *AC: Mathematics* aims to ensure that students develop these proficiencies within all content domains, including algebra.

Algebra is commonly accepted as an activity (Kieran, 2007) in which two aspects have been distinguished: "(a) algebra as a systematic way of expressing generality and abstraction; and (b) algebra as syntactically guided transformations of symbols" (Kilpatrick et al., 2001, p. 256). According to Kilpatrick et al. these two main aspects of algebra have led to various activities in school algebra, including representational activities, transformational or rule-based activities, and generalizing and justifying activities. This classification by Kilpatrick et al. (2001) is echoed in Kieran's (2007) GTG model, where the activities of school algebra are grouped into three aspects: Generational, Transformational, and Global/meta-level. If teachers possess a better understanding of these algebraic activities, it can benefit both the students and the teachers in creating a better classroom environment to learn. Supporting teachers to meet curricula expectations can sometimes be difficult, however, because of the breadth and complexity of required teacher learning (Kazemi & Franke, 2004). Our workshop aimed to guide teachers to assist their students in formulating, representing, and solving algebraic problems. We provided three problems, according to the algebraic activities of representing, transforming, and generalising and justifying. Each of these is introduced in the relevant sections that follow (Problems #1–3).

Representational Activities of Algebra

The representational activities of algebra involve translating verbal statements into symbolic expressions and equations. Generally, they include "generating (a) equations that represent quantitative problem situations in which one or more of the quantities are unknown, (b) functions describing geometric patterns or numerical sequences, and (c) expressions of the rules governing numerical relationships" (Kilpatrick et al., 2001, pp. 256-257). Facility with representational activities requires both conceptual understanding of mathematical concepts, and ideas stated verbally, and strategic competence to represent statements in algebraic expressions and equations (Kilpatrick et al., 2001).

Problem #1 is an example of a worded problem where there are two unknown quantities: the number of cows (say x) and the number of chickens (say y). The problem may be solved algebraically by generating two equations representing the problem situation: $x + y = 19$ (number of animals) and $4x + 2y = 62$ (number of legs). The two equations can then be solved simultaneously: four times the first equation is $4x + 4y = 76$, and the difference then obtained by subtracting the second equation is $2y = 14$. Therefore, the number of chickens is $y = 7$. As $x = 19 - y$, the number of cows is $x = 12$. There are, of course, other ways of solving the problem, for example using a table, drawing a pictorial model, using a graph (see Tripathi, 2008), or guessing and checking.

> Problem #1: A farmer had 19 animals on his farm - some chickens and some cows. He also knew that there was a total of 62 legs on the animals on the farm. How many of each kind of animal did he have? (Tripathi, 2008)

Transformational Activities of Algebra

Transformational or rule-based activities include collecting like terms, factoring, expanding, substituting, simplifying expressions, and solving equations. In transformational activities, the rules for manipulating algebraic symbols are mainly used to change the form of an expression or equation to an equivalent one (Kilpatrick et al., 2001). In other words, the majority of these types of activities are concerned with changing the symbolic form of an expression or equation in order to maintain equivalence (Kieran, 2007) (e.g., see Figure 1).

Problem #2 (Star & Seifert, 2006)

Find a, if $0.3a + 0.2 = 1.1$

Solution 1:

$$\begin{gathered} 0.3a + 0.2 = 1.1 \\ 0.3a = 1.1 - 0.2 \\ \frac{0.3a}{0.3} = \frac{0.9}{0.3} \\ a = 3 \end{gathered}$$

Solution 2:

$$\begin{gathered} 0.3a + 0.2 = 0.9 + 0.2 \\ 0.3a = 0.9 \\ 0.3a = 0.3 \times 3 \\ a = 3 \end{gathered}$$

Solution 3:

$$\begin{gathered} 0.1(3a + 2) = 0.1 \times 11 \\ 3a + 2 = 11 \\ 3a + 2 = 9 + 2 \\ 3a = 9 \\ a = 3 \end{gathered}$$

Solution 4:

$$\begin{gathered} \frac{0.3a}{0.1} + \frac{0.2}{0.1} = \frac{1.1}{0.1} \\ 3a + 2 = 11 \\ 3a = 9 \\ a = 3 \end{gathered}$$

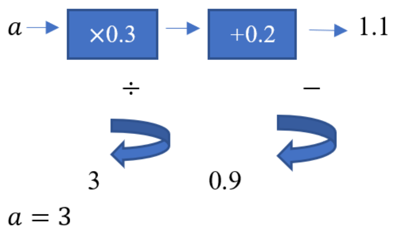
Solution 5a:

$$\begin{gathered} 10 \times (0.3a + 0.2) = 10 \times 1.1 \\ 3a + 2 = 11 \\ \Delta + 2 = 11 \\ \Delta = 9 \\ a = 3 \end{gathered}$$

Solution 6a:

$$\begin{gathered} \frac{3}{10}a + \frac{2}{10} = \frac{11}{10} \\ \frac{3a + 2}{10} = \frac{11}{10} \\ 3a + 2 = 11 \\ 3a = 9 \\ a = 3 \end{gathered}$$

Solution 7:



Solution 5b:

$$\begin{gathered} 10 \times (0.3a + 0.2) = 10 \times 1.1 \\ 3a + 2 = 11 \\ 3a = 9 \\ a = 3 \end{gathered}$$

Solution 6b:

$$\begin{gathered} \frac{3}{10}a + \frac{2}{10} = \frac{11}{10} \\ 3a + 2 = 11 \\ 3a = 9 \\ a = 3 \end{gathered}$$

Figure 1. Possible solutions to Problem #2 (reproduced from Hatisaru, 2021).

Charles (2005) suggests the idea of equivalence is one of the *big ideas* in mathematics. Examples of mathematical understanding with algebraic expressions and equations include:

> Algebraic expressions can be named in an infinite number of different but equivalent ways. For example: $2(x - 12) = 2x - 24 = 2x - (28 - 4)$.
>
> A given equation can be represented in an infinite number of different ways that have the same solution. For instance, $3x - 5 = 16$ and $3x = 21$ are equivalent equations; they have the same solution, 7. (Charles, 2005, p. 14).

In order to solve a given equation, the problem solver must engage with equivalent representations of the given expression or equation. Consider the linear equation below:

Problem #2: Find a, if $0.3a + 0.2 = 1.1$ (see Figure 1).

Figure 1 presents a number of different solutions to this equation, where the terms in the equation, and accordingly the equation itself, are represented by its various equivalents. The numerical expression 1.1, for example, is represented as $0.9 + 0.2$ in Solution 2, as 0.1×11 in Solution 3, as $1.1/0.1$ in Solution 4, as 10×1.1 in Solution 5 and as $11/10$ in Solution 6. Similarly, the algebraic expression $0.3a$ is represented with its equivalent forms including $0.1 \times 3a$ (Solution 3) and $3a/10$ (Solution 6). Naming these equivalents yields various solutions, each of which include internal mathematical connections (Hatisaru, 2021).

Facility with transformational activities in algebra is important (McCallum et al., 2010). In these activities, aspects of conceptual understanding and strategic competence interact with each other along with procedural fluency (Kilpatrick et al., 2001). Although there might be a temptation to equate representational activities with the conceptual aspects of algebra and transformational activities with skill-based aspects of algebra, conceptual work and meaning building occur within both types of activities of algebra (Kieran, 2007).

Generalising and Justifying Activities of Algebra

Generalising and justifying activities include problem solving, modelling, justifying, proving, and predicting (Kilpatrick et al., 2001). Although they often use the language and tools of algebra, they are not exclusive to algebra (Kieran, 2007). These activities usually involve examining and interpreting representations that have already been generated or manipulated, and they can generate answers to particular questions or conjectures. In these activities, all aspects of mathematical proficiency come together, but especially adaptive reasoning (Kilpatrick et al., 2001). Problem #3 below illustrates how algebra is used to generalise and justify (see Table 1).

> Problem #3: If you are given the sum and difference of any two numbers, show that you can always find out what the numbers are. (Harper, 1987, as cited in Kieran, 1992)

Table 1
Solution Methods to Problem #3 (adapted from Kieran, 2007)

	Examples
Rhetorical method	Divide the sum by 2, then divide the difference by 2. To get the first number, add the sum divided by 2 to the difference divided by 2. To get the second number, take the difference divided by 2 away from the sum divided by 2.
Diophantine method	Given x is the first number, and y is the second number, assume that $x–y = 2$ and $x + y = 8$. x and y can be found by solving these two equations for x and y, and it is clear this can be applied for any numbers.
Vietan method	Assume the numbers are x and y. m: the sum of x and y. Then, $m = x + y$ n: difference of x and y. Then, $n = x – y$ Add together: $m + n = 2x$ Find x and substitute back for y. That is, $x = (m + n)/2$ and $y = (m – n)/2$

Harper (1987, as cited in Kieran, 2007) used Problem #3 to investigate the stages that algebra students pass through in their development of algebraic symbolism. According to Kieran, Harper interviewed 144 secondary school students and found evidence of three types of solutions identified as being used historically in solving such generalisation questions: the *Rhetorical method*, the *Diophantine method*, and the *Vietan method* (Table 1). It is notable that, while in the *Diophantine method* letters are used to represent unknowns, in the *Vietan method* letters are used for both unknown and given quantities. In the *Rhetorical method*, algebraic symbolism is not used but a procedure that is general is specified (Kieran, 2007).

Teachers' Solutions to the Workshop Problems

As mentioned earlier, the workshop aimed to guide teachers to assist their students in solving algebraic problems representing the three types of algebraic activities presented (GTG, Kieran, 2007). Participants were also asked to what extent they had opportunities to discuss such problems in a PL capacity in their schools, and if so, what they thought about the value of considering such aspects or approaches. At the commencement of the workshop, participants were given a problem sheet and 20–25 minutes to complete it. The sheet included the three problems sketched above and a prompting statement:

> *Find and explain as many different possible solutions to each of the problems as you can. Name the solutions as Solution A, Solution B, Solution C and so on.*

Participants were also given the opportunity to respond the questions below.

> *Assume you are teaching these problems in the class. For each problem identify: (a) the solutions that you would use to solve and why; (b) the solutions that your students might use; and (c) the solutions you hope your students would use.*

Once completed, all participants attached their solutions to the problems (see Table 2 and next page, more details to follow) to a wall in the workshop room and were given time to peruse each other's solutions. This provided participants time to reflect on the variation in solutions of each problem within the group, and upon themselves as teachers of mathematics. We did not formally record the discussions at this point, but there were some rich interactions, which focused predominantly on the solutions which were different to their own. Many participants commented that they had limited time for such in-depth exploration of problems in their usual school-based PL. Nine participant teachers gave their consent for the research, and they were assigned codes P1, P2, P3, etc. to protect their anonymity.

Table 2
Teachers' Solution Strategies to the Workshop Problems

Problem	Solution Strategies	Participants (correct, incomplete, incorrect)
Problem #1	Use a table	P2, P5
	Use simultaneous equations	P1, P4, P5, P7, P8, P9
	Guess and check	P1, P3, P6, P9
	Pictorial	P5
Problem #2	Only one solution	P2, P3, P4, P5, P6, P7, P9
	Two solutions	P1, P8
Problem #3	Using numerical examples	P9
	Diophantine method	P2, P3, P8
	Vietan method	P1, P4, P5, P6, P7

We examined participants' responses according to the solutions presented in the previous section, and also recorded any additional solutions that emerged in the data (P9's solution in Table 3). Table 2 presents the results based on our assessments. We received a total of thirty-three solutions to consider and assess (a few participants provided more than one solution). While most responses included evidence of the types of solutions presented above, a few responses were incomplete, and a few were incorrect. We used traffic light colours to represent them, yellow indicating incomplete, and red indicating incorrect solutions.

In general, the participants solved Problem #1 by using simultaneous equations, and to a lesser extent by the use of a guess and check method (e.g., Figure 2). P1 provided two and P5 provided three different methods to solve the problem, while P4, P6, P7, and P8 gave only one solution. P2's solution was incorrect, and P3 and P9 had incomplete solutions.

11 cows = 44 legs.
18 legs leftover = 9 chickens. $11+9=20$ ✗

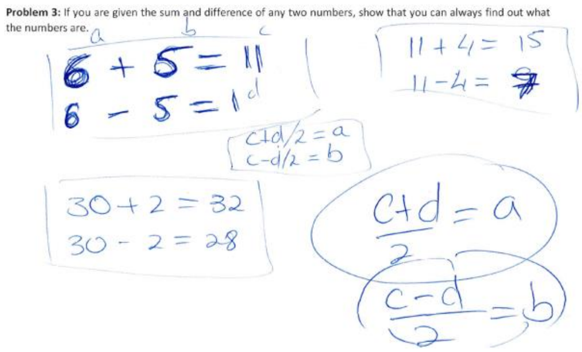
12 cows = 48 legs.
14 legs leftover = 7 chickens. ~~tt~~ $12+7=19$ ✓

Figure 2. P6's solution to Problem #1.

In responding to Problem #2, seven participants gave only one solution, and their solutions refer to either Solution 1 (five occurrences) or Solution 5 (two occurrences) presented in Figure 1. P1 and P8 solved the equation also by the guess and check method, in addition to using Solution 1. None of the other types of solutions presented in Figure 1 were found in our workshop participants' responses.

Table 3
Example Teacher Solutions to Problem #3

P8's solution: Diophantine method	A $x+y=20$ $x-y=10$ $2x=30$ $\Rightarrow x=15$ x & y can be any number. ∴ we can find out the number ~~a~~ if we know all the
P4's solution: Vietan method	let first no = a and second no = b. let ~~So,~~ $a+b=x$ and $a-b=y$. if $a+b=x$ then $a=x-b$ substituting $a=x-b$ into $a-b=y$, we have $(x-b)-b=y$ $x-2b=y$ $\therefore 2b=x-y$ So, can find value of b: $\frac{x-y}{2}$ + then locate a by substitution

P9's solution: Using numerical examples	

Among the three, Problem #3 seemed to be the most challenging problem to the participants as there were comparatively more incomplete or incorrect solutions. From the responses of the nine participants, we found evidence of the two types of solutions recorded in previous research—the Diophantine and the Vietan methods (see Table 3)—while none of the participants used the Rhetorical method. P9 worked with numerical examples in solving the problem and concluded from these examples (Kieran, 1992) as shown in Table 3. That is, P9 assumed that the numbers are a = 6 and b = 5. Their sum, c, is 11 then, and their difference, d, is 1. P9 next tried if a = 11 and b = 4 (c = 15, d = 7), and next if a = 30 and b = 2 (c = 32, d = 28). Based on observations on the sum and difference in each case, P9 concluded the result as: $(c + d)/2 = a$ and $(c – d)/2 = b$. P9 may have had difficulty in using letters to express the general equation (Kieran, 1992), or P9's conceptions of generalisation and justification may be somewhere between "a procedural conception, which derives support from numerical operations, and a structural conception" (p. 407). With the absence of interview data, however, we would be cautious to make these judgements.

Reflections on Teachers' Solutions and Conclusions

It is interesting that for all three problems, the participants' favoured an algebraic response, for example simultaneous equations in Problem #1, and most did not consider additional solutions (maybe time played a part). While such an approach is explicitly prompted by Problem #2, only two participants provided more than one solution, despite the varied possibilities suggested in Figure 1. This may be because they felt that they had exhausted the possibilities for other algebraic manipulation and did not consider the possibility of numerical manipulation. It is, perhaps, not surprising that Problem #3 was the most challenging, because the absence of explicit numerical values makes the problem more abstract. The Vietan method involves pronumerals that are *unknown knowns* (the sum and difference) and *unknown unknowns* (the original two numbers). In the Vietan method case, the method used to find the solution is sufficient to prove that it is the solution, but in other cases, such as in P9's solution, the values have been found but are unproven.

To conclude, this paper analysed the solutions of this sample of teachers to the workshop problems to identify the possible needs of teachers in teaching algebra. Most participants considered only one solution, and this tended to favour more traditional, algebraically procedural methods. Although we have not reported extensively on the discussions that teachers had, they found value in analysing alternative solutions, and identifying the different ways in which algebra is used to represent, transform, and generalise. We anticipate that—with exposure to different forms of algebraic activities, and time to discuss various solutions with their peers—teachers' understanding of algebraic activity, and expectations for students, would grow. We look forward to further developing this PL approach based on these findings.

Acknowledgements. We thank the Mathematical Association of Tasmania (MAT) for supporting the implementation of the teacher working group research, and the teachers who participated in the workshop and provided their solutions to the problems for this study.

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RESEARCH PAPER

Regarding STEM: Perceptions of Academics Revealed in Their Drawings and Text

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How a sample of university educators described STEM, expected outcomes of STEM, expertise of STEM educators, and STEM learning environments were investigated through drawing- and text- based responses. Data were examined by applying the *Legitimation Code Theory (LCT) Specialisation* plane (Maton, 2014). Participants generally held *knowledge-code* (epistemic relations are foregrounded) or *mixed-code* (sometimes epistemic and sometimes social relations are foregrounded) perceptions. Further analysis showed that participants value both disciplinary knowledge and discipline-related practices such as analysing data and providing evidence-based discussions. The LCT approach has been found powerful in its ability to represent the kind of knowledge that might be valued, and the kind of knowers that might be desired by educators of STEM or individual STEM disciplines including mathematics.

A group of researchers involving the first and third authors of this paper patterned an instrument from the literature entitled, *Draw a STEM Learning Environment* (D-STEM), and implemented it with a sample of national project workshop participants to explore their perceptions of STEM learning environments in 2018 with a selection of school principals and education researchers. Next, the group developed the D-STEM Rubric to analyse the data and reported the participant researchers' (Hatisaru et al., 2019) and principals' (Hatisaru et al., 2020) perceptions of STEM learning environments including the presence of subject integration, the use of realistic problems, and student-centred instruction. The D-STEM instruments have been found to be powerful tools for studying individuals' understanding about STEM and STEM learning environments. Within a follow-up research project funded by the *University of Tasmania* (UTAS) *Research Enhancement Program*, we expanded the D-STEM research to investigate the perceptions of a sample of university academics (n = 15) about teaching and learning of STEM (Hatisaru et al., in press) in that context. In this paper, we use *Legitimation Code Theory (LCT)* (Maton, 2014), a sociology of knowledge approach, to describe the kind of epistemic relations (knowledge practices) and social relations (who enacts them) might be valued and emphasised by participant academics in STEM education. The research question underpinning this analysis is:

> *What perceptions of STEM education are evident in participant academics' drawings and descriptions on STEM?*

The study is significant for three reasons. First, for more than a decade now, there has been increased interest in schools and at universities in STEM education. Less is known, however, about what is valued and put emphasis on in the teaching and learning of STEM. Popular misconceptions about STEM practices reported years ago, such as equating STEM practices to hands-on activities, still exist (Morrison, 2006). For example, some educators view STEM practices as teamwork and communication, while some others think that teachers of STEM do not need to be expert but can be co-learners with the students (Hatisaru et al., 2020). There have been concerns that less attention is given to STEM discipline-specific content knowledge and practices in STEM professional learning activities (e.g., Winberg et al., 2019). The findings

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 258–265. Launceston: MERGA.

of this study extend the results of existing studies and contain valuable insights into STEM educators' perceptions of STEM education. Second, past research (e.g., Breiner et al., 2012) indicates that for individuals within an educational institution, a common operational definition of STEM may be helpful for fostering a clearer understanding about how to address issues in STEM to achieve intended learning outcomes. In this study, we explore how a group of academics within a single institution perceive STEM, expected outcomes of STEM, expertise of STEM educators, and STEM learning environments. Finally, the study is methodologically significant, as it presents an innovative conceptual framework (LCT) and a method (visual and textual data) for investigating individuals' perceptions of STEM and STEM education, and that can be used in mathematics education and elsewhere.

Production of the Drawings and Text

Academics from College of Arts, Law and Education and the College of Sciences and Engineering were invited to participate in the research, through their attendance at a workshop (2–3 hours in duration) run by the research team. The workshop focussed on unearthing understanding and discussing aspects of effective STEM learning environments. Fifteen academics from the disciplines of Architecture and Design, Biology, Education, Information and Communication Technology (ICT), Management, Medicine, Pharmacy, and Physics. Participation in the research was based on interest in being involved, which was indicated by attendance at the workshop. Participation was voluntarily and all workshop attendees gave consent for their responses to be used for research purposes. At the commencement of the workshop, the participants were provided an adaptation of the D-STEM instrument, which was contextualised for higher education, and given 25–30 minutes to complete it. The instrument was comprised of three prompts with space provided for visual (Prompt 1) and written descriptions (Prompt 2; Prompt 3).

> Prompt 1: A learning environment is the diverse physical location, context and culture in which students learn. Think about STEM classes and the kinds of things that would be done in those classes. Draw a STEM learning environment.
>
> Prompt 2: Look back at the drawing and explain it so that anyone looking at it could understand what it means.

The descriptive narrative requested in Prompt 2 was to clarify and/or expand upon the information contained in the drawings and to assist in subsequent coding using the D-STEM Rubric (Hatisaru et al., 2020). The data from this section would enable the exploration of participant academics' perceptions of teaching and learning of STEM in their context. Also included was Prompt 3 with three prompt stems, which aimed to explore participants' understanding of STEM, STEM teaching and learning, and STEM education expertise:

> Prompt 3: Think about STEM education, learning and teaching. Please complete the sentences below. To me: (1) STEM is ...; (2) The goals and outcomes of STEM education for individuals involves ...; and (3) An educator of STEM knows ...

An Analytical Tool for Analysing the Data: Legitimation Code Theory (LCT)

LCT provides a conceptual tool for analysing knowledge or knowledge practices within academic disciplines, including STEM (e.g., Winberg et al., 2019). We used the *LCT specialisation codes* as an analytical tool to analyse the data generated in this study. As also described in Hatisaru (2021), in LCT, *specialisation* is about what makes someone or something distinct, special, or different (Carvalho et al., 2009). Its premise is that all knowledge, beliefs, or practices are about or oriented towards something, and are practiced by someone. It sets up *epistemic relations* (ER) to an object (e.g., STEM disciplinary knowledge) and *social relations* (SR) to a subject (e.g., STEM dispositions) (Maton, 2014). These relations consider what can be objectively described as knowledge and who can claim to be an ideal knower (e.g., a student or teacher). Epistemic and social relations may be more strongly (+) or weakly (–) emphasised; the strength of the relations originates specialisation codes (ER+/–, SR+/–) (Maton & Chen, 2020). The relative strengths can be placed into four quadrants in the specialisation plane (see Figure 1) at numerous positions (Maton, 2014) and encapsulate the basis of legitimation—or focus or success—in a particular field, event, or practice (Winberg et al., 2019). The *x*-axis represents a continuum of weaker to stronger social relations and the *y*-axis represents weaker to stronger epistemic relations. The relative strength of these relations gives rise to four principal codes: *knowledge code* (ER+, SR–); *élite code* (ER+, SR+); *knower code* (ER–, SR+); and *relativist code* (ER–, SR–).

To operationalise the analysis of the data using the *specialisation plane*, a *translation device* is necessary (Maton, 2014). We used the translation device presented in Figure 1 for data analysis in this study, which was based on Maton (2014), Carvalho et al. (2009), and Ellery (2009).

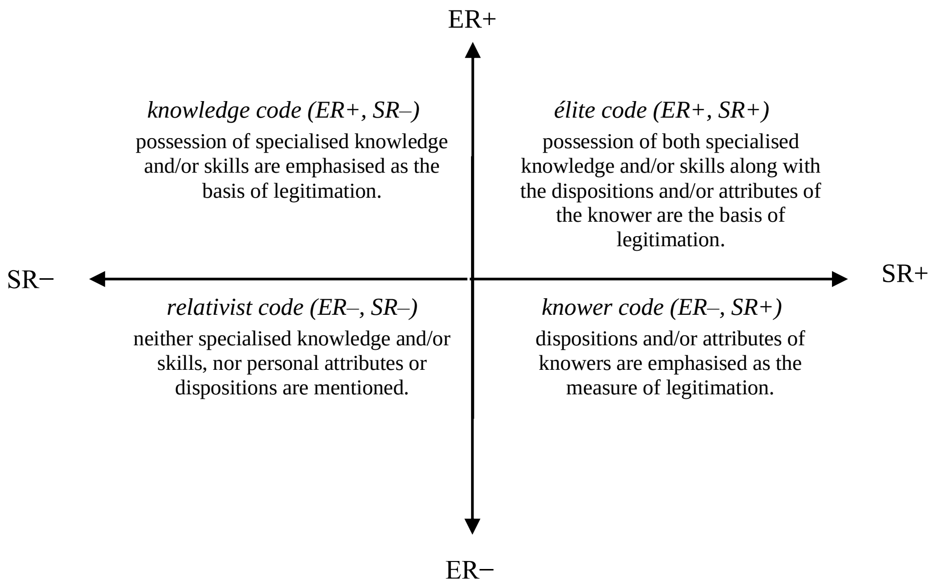

Figure 1. The translation device used in this study.

According to this device, the participant responses located in the *knowledge quadrant* may focus on discipline specific knowledge relating to STEM. In STEM education, however, the *knowledge code* not only incorporates the content knowledge of the associated disciplines (e.g., science and mathematics), but it also extends to the skills and practices that are associated with them such as scientific inquiry, investigations, analysing and interpreting data, and designing solutions (Ellery, 2019). Responses in the *knower quadrant* may focus on dispositions and/or attributes of the knowers (e.g., students) including being self-directed and confident, willingness to explore, share experiences, consider multiple views, and be reflective (an ideal knower) (Maton & Chen, 2020). Responses in the *élite quadrant* may emphasise both possessing specialist knowledge and being the right kind of knower as the measure of achievement, and these responses therefore may highlight the necessity of possessing both legitimate knowledge and legitimate attributes or dispositions. Finally, responses in the relativist quadrant would have no or little STEM content and no or little STEM dispositions or attributes. By examining these codes, the underlying perceptions in participant responses could be made explicit.

An Overview of Data Analysis

We implemented a content analysis of the statements or words that the participants used to respond to three prompts given, and their D-STEM depictions and descriptions. We were interested in the way participants described STEM, expected outcomes of STEM for individuals, expertise of STEM educators, and a STEM learning environment. A code book, mapping specialisation codes to data, using *specialisation codes* as presented in Figure 1, was utilised in the analysis of data. Participants were assigned identifiers (P1, P2, P3, etc.) to maintain their anonymity. The data were analysed by the first two authors manually using excel spreadsheets. By way of illustration of the coding process, examples of each of the specialisation codes identified in the participants' responses are provided in Table 1.

The same specialisation codes were used in the analysis of the participants' STEM drawings and associated descriptions. Adapted from Maton and Chen (2020), when a STEM drawing or description included more indicators of specialist knowledge and/or skills, and less or no indication of personal beliefs, personal dimensions of learning, collaborative learning or of personal skills (e.g., teamwork, collaboration), they were assigned the *knowledge code* (ER+, SR–), while they were assigned the *knower code* (ER–, SR+) when the emphasis was vice versa. When the drawing or associated description included emphases to both, it was coded as *élite* (ER+, SR+). Some responses were coded as *relativist* (ER–, SR–) as neither epistemic nor social relations were mentioned. In Figure 2, two responses are presented representing the *knowledge-code* (P6) and *knower-code* (P3) drawings.

Table 1
LCT Specialisation Codes Enacted in the Participants' Responses (emphasis added)

Codes	(1) STEM is ...	(2) Outcomes of STEM includes ...	(3) A teacher of STEM knows ...
Knowledge (ER+, SR–)	Cross curricular *understanding of all related science* to help detailed meaning of topic being investigated. (P9)	Offering opportunities to explore real world *based upon existing knowledge, models and techniques*. Enhance *scientific knowledge, skills and logical thinking*. (P1)	Two skills: - Subtle level - Gross level – *topics specific to subject being taught*. (P11)
Knower (ER–, SR+)	*Thinking about our world, our place in it*, understanding and developing ways to further *think about it and interact differently*. (P4)	Enabling the student to *think outside the box* about a problem. (P15)	That *we don't know everything, but we know* how to try to know everything. (P3)
Élite (ER+, SR+)	*Science, engineering, technology and maths*. Forms the basis of most learnings. *Allows a broader view*. (P15)	*Literacy acquisition* to be able to judge information sceptically, requiring evidence, *being open and transparent*. (P14)	Each of *the big ideas in each discipline*; knows how to make experiences authentic; *make the learning meaningful*; what evidence constitutes learning in each of the disciplines. (P5)
Relativist (ER–, SR–)	A *learning platform*. (P13)	-	*To give* immediate *feedback*. (P13)

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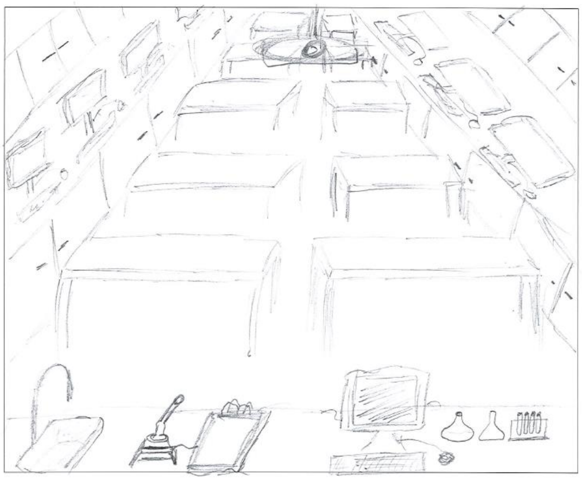
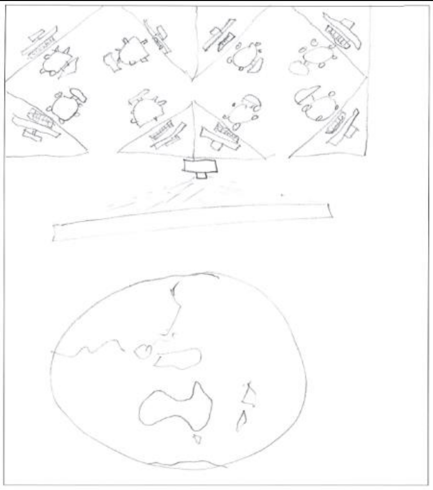
Further elaboration of the *knowledge code* was warranted because, in STEM, the *knowledge code* not only incorporates content knowledge of the associated disciplines, but it also extends to the skills and practices that we associate with these disciplines. When referring to Maton's (2014) model of specialisation, Maton and Howard (2016) noted that "the model distinguishes epistemic relations into ontic relations that specialize the known and discursive relations that specialize the discursive practices whereby it is known" (p. 64), but that these two relations are collapsed into a single *knowledge and skills* scale within the LCT framework. They suggested the creation of "two scales that addressed knowledge and skills separately" (p. 64), and after further consideration, they expanded the *knowledge and skills* labels to the *Knowledge, Theory and Concepts (KTC)* and *Skills and Practices (SP)*, respectively, to increase the clarity and understanding of these two labels.



Lab partner tables (standing, no seats) to encourage movement. Side walls have cabinet space full of lab equipment, and networked computers for inquiry.

Teacher desk in front with relevant equipment (e.g., microscope) linked to digital outputs to share with students through the projector. (P6)



Computers set up so that students can work independently on computer-based individual tasks. Students can turn around and work in small groups to achieve things together and discuss their problems. A big ocean so that all students can learn together and share what they know. Easy access outside to the rest of the world. (P3)

Figure 2. Examples of responses representing the *knowledge-code* (P6) and *knower-code* (P3) drawings.

In STEM, KTC is the disciplinary content knowledge or the theory underpinning the relevant discipline(s), what we might call the 'facts and figures', whereas SP includes the skills, practices and methodological approaches that we consider as part of the training within each STEM discipline. Skills and practices cover those used in scientific inquiry (e.g., analysing data and providing evidence-based discussions) along with the disciplinary skills that students of STEM are trained in (e.g., making observations in the field, designing new technology). Assignment of each of the relevant measures to KTC+/– and SP+/– considered the context of the response and the participant's disciplinary focus (or otherwise):

(KTC+, SP–): Related areas of knowledge (P9, Prompt 3)

(KTC–, SP+): Teaching students to apply real scientific approach where science, mathematics are instruments. (P11, Prompt 1)

(KTC+, SP+): Beginning with a problem or question and then using appropriate disciplines to answer/address the problem. (P5, Prompt 2)

Findings: Perceptions of STEM Education in Participant Academics

With fifteen participants and four measures per participant, there were a total of 60 responses (*f*: frequency) to analyse. Participants' responses to three prompts and their STEM drawings grounded on *specialisation codes* are presented in Table 2. In general, the participants described STEM through using *knowledge code* (*f* = 26) or *knower code* (*f* = 18), and to a lesser extent *élite code* (*f* = 12). Two participants (P12 and P13) had responses assigned as *relativist code* (*f* = 4) with P13 the only participant showing consistent responses assigned as *relativist code* (*f* = 3).

Table 2

Participants' Responses Grounded on the Specialisation Codes (f = 60)

	Knowledge (ER+, SR−)	Knower (ER−, SR+)	Élite (ER+, SR+)	Relativist (ER−, SR−)
Drawing and its description	P2, P6, P7, P9	P1, P3, P10, P11, P14	P4, P5, P8, P12, P15	P13
(1) STEM is...	P1, P2, P3, P5, P6, P9, P10, P11, P12, P14	P4, P7, P8	P15	P13
(2) Outcomes of STEM includes ...	P1, P2, P5, P6, P8, P9, P11	P3, P4, P7, P10, P13, P15	P12, P14	-
(3) A teacher of STEM knows ...	P2, P6, P9, P10, P11	P3, P4, P8, P15	P1, P5, P7, P14	P12, P13

We next located participants' responses in the *specialisation plane* based on the frequency of responses in each quadrant (see Figure 3). When the frequency of responses of a participant in a particular quadrant was three or more than three ($f \geq 3$), we assumed that the participant held the relevant quadrant perception. When their responses spread across two quadrants, we considered that these participants show *mixed-code* perceptions, and when responses were spread around more than two quadrants, we decided not to group these participants as their dominant views were indeterminant (* in Figure 3).

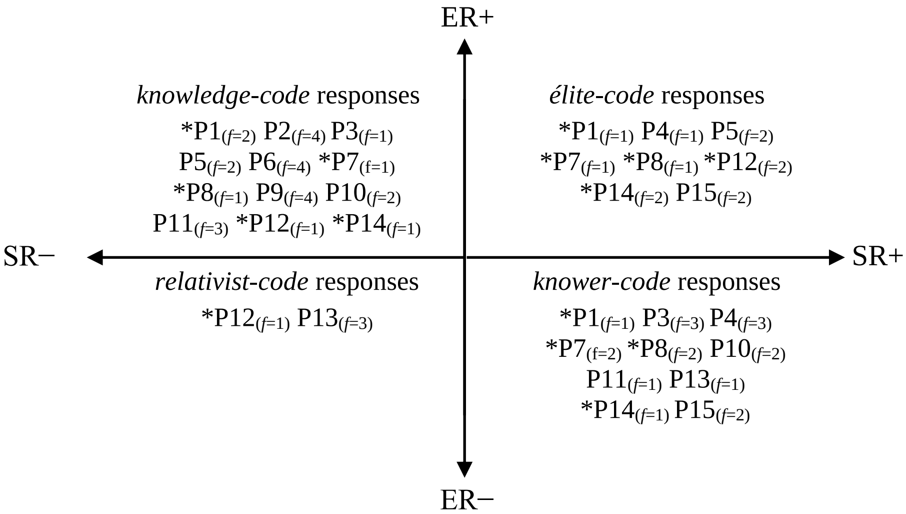


Figure 3. Participants' responses located into the specialisation plane.

Based on these judgements, none of the participants showed consistent *élite-code* perceptions. P2, P6, P9, and P11 displayed *knowledge-code* perceptions as they chiefly emphasised possession of knowledge and/or skills of STEM, while the dispositions or attributes of the knower were less evident or completely absent in their responses. Both P3 and P4 showed *knower-code* perceptions as they mostly downplayed specialist STEM knowledge and/or skills and emphasised social relations. Some of the participants (P5, P10, and P15) reflected *mixed-code* perceptions—sometimes emphasising STEM disciplinary knowledge and/or skills, and other times dispositions and/or attributes of students or educators; and sometimes both. Within this group, P10 displayed *knowledge-knower* code, P5 showed *knowledge-élite* and P15 showed *knower-élite* code perceptions.

The distribution of participants across the various *specialisation codes*, with only two participants holding *knower-code* perceptions, is not surprising considering that the participants were academics from various STEM-related disciplines. It might be expected that they would value or promote an understanding of disciplinary knowledge and/or skills in their teaching and student learning. Having three participants in the *mixed-code* perceptions group suggests that both the attributes of STEM knowers and STEM knowledge and/or skills are valued by some participant academics. For example, as described by participants when responding to the D-STEM prompts, STEM includes "related areas of knowledge" (P9) and "the scientific method" (P6). It is "about incorporating science, technology, engineering and mathematics—two or more of these disciplines" (P5). STEM-educated persons are "curious" (P8), "interested in science and mathematics" (P10), and show "proactive attitude" (P13) and "empathy, awareness and openness to the unexpected" (P4). They find STEM as "an important part of understanding the world and making it better" (P3), "[work] on important issues relevant to the community" (P7) and think "outside the box about a problem" (P15).

We undertook further analysis of the *stronger-epistemic-relations* assignments (i.e., ER+, SR–; ER+, SR+) for each of four measures to explore these participants' perceptions in more depth. As shown in Table 3, of these twenty-five assignments, twelve were coded as (KTC+, SP–) and nine (KTC–, SP+), with four responses judged as (KTC+, SP+). The split between (KTC+, SP–) and (KTC–, SP+) responses indicate a focus on either KTC or SP in these participants, indicating that while some may give importance to facts and figures, others may value the skills and practical aspects of the relevant disciplines.

Table 3
Stronger-epistemic-relations Assignments (f = 25) Differentiated as KTC (+/–) or SP (+/–)

	Knowledge (ER+, SR–)		Élite (ER+, SR+)
Drawing and its description	P2 (KTC–, SP+) P6 (KTC–, SP+) P9 (KTC+, SP+)		P4 (KTC–, SP+) P5 (KTC–, SP+) P15 (KTC+, SP+)
(1) STEM is ...	P2 (KTC+, SP–) P3 (KTC+, SP–) P5 (KTC+, SP+) P6 (KTC–, SP+)	P9 (KTC+, SP–) P10 (KTC+, SP–) P11 (KTC–, SP+)	P15 (KTC+, SP–)
(2) Outcomes of STEM includes ...	P2 (KTC+, SP–) P5 (KTC+, SP+) P6 (KTC–, SP+)	P9 (KTC+, SP–) P11 (KTC–, SP+)	
(3) A teacher of STEM knows ...	P2 (KTC+, SP–) P6 (KTC–, SP+) P9 (KTC+, SP–)	P10 (KTC+, SP–) P11 (KTC+, SP–)	P5 (KTC+, SP–)

Across the *knowledge-code* (P2, P6, P9, and P11) and *mixed-code* (P5, P10, and P15) perceptions participants, the split between KTC (+/–) and/or SP (+/–) was also interesting. That is, within the *knowledge-code* perceptions group (*f* = 15), seven (KTC+, SP–) and seven (KTC–, SP+) were observed, and within the *mixed-code* perceptions group (*f* = 8), four (KTC+, SP–) and three (KTC+, SP+) were recorded. The *mixed-code* perceptions participants provided a greater proportion of *élite-code* responses than the *knowledge-code* perceptions participants. This difference most likely reflects the different foci the two groups have with regards to the *knower code*, with the *mixed-code* perceptions participants more likely to focus on both content knowledge and skills and practical aspects of the relevant disciplines when providing a response consistent with stronger *epistemic relations* coding.

Concluding Remarks

In answer to the research question, what perceptions of STEM education are evident in participant academics' drawings and descriptions on STEM, we have been able to apply the LCT framework to identify a diversity of *knowledge-code* and *knower-code* perceptions of STEM amongst academic participants. *Knowledge-code perceptions*—or the valuing of disciplinary knowledge and skills—dominated, while the relative value or importance of knowledge versus skills in STEM education was mixed. Understandings of STEM and the ideal knower, as requiring or encompassing both specialised knowledge and skills as well as dispositions and/or attributes (*élite code*) were not common. These data highlight two things; firstly, perceptions of STEM varied across the group of academics and secondly, individuals within the group did not hold coherent perceptions across each response type (i.e., drawings; and three prompts). Both findings indicate the need for further investigation of the perceptions of STEM within academic contexts, with larger sample size and which consider individuals' discipline background, research and teaching focus and experience, with a particular focus on the relevance and/or importance of interdisciplinarity. Most importantly, the methodological significance of the LCT when used in the analysis of diagrams and text, has been highlighted. It is powerful in its ability to represent the kind of knowledge that might be valued, and the kind of knowers that might be desired by educators of STEM or of individual STEM disciplines including mathematics.

Acknowledgements. An early version of the paper was presented by the first author at the British Society for Research into Learning Mathematics (BSRLM) 2021 Online Day Conference.

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RESEARCH PAPER

Development of the *Self-Efficacy-Effort* in Mathematics Scale and its Relationship to Gender, Achievement, and Self-concept

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Mathematics self-efficacy is considered an important variable in mathematics education because of its links to mathematics achievement. This paper reports on the development of the *Self-efficacy-effort in Mathematics Scale* (SEEMS), an instrument that has strong theoretical and psychometric properties. Based on a sample of n = 224 Australian primary students and n =133 secondary students, the instrument demonstrated positive correlations with related measures of mathematics achievement and mathematics self-concept. Girls had higher mathematics achievement within the primary and secondary school data. There were no significant gender difference in mathematic self-concept scores for the primary and the secondary school students. For mathematics self-efficacy, a gender difference was only identified in the high school data. The implications of this research to mathematics education practices are reviewed in the paper.

Self-efficacy beliefs are people's judgments of their capability to accomplish a task or succeed in an activity (Bandura, 1997). Self-efficacy and self-concept are considered to be multidimensional constructs. The claim by Lau et al. (2018) is that students' academic self-efficacy beliefs influence: (1) their academic attainments; (2) how long students will persist and persevere when confronting difficulties; and (3) how resilient students will be in the face of adverse situations. Mathematics self-efficacy is considered an important agent in students' learning because mathematics self-efficacy beliefs help determine how much effort persistence, and resilience students will expend on a mathematics problem and/or activity (Mozahem et al., 2021; Zimmerman, 2000).

Self-efficacy, self-concept, and self-esteem belong to a group of self-belief variables identified as self-perception variables (Hay & Ashman, 2018). There are, however, subtle differences between these variables: self-concept is more descriptive (I am good at mathematics); self-esteem is more evaluative (I like mathematics) (Watkins & Dhawan, 1989); and self-efficacy is more effort (the doing) (I work hard at mathematics) (Sachitra & Bandara, 2017). Students' self-perception variables and their academic achievement are considered to be significantly correlated (Marsh et al., 2004). Mathematics self-efficacy is, however, deemed to have a higher correlation with mathematics achievement, than does mathematics self-concept (Pietsch et al., 2003). Certainly, mathematics self-efficacy is predictive of mathematics achievement and mathematics problem-solving (Pajares & Kranzler, 1995).

The evidence indicates there are gender differences related to mathematics self-concept, self-efficacy, and interest for both primary and secondary school students (Rodriguez et al., 2020). Boys generally have better motivation and self-perception profiles in mathematics, than do girls (Kurtz-Costes et al., 2008). In contrast, girls tend to exhibit fewer positive attitudes about mathematics and have higher rates of mathematics anxiety than their male classmates (Rodriguez et al., 2020). Within mathematics self-concept research, boys are more likely to rate themselves higher than girls, even when girls had equivalent or even higher mathematics achievement scores (Hay et al., 1989).

In terms of the assessment of a person's self-efficacy there has been the development of: (1) general measures, such as the Schwarzer and Jerusalem (1995) *General Self-Efficacy Scale* (example item "I can solve difficult problems if I try hard enough"); (2) academic self-efficacy measures, such as the Sachitra and Bandara (2017) *Academic Self-Efficacy Scale* (example item, "I make an attempt to meet the deadline for group assignments"); (3) and specific domain

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 266–273. Launceston: MERGA.

measures, such as in mathematics, “I work hard at mathematics” (Marsh et al., 2004).

Students’ mathematics self-efficacy beliefs are assumed to form as students interpret and accept information from four sources (Lau et al., 2018): (1) mastery experiences (i.e., interpretation of one’s performance in relationship to previous efforts and outcomes); (2) vicarious experiences (i.e., observing the actions and performance of peers and others who provide comparison information regarding one’s capabilities); (3) social persuasions (i.e., verbal persuasion of capability from others in the social environment, such as teachers); and (4) physiological states (i.e., the interpretation of one’s own physiological and emotional states as an indicator of capability, often associated with levels of anxiety). Self-efficacious students are considered to perceive themselves, rather than their teachers, as more responsible for their own academic learning outcomes (Zimmerman & Kitsantas, 2014). This supports the notion that students’ self-efficacy beliefs are linked to their use of effort, as part of their mastery orientation to learning and their ability to self-regulate (Mozahem et al., 2021).

Rationale

There are a number of concerns associated with the assessment of students’ self-efficacy in mathematics that have influenced the conceptualising of this research study. The first is, much of the previous self-efficacy research has focused on adults in the workplace, university students, and secondary school students (Bandura, 1997; Joët al., 2011). There has been less of a focus on specific subject domains, such as mathematics self-efficacy (Sachitra & Bandara, 2017). The second is more recent with self-efficacy researchers, such as Toland and Usher (2016) and Moriarty (2014) tended to shift the stem of the self-efficacy question from effort (I try hard) to confidence (I could). This was done, in part, to identify the student’s confidence to complete a mathematical task just before doing that task. Confidence was considered an aspect of self-efficacy along with effort and persistence by Bandura (1997) but there is some unease associated with this perception. The first is, self-efficacy is an important determinant of academic performance, but the counter-position is that self-efficacy is merely a reflection of past performance (Talsma et al., 2018). The second is, academic achievement is influenced by effort plus high academic self-concept (Marsh et al., 2016). The third, relates to the notion that confidence is not the same construct as effort, with self-evaluation statements, such as “I am confident is mathematics” more related to self-esteem (Watkins & Dhawan, 1989).

The ability of an individual to self-evaluate and self-monitor their thinking and behaviour is considered a metacognitive skill (Weil et al., 2013) that involves students making a comparison of their performances and their emotional reactions over time (Schraw, 1998). When self-efficacy researchers shift the focus question to students’ confidence with a task, they may be indirectly by asking students to make more of “higher order” metacognitive evaluation of their performance, a judgement which is considered more cognitively difficult for younger students to grasp (Weil et al., 2013).

The research evidence, particularly for younger students, lends more support to the use of effort questions when investigating self-efficacy. In addition, students’ previous experiences and feedback with mathematics tasks has the strongest predictive influence on the formation of students’ mathematics self-efficacy (Usher & Pajares, 2009). The main sources of this formation information comes from the social messaging provided by teachers to their students and the individual student’s previous emotional and anxiety reactions as they engage with mathematics (Bandura, 1997; Joët et al., 2011).

The core aim of this paper then is to report on the development and the psychometric properties of a mathematics self-efficacy scale, the *Self-Efficacy-Effort in Mathematics Scale* (SEEMS), designed to address the concerns identified above. The items selected within this scale are orientated towards students’ level of effort and persistence with mathematics and their participation in mathematics classes (Schwarzer & Jerusalem, 1995; Zimmerman, 2000).

Students' self-efficacy, self-concept, and academic achievement measures are correlated (Marsh et al., 2004), with gender having an influence on students' self-beliefs (Fyffe & Hay, 2021; Marsh, 1988) along with age (Mozahem et al., 2021). Thus, as part of the development and evaluation of SEEMS the following two research questions will be investigated:

How does the *Self-Efficacy-Effort in Mathematics Scale* (SEEMS) relate to students' mathematics self-concept and their academic achievement in mathematics? Does the *Self-Efficacy-Effort in Mathematics Scale* (SEEMS) identify any gender and or age differences within a cross-section of primary and secondary students?

Method

Ethical permission to conduct the research was provided by the relevant University and school authorities, with parents and teachers of the participating students also providing informed consent.

Participants: There were two participating groups. One group of participants were primary school students who were drawn from two co-education, non-government primary schools located in provincial Australian towns (n = 224, Years 3 to 6, mean age 10 years 5 months). The other group were secondary school participants who were drawn from one government high school in a provincial Australian town (n = 133, Year 7, mean age 12 years 4 months). All three schools clustered around the national socio-economic status (SES) school average, mean of 1000 as identified using the Australian Index of Community Socio Education Advantage (ICSEA, 2016). Data were collected in the mid to later part of the school year.

Instruments: The SEEMS was designed using relevant literature and the theoretical framework that: (1) self-efficacy pertains to students' use of effort and persistence with a task (Schwarzer & Jerusalem, 1995; Zimmerman, 2000); and (2) students' self-belief tests need to be designed so they are sympathetic to the students' ability to comprehend, interpret, and understand all items (Hay & Ashman, 2018; Weil et al., 2013). An initial bank of 15 survey items was generated and trialed with four teachers and 15 students from an independent urban primary school located in a middle socio-economic school district. Based on this feedback, items that were too similar to other items, or were not clearly interpreted by the students were either taken out or reworked. The end result was, the SEEMS, contained 12 items, focusing on students' effort and involvement in mathematics. The participants completed the SEEMS by selecting one of four Likert responses to each question (scored 1 to 4): Almost never like me; Usually not like me; Usually like me; Almost always like me.

The mathematics self-concept measure was extracted from the *Self-Description Questionnaire-1* (M-SDQ-1) (Marsh, 1988). The students read 10 declarative sentences related to mathematics (e.g., I'm good at mathematics) and selected one of five Likert responses: False; Mostly false; Sometimes false/sometimes true; Mostly true; or True. Marsh (1988) reported a Cronbach alpha reliability score of 0.89 for the M-SDQ-1.

The participating primary school teachers were asked to rate the mathematics ability of each participating student in their class, in relationship to the students' grade level achievements. To do this the teachers were provided with a rating scale from 1 to 10; 1 = poor performance, 5 = sound achievement, 10 = extremely high achievement. The participating secondary students' mathematics results were obtained from their school semester reports and converted onto the same rating scale identified above.

Procedure: The SEEMS and the M-SDQ were administered in-class time to the participating students in their regular class groups. Four weeks later, the SEEMS was re-administered to a cohort of 50 primary and 25 secondary students to investigate its reliability. All data were coded into SPSS (2016) for analysis.

Results

The SEEMS produced a high level of internal consistency (Cronbach alpha = 0.92) and a high level of test re-test reliability (r = 0.84) (Tabachnick et al., 2007). An exploratory factor analysis was conducted on the student data. The extraction method was principal component analysis, and the rotation method varimax with Kaiser normalization. The Eigenvalues scree plot identified that the 12 self-efficacy items formed two factors. Interpreting the output, the first factor identified (9 items) related to effort and trying hard in mathematics. The second factor related to students' involvement and participation in mathematics classes (3 items), as shown in Table 1.

Table 1
The SEEMS Factor Structure

SEEMS Items	Factors#	
	1	2
1. In mathematics I try hard.	.741	
2. I try hard to complete mathematics homework.	.772	
3. I try hard when given mathematics tests.	.808	
4. I am keen to answer teacher's questions asked of me about mathematics.		.787
5. I try hard to solve mathematics problems.	.749	
6. If I have a difficulty with a mathematics problem I keep working on it until it is solved.	.552	
7. I enjoy helping other students with mathematics.		.772
8. I want to finish class mathematics activities.	.612	
9. I like talking to the teacher about mathematics.		.822
10. I try hard to pay attention when the teacher is talking about mathematics.	.692	
11. I try hard doing workbook activities in mathematics	.789	
12. I study hard before a mathematics test	.575	

Note: #Factor loadings less than Eigenvalues 0.35 were suppressed

To further validate the internal structure of the SEEMS a Confirmatory Factor Analysis (CFA) was conducted using AMOS in SSSP (2016). The fit indexes for the two factor model (reported above) were all high and above the 0.9 criteria point (Schreiber et al., 2006): Norm Fit Index = 0.968, Comparative Fit Index = 0.976. The Root Mean Square of Error of Approximation was low = 0.052 and below the 0.06 criteria point (Schreiber et al., 2006).

The students' SEEMS scores, their M-SDQ scores, along and their teachers' ratings their mathematical achievement in class were significantly correlated for both the primary school and secondary school students (Tables 2 and 3).

Comparing cohorts, secondary school students demonstrated somewhat higher correlation levels between mathematics achievement, self-concept self-efficacy (r around 0.4), than the primary school students (achievement, self-efficacy, and self-concept r around 0.3). Mathematics self-concept and mathematics self-efficacy were significantly well correlated for both the primary and secondary school students (r around 0.6).

Table 2
Primary Students' Correlation: Mathematics Achievement, Self-concept, and Self-Efficacy
n = 224 (Years 3–6) (** $p > 0.01$)

Mathematics	Achievement (in class)	Self-Concept (M-SDQ)	Self-Efficacy (SEEMS)
Achievement (in class)	1.00		
Self-Concept (M-SDQ)	0.28**	1.00	
Self-Efficacy (SEEMS)	0.35**	0.60**	1.00

Table 3
Secondary Students' Correlation: Mathematics Achievement, Self-Concept, and Self-Efficacy
n = 133 (Year 7) (** $p > 0.01$)

Mathematics	Achievement (in-class)	Self-Concept (M-SDQ)	Self-Efficacy (SEEMS)
Achievement (in class)	1.00		
Self-Concept (M-SDQ)	0.48**	1.00	
Self-Efficacy (SEEMS)	0.47**	0.56**	1.00

Gender was explored within the student data using independent ANOVAs with a Bonferroni adjustment. Girls in primary and secondary school were rated by their teachers as having higher mathematics achievement, compared to boys. In primary school, girls rated their mathematics self-concept and self-efficacy similar to the boys (Table 4). In secondary school, compared to boys, girls had higher self-efficacy but similar self-concepts scores to the boys (Table 5). Reviewing mean scores, boys dropped in mathematics self-concept and self-efficacy in the first year of high school, compared to their primary school scores.

Table 4
Gender Difference Primary Students: Mathematics Achievement, Self-Concept, Self-Efficacy

Mathematics	Boys n = 104		Girls n = 121		F (1,223)	Sig ** $p > 0.01$
	M	SD	M	SD		
Achievement (in class)	5.65	2.09	6.36	1.92	7.19	0.008**
Self-Concept (M-SDQ)	30.12	9.29	28.87	8.91	1.06	0.305
Self-Efficacy (SEEMS)	40.45	6.84	41.63	5.53	2.25	0.135

Table 5
Gender Difference Secondary Students: Mathematics Achievement, Self-Concept, Self-Efficacy

Mathematics	Boys n =74		Girls n = 59		F (1,131)	Sig ** $p > 0.01$
	M	SD	M	SD		
Achievement (in class)	3.55	1.48	4.21	.29	7.14	0.009**
Self-Concept (M-SDQ)	25.52	8.53	28.10	7.89	3.19	0.076
Self-Efficacy (SEEMS)	34.22	8.10	39.02	5.59	14.44	0.000**

Discussion

The psychometrics of the SEEMS were identified to be strong (Tabachnick et al., 2007). The exploratory factor structure of the instrument was reflective of its theoretical framework that associates mathematics self-efficacy with students' level of effort and participation in-class (Schwarzer & Jerusalem, 1995; Zimmerman, 2000). A confirmatory factor analysis of the SEEMS demonstrated strong fit indexes for the obtained two factor model (Schreiber et al., 2006). This research demonstrated that for mathematics there was a positive correlation between academic achievement, self-concept, and self-efficacy, which supports the notion that these three variables are interactive (Lau et al., 2018). Of note, mathematics self-efficacy had a higher correlation with mathematics achievement than with mathematics self-concept in the primary school data, which is similar to the findings of Pietsch et al. (2003).

For both primary and secondary school students a significant correlation was identified between SEEMS, M-SDQ, and students' mathematics achievement. Such a finding between self-perception and achievement is consistent with previous research (Marsh et al., 2004). The correlation between mathematics self-efficacy and mathematics achievement was stronger in the first year of the high school student data, compared to the correlation within the primary school student data. This may reflect that in the secondary school setting, first year students have to engage in higher levels of self-efficacy to adapt to a new and faster delivery of curriculum, to new teaching and assessment practices, to more demands in terms of homework and assessment, and higher expectations (Hopwood et al., 2016). This links to previous self-efficacy research that as learning tasks become more difficult, students have to use more mastery orientation and self-regulatory strategies (Mozahem et al., 2021).

Gender was identified as a variable in this study. In mathematics self-concept research, boys are more likely to rate themselves higher than girls, even when girls have equivalent or higher mathematics scores (Hay & Ashman, 2018; Marsh, 1988). Watt (2004) noted that girls typically reported higher self-perceptions in English and boys typically reported high self-perceptions in mathematics. In this study, girls had higher mathematics achievement to boys but not higher mathematics self-concept. The gender results were more pronounced in the first year of high school, with girls having significantly higher achievement and self-efficacy scores. The hypothesis is that because mathematics self-efficacy and mathematics achievement are interactive and positively correlated, a change in one of these variables will influence the other (Mozahem et al., 2021; Zimmerman, 2000). On this point, Hyde et al. (1990) noted that over time, as male secondary students improved in their mathematics achievement scores, so too did their level of self-efficacy in mathematics improve. In this study girls had higher levels of mathematics achievement in primary and first year of secondary school, but similar mathematics self-concepts to the boys. The concern is that girls are more likely to transition away from engaging with mathematics and associated careers, not because they lack the ability but because of more negative gender social messaging and lower expectations associated with girls and mathematics (Goetz et al., 2013). This concern reflects Watt's (2004) statement that cautioned against concluding that in terms of academic and psychosocial well-being, it is the boys who are necessarily more at risk through secondary school, than the girls. Thus, helping all students to develop and grow their mathematics self-efficacy and engagement with mathematics needs to be a goal of education, with additional attention provided to students as they transition into their secondary school education.

Teachers have to actively teach mathematics content in a way that facilitates students' interest, motivation, and engagement with that content, which in turn facilitates students' self-efficacy and knowledge development (Hay et al., 2015). In addition, teachers also need to actively assist students to develop positive work, study and homework strategies that can assist students develop more of a mastery orientation to their learning (Schunk, 2012). Reducing students' levels of anxiety associated with mathematics also needs to be a consideration, because mathematics anxiety has a detrimental influence on the development of a positive self-efficacy in mathematics (Joët et al., 2011). It is the negative and/or neutral social messaging that teachers, may unknowingly provide to students, that has the corrosive influence on their level of mathematics effort, performance, and anxiety (Goetz et al., 2013; Krispenz et al., 2019). Helping students to manage negative self-beliefs, such as anxiety and low self-efficacy typically involves a combination of social, behavioural, and cognitive interventions (Krispenz et al., 2019). Social messaging influences the formation of self-efficacy (Lau et al., 2018). Thus, teachers need to be mindful of their in-class conversations with their students, with less of a focus on the "the correct answer" and a greater focus on dialogue and constructive feedback with students about the processes of learning and students' use of strategies and effort (Callingham et al., 2019; Moni & Hay, 2019). At different points in their delivery of mathematics content, teachers need to raise topics, such as: students' fearfulness of making a mistake; students not wanting to be seen by peers as different or better; asking for help rather than giving up; homework and study strategies; and the application of mathematics across society. Positive messaging and informed feedback are for all students and certainly not restricted to the so called "good" mathematics students.

There are limitations associated with this study. The participants were drawn from three co-education schools located in a regional town in Australia, therefore, there may be specific factors associated with these students that may influence the generalisability of the findings. Different curriculum domains, such as English or science may react differently to mathematics in terms of students' self-perceptions and so this needs to be considered when interpreting and extending these findings. Future researchers could investigate how the SEEMS correlates to a mathematics confidence self-efficacy measure, as used by Moriarty (2014).

In conclusion, for students to be successful in mathematics, teachers need to engage in teaching the mathematics content and assisting all students to develop positive self-perceptions associated with mathematics. The evidence is students' mathematics self-efficacy is an important agent that directly and indirectly influences students' mathematics achievement. Thus, mathematics educators need to continue to actively encourage and support the development of positive self-efficacy in all of their students.

Acknowledgement: Dr Rosemary Callingham AM, University of Tasmania for advice when drafting the paper.

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Teacher STEM Capability Sets that Support the Implementation of Mathematics Active STEM Tasks

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In this paper we report on an aspect of a larger study and explore whether a framework for teacher STEM Capability Sets (SCS) enhanced teacher planning and the implementation of mathematics active STEM tasks. A case study approach was employed to understand how a classroom teacher used the digital resource, Gapminder, to teach a Year 5/6 cohort of students to interpret data as the basis for a STEM assessment. The research found that using the SCS as a guide, and emphasising mathematics in STEM learning and assessment, allowed the teacher to: reflect on existing STEM capabilities; observe the enhancement of other capabilities; and identify areas for future development.

Teacher capability that supports mathematics active STEM learning is vital to the development of responsible, discerning citizens who can engage with, and interpret, the large quantities of data that characterise modern society (Maass et al., 2019). The purpose of this paper is to report on an aspect of a two-year study (*Principals as STEM Leaders (PASL)*—DESE, 2018–2020) that aimed to define and develop the STEM Capability Sets (SCS) for school leaders, teachers, and students. Hence, a component of the study was devoted to the development of a model for teacher STEM capability. In this paper, we focus on mathematics active instruction and the development of teacher STEM capability—an aspect that has been often underrepresented (English, 2015). To achieve this, the following research question will be addressed:

> *To what extent does a framework for STEM Capability Sets support teachers' planning and implementation of mathematics active STEM tasks?*

In responding to the question, this paper first presents a concise review of relevant literature. Second, descriptions of the theoretical model and methodological approach undertaken are presented. Third, we present a case study in which a freely available digital resource, Gapminder (https://www.gapminder.org/data/), is used to integrate mathematical ideas and procedures into a STEM assessment item. Finally, the teacher STEM capabilities required to implement the assessment item are identified and aligned with the SCS.

A Synthesis of Relevant Literature

Technological advancement, such as datafication, and economic, social and environmental factors are rapidly changing the way we live our lives (Blackley & Howell, 2015). The datafication of society drives all aspects of business and social enterprise. Digital technologies are used to facilitate activities such as work, schooling, social entertainment, shopping and healthcare. The viability of these services is reliant on businesses anticipating consumer needs through the collection of consumer data (Saha, 2020). As such, citizens require mathematical and digital skills to effectively: collect, interpret, and use data to drive economic growth; and to monitor and protect their shared consumer data.

Increasingly, data literacy also centres around the ability to discern meaning from data presented in the media or used by authority to influence the behaviour of citizens, such as seen

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 274–281. Launceston: MERGA.

during the COVID-19 pandemic (Nguyen, 2020). Such data are sometimes presented in ways that display bias and may be contradictory or ambiguous. This means that citizens require essential skills to critically engage and cope with the rapid mediatisation, digitalisation and social discourse surrounding COVID-19 data. Thus, there is an urgent requirement for citizens to have the mathematical capacity to critically evaluate data and associated information so that they can form balanced judgements and make informed decisions (Maass et al., 2019).

Despite the need for a focused emphasis on mathematics within STEM teaching and learning, concerns have been raised about the lack of balance in the way STEM disciplines have been represented in integrated STEM approaches. English (2015), for example, commented that in current conceptualisations of integrated STEM education, science is over-represented and dominates the field. The over-representation of science is arguably due to national policies that have emphasised the importance of scientific innovation for economic security. This implies that scientific inquiry may be taking precedence over the important role of mathematics in society. This is echoed by Geiger et al. (2015), who emphasised the importance of numeracy capability so that students can become informed, responsible, and participative citizens who are able to cope with the mathematical demands of modern life.

Blackley and Howell (2015) have suggested that the underrepresentation of mathematics within STEM education is related to teachers' lack the capability and confidence to teach mathematics within other STEM disciplines. Despite significant initiatives being undertaken to support teachers, there remains a need for increased national collaboration to increase teacher capability through identifying areas of concern, providing teachers with access to resources and knowledge, and transforming teacher practice and identity (Blackley & Howell, 2015). This is necessary to enhance best practice teaching of mathematics as a foundation for STEM learning within the context of solving real-world problems, such as through the critical scrutiny of data.

One way mathematics can be elevated within STEM learning is through mathematics active learning. This type of instruction promotes autonomous and collaborative student learning through problem solving, experiential learning and investigative work (Singh et al., 2018). Using mathematics active learning tasks as the foundation for integrated STEM education can provide students with the opportunity to apply mathematics when solving real-world problems (Singh et al., 2018). The impact of this on student development is highlighted by Maass et al. (2019), who argued that STEM education programmes grounded in mathematics are key to the civic, cultural, intercultural, and socio-critical development of citizens.

While arguments for increasing the primacy of mathematics within STEM learning are compelling (e.g., English, 2015), there appears to be limited research on how this can be achieved within school classrooms. In taking one step towards addressing this gap in research, the following section will outline the STEM capabilities of effective teachers. This will be followed by a case study of how one teacher's STEM capability was developed through teaching students to use a data analysis tool as a foundation for their STEM learning.

Conceptual Model

The study was underpinned by a conceptual model developed from a synthesis of relevant literature—the SCS. These are an extension of the 21st Century Model of Numeracy developed by Goos et al. (2014). The SCS identify STEM capabilities required for leading, implementing and sustaining STEM teaching and learning initiatives. They consist of five dimensions, which are aligned with sets of capabilities for principals, teachers, students, the community, and researchers. The five SCS dimensions are: STEM discipline specific and integrated knowledge and practices; contexts; dispositions; and tools, which are embedded within an overarching dimension—a critical orientation to STEM see (Figure 1).

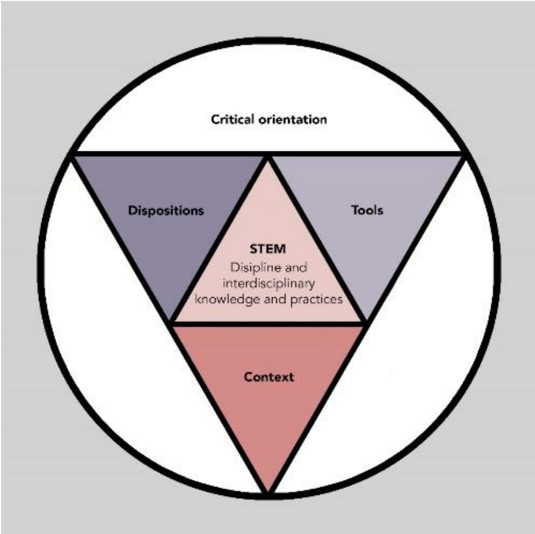

Figure 1. STEM Capability Sets (Geiger et al., 2017).

Within the SCS, discipline-specific (e.g., mathematics) and integrated STEM knowledge and practices are viewed as essential to effective teaching and learning. Teachers are required to operate across numerous contexts to meet individual student needs within the constraints of available school resources. In addition, positive teacher dispositions are necessary for effective problem solving to occur. Teachers also require capability in their ability to resource and use tools when teaching STEM. Tools are mediators of meaning-making and are central to mathematics and STEM practice. Examples include physical (e.g., rulers), digital (calculators, computers, applications—Gapminder), and representational (maps, diagrams) tools. Within the SCS, teachers must both possess, and foster in students, the adoption of a critical orientation to real-world tasks, which involves the making evidence-based judgements and decisions. Descriptions of the SCS for effective practice in STEM teaching, in alignment with the five dimensions, are presented in Table 1.

Table 1
STEM Capability Sets: Teachers

STEM discipline and integrated knowledge and practices (DIKP)	Competence with concepts, skills, and practices of at least one STEM discipline (DIKP1).
	Awareness of the range of careers that require STEM skills, and the role STEM plays in both individual well-being and national economic growth (DIKP2).
	Knowledge of relevant components of the Australian Curriculum and national policy documents (DIKP3).
	Capacity to provide leadership within STEM teaching and learning initiatives (e.g., teacher professional learning groups) (DIKP4).
	Capacity to select/design/design and implement STEM capability promoting curriculum (e.g., learning programs, tasks, pedagogies) – within at least one discipline and in an integrated mode working with others (DIKP5).
Contexts (C)	Capacity to identify and utilise STEM relevant real-world situations suitable for students' teaching and learning experiences (C1).

	Capacity to fit STEM learning experiences to the learning circumstance of students (e.g., learning needs, level of development) C2).
	Capacity to make links to students' diverse cultural backgrounds and acknowledge their contributions (e.g., Aboriginal and/or Torres Straight perspectives) (C3).
	Capacity to fit STEM learning experiences to the available school resources (e.g., digital technologies, built environment) (C4).
	Capacity to establish or be involved in partnerships (e.g., with industry, business, tertiary education etc.) (C5).
Dispositions (D)	Belief that they themselves can develop STEM capability (D1).
	Belief that all students can develop STEM capability (D2).
	Motivation to enhance STEM teaching practice (D3).
	Willingness and confidence to adopt STEM based approaches to teaching (D4).
	Preparedness to adopt flexible and adaptive use of STEM teaching practices (D5).
Tools (T)	Confidence and capacity to teach how to use material (e.g., models, measuring instruments), representational (e.g., symbol systems, graphs, maps, diagrams, drawings, tables) and digital (e.g., computers, robots, internet of things) tools to mediate and shape thinking (T1).
	Possess strategies to identify resources that support effective STEM teaching and learning within their school context (T2).
Critical orientation (CO)	Capacity to develop their own, and promote their students' capacity to gather and analyse evidence to: make judgements and decisions (CO1); add support to arguments (CO2); challenge an argument or position (CO3); and defend an argument of position (CO4).

Research Design

The Larger Study

The larger study, from which the aspect reported here was drawn, was conducted within a design-based research (DBR) framework (Cobb et al., 2003). Cobb et al. (2003) argue that DBR is particularly pertinent to testing innovations within educational contexts. This approach was suitable to the study because the researchers were testing the efficacy of the SCS, as an innovation for educational improvement, through the iterative provision of professional learning (PL) to participants.

One aspect of the PL provided to participants was the notion of becoming conversant with the use of digital tools as an essential component of the techno-mathematical literacies required to be an effective employee within the modern workforce. Throughout the PL sessions, the researchers made participants aware of freely accessible resources. One such resource was Gapminder—an online non-profit, independent educational research tool that is a database of reliable global data used to not only report on international trends, but also challenge misconceptions about environmental, social, and health issues prevalent in society (Gapminder 2022). During PL, principals were shown how Gapminder could be used to analyse real-world data and apply findings in a critical way.

The Current Focus for Investigation

Following the PL sessions, many principals conducted their own in-school PL and shared resources with staff, such as the SCS and Gapminder. One school used these resources to guide an assessment piece. Data from a classroom teacher at this school, provided insight into the development of teacher STEM capability and how mathematics can be highlighted within STEM learning contexts.

Participants from the school that is the focus of this study included the classroom teacher and 55 Year 5/6 students, spread across two classes. The teacher had a dual role, acting as both a classroom teacher and as the STEM leader within the school. The participating school was a non-government primary school (Prep to Year 6) in an educationally advantaged metropolitan region of an Australian city. There were 460 students enrolled in the school.

The paper is based on two iterations of interview transcripts, supplementary materials, and video data of student assessment submissions, which were provided by the teacher. The video data were of student group “pitches”, where they described how Gapminder data informed their prototype design and presented a “model” of what their prototype could look like. During researcher visits, interviews with the classroom teacher were conducted and audio recorded. These were transcribed as soon as possible after the visit. The transcript data w then analysed deductively, by way of content analysis (Mayring, 2015), using the SCS as an analytic lens.

Case Study

Renee is a primary school teacher with more than 20-years’ experience. Following attendance at the PASL PL sessions, the principal at Renee’s school conducted her own PL with teaching staff. An outcome of this PL was that Renee and her principal decided to modify an existing open-ended STEM assessment piece for their Year 5/6 classes to include Gapminder as a key resource. Using Gapminder was seen to emphasise the mathematical underpinning of STEM learning by teaching students how to analyse and interpret real-world data.

The modified assessment integrated Mathematics, Science and Humanities and Social Sciences into one STEM project. Within the assessment, students were required to identify real-world problems facing developing nations in Asia and to design sustainable solutions for one of the countries. Over a six-week period, students used Gapminder to analyse global data and inform their thinking about issues affecting these countries, such as access to water, electricity, food, and the impacts of war or trade policies. In small groups, students selected one country to focus on for the assessment. Students were asked to design a prototype for a product that would assist citizens to address an identified problem in a sustainable way.

As an example, one group of students first identified the Asian countries affected by poverty. After learning how to interpret trends in the data, the students investigated why each country had experienced changes to the number of people in poverty at specific timepoints (e.g., because of war, famine etc.). The students in this group then examined issues relevant to their selected country (Bangladesh), such as extreme temperature and access to electricity. Based on these findings, they created, tested, and refined a prototype for a solar power backpack that could be used to power handheld fans, solar lights, and other devices.

The assessment piece allowed Renee to emphasise the importance and relevance of mathematics active learning, by engaging with data (e.g., assumptions, graphs, trend lines, longitudinal data), as a foundation to STEM.

Results

Both Renee and the researchers observed development in her teaching capabilities within mathematics, the other individual STEM disciplines, and integration of STEM learning through her facilitation of the assessment. Her data were analysed, using the SCS (Figure 1 and Table 1) as the theoretical lens to understand her development. Changes to her understanding of mathematics-active STEM are documented against the dimensions of the SCS in the following sections.

STEM Discipline and Integrated Knowledge and Practices

After the assessment concluded, Renee observed she had attained a deeper understanding about both mathematics and integrated STEM knowledge, skills, and practices:

> I really understand mathematics and STEM better than I did last year… It has to be purposeful. It has to have a connection with the kids. It has to be a transdisciplinary approach to all of the subjects and the crossover of all of them, it has to be rich and authentic. [DIKP1; C1]

Renee also reported enhanced knowledge about using the Australian Curriculum to teach mathematics within STEM. This allowed her to design and implement an integrated assessment piece that promoted the goals of the curriculum:

> STEM is maths driven, with your critical and creative thinking at the core, driven through general capabilities. Whilst the activity, comes from understanding your curriculum, we need to recognise the general capabilities the kids are at and what we want from them. [DIKP3; DIKP5]

Contexts

Throughout the assessment, Renee developed her capabilities in using real-world situations and linking these to diversity of students' cultural backgrounds.

> We linked Gapminder to our Aboriginal Torres Strait Islander connections and compared to landlocked countries like Afghanistan. And they were saying "Oh, we'll just use the sea water" and we said, "How can you if it's landlocked?" And then "what are the tribal, what are the native people of that country? What were their sustainable ways of living and how can we borrow and use?" [C1; C3]

Renee also demonstrated some capability in tailoring instruction to fit the learning circumstances of individual students. She was able to:

> … sit with the kids and go through the process where they were stuck at their time, at their learning. So, some of these groups were here … and then some groups were down here but they're directing their own learning. [C2]

Dispositions

The researchers noted that Renee displayed positive beliefs about: students' capabilities, her capabilities, and high levels of motivation to enhance her teaching practice. Consistent with this, she was undertaking further study in a Master of Education programme.

> I am very passionate about it, and I am studying it. So, I think, not from the school's perspective—the principal supported me to get the scholarship for STEM so that's been brilliant. [D1; D3]

Throughout the assessment piece, Renee also saw the value of becoming more willing and confident to adopt STEM-based approaches to her teaching practice.

> I'm more aware of how I can embed mathematics into the curriculum, which means it's not an added extra. It makes us think more and ask questions. Ultimately, they're the core skills we're trying to develop - the general capabilities of deeper thinking, asking questions, reflecting. [DIKP3; D3; D4]

Tools

Renee demonstrated capability in guiding students in the use of a range of tools, particularly digital and representational, throughout the assessment. She learnt how to use Gapminder alongside the students, guided them to conduct further research online, and oversaw students using physical and digital tools to design their prototypes. Through this, Renee also demonstrated her ability to source and use tools that were available in her school context.

> They had to research. They had to find out "What is it that we want to find out about that country?" They had to articulate it and then look at a mind map of possible solutions…we've had everything from backpacks with solar panels, to using Minecraft to create their prototypes. [T1; T2]

Critical Orientation

The assessment task provided Renee with the opportunity to develop students' critical capabilities. Students developed their capability to critically analyse the data and conduct further research so they could effectively form judgements and make decisions about their countries of interest.

> The students took it upon themselves to go "What happened in 1988? Why did that graph go down?" So they researched the history of that country to find out actually it was war torn, or "They made trade links there Miss. The trade links mean they could export which meant the government had more money." This was the sort of stuff that was coming out. [CO1; CO2]

Renee took a focussed approach to encourage students in their critical thinking and reflection about their data research, which in turn demonstrated her own critical orientation to both the data, and to the wider goals of the assessment activity.

> I really focused on making sure the kids can think for themselves critically. We are really wanting them to pose open-ended questions about the data. So, at each stage I said to them "What do we know? What do we want to? What have I learnt?" [CO2; CO3; CO4]

Discussion

The Gapminder assessment, when analysed with the SCS, provides insight into Renee's existing, and developing, STEM capabilities. Renee demonstrated STEM capabilities across all five dimensions of the SCS, whilst also observing new learning. Renee's *STEM discipline and integrated knowledge and practices* were evident in her competence with the concepts, skills, and practices across several STEM disciplies and ability to design and implement a STEM learning programe. She also came to better understand that mathematics had to drive STEM education and that a thorough understanding of Curriculum requirements and general capabilities were essential.

Context capabilities were also evident in Renee's teaching. She was able to construct and use a real-world situation where students could integrate their STEM skills that was appropriate to the their learning and cultural experiences. Renee appeared to experience some difficulties, however, in tailoring the learning to the circumstances of all students, reporting that some students struggled for weeks to identify an issue from the data they had collected.

One area where Renee reported a high level of capability was in the dimension of her *Dispositions*. Renee consistently demonstrated her belief that STEM was for all and that everyone (both students and herself) could develop their STEM capacity. The research team also noted how willing she was to adopt STEM-based approaches to teaching.

Renee was also able to guide students to use a variety of material, digital and representational *Tools*, commenting that her own capacity was enhanced through the assessment. These included the Gapminder database, other web-based search engines, graphs, diagrams, video cameras and other software applications. Renee's ability to identify other resources that could support STEM learning was not observed during this study. An area for future investigation would be to explore how teachers identify resources that enhance the instruction of mathematics in STEM contexts.

The development of a *Critical Orientation* to STEM surrounds all STEM capabilities. Renee gave insight into her critical orientation to STEM, and her ability to foster a critical orientation in students, for example, by challenging students to defend their judgements. The scope of this study, however, did not afford the research team the opportunity to more broadly understand the depth and extent of Renee's critical orientation. This is an area for future investigation.

Conclusion

The teacher capabilities required to effectively advance mathematics active learning in STEM are described in the SCS. In this paper we have highlighted how an activity, aimed at emphasising mathematics as the foundation for STEM inquiry, can assist teachers to develop their own STEM capability. Renee's responses demonstrated that she had developed her thinking about planning for, and implementing tasks, across the dimensions of the SCS. While this preliminary research provides some tentative evidence about the potential to guide the planning and implementation of mathematics active STEM tasks in school contexts, further studies into the capabilities required of teachers in mathematics and STEM contexts are needed. For example, one area needing research attention is in teacher STEM leadership. Renee noted, in her final interview, that she was facing difficulties in leading school-wide STEM change. She faced challenges associated with finding time, financial constrants, and trying to encourage teachers who were not confident in their STEM ability, or who were unwilling to adopt STEM-based approaches. Whilst this is an area for future investigation, the SCS provide guidance that could assist teachers to enhance their capability in: STEM discipline knowledge, becoming aware of different contexts pertinent to STEM, selection and use of tools and other resources, and the development of a critical orientation to STEM.

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Assessing Mathematical Competence Through Challenging Tasks

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Making accurate judgements and interpretations about student growth and progress in mathematics can be problematic when using open-ended assessments. This study reports on the development of a class-based assessment instrument and marking key designed to assess Year 2 students' mathematics competence to reflect their learning of mathematics through a challenging tasks approach. A qualitative coding process was undertaken to analyse the written responses of 59 Year 2 students resulting in the development of a 7-point marking key to identify levels of progress. The marking key proved effective in supporting the interpretation of the written responses and identifying future learning pathways.

Introduction

Mathematics assessment is a central component within the teaching and learning cycle that can provide insight into student progress and inform future instructional decisions (National Council of Teachers of Mathematics [NCTM], 2014). For teachers to be effective in meeting this intention, it is recommended that assessment practices more accurately reflect students' learning experiences (Wiliam, 2007). An ongoing challenge in mathematics education is finding suitable methods of assessment that authentically reflect student learning and align to the reform orientations evident in contemporary classrooms. Despite a shift in mathematics education that encourages breadth and depth of curriculum knowledge, Dong et al. (2021) identified that many current assessment practices used for young students continue to rely on formal, narrow, skill-based tests. Clarke (2011) described these traditional assessment practices as misrepresenting both mathematics as a discipline as well as the actual student learning that has been experienced. Rather, class-based assessments based on open-ended or rich tasks enable teachers to ascertain what and how students are learning (Yeo, 2011). In a review of mathematics assessment throughout Australasian literature, Serow et al. (2016) identified that further research reporting class-based assessment is required across all levels of schooling. In particular, they recommended that such assessment should strive to provide students with adequate opportunities to comprehensively demonstrate their knowledge and understanding.

Context

The study reported in this paper was conducted as part of a larger project, Exploring Mathematical Sequences of Connected, Cumulative and Challenging Tasks (EMC^3), led by Professor Peter Sullivan and colleagues. Building on previous research of challenging tasks (see Sullivan et al., 2015), EMC^3 aimed to investigate how the sustained use of challenging tasks and the associated pedagogies supports Foundation to Year 2 students' mathematics development (Sullivan et al., 2020). Most of the research reporting on the benefits of learning mathematics through challenging tasks is limited to teacher professional development (e.g., Ingram et al., 2020; Sullivan et al., 2015) or student achievement in the middle to high school years (e.g., Sullivan et al., 2016), with some exceptions (e.g., Russo & Hopkins, 2017). Further investigations about assessing students and interpreting their mathematics competence when learning through challenging tasks in the Early Years will contribute to the literature.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 282–289. Launceston: MERGA.

Literature Review

The interpretation of what and how mathematics should be taught influences teacher assessment choices and therefore, perceptions of student competence (Nortvedt & Buchholtz, 2018). In Australia, the current review of the national curriculum has highlighted the ongoing debate on what constitutes effective approaches for teaching mathematics for students including the Early Years of schooling (Foundation to Year 2). One perspective is that students learn mathematics best when experiencing direct instruction and having time to practise skills and facts before they encounter problem-solving situations (Kirschner et al., 2006). Such approaches are aligned with traditional notions of mathematics achievement and progress is often measured by assessments where students are required to reproduce facts and procedural knowledge (Clarke, 2011). Others believe young students can attain meaningful conceptual knowledge and foundational skills through pedagogies that centralise problem-solving approaches (Chan & Clarke, 2017). Educators aligned with inquiry approaches believe that the opportunities to engage students with problem-solving can strengthen mathematical understanding and support them to adapt their knowledge to a range of contexts (Schoenfeld, 2007). To ensure this approach rigorously improves student outcomes, it is recommended that teachers take an active instructional approach and scaffold student learning with the use of guided questions and rich classroom discussions (Chan & Clarke, 2017).

Regardless of the mathematics teaching orientation with which teachers choose to align, Wiliam (2007) recommends that for assessment practices to be deemed effective and helpful for informing future instruction, instruments and processes should accurately reflect the learning experiences of students. One of the criticisms of inquiry approaches in the Early Years is the lack of substantial evidence reporting its effectiveness for developing sufficient mathematical competence. One reason why student competence may not be readily apparent could be due to the mismatch in pedagogies and assessment practices used in schools (NCTM, 2014). The intention of this study is to investigate assessment practices that can effectively identify student progress while also aligning with student experiences of learning mathematics through a challenging tasks approach.

Developing Mathematics Competence Using Challenging Tasks

Teaching mathematics through problem-solving is one way students' experience of school mathematics can accurately reflect comprehensive notions of mathematics competence. Schoenfeld (2007) believes that through the act of problem-solving, students are afforded opportunities to demonstrate flexible and resourceful thinking; use efficient and productive strategies; and develop persistence and resilience. The thinking and reflection that is required to successfully solve problems also supports students in developing conceptual networks that help them to make sense of basic facts (Baroody et al., 2009).

Problem-solving is considered effective when students are encouraged to engage with mathematics tasks that are considered cognitively demanding (Smith & Stein, 2018). A challenging tasks approach (Sullivan et al., 2015) is one example of how students can experience cognitively demanding mathematics in the pursuit of developing mathematics competence. Challenging tasks, often characterised as being open-ended, encourage students to solve non-routine problems by eliciting their prior knowledge and exploring multiple solutions (Sullivan et al., 2020). Student learning is supported throughout the experience with consistent pedagogies and structures that include clear lesson foci; the use of probing questions; and opportunities to develop reasoning through class discussions. To account for the various levels of student readiness and to increase accessibility for all students, this approach also advocates differentiating the main task through the use of enabling and extending prompts (Sullivan et al., 2020). Pertinent to challenging tasks is accepting that there are multiple interpretations of success with the belief that students demonstrate competence when they can apply and transfer their mathematical knowledge across various contexts (Sullivan et al., 2020). However, with so many manifestations of mathematics competence, it can be difficult for teachers to make appropriate interpretations about student progress as well as determine accurate directions for future learning.

Assessing Mathematics Competence to Align with Challenging Tasks

The literature suggests that class-based assessments should allow for the interpretation of student growth and support teachers to adequately prepare for future learning experiences (Clarke, 2011; Wiliam, 2007). This has seen the development of alternative assessment processes such as rich assessment tasks where student growth is evaluated through marking keys or scoring rubrics (e.g., Downton et al., 2006). In a recent study conducted in the Netherlands, assessment techniques were evaluated to determine their effectiveness (Veldhuis & van den Heuvel-Panhuizen, 2020). The findings indicated that alternative class-based assessments can be productive when teachers possess sufficient levels of pedagogical content knowledge to support the interpretation of results. This reiterates the necessity for teachers to have a clear focus or purpose when administering assessment in the first instance, as well as being able to use assessment data to support improvement of student learning (Wiliam, 2007).

Making judgements about the divergent outcomes that result from open-ended, class-based assessments appears to cause difficulty for teachers (Yeo, 2011). To support teachers in the interpretation of work samples, Tomlinson et al. (2015) suggested that grouping responses according to patterns may be helpful. Such processes are regarded as response coding (Clarke, 2011) or comparative judgement (Jones et al., 2015). In essence, grouping student work according to similar responses or a marked point of difference allows for the scaffolding and then categorising of distinct levels of competence. These processes are most effective when teachers have a clear understanding of the task's intention as well as an ability to make inferences beyond a particular solution (Tomlinson et al., 2015). Similar analytical approaches to interpreting mathematical competence could be helpful when assessing student learning experiences through challenging tasks.

Research Question

The purpose of this paper is to report on the development of a class-based assessment instrument and marking key created to analyse and interpret the mathematics competence of Year 2 students when solving challenging tasks. The instruments were designed to answer the following research question:

> *How can Year 2 students' mathematical competence be analysed and interpreted using an assessment based on a challenging task?*

Methodology

The study reported in this paper is part of a larger PhD designed to investigate Year 2 student perceptions of challenging mathematical tasks. The mixed method study included a qualitative design for developing a marking key, whilst quantitative data were used to analyse and make interpretations about students' responses to the assessment task.

Participants, Data Collection Instrument and Administration

Participants were selected from one of the EMC^3 project schools. Three classes of Year 2 students (N = 59) from a Catholic primary school in metropolitan Melbourne participated in this assessment investigation. Students' prior experiences of learning mathematics was mostly through traditional instruction with limited exposure to open-ended tasks that encouraged student reasoning.

The assessment instrument consisted of three items designed to determine the extent to which students could competently find all possible solutions when partitioning two-digit numbers. Figure 1 provides an example of an item from the assessment instrument. The item had a similar mathematics focus to tasks from teacher resource materials for the EMC^3 project.

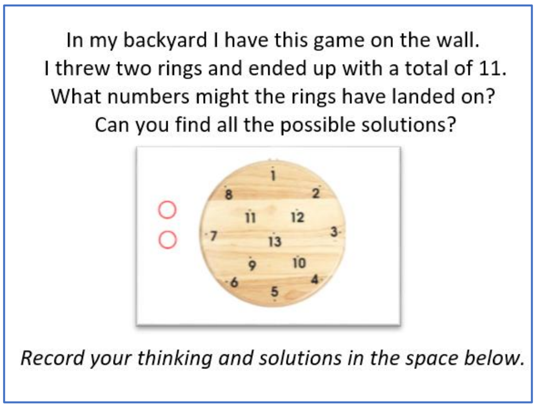

Figure 1. Assessment Item 1.

The item shown in Figure 1 can be described as non-routine having multiple possible solutions (for example, 6 & 5; 7 & 4; 8 & 3). Importantly, the context could be considered engaging and realistic for students in Year 2. The subsequent items were slightly more difficult. Item 2 required students to use two rings to find the possible ways to partition 15; and Item 3 asked students to partition 19 using three rings, with one ring already positioned on number seven. While this instrument focused on number partitioning, the open-ended components such as finding all solutions; recording systematically; and communicating mathematical thinking were all considered important elements to successfully demonstrate mathematics competence when solving challenging tasks.

The instrument was administered twice, using the same items each time. The first assessment occurred early in Term 1 and the second assessment occurred approximately nine months later. The specific items were not reviewed with the students between assessments. However, over the study students completed a range of other tasks from the project including suggestions from the Structure of Number Sequence to support their learning of the assessment concepts. It is important to note that during the timeframe of this study, school disruptions occurred in Melbourne as a result of COVID-19 restrictions.

Students were given 30 minutes to attempt to solve all three items. During this time, students were encouraged to read and attempt the items on their own. Students who experienced reading difficulties were able to have the task read to them although no additional guidance or prompts were provided about how to solve the task. Those who finished early were encouraged to review their responses. Such implementation is consistent with the introductory phase of the pedagogical approach used when implementing EMC^3 lessons.

Data Analysis

An initial marking key, created prior to any data collection, consisted of a general 4-point scale anticipating the range of possible responses. During the analysis it became apparent that these initial codes were insufficient in adequately identifying and categorising the range of responses. The coding of student responses was repeated using a combination of processes from the literature such as response coding (Clarke, 2011) and comparative judgement (Jones et al., 2015) to better distinguish and identify the different stages of progress evident within the data. Further details about the coding process are reported in the results

Results

The results are reported in two parts. The first section compares the initial marking key with the final marking key developed through the interpretation of student results. The second section reports both assessments scored according to the final marking key.

Developing the Marking Key

All written responses were read, sorted and coded according to an initial marking key which had a 4-point score range: 0 point (no response/ incorrect response); 1 point (one correct solution); 2 points (more than one correct solution) and 3 points (all correct solutions identified). When different examples of competence were identified throughout the analysis that was not specified within the initial marking key, the coding descriptions were modified and the responses were re-coded. This process was repeated until a final marking key was created that sufficiently represented the different levels of student competence within the data. Before a new code was included, the data were reviewed to determine if any other written responses also fitted in the alternative categories. Codes were changed only when there were at least three examples across all items that could verify the addition of a new category.

The final 7-point marking key (scores 0 to 6) in Table 1 shows the range in skill and knowledge development demonstrated by Year 2 students when participating in a written assessment reflective of a challenging task involving multiple solutions. The progression extends beyond content knowledge and accounts for broader mathematical skills such as comprehension of the context and pattern recognition. That all adjustments to the marking key were derived from the student data itself supports the validity of the categories within the marking key. Furthermore, when the marking key was presented to the EMC^3 project team (N = 8) there was consensus that the seven levels represented distinct developments in students' mathematical competence when solving open-ended, challenging tasks. It is also noteworthy that applying the revised marking key (with 7 levels) to the student response data generated a somewhat more reliable 3-item measure of student mathematical competence compared with applying the original marking key (with 4 levels). For example, the Cronbach alpha for the pre-assessment data applying the revised marking key was 0.70, whereas using the original marking key it was 0.62.

One of the more noticeable modifications to the marking key occurred for responses scored 0 and 1. For students who provided an incomplete and seemingly incorrect solution, some demonstrated a level of progress within their response that suggested that they had understood the context of the task and were accessing prior knowledge. For example, Student A who circled two digits on the ring board image that when added, made the target of 11 (e.g., 7 & 4) clearly demonstrated more comprehension than Student B who left it blank or wrote something irrelevant. Therefore, even though Student A made no attempt to formally record a solution (e.g., "7 + 4 = 11"; "7 and 4 is 11") their response was scored 1 point and the response by Student B was scored 0 points. The other prominent change in the marking key focussed on further delineating between responses that included all of the possible solutions. For this level of response, distinguishing between all correct solutions ordered systematically and those that were correct but recorded randomly, or that contained additional 'impossible solutions' (e.g., "12 – 1") became important.

Table 1
Final Marking Key

Score	Description	Elaborations
0	No attempt / completely incorrect attempt	
1	Some evidence of comprehending the task	Circling digits on the ring board that relate to a possible solution
2	One correct solution	
3	More than one correct solution	More than one solution but not all solutions
4	All solutions identified but additional incorrect/irrelevant solutions also included	Including subtraction: 12 – 1; 13 – 2 Including more partitions: 4 + 4 + 3
5	All solutions identified but unsystematic recording	Solutions randomly recorded No order or pattern identifiable
6	All solutions identified and correct, systematic recording and clear communication of solutions	Responses clearly demonstrating patterns, no errors evident

Cohort Data

Using the final marking key as our guide for all items, data were collated according to the frequency of different scores for both the pre and post assessment (Table 2).

Table 2
Year 2 Pre- and Post-assessment Responses (N = 59)

Item	/Score	0	1	2	3	4	5	6	Total
1	Pre	17	2	10	14	2	7	7	59
	Post	0	0	2	19	2	13	23	59
2	Pre	24	2	16	10	4	3	0	59
	Post	2	0	5	26	12	10	4	59
3	Pre	41	10	5	2	0	1	0	59
	Post	13	12	13	17	0	1	3	59

The frequency of scores obtained by students demonstrates various levels of student progress across the three items. Comparisons between pre and post assessments can be made at a particular score level, allowing for judgements to be made about specific progress of mathematics competence. For example, it is evident from the pre assessment that 17 students were unable to answer item 1 in any capacity. In the post assessment, all students were able to provide at least one solution to the task, demonstrating an improvement in students' ability to independently comprehend the task as well as the appropriate mathematics required.

Combining scoring groups can also help make judgements about mathematical competence. For example, in the pre assessment for item 1, around half of students (n = 29; 49%) found only a single solution to the task, which was indicated by a score of 2 or less. In comparison, the post assessment data revealed that only a small minority of students (n = 2; 3%) recorded a score of 2 or less, with almost all students (n = 57; 97%) now identifying multiple solutions to the task indicated by a score of 3 or more. These results show not only did students develop the core skills associated with scores 0-3 but also many were also able to demonstrate higher thinking skills with scores ranging from 4-6. Such insights can support teachers in the interpretation of student progress and necessary directions for future learning. In this instance, the emphasis on future learning might shift from focussing on the need to find more than one solution (following the pre assessment) to demonstrating knowledge that all solutions have been found (following the post assessment).

Discussion and Conclusion

The assessment instruments reported in this study were reflective of students' learning experiences, which according to Wiliam (2007), is necessary for establishing whether learning outcomes are met as intended. The inclusion of three similar assessment items within the instrument simulated the challenging tasks approach through which students had been learning mathematics throughout the year. Moreover, each item required students to demonstrate knowledge and skills beyond a single numerical response. In this particular assessment, students were required to comprehend the context of the task; access prior knowledge to solve the task; demonstrate flexibility with their conceptual understanding by providing multiple solutions; and communicate their solutions clearly. Demonstration of such knowledge and skills reinforce and reflect the mathematics competence intended by the EMC^3 project (Sullivan et al., 2020).

Developing and refining the marking key from the raw data enabled the identification of nuanced differences about how student mathematics competence progresses when learning through challenging tasks. The identification of different scoring codes within the marking key provided a clear scaffold through which interpretations about student learning with challenging tasks could be made. Furthermore, the results demonstrate the versatility in which a marking key can be applied to support teachers in making inferences about mathematics competence on many levels. These processes may alleviate the confusion and uncertainty that comes with interpreting open-ended assessments (Tomlinson et al., 2015) and support teachers in strengthening their knowledge for teaching mathematics (Veldhuis & van den Heuvel-Panhuizen, 2020; Yeo, 2011).

This study aimed to investigate how assessment of student progress can be designed to align with their experiences of learning mathematics through a challenging tasks approach. The marking key derived from student responses captured the diverse range of knowledge and skills that Year 2 students demonstrated when they solve non-routine, cognitively demanding tasks. While limitations of this study include the small sample size across one school setting, the processes described here may provide a template to initiate further research into assessment of challenging tasks in the Early Years.

Acknowledgements. EMC^3 was funded by the Australian Research Council, Catholic Education Diocese of Parramatta and Melbourne Archdiocese Catholic Schools (LP 180100611).

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Developing Equitable Participation Structures

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Participation structures are important in relation to who gets equitable access to mathematics within classrooms premised on co-construction of mathematical reasoning. This paper takes a strength-based focus to explore how two teachers extended their Pāsifika students' known repertoires of practice to encompass others, which supported them to better engage in mathematics. The data illustrates the importance of teachers drawing on task contexts and student ways of knowing and being to facilitate student engagement in reasoned discourse and argumentation. The teacher actions provide a model which other teachers could draw on when teaching mathematics to students from predominantly collectivist groupings.

In mathematics who participates and how is closely aligned with who and who does not achieve mathematically. Aotearoa/New Zealand like many other countries have particular groups of students who fit into this category of underachievement, including a large number of Pāsifika and Māori learners (Hunter & Hunter, 2018). Within this paper we propose that these low levels of achievement can be directly attributed to the participation structures many Pāsifika and Māori students encounter in our mathematics classrooms. We contend that these should be addressed as a key equity issue.

Core to participation structures is recognition of the significance of all students being able to actively engage in the reasoning of others through participating in collaborative mathematical discussions. Educators, researchers, and policy makers alike verify the importance of co-construction of collective understandings to advance rich and deep mathematical knowledge (Civil, 2014; Franke et al., 2015). Co-construction in mathematical activity asks for interdependence of students contributing both individual and shared knowledge, strengths, and perspectives (Calor et al., 2019; Kaendler et al., 2015). However, when considering the required interactive collaboration many factors need to be considered, not the least being close examination of equitable ways for all students participate. There appear to be a paucity of papers which specifically explore equitable participation structures which draw on the background of students from collectivist societies' ways of *knowing and being*. To this end, in this paper we take a sociocultural perspective to examine and explore how diverse learners from collectivist societies can be scaffolded to draw on their ways of knowing and being to collaboratively co-construct mathematical understandings. The question we ask is; what are the actions teachers of students from collectivist societies (in this case Pāsifika students) take, to scaffold these students to build on their own collectivist strength-based participation patterns in mathematics lessons?

Literature Review

In this paper, we take a sociocultural and situative frame and a strength-based perspective rather than what Valenzuela (1999) describes as a subtractive focus. Valenzuela suggests that within a subtractive model the dominant cultural group's ways are assumed to be the only ways of *knowing and being*. As a result, the cultural and language backgrounds of those from non-dominant groups are seen as problems not resources. As a direct result many non-dominant communities adapt to ways of *knowing and being* of the dominant cultural group giving up their own known and successful interaction patterns. In a subtractive model subtle messaging related to what represents a normatively appropriate person in schools and classrooms suggests concepts of self as independent and self-reliant (Stephens et al., 2012). For many Pāsifika

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 290–297. Launceston: MERGA.

students in mathematics classrooms in Aotearoa their concept of self, causes them dissonance with normative expectations as independent beings in the school environment and interconnected and interdependent beings in the home context (Hunter & Hunter, 2018). Hunter and her colleague reported how these students described a negative sense of self and their culture when expected to work within ability mathematics grouping premised on individualism and competitiveness within the school setting. In contrast, these same students reported an increased positive sense of well-being when working within heterogenous multi-strength-based groupings in collaborative co-construction of mathematical reasoning.

A strength-based approach supports notions of drawing on communities' cultural strengths resulting in building towards more flexibility in ways of *knowing and being*. This includes not only drawing on mathematics tasks which are contextually appropriate but also developing different ways for interacting while doing mathematics. Supporting students to build on and extend from what Rogoff and colleagues (2017) describe as their repertoires of practice (as those known within their community setting) the students are enabled to be more adept at selecting which repertoire of practice is appropriate to other contexts. In examining the cultural strengths of Pāsifika children raised within a collectivist world, a key emergent element is an expectation of them working together in collaborative ways. From a very young age these children, within their homes are expected to take the initiative in many key aspects of family life including cleaning, food preparation as well as some economic activity (Hunter, 2021). Most often these collaborative interactions and reciprocal responsibilities are learnt at the knees of elders (Alefaio, 2019). Alefaio describes how respect of elders underpins close listening and watching by younger community members as they learn communal patterns of interactions. Alefaio outlined how very young Samoan children took responsibility for younger siblings and inducting them into learning how to do household tasks as integral to supporting the family.

Family is a core value. Encompassed within this Pāsifika value is a sense of family, as thinking and working as one unit rather than as individual members of the unit (Hunter & Hunter, 2018). Rogoff and colleagues (2017) describe these ways of *knowing and being* as sophisticated collaboration. These researchers describe how they observed children from indigenous backgrounds in the Americas as they co-constructed thinking through building on each other's ideas to progress a collective goal. Alefaio (2019) also described how Pāsifika children achieved collective progress with little discussion but through close watching and noticing the activity of each other. All these researchers describe how such actions supported the children to adapt and align their own activity towards a shared goal almost synchronously as one. This contrasts with observations made by Rogoff and colleagues of middle-class European American children. Although these students were tasked with working collectively, they tended to work as individuals within the group, allocating separate tasks and not attending to the activity or thinking of each other. Therefore, conflict developed when asked for a shared response as each individual child competed for acceptance of their own ideas.

Establishing mathematics classrooms where all students actively participate in collaborative co-construction of reasoning requires social norms based within the relational trust and reciprocal respect found within families within the collectivist frame. Researchers (e.g., Fletcher et al., 2011; Tamarua, 2006) described how Pāsifika students in their homes learn through collaborative reciprocal relationships and when interviewed stated that they would prefer to learn in this way at school. Hunter and Hunter (2018) and Civil and Hunter (2015) illustrated how Pāsifika values set within this perspective of family, support other important interactive factors needed in mathematics classrooms structured in that way. They showed how mutual trust and respect founded within notions of how families interacted with each other, provided students with room to ask each other challenging questions, to engage in mathematical agreement and disagreement, be safe to take risks and make mistakes, and use humour and small talk to reduce tension. Hunter and Hunter showed the importance of establishing productive mathematical discourse within Pāsifika values. They illustrated how through building on the students' own language, mathematical argumentation became 'friendly arguing'. They explained how this resulted in students perceiving this type of arguing as positive and focused within on the mathematics, and not on a person.

Nevertheless, we know that asking teachers to build such mathematics classroom environments is challenging particularly given the paucity of models, teachers have access to, of how these might look. However, their importance is critical as a key equity issue to support all students to access participation structures in mathematics classrooms. So, our aim in this paper is to provide one possible model of how two teachers scaffolded their students to build and extend their repertoires of practice through using their family-established strength-based interaction patterns to co-construct mathematical reasoning collectively in the school setting.

Methods

The research reported in this paper is a small section of a large on-going longitudinal project which has spanned more than fifteen years in Aotearoa/ New Zealand. Participants in the larger project over that time, have included more than 2000 teachers and their students in urban primary schools. These schools are set within low socio-economic communities with a high proportion of Pāsifika nations' students in attendance. The teachers come from diverse backgrounds and have varying teaching experience. The two teachers selected to report on in this paper, were both experienced and of mixed European and Pāsifika nation's heritage but born and schooled in Aotearoa/ New Zealand. The students were predominantly Pāsifika and aged 9–10 years.

Data collected across the project included video recorded classroom observations, field notes, questionnaires and teacher interviews.

The two teachers selected and reported on in this paper were chosen as a representative sample of the common culturally sustaining (Paris, 2012) pedagogical practices observed in a larger set of classrooms. To select the initial set of teachers the first researcher had reviewed a wider selection of teacher video records and from these she narrowed the focus to a smaller group of teachers and students who showed specific attributes the two researchers had agreed upon. The analysis of the larger teacher video records consisted of comparing and contrasting actions and responses and from this developing codes. From the codes emerging themes and patterns were identified which directly related to culturally sustaining pedagogical practices and which were consistent actions across the different teachers and their students.

Findings and Discussion

The classroom mathematics lessons consisted of pedagogical practices embedded in ambitious mathematics teaching (Kazemi et al., 2009) and culturally sustaining practices (Paris, 2012). This open and flexible pedagogy was premised in teacher noticing and responding to both student's reasoning and their participation patterns. High level, challenging and group worthy mathematics tasks (Featherstone et al., 2011) were used, rewritten within known Pāsifika contexts. Lessons followed a pattern promoted by Smith and Stein (2011). Each lesson followed a similar format where problems were launched and social norms discussed, then students worked in small groups of four to co-construct mathematical solutions. Lessons ended with large group sharing session of sequenced student explanations, discussion founded in inquiry and argumentation and ended with connecting to a big mathematical idea.

Making Cultural Connections Through Tasks

The two tasks selected both had close links to activities in the local community, to ensure that students did not struggle with both context and the mathematics. Teacher 1 used a task that linked to a local night market and food commonly sold at this market as family fundraising.

Task 1: Aliyah and her cousins are helping their family sell topoi (Fijian Indian dumplings) at the market. They sell the topoi in big polystyrene boxes to people who are having a family celebration. In each box the girls are told to put 26 topoi. If Aliyah's mother and her aunties between them have cooked 425 topoi. How many boxes of topoi will they have to sell? How many will be left over for the girls to eat?

Teacher 2 selected a task that involved the making of a tivaevae (bed quilt). Making these is an enterprise embedded in family and wider communally based activity well known within Pāsifika nations peoples and is an item with patterns common within most of their Pāsifika students' homes.

Task 2: A group of Mamas are working on a tivaevae design.

This is the 1st position

This is the 2nd position

This is the 3rd position

How many leaves does it have? How many leaves does it have? How many leaves does it have?

They would like to turn the pattern from the cushion cover into a bedspread. Can you help them work out how to keep the pattern the same?

Talk in your group about how you notice the pattern growing. Can you represent what it looks like using numbers? Think about how many leaves would the 6th position have?How many leaves would the 12th position have? How many leaves would the 51st position have? Can you come up with a rule to find out how many leaves there would be in any position?

The two teachers began by launching the task with an immediate emphasis on ensuring that all students had access to the context. Both ensured that they gave voice, as well as intellectual ownership of the cultural context, to specific students who came from the island nation the problem related to. For example, teacher 1 asked a Fijian Indian student to elaborate on making and eating topoi. Then she extended discussion to have Vietnamese students describe similar Vietnamese dumplings and Cook Island students describe coconut buns. She included other students sharing their experiences of buying and selling food at the night market and the way in which rectangular polystyrene boxes were used for transport and storage. Through close listening and responding to student explanations she gradually drew out important points which would support the students to construct a mathematical explanation. Finally, she shifted discussion to the mathematics and collectively the group agreed on what the problem was asking them to do (find out) without her providing any hints or suggestions of procedures.

Using similar actions, the two teachers ensured that all students could not only relate to the contexts but were able to see mathematics as alive and relevant within their lives beyond school mathematics. The challenge of the tasks was maintained but at the same time their group-worthy features required a multi-strength approach to solve (Featherstone et al., 2012). Featherstone and colleagues promote need for tasks which call on a range of different attributes from group members and which need to be pooled to gain collective solutions. These tasks fitted well within this criterion. Using tasks in this way go some way towards constructing a learning environment which draws closer to the home learning environment of many Pāsifika learners described by Fletcher and colleagues (2011) but which also build their mathematical disposition.

Using the Contexts of Tasks to Establish Social and Mathematical Norms

The very essence of the problems was set within family and community contexts which involved collective action and modelled communal problem solving. This gave both teachers an opening to draw on known ways these Pāsifika students worked together at home to establish the required social norms for group interactions in the mathematics lessons.

Both teachers referred to how the families in the problems worked together as a collective. Teacher 1 used the context of the problem in establishing her independent mathematics groups: "Today you are going to work together like you do in your family, like Aliyah is doing." Then she discussed with them in more depth the different roles the individual members had in making, packing, and selling the topoi and the way in which they were all dependent on each other for the final product and sale.

Teacher 2 also referred the students back to the way in which all the mamas worked together in making the tivaevae from cutting through to sewing the complete bedspread. Both teachers emphasised that at the completion of the tasks the ownership was shared rather than individual. For example, teacher 2 told her students:

> So, think about how you are going to work together today in your small mathematics groups. You are going to really need to listen and watch really carefully and depend on each other to explain and answer questions and get your shared solution like your mamas do when making things like mats and a tivaevae. That is because what your group explains in our sharing group it will be all your shared thinking put together just like those beautiful tivaevae share who your family are.

These actions provided the students with opportunities to explore the mathematics through drawing on a repertoire of practice (Rogoff et al., 2017) they were familiar with in their home context, but which is not common in New Zealand/Aotearoa classrooms. The close watching and listening as another way to learn mathematics, Alefaio (2019) described as the way Pāsifika children from a very young age learnt to participate and contribute within collective family activities.

Both teachers also drew on aspects within the contexts of the problems to establish other mathematical behaviour they were currently promoting in mathematical activity. For example, Teacher 1 extended the discussion to include a focus on student need to ask challenging questions and their need to engage in mathematical argumentation when she asked: "But if one of the Aunties doesn't put enough flour in, or Aliyah starts to put topoi just anywhere in the box, what would you do?" After listening to the students as they offered their ideas, she pressed the focus towards how she wanted them to engage in the mathematics:

> I know that all of you sometimes hold back when asking each other hard questions or challenging thinking because you worry about losing mana (status or loss of face) or them losing mana. But you do not need to worry, it's not about anyone it is about the maths and the thinking, and you do it in a respectful way, a polite way like you would at home. You really need to ask for more explanation or justification because that is what doing maths means but do it in a way that Aliyah's family or Aliyah would do with friendly arguing, but you are doing it about the mathematics.

In this way, she established their need to shift past their own reticence to ask each other questions that challenged reasoning even when they worried that the explainer might not be able to respond.

In a similar manner Teacher 2 used an aspect of the problem to engage the students in thinking about how they might consider making errors as a learning opportunity for the whole community. She began by talking about a time when she was involved in sewing a section of the tivaevae. Her stitches were uneven and one of her aunties had got her to unpick a section and resew it but in doing so had modelled and explained carefully how to redo it so that she felt that her mana was intact. After she explored what would happen if she had not been supported in this way, she asked "how is this similar to working with your group during maths?" The students identified that if they disagreed with an idea, they could then rework the solution. Continuing the discussion, she then outlined the similarity between making the tivaevae and co-constructing a mathematics solution in saying:

> Remember a tivaevae is a taonga (treasure) and so if you do not tell someone that they are making a mistake it ruins the beautiful gift. Well, it is the same in maths and when you are working together. If you don't tell someone when there is a mistake, then your explanation, your gift to all of us will be ruined and you have lost the opportunity to learn more together.

Through such actions these teachers inducted their students into use of productive discourse while all the time they remained cognisant of their need for comfit to shift between culturally accepted interactions and those required within productive mathematical interactions. Both teachers were grounding the need for productive mathematical discourse within relational trust and reciprocal respect (Fletcher et al., 2011; Tamarua, 2006). As Civil and Hunter (2015) and Hunter and Hunter (2018) previously described, these values are key to providing Pāsifika students with opportunities to engage in participation structures which support them learning rich and deep conceptual mathematics.

Maintaining the Social and Mathematical Norms during Group Activity

As the students worked together collectively within their independent small groups of four the teachers monitored both the mathematics and the participation of the students. Focus was maintained on how they engaged in questioning and challenging their reasoning. They were frequently heard to comment out loud at what they observed. For example, Teacher 1 says:

> Great, Tasa I just heard you ask a good question and that helped everyone make sense of what Katalina is saying. I saw you hesitate for a moment and that is okay because then you asked in a way that was respectful of Katalina and gave her a chance to explain or have others in your little family add bits and altogether you had some great thinking.

In this way, the press on productive discourse was maintained but students were able to see that it was all right for them to feel a sense of reticence, but they needed to work through it.

Teacher 2 also continued to monitor the participation, but she also built on their patterns of interaction which supported co-construction of reasoning embedded within the mathematics as it was developing. For example, she closely observed them as they examined the pattern:

> Student 1 records +8 on their sheet as she says: It adds by 8.
>
> Student 2: But it has 4 in the middle.
>
> At this point student 3 takes the pencil and draws the pattern as he explains: Every time you add on you plus 8.
>
> Student 2: What about for position 7?
>
> Student 3 explains as he circles one group and adds dots for more: It would be 7 groups of 8.
>
> Student 4 has been silent but watching closely and now he takes the pencil, and records 7 x 8.
>
> Student 1: But what about the four in the middle?
>
> In response student 4 writes + 4
>
> Student 2 then points at the final recording on the sheet, and she asks: So what were we thinking there?
>
> At this student 1 picks up the pencil and uses it to link the pattern student 3 drew to the recording of +8 as she explains: Well, we started by saying that it was plus eight and plus 4 but we realised that we can just as easily go seven times eight…

Student 3: And we didn't forget the middle so plus four all the time.

The teacher has been watching without speaking and now she says loudly: It has been great watching how you are working together and really watching so closely everything anyone else says and does and listening to each other but also great question about what you were thinking because then you all had to check out your understanding of your explanation together.

It was clear that these students were almost acting together in a symbiotic way evidenced by the use of the word "we" used normatively. Rogoff and colleagues (2017) described such interactions as sophisticated collaboration which they observed within students from other communal societies. Few words were spoken but each student totally engaged in the actions of other's and through this process co-constructed their group explanation. The teacher recognised this way of interacting as important and chose to highlight this for the other students. In this way she provided permission for all students to work together collectively in whatever way they found comfortable but also constructive in their mathematics learning.

Conclusions and Implications

We began this paper with the assertion that one possible reason for lack of achievement for many Pāsifika students can be attributed to the participation structures of mathematics classrooms in New Zealand/Aotearoa structured on those of the dominant cultural groups. As part of embracing equitable practices which are culturally embedded, we argue the need for teachers to also consider other forms of participation structures possible. This requires teachers to consider what is often outside their own experiences, but as this paper shows is needed and can be achieved.

The concept of family within a Pāsifika view has an important role. As the students stated to Fletcher et al. (2011) their preferred style of learning was within and as a group. Like the students in this study this reflected their preference for their own ways of *knowing and being*. In this form of learning this was comprised of them acting as one organism and co-constructing mathematical reasoning through sophisticated collaboration (Rogoff et al., 2017). Through the teachers supporting their interactions based on their home and community ways of being, they were able to engage in a positive strength-based approach rather than what Valenzuela (1999) described as a subtractive one. Their normative sense of being as a symbiotically interconnected and interrelated units was maintained, rather than them experiencing the more commonly known within mathematics classrooms in New Zealand/Aotearoa of a normative sense of sense of self as independent and unrelated (Stephens et al., 2012). Considering family as a collective operating as a single cell or organism calls for teachers to integrate their own views of how the family operates as a unit. Implications suggest that the expected norms of students working independently within mathematics activity needs direct addressing so that all students have opportunities to engage in ways which better match their own cultural values.

Throughout both lessons the teachers pressed for productive mathematical behaviour by building on their known ways of *knowing and being* embedded within aspects of the problems. Both teachers adapted appropriately challenging mathematics problems from a website which they then rewrote within their students' cultural context. This needs to be part of teacher planning where an equitable action would be for them to consider both who the context relates to, and how they can use the problem context to engage all students. Careful consideration of this would avoid a situation often seen in classrooms where Pāsifika are often left to struggle with both the context and the mathematics (Hunter & Hunter, 2018).

The teachers' explicit talk supported the students to engage more readily in reasoned mathematical discourse and argumentation. In previous research Hunter and Hunter (2018) outlined the diffidence many Pāsifika students showed when asked to question or disagree with the reasoning of their peers. In these classrooms, the teachers clear focus on issues of mana allowed students to move past these feelings of whakamā (sense of shame and loss of face). As

Rogoff and colleagues (2017) argue, all students need to be able to select an appropriate repertoire of practice flexibly and adeptly in classrooms. This is what these teachers were ensuring through their deliberate talk and actions; the students were being given opportunities to extend from their known repertoire of practice to become more adaptive within those most often encountered within classrooms in New Zealand/Aotearoa. However, for both groups of students it was made clear that their ways of *knowing and being* were seen as strengths and this enabled them to select how they functioned within the mathematical discourse. Taking a strength-based focus has a lot to offer those students so often inequitably positioned. There is a lot of work to be done but as this paper shows, it is achievable.

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“It has the same numbers, just in a different order”: Middle School Students Noticing Algebraic Structures Within Equivalent Equations

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In this paper, we explore student solutions to a free response mathematical assessment task which had opportunities for students to notice structural properties in the context of number systems. In total, 308 students aged between 10 years to 13 years participated in the study. Their responses were analysed to determine whether they noticed algebraic structures in a task using equivalent equations. Findings indicate that students were able to recognise equation pairs that drew on both the associative and distributive properties. A limited number of students were able to notice the general structure and draw on number properties to support their claims, moving beyond using algorithmic thinking.

Introduction

In recent years, there has been an increased recognition of the importance of developing algebraic reasoning in primary and middle school classrooms. This has included a focus on growing patterns and the development of functional thinking, supporting student capability to engage in the generalisation process and the unification of arithmetic and algebra as a unified curricula strand (Chimoni et al., 2018; Fonger et al., 2018; Hunter & Miller, 2021). Across these different areas, positioning students to notice mathematical structure is a key aspect of supporting students to make sense of mathematics, understand operations as mathematical objects, and both engage in algebraic transformational activity and make sense of transformations (Kieran, 2018; Schifter, 2018). We draw on Schifter’s (2018, p. 310) definition of mathematical structure as “behaviors, characteristics, or properties that remain constant across specific instances”. Of interest, is students’ capacity to work flexibly with numbers and equations and to notice relationships and mathematical structure. In this paper, we explore student solutions to a free response mathematical assessment task which had opportunities for students to notice structural properties in the context of number systems. We examine the responses of students within the age band of 10 years to 13 years old in relation to whether they noticed algebraic structures in a task using equivalent equations. We also investigate the properties that they identified and the explanations provided by the students. This paper contributes to previous research in relation to interrogating student capability to use structure and relationships when working with equations.

Literature review

Noticing Mathematical Structures Across Equations

The ability to notice and identify structure is central to mathematics and one’s ability to work with generalised forms. Mason et al. (2009) articulated that mathematical structure is “the identification of general properties which are instantiated in particular situations as relationships between elements” (p. 10). In the context of number systems, structure is typically referred to as the properties of arithmetic such as commutativity, associativity, and

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 298–305. Launceston: MERGA.

distributivity properties, identity law, and an understanding of inverse relationships; and these properties are generalisable (Kieran et al., 2016). Warren (2003), further expands this definition of structure in number systems to include: "(i) relationships between quantities (for example, are the quantities equivalent, is one less than or greater than the other); (ii) group properties of operations (for example, is the operation associative and/or commutative?; do inverses and identities exist?); (iii) relationships between the operations (for example, does one operation distribute over the other?); and (iv) relationships across the quantities (for example, transitivity of equality and inequality)" (p. 123–124). However, in the teaching of arithmetic, it appears that students are rarely given the opportunities to focus on and build an appreciation of these structures when forming generalisations (Arcavi et al., 2017).

In primary school mathematics, considerable attention is given to providing students with opportunities to engage with a range of equations across number systems. Despite this, little attention is given to the structure of these equations, with the majority of the focus being on teaching the procedures of how to calculate (Schifter, 2018). It can be argued that students frequently do not notice the structural differences between each of the equations to form generalities. Schifter argues that a "consequence of such absence is the lack of salience of the operations in students' minds. The operations are interpreted as instructions to perform a set of steps rather than as objects, each with its own set of characteristics and properties" (p. 325). Moving away from a procedures/operations focus on arithmetic, to one that examines addition, subtraction, multiplication and division properties, gives space for these to be stand-alone mathematical objects (Kieran, 1989; Slavit, 1999). This is where students demonstrate relational understanding and reasoning (Schifter, 2018). However, research conducted by Arcavi et al. (2017), demonstrates that when students' have a compulsion to calculate numerical answers it presents a barrier for them to recognise the patterns and mathematical structures. This presents a challenge to shift students' attention from "calculating" to that of noticing the underlying structures of operations to see these as mathematical objects, which is essential for algebra.

Making the shift from arithmetic to algebraic thinking across number properties and equations requires students to understand equivalence and equality. Research that examines this shift typically focuses on equality and how students understand the equal sign. Matthews et al. (2012) highlight that tasks typically used in this research fall into the following categories: (i) solving open equations, such as $9 + 4 = * + 6$ (e.g., McNeil, 2007; Powell & Fuchs, 2010); (ii) equations focusing on true or false statements (e.g., Molina & Ambrose, 2006; Seo & Ginsburg, 2003); (iii) students verbally articulating what the equal sign means (Knuth et al., 2006; Seo & Ginsburg, 2003); and, finally, though typically not as common as the other tasks mentioned (iv) advanced relational reasoning items, such as asking children to solve the equation $24 + 57 = * + 55$ without having to add $24 + 57$ (e.g., Blanton, et al., 2011; Carpenter et al., 2003). We note that much of this research has informed the initial underpinnings of how to begin to support students to make this shift from arithmetic to algebraic thinking. However, to date, there appears to be little research that focuses on how young students articulate the structures and relationships they notice when considering how two equations are equivalent and which generalised number properties they are drawing from.

Methodology

This study was exploratory in nature and used a qualitative case study design. We were interested in examining student solution strategies to a free-response mathematical assessment task that involved structural properties and relationships in the context of number systems. Our research aligned with better understanding the advanced relational structures students notice and the reasoning they provide to articulate these relations. In particular, the study addresses the following research questions:

1) *What do students notice in a task involving algebraic structures with equivalent equations?*
2) *How do students notice and explain structural properties and relationships in the context of number systems?*

Participants

The participants were 308 middle school Year 7-8 students aged between 10 years and 13 years old. The data were collected from two low decile schools. Decile ratings in New Zealand are based on census data of households with school-aged children and use household measures such as income, government assistance, occupation, and education with a low decile rating indicating that students live in low socio-economic communities. The students were from a range of ethnic groupings with most being Pacific nations ethnic grouping (55%), followed by Māori (23%), and Pakeha/European (13%), and included 151 male students and 157 female students.

Data Collection

The students were given a free-response task developed by the first author. This consisted of a set of equations (see Figure 1), follow-up prompts and two blank pages for the students to comprise their response. The equations were designed to be matching pairs which could be identified through noticing structural properties and relationships in the context of number. This included aspects such as the associative property of addition and multiplication, the distributive property of multiplication, and exponents, with an over-arching focus on equivalent relationships. Each pair of equations was developed to match a specific property or relationship. The prompts following the number sentences were provided to position the students to notice, describe, explain, and generalise the structural properties without the need for calculation.

76 x 15 = 37 + 43 + 40 + 36 = 99 ÷ 3 ÷ 3 =

7 x 86 = 99 ÷ 9 =

6^3= (70 x 5) + (70 x 10) + (6 x 10) + (6 x 5) =

37 + 40 + 36 + 43 = 12 x 22 = 6 x 6 x 6 =

(7 x 90) - (7 x 4) = 4 x 66 =

Look at the number sentences above:

- Describe what patterns you can find
- Why do your patterns work?
- Do they work with other numbers?

Figure 1. Number sentence task

The task also aligned with the New Zealand Curriculum (NZC) (MoE, 2007) elaborations for Level 4 which relates to Year 7–8 students. The NZC outlines an expectation that students are both expected to generalise properties of multiplication and division and describe these using appropriate mathematical terminology and/or symbols. Additionally, students should be able to use the properties when operating on numbers. There is also a developing expectation at these year levels that students should be able to understand and use mathematical notation.

The item related to exponents and mathematical notation was included to address this expectation.

The task was administered by the classroom teacher and completed by all students independently and individually during their regular mathematics lesson. Students were provided with adequate time to complete it. The students were advised that the assessment task was not a test but an opportunity for them to show what they knew in mathematics. Student responses on the two blank pages were collected and wholly scanned for analysis by the research team.

Data Coding and Analysis

In the first instance, all student responses were coded as either: (i) identifying structure and relationships in the task; or (ii) as not identifying structure or relationships in the task. Those responses that were coded as student being able to identify structure and/or relationships, required an explicit response that included the identification of one or more mathematical structures in the number sentences. These explicit responses may have been represented as drawing arrows to represent matched equations, re-writing the equations together, or providing a more detailed written description (see Figure 2). Responses coded as no identification of structure or relationships were those where the student did not explicitly identify any mathematical structures in the number sentences.

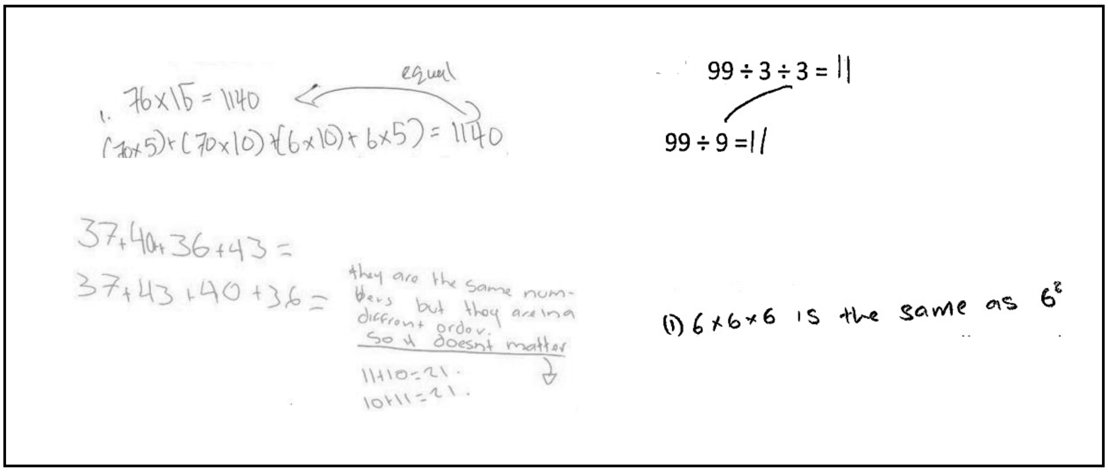

Figure 2. Student responses coded as identifying structure.

The dataset of student responses that were coded as identifying structure and relationships were then re-analysed. The second phase of analysis consisted of three aspects. First, the responses were analysed in relation to the pairs of equation (based on mathematical structures and relationships) that were identified. Second, the response for each pair of equations were classified as calculation or relational dependent on whether the student had calculated the answers when identifying the mathematical structure or had used relational strategies without calculation. Finally, student explanations were analysed qualitatively to identify themes and to examine the differing levels of sophistication in the explanations that were provided.

Results and Discussion

Identifying Structural Properties in the Context of Number Systems

From our initial analysis, we found that the majority of students (n = 204 or 66%) treated the task as a computation activity and solved one or more of the equations without responding to the prompts in relation to the patterns. This aligned with past findings from research (Arcavi et al., 2017). Instead, the students recorded calculations and solution strategies for the equations without referring to the structural properties. In contrast, a relatively small proportion of students (*n* = 104 or 34%) identified structural properties and described these and the patterns that were evident in the task.

The second layer of analysis re-examined the responses from the 34% of students who identified structural properties. In the first instance, we examined the equations and properties that were identified by the students as shown on Table One.

Table 1
Number of Students Identifying Each Pair of Equations Representing Structural Properties (N = 104)

Equation pair	Property	Number of students who identified the pair
37 + 43 + 40 + 36 = 37 + 40 + 36 + 43 =	Associative	87% (*n* = 90)
6^3 = 6 x 6 x 6 =	Exponents	69% (*n* = 72)
99 ÷ 3 ÷ 3 = 99 ÷ 9 =	Associative	63% (*n* = 65)
12 x 22 = 4 x 66 =	Associative	34% (*n* = 35)
76 x 15 = (70 x 5) + (70 x 10) + (6 x 10) + (6 x 5) =	Distributive	30% (*n* = 31)
7 x 86 = (7 x 90) – (7 x 4) =	Distributive	19% (*n* = 20)

As illustrated on the table, we found that most commonly, the students were able to identify the associative property as represented in addition. Many students also recognised the connection between mathematical notation (y^3) (powers) and expanded multiplicative relationships. Less than half of the students were able to recognise the structural relationships between multiplication equations. Multiplication equations represented, using the distributive property, were particularly challenging for these students to recognise, with only 30% of students recognising this pair to equations.

How do Students Notice and Explain Structural Properties and Relationships in the Context of Number Systems?

This section of the findings will use a sub-set of the related equations and student responses to illustrate the ways in which students noticed and explained structural properties and relationships in the context of number systems.

Associative property in addition. We begin with a focus on the associative property in addition (37 + 43 + 40 + 36 = 37 + 40 + 36 + 43) given that this was the property most identified by students. Many of the students (*n* = 68/90) undertook calculations in responding to this aspect of the task. In some cases, the student calculated both equations separately and then recognised that they were equivalent and identified the relationship. In other examples, the student undertook one calculation and then proceeded to write the same sum for both equations.

In this case, it appeared that the student was able to recognise equivalence in the equations by using knowledge of the associative property. A smaller group of students (*n* = 22/90) did not record any sum or calculations and appeared to identify the equivalence of the equations by recognising the associative property. For example, one student drew an arrow between the equations without recording any calculations and wrote: "37 + 43 + 40 + 36 = is the same as 37 + 40 + 36 + 43 = but just the 40 and the 43 are swapped."

The sophistication of the explanations provided by the students varied with some students (*n* = 34/90) providing no explanation or simply writing their workings to show how they had solved the equations. Other students (*n* = 15/90) also provided limited explanation of the associative property and connected the equations by identifying that the answers were the same or the equations were equal. The largest group of student responses (*n* = 37/90) began to explain the relationship between the two equations using informal mathematical language and related this to the specific example provided in the task:

> Student: The same numbers, they are just jumbled up.
>
> Student: It has the same numbers, just in a different order.

Finally, a small group of students (*n* = 4/90) began to provide an explanation that moved beyond the specific to provide general examples of the structure of the associative property. Interestingly, several of these students gave examples using informal language that referred to the commutative property: "they are the same numbers but in a different way so it doesn't matter like 11 + 10 = 21 and 10 + 11 = 21." Other students provided explanations with further examples of the associative property: "they are the same numbers but just mixed up but because it's addition they will add up as the same number, 1 + 5 + 6 is the same as 5 + 6 + 1."

Associative property in multiplication. In contrast to recognition of the associative property in addition, students appeared to have greater difficulty in recognising the associative property in multiplication. Most students (*n* = 29/35) undertook a calculation and then identified the equations as related. It was evident in the student responses that they typically calculated the solution for each equation rather than only completing one calculation as many did for the addition equivalence equations. A small group of students (*n* = 6/35) did not record any calculations with most of these students simply drawing an arrow or circle to connect the equations.

Similar to responses related to the addition equations and associative property, there was variation in student explanations. Overall, for this aspect of the task, many students (*n* = 17/35) provided no explanation or only recorded their solution strategy for each equation. It was evident that students found it difficult to construct an explanation with an additional group of students (*n* = 9/35) giving a limited explanation indicating that the answers were the same or equal. A further group of students (*n* = 9/35) provided explanations related to the associative property of why the specific equations in the task were equivalent:

> Student: The two equations have a pattern because 4 and 12 are both a common multiple of three.
>
> Student: They equal the same number. Also the 66 is 3 times bigger than the 22 and the 12 is also 3 times bigger than the 4.

One of these students provided an explanation that included a further example of the associative property; however, this was directly related to the example in the task: "Look, 4 x 6, 8 x 33, 12 x 22, that's the pattern."

Distributive property in multiplication. The relationship developed from the distributive property in multiplication appeared to be difficult for the students to identify in regards to equivalence. This final section examines the student responses to the equations 7 x 86 = (7 x 90) – (7 x 4) as this was the pair of equations least often identified as equivalent by the students.

Analysis of the student responses revealed similar findings to the previous pairs of equations that have been discussed, with most students (*n* = 16/20) undertaking a calculation to identify that the equations were equivalent. Interestingly, in parallel with the responses to the associative property of addition, a number of students calculated the product of one equation, generally (7 x 90) – (7 x 4) and wrote the same product for both equations or used one calculation to connect the equations. We assume that in this case, the students were able to recognise the relationship of the distributive property without undertaking both calculations. For example, one student solved the first equation and then wrote below: "7 x 86 is the same as (7 x 90) – (7 x 4) because 7 x 90 is rounded up from 7 x 86 and subtracted by 7 x 4". A small group of students (*n* = 4) did not record any calculations and appeared to use their understanding of the distributive property to identify the equivalence of both equations.

In relation to student explanations, most students (*n* = 8/20) again either provided no explanation or simply recorded calculations. A small group of students (*n* = 5/20) gave a simple explanation referring to the equations having the same answer. Finally, the other group of students (*n* = 7/20) provided explanations linked to their understanding of the distributive property as represented in the specific equations in the task:

Student: (7 x 90) – (7 x 4) is the same as 7 x 86 because 90 – 4 = 86

This section has illustrated the ways in which the students in this study noticed and explained structural properties and relationships in the context of number systems.

Conclusion and Implications

In both curriculum documents (MoE, 2007) and research studies (Chimoni et al., 2018; Fonger et al., 2018; Schifter, 2018), there has been increasing emphasis placed on early algebra and the need to facilitate students to work flexibly with numbers and notice relationships and mathematical structure. The aim of this exploratory study was to begin to address the gap in the literature in relation to better understand which number properties students use to notice the general structure of equivalent equations. Prior to this study, the majority of research has typically focused on tasks that require students to identify true and false equations statements, solve open-ended tasks, and or verbalise their understanding of the equal sign (Matthews et al., 2012).

Findings from this study indicate that students noticed equivalent equations underpinned by the associative and distributive properties, by matching paired equations. It was apparent that it was easier for students to notice equations underpinned by the associative property than it was to recognise the distributive property. While students noticed the pairs, relatively few students appeared to approach the task in structural way. Students typically performed calculations, rather than seeing the equations as mathematical objects (Kieran, 1989; Slavit, 1999; Schifter, 2017). This meant that there was an ongoing focus on arithmetic solutions rather than engaging in algebraic thinking by generalising the common structures across the equation pairs. This may have been due to the fact that the task included an equal sign and students interpreting this as "calculate" or "find the answer." Many of the responses provided by the students, demonstrated that they could provide a simple explanation. It may be that, if in fact students were interviewed, the opportunity to verbalise their response with accompanying gestures may have given more insight into their thinking. Implications from this study include that there needs to be a greater focus on supporting students to notice the structure of number properties across different equations and then to use this to form generalities. It is evident, future research is needed in this field to more deeply understand how students notice algebraic structures within equivalent equations.

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Using Waiata in Mathematics Teaching: Te Whakamahia o te Waiata i roto i te Pākarau

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Singing, and specifically, the singing of waiata (a song or chant related to the Māori world), is a useful and important mathematics classroom practice in any culture. Focussing on teachers in Aotearoa me Te Waipounamu New Zealand, we suggest waiata is one step teachers can take towards honouring their obligations under Te Tiriti partnership. We discuss how the singing of waiata during mathematics can support Māori students to achieve success as Māori. We outline how the use of waiata can retain focus on the mathematics while supporting the building of the classroom community, and we describe ways waiata is good for students' learning processes and how it can support the teaching of mathematical content. Lastly, we explore how the use of waiata can support the teaching of mathematics when using contexts related to the Māori world.

The catalyst for this paper came when the authors, Curtis (Kāi Tahu, Kāti Māmoe, Waitaha me Ngāti Kahungunu) and Ingram (Kōtarani, Ikarihi), discovered a shared love of waiata. Soon after, Curtis, an early childhood educator, challenged Ingram, a secondary mathematics educator, to deliver a series of workshops on the use of waiata to secondary preservice teachers. This mahi (work) resulted in shared discussion and discovery both between Curtis and Ingram and within the preservice students' curriculum classes. This paper captures that discussion and discovery and promotes the use of waiata at all levels of early childhood and school mathematics teaching and learning.

Waiata is a Māori word, directly defined as a song or chant, but this simple definition does not represent the importance of waiata in te ao Māori (the Māori world) in Aotearoa me Te Waipounamu New Zealand. *Waiata*, includes lullabies, laments, songs of love, chants, action songs and poi (Trinick & Dale, 2015). They are oral expressions embedded in every aspect of Māori society and vehicles for the intergenerational transfer of linguistic and cultural knowledge (Paringatai, 2019). Waiata are an important part of tikaka Māori (Māori practices and protocols), such as supporting a whaikōrero (speech). They are also a way to express "the complete gamut of human emotion: joy, sorrow, pain, regret, love, understanding, hate, anger" (Paringatai, 2019, p. 206).

In this paper, we focus on the use of waiata in mathematics teaching as a positive step teachers can take towards embracing a te ao Māori perspective in terms of both te reo Māori (the language) and tikaka Māori. As this whakataukī (proverb) suggests, even small steps can make a big impact.

He iti te mokoroa, nāna i kati te kahikatea
The mokoroa (grub) may be small, but it cuts through the Kahikatea (native pine)

We discuss the use of waiata in mathematics teaching generally, and then focus on what teachers in early childhood settings, primary and secondary schools can do to engage their students in te ao Māori using waiata, while also focussing on mathematics.

Te Ao Māori

Defining te ao Māori is not an easy task as it has been said that "only those with the soul of a poet can enter into the existential dimension of Māori life" (Marsden, 2003, p. 23). Te iwi Māori (the Māori people) are the indigenous people of Aotearoa me Te Waipounamu New Zealand and share whakapapa (genealogy) with whānau (families) from across the Pacific and therefore are a part of what is known as Te Moana Nui a Kiwa (the Pacific Ocean).

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 306–313. Launceston: MERGA.

Te ao Māori simply translated means the Māori world, but to get true depth of understanding you need to understand the important concepts of te reo Māori, tikaka Māori and the inextricable whakapapa links between Māori and all things, living and non-living. For centuries prior to the arrival of Europeans in the late 1700s to early 1800s, Māori had created a way of life that focussed on sustainability of environment through their understanding of the maramataka (the Māori lunar calendar used to understand the best times for planting, fishing, and harvesting), te reo Māori, tikaka Māori, whakapapa and innovation.

It was the innovation, new technologies and a window into a new world that Europeans brought with them that provided the basis of the relationship that formed between the two groups. In February 1840, two different versions of a treaty, *Te Tiriti o Waitangi* was written and signed in Waitangi, and then signed across the North Island as well as three locations in the South Island. The original version was written in te reo Māori and the second was written in English with most Māori chiefs signing the te reo version and the Crown representatives signing the English. Te Tiriti o Waitangi was made up of four articles (three written and one verbal) which, due to the translations, have different meanings in the different languages. These mistranslations have been the root of many historical and present-day claims from iwi Māori to the Treaty of Waitangi Tribunal in Aotearoa me Te Waipounamu New Zealand.

Te Tiriti has implications for the education system in the country, a system which emphasises principles of knowledge deeply rooted in Western values and traditions (Krakouer, 2015). There is continual underachievement of Māori students compared with non-Māori (Rajagopal, 2021), including in mathematics (National Monitoring Study of Student Achievement, 2018). As with other indigenous students, this is not because of any lack of intelligence or skill in problem-solving (Battiste, 2002). Rather it is because Māori students' identities are often not valued and honoured within classrooms. To honour Te Tiriti, equitable outcomes for Māori in traditional school subjects, including mathematics, need to be sought so Māori can fully engage with the wider world. These outcomes should not, however, be at the expense of students identifying as strong members of their own culture (Meaney et al., 2013). Rather than adapting to an education system in which they do not feel they have a place, Māori students need to achieve success as Māori, whatever that looks like for them and their whanau (family).

The expectation to include te ao Māori into teaching programmes is evident in the current curriculum documents and in the standards expected of teachers in English medium education settings in Aotearoa me Te Waipounamu New Zealand. Te Whāriki (Ministry of Education, 2017), the early childhood curriculum, draws on traditional Māori concepts and describes how, in Māori tradition, children are seen as important living reflections of their ancestors and therefore enter this world as competent, capable, and complete. The vision of the New Zealand curriculum for Years 1-13 (Ministry of Education, 2007) includes a vision for young people who will recognise the bicultural foundations of Aotearoa me Te Waipounamu New Zealand. Young people need to have the opportunity to acquire and thus nurture te reo Māori so that it "not only survives, but thrives" (Ministry of Education 2017, p. 3). Furthermore, to maintain status as registered teachers, all teachers in the country need to demonstrate a commitment to Te Tiriti partnership, which includes acknowledging the histories and cultures of treaty partners and practices, including developing their own use of te reo and tikaka Māori (Education Council, 2017).

The implications of these imperatives are that teachers need to be committed to understanding a Māori world view to be able to weave te reo Māori and tikaka Māori into their everyday curriculum meaningfully. To support teachers with this, there have been a range of initiatives that seek to align the education system more closely with the world of Māori. These initiatives include an expectation that achievement statistics are carefully monitored for priority learners, including Māori students (Ministry of Education, 2007) and the availability of professional learning for teachers. For example, Tāitaiako (Ministry of Education, 2011) is a resource that supports teachers to understand and value what is important when taking a Māori world view in relation to teaching Māori learners. More recently, Te Ahu o te reo Māori is an initiative that aims for an education workforce that has the capability of implementing te reo Māori into the learning of all students (Ministry of Education, 2022).

Integrating Te Ao Māori into Mathematics Teaching

Mathematics is an important subject for students to learn and achieve success in so they can fully engage in our technologically rich, global society (Leder et al., 2002). A significant problem in mathematics education is that, despite the importance placed on it, many students do not have quality engagement in the subject, or do not participate in it when it becomes non-compulsory later in their schooling. This is often because of their perceptions of mathematics as difficult, and their negative affective relationships with the subject (Ingram, 2017). For Māori students, as well as potentially experiencing these barriers to success, they can also be impacted when their cultural capital is not considered to be a resource (Averill et al., 2009). C*ultural capital* is culturally located expertise, knowledge, interests, and experience (Barnhardt & Kwagley, 2005). When students' cultural capital is acknowledged, the learning environment within the classroom is enriched by the knowledge, behaviours and skills that have accumulated over the students' lifetimes (Paringatai, 2019)

Te ao Māori has the potential to provide solutions to addressing Māori students' academic needs, while delivering enriching mathematics education for all students (Meaney et al., 2013). Mathematical success includes achieving in mathematics, developing confidence in its use, as well as growing students in terms of their social consciousness and cultural competence (Averill et al., 2009). By integrating mathematics programmes into te ao Māori, students have the opportunity to experience a holistic and relational learning environment (Barnhardt & Kawagley, 2005).

The connections between te ao Māori and mathematics education are exemplified in the useful resource documents Tātaiako (Education Council, 2011), and Effective Pedagogy of Mathematics (Anthony & Walshaw, 2007). Both emphasised the importance of teachers establishing an ethic of care within their classrooms, where both mathematics and culture that students bring to the classroom are acknowledged and respected. Problem solving, community and discussion are also at the heart of both documents.

There are a range of pedagogies potentially useful in mathematics teaching which enact te ao Māori while maintaining focus on the mathematics. These include using parents and students as resources in students' learning (Bishop & Glynn, 2000), using local contexts, connection to the taiao (the natural environment that connects and surrounds us), rote learning and repetition (Paringatai, 2019), and using storytelling, metaphor, and waiata (Averill et al., 2009). We focus in this paper on the use of waiata in mathematics teaching as one way of building the cultural community of the classroom and thus supporting Māori students' achievement.

Using Waiata in Mathematics Teaching

The potential use of waiata as a tool to teach mathematics is not a new idea. Music and mathematics have well-established connections (Perger et al., 2018), and music is seen as an appealing lens within which to teach the subject (Trinick & Dale, 2015) and a positive way of meeting the requirements of Te Tiriti (Joseph & Trinick, 2018, p. 2518). Averill et al., (2009) included waiata as one in a repertoire of strategies they used in their preservice primary mathematics courses to model ways that cultural perspectives can be woven into teaching. The students identified both waiata and music as a cultural activity they could use in their teaching of measurement and number.

The use of waiata is prevalent in early childhood settings and in the early years of primary schooling (Trinick & Dale, 2018). However, as students grow into the later primary and secondary years of schooling, the mathematics becomes more formalised (Perger et al., 2018) and more content-rich and assessment-oriented. Students are usually taught mathematics separately and sometimes only participate in or observe waiata during formal school events such as the greeting of a new member of staff. Pascale (2013), in extolling the benefits of integrating singing within curriculum, suggested it must not be saved for unusual or special events and should not be avoided by teachers who do not consider themselves music specialists. Rather, it should be a common phenomenon and a usual practice for classroom teachers because of its benefits. Indeed, Trinick and Dale (2015) emphasised that teaching using waiata is more beneficial than many classroom teachers realise.

Informed by the literature and our knowledge of Te Tiriti and mathematics pedagogy, we argue that using waiata as a pedagogical tool for teaching mathematics has five benefits. Using waiata potentially enables teachers of mathematics to enact Te Tiriti partnership, waiata can support the building of the classroom community, it is potentially good for students' learning processes, it can support the teaching of content, and singing waiata can support the teaching of mathematics within Māori contexts.

Using waiata in Mathematics Teaching Enacts Te Tiriti Partnership

By meaningfully including waiata in their repertoire of teaching pedagogies, teachers can enact the partnership expectations of Te Tiriti, especially related to te reo Māori and tikaka Māori, if done correctly. Exploring the meaning of the te reo Māori within the waiata is important so the students can understand what they are singing. To introduce te reo Māori within a waiata, Trinick & Dale (2015) suggest asking students to circle any words they know or have heard before and use the knowledge of the class to build community knowledge of the words. Being able to discuss the meaning embedded in the lyrics is important (McDowell, 2017), and through practising waiata, students have an opportunity to improve their te reo Māori pronunciation as well as deepening their understanding of the words.

Choosing an appropriate waiata for the occasion and being explicit about its origins and associated tikaka is important. For example, students should stand when waiata is done within the classroom and ensure that they sing the waiata with pride as a sign of respect for the meaning and the whakapapa (reason), as well as for its composers. One opportunity for performing waiata together and observing tikaka is at the beginning of the mathematics lesson, perhaps after a karakia. *Karakia* are ritual chants invoking spiritual guidance and protection and are used to ensure a warm welcome to the class and a favourable outcome to the lesson. An appropriate waiata at this stage can complement the karakia.

Asking mana whenua (indigenous people who hold authority over their tribal land)) for locally sung waiata and tikaka is helpful. For example, the Kāi Tahu website, *Kotahi Māno Kāika* (www.kmk.maori.nz) is an excellent resource. Using local waiata is an important tool in learning about the local rohe (area) because many waiata are written to recount local legends, remember ancestors or connect with the mana whenua, flora, fauna or geography of a region. The language of waiata is therefore often strongly linked to taiao and mahika kai (food gathering practices).

Waiata Builds Community

Waiata is useful in building a community of learners. When everyone is actively participating, singing waiata together can create a bond known in te ao Māori as whanaukataka (a shared relationship or experience that brings people together). By singing waiata, the class are building a living, feeling community and this establishes the framework of a class identity and purpose. A good example of this is the use of the song *Tūtira mai ngā iwi* (written by Canon Wiremu Te Tau Huata) as a song that brought the country together to support the All Blacks (New Zealand Rugby team) during the Lion's Rugby Tour in 2017. Singing builds creativity and compassion, enhances learning, and connects to the emotional frame of mind of the students (Pascale, 2013).

By singing waiata that has meaning for both the group and individuals, particularly for Māori students, their identities as valued members of that classroom and mathematics learners can be enhanced. Māori students have a sense of belonging and waiata supports the link they have with the past. They do not feel 'othered' as their culture is being normalised through waiata. Furthermore, by waiata reinforcing a sense of community and whanaukataka, students do not feel alone if they are struggling with mathematics and may be more inclined and confident to discuss the mathematics as peers or ask for support.

Waiata is Good for Learning Processes

Singing is an important human activity that develops students mentally, physically, and emotionally. It has the potential to influence various parts of the brain, including connections between the left and right sides (Neumann, 2018). Trinick and Dale (2015) emphasise that the learning processes involved in waiata are complex and develop students' skills in coordination, listening and creativity.

Particularly useful for the secondary classroom, waiata is an example of a tool that can be used as a brain break (Came et al., 2020), A *brain break* is a break from the current learning task that students are working on and is often needed for students with ADHD and anxiety (Smith-Nelson, 2016; Wen et al., 2021). Furthermore, students experience a range of affective responses to mathematics, including anxiety (Ingram, 2017) and students need emotional and mental release to remind them they are not alone in the classroom community.

Having a brain break can also help the problem-solving process. In mathematical problem solving, where students are encouraged to embrace confusion (Ingram, 2017), taking a brain break is useful so they can think more clearly after taking a few minutes away from the problem. Smith-Nelson (2016) suggests having a planned class-wide brain break in the middle of the most mathematically intense portion of the period. Waiata is ideal for taking a brain break, particularly waiata that is active, includes actions and is uplifting. Averill's (2017) use of singing the positions of the compass is a good waiata for this purpose. Singing the compass directions in te reo Māori as a gloriously chaotic round, while the students point in the compass direction they are singing, is a waiata that can be used with students of all ages.

Tokerau, tokerau
Toka, toka,
Rāwhiti rāwhiti
Hauāru

Waiata Supports the Teaching of Mathematical Content

Particularly useful at an early childhood or primary level, waiata can be used to teach concepts such as numbers or shapes (Trinick & Dale, 2015). Through the waiata that are sung within the classrooms, teachers and students can identify and connect the content of the waiata to mathematical knowledge or concepts. It is common to use waiata (using melody, chanting stamping, and clapping) to reinforce the learning of mathematical knowledge and recognise, reproduce, and create number patterns (Perger et al., 2018). Rote learning is a familiar concept in te ao Māori because traditionally Māori was only an oral language and the constant reciting of whakapapa, waiata and pūrākau (cultural narratives) ensured the passing down of important messages and learning to the next generation. Using waiata to support counting song rhymes can help to memorise counting patterns (Perger et al., 2018). Infants and toddlers, for example, can explore their working theories about time and pattern using rhymes, song and by listening to the intonation of voice. Māori Television have produced resources to support students' learning (e.g., Numbers in te reo Māori–Waiata Mai). The popular colour song *Mā is white,* continuing with numbers is also a good waiata for this purpose.

There are a number of waiata that are useful to teach fractions. Most waiata used in Kapa Haka (action songs) have 3/4 timing. *Tahi,* a waiata by Moana and the Moa Hunters is a good example of 4/4 timing. The use of tī rākau (a traditional game of sticks) is a good source of number patterning, and this game is often accompanied by the waiata *E pāpā waiari* which has 6/8 timing. Resources available on *nzmaths* (https://nzmaths.co.nz/resource/matariki-level-1) describe how to be explicit about the mathematics within the waiata, which is also useful to use with early number and algebraic patterning. An example of a good waiata that directly teaches young students about position and orientation through song and action is *E rere taku poi.*

Table 1

E rere taku poi, adapted from Royal-Tangaere (1997, pp. 40–41) and Reproduced in Anthony & Walshaw (2007, p. 26).

Original dialogue	Translation	Nonverbal Actions
Toru whā	Three four	Hands on hips
E rere taku poi	Fly my poi	Swings poi in front of her and above her
E rere taku poi ki runga	Fly my poi above (me)	
Ki runga	Above (me)	
E rere taku poi	Fly my poi	Brings poi back in front of her
E rere taku poi	Fly my poi	
Ki raro	Below (me)	Swings poi down below
Ki raro	Below (me)	
E rere runga	Fly above (me)	Swings poi up
E rere raro	Fly below (me)	Swings poi down
E rere roto	Fly inside	
E rere waho	Fly outside	
E rere taku poi	Fly my poi	Swings poi in front of her and then
E rere taku poi	Fly my poi	above her
Ki runga	Above (me)	
Ki runga	Above (me)	

Waiata is Good for Connecting with Māori Contexts

A context-based approach is useful in the mathematics classroom to enhance students' engagement and learning. Contexts locate the learning of mathematics within the social practices of the classroom, which are the norms and relationships that are important to a community (Cobb et al., 2001).

Choosing authentic contexts that relate to Māori are important in infusing the mathematics curriculum with te ao Māori. From early childhood to senior secondary, there are many te ao

Māori contexts that can be used in learning mathematics. Calculating the capacity needed for digging a hole to bury the hangi baskets of particular dimensions can be linked to volume and capacity within Measurement. The long history of Māori navigation and migration and the building of waka (canoes) is filled with potential for mathematics and statistics. Tracking the traditional pounamu (greenstone) trails and recounting the pūrākau of Poutini the Taniwhā (powerful water creature), guardian of pounamu, is useful in a position and orientation unit. There are many mathematical concepts involved in the legend of Matau whose body formed the breathing Lake Whakatipu wai Māori (Lake Wakatipu), which rises and falls 10 cm every 27 minutes due to a seiche. There is also the more prevalent use of weaving harakeke (flax) and tukutuku (woven panels) to connect with linear and quadratic patterns, as well as the transformational geometry found in kōwhaiwhai and whakairo (carving) patterns.

Each of these contexts have great potential for mathematical learning and they also contain mātauraka Māori (Māori wisdom and knowledge). To honour each of these knowledge systems, each context needs to be presented to the students using a te ao Māori approach, with the storytelling, history, te reo Māori and tikaka intertwined. We suggest the meaningful incorporation of waiata is a good way to add to these contexts to honour te ao Māori. Waiata, when used to enrich a context may not include mathematics directly but adds another layer of richness to that context and supports the students' engagement. Talking to mana whenua and researching these contexts with an ear for associated waiata will add value to these contexts. For example, just as Came et al. (2020) suggested, the use of the Split Enz song, *Six Months in a Leaky Boat* to add richness to a unit on European migration, the same could be done using waiata about waka, such as *Utaina mai* to add richness when teaching a statistics or position and orientation unit in the context of the history and navigation about Māori migration.

Whakawhāiti - Conclusion

By understanding and using waiata in meaningful ways when teaching and learning mathematics, this has the potential to provide a connection and a sense of whanaukataka for Māori students amongst their peers. For Māori to succeed as Māori they need to see themselves and their culture genuinely reflected in the spaces and curriculum that is being provided to them by their teacher and for their knowledge to be seen as valuable. Education needs to strengthen, rather than weaken students' connection to their culture (Meaney et al., 2013). Teaching mathematics using a te aō Māori perspective not only meets the obligations of teachers within Aotearoa me Te Waipounamu New Zealand and enhances the education experience for Māori students, but it also enhances the quality of all students' learning (Averill et al., 2009).

Ka waiata tatou! Let us sing.

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RESEARCH PAPER

Influence of the COVID-19 Lockdown on High School Mathematics Teachers' Beliefs About Using Digital Resources

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This study examined the impact of COVID-19 on the beliefs of high school mathematics teachers in Pakistan regarding the use of digital resources. Participants were six mathematics teachers from both private and government schools with varying years of teaching experience. The responses indicate that despite facing issues, teachers' beliefs have significantly changed due to COVID-19 online teaching. Teachers' beliefs are subject to the opportunities and constraints that exist in the social context of teaching mathematics, which includes the influence of students, parents, schools, peers, and senior teachers. The regular practice and use of digital resources may further deepen teachers' beliefs and their use in mathematics teaching.

Introduction

During the COVID-19 pandemic many teachers became more interested in digital resources (Scully et al., 2021). Their positive beliefs and attitudes supported the transition and substantially increasing the use of digital resources (Alberola-Mulet et al., 2021). However, the transition was not easy. Especially, for developing countries such as Pakistan the challenges were gigantic. The quick transition was largely dependent on the availability of digital resources and internet connectivity at both ends of the teaching and learning spectrum. Secondly, it required changes in curriculum, content, teaching methods, and teaching skills (Clark-Wilson et al., 2020). Later, however, more essential and "deeper changes" were required i.e., changes in "teachers' beliefs" towards using digital technologies.

In Pakistan, mathematics teachers traditionally rely on direct teaching approaches (Amirali & Halai, 2021). Due to the COVID-19 lockdown, there was a sudden and significant surge in online learning. Teachers were rushed into adopting and transitioning to online education and ensuring students' educational needs were taken care of during the pandemic. Principally, the school closures stimulated the use. However, teachers' social contexts, digital skills, and their confidence in working in an online environment may also have played an important role. Which poses a significant question for researchers: how, and if so to what extent, has the sudden transition to online education and use of digital resources influenced teachers' beliefs about digital resources in Pakistan.

Historically, at the school level, the use of technology is not common. Mathematics teachers may possess strong subject content knowledge, but they have been inadequately trained in the use of technologies and lack the confidence to integrate digital resources into teaching and learning mathematics (Asad et al., 2020; Jafri, 2020). Additionally, previous literature (pre-COVID-19) about Pakistan's high school mathematics teachers never predicted or measured the unprecedented level of mental readiness that could support a fully online teaching environment. Limited literature exists that measures teachers' beliefs related to curriculum, assessments and the nature of mathematics (Amirali & Halai, 2021), and there is also a lack of research on teachers' beliefs and the changes in beliefs relevant to the integration of digital technology. Therefore, after almost 18 months of school closure and managing to teach and learn online, it appears to be an appropriate time to understand Pakistani mathematics teachers' beliefs about digital resources.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 314–321. Launceston: MERGA.

The Purpose of the Paper

The purpose of this paper is to present the qualitative findings of mathematics teachers' beliefs and changes in beliefs towards digital resources due to COVID-19 online teaching and learning. The findings are part of ongoing doctoral research designed to understand the digital competencies of mathematics teachers in Pakistan. The main study has two phases: Phase 1–an online survey and Phase 2–semi-structured interviews of mathematics teachers. This paper focuses on Phase 2 data obtained from interviews conducted with six mathematics teachers. The following section presents a brief literature review concerning teachers' beliefs towards mathematics and the use of digital resources in mathematics. After a brief description of the methodology, the paper proceeds to the findings and discussion.

Literature Review

Understanding teachers' beliefs has been a focus of attention in educational psychology (Ernest, 1989; Pajares, 1992; Schoenfeld, 2011). To understand why teachers do what they do and how they behave in response to different pedagogical situations, there is a need to understand teachers' system of beliefs (Schoenfeld, 2011). Teachers may hold identical or different beliefs based on their personal life experiences, socio-cultural and religious contexts. Teachers' beliefs provide understandings, judgments, evaluations, and justification of their teaching practices (Pajares, 1992).

Partly integrating differing characteristics of teachers beliefs articulated by Ernest (1989) and Pajares (1992), mathematics teachers' beliefs may be defined as, opinions, dispositions, (pre)conceptions and philosophies that mathematics teachers hold about the nature of mathematics and its teaching as a whole, and which influence their approach to mathematics teaching and learning. For example, a teacher could develop a self-confirming bias about a particular teaching practice or give preference to mathematics textbook because this is how they learned as a student. Such biases and beliefs may inhibit their desire to changes teaching practices (Pajares, 1992) and are normally unaffected by new information (Karatas, 2014). Beliefs change when the social context of the teaching situation and the teacher's level of thought processes and reflection change (Ernest, 1989). Influenced by the social context, teachers are likely to adopt the same teaching methods despite holding differing beliefs about mathematics. It also plays an important role in the adoption of technology. As such, witnessed during the pandemic, the change in social context and settings (COVID-19 restrictions—social isolation and distancing) intensified the personal and professional uses of video conferencing, virtual collaboration, and social networks, and other digital resources. Teachers' views also changed, for example, Morge (2020) used screen-sharing during online video conferencing for problem-solving that improved teachers' confidence in digital resources. Similarly, Carey et al. (2020) observed changes in teachers' beliefs towards online teaching when they employed breakout rooms for collaborative problem-solving. This suggests transformation of beliefs happen when changes in teachers' practice produce expected teaching/learning outcomes consistent with their social context.

Pajares (1992) argued that whoever uses "beliefs" to define teachers' practices and decisions need to find a clear distinction between teachers' beliefs and knowledge because "it is difficult to pinpoint where knowledge ends and belief begins" (p. 309). When compared with knowledge, teachers' beliefs are disputable, inflexible, and less dynamic. Therefore, they are not easily persuaded or changed by reason or argument. Beliefs changed when teachers saw the pattern of pedagogical reality change or when teachers' fundamental approach or underlying assumptions were changed, i.e., a shift in teaching paradigm (Pajares, 1992), whereas the knowledge provided both affordances and constraints regarding what a teacher can and cannot do in a pedagogical situation (Schoenfeld, 2011).

Negative or positive beliefs about digital technology can influence its use accordingly. The evidence suggested that during the pandemic positive beliefs and affordances increased teachers' confidence in digital technology and its potential to enhance outcomes for learners (Scully et al., 2021). Teachers were able to create innovative and supportive communities, collaborate for problem-solving, hold online conversations, and share resources thanks to the affordances of persistence, visibility, spreadability, and searchability made available by accessible technologies (Carey et al., 2020). However, Christopoulos and Sprangers (2021) suggested that although the pandemic served well to enhance teachers' digital skills, it was important to keep an eye on concerns and constraints as they affect teachers' beliefs. Calder et al. (2021) found that during the pandemic, teachers were unable to find support related to digital applications. Reich et al. (2020) identified that many teachers struggled to motivate students during online sessions. Teachers who were unfamiliar with digital tools felt isolated and suffered a loss of self-efficacy and professional identity. Also, less privileged students from developing or poor countries with limited or no access to digital resources, became victims of social and economic inequalities and were unable to continue education during the pandemic (Ndambakuwa & Brand, 2020). These are important social contexts and constraints relevant to teaching online, which may affect the beliefs of teachers who experienced similar issues during the pandemic.

Pakistani Mathematics Teachers' Beliefs

In the context of Pakistani mathematics teachers, mainly socio-religious experiences, school education, and pre-service training experiences shape their beliefs about teaching mathematics (Amirali & Halai, 2010). They regard mathematics as a constantly evolving discipline, where mathematicians continuously revise their invented body of knowledge while everyone else consumes it (Amirali & Halai, 2010). They use direct teaching methods with a strong emphasis on delivering textbook content (Amirali & Halai, 2021). Technology is infrequently used in teaching because teachers believe it to be complex, expensive, and scarce (Dundar et al., 2014). Whether good or bad, these are traditional beliefs that contribute to low student achievement levels throughout Pakistan, as evidenced by the fact that a significant proportion of school leavers (especially from government schools) do not achieve the minimum standards in mathematics that are required by the curriculum (Dundar et al., 2014). Interestingly, most teachers regardless of their professional role (government or private), gender, or teaching experience, hold identical beliefs (Amirali & Halai, 2021).

In a comparative study of Pakistani government and private school mathematics teachers, Shiraz and Qaisar (2017) found that teachers' beliefs are not entirely aligned with their teaching practices. These factors, such as classroom environment, resource availability, senior teachers and career opportunities, influence Pakistani teachers' teaching practices, hence they make decisions against their beliefs (Shiraz & Qaisar, 2017). Christopoulos and Sprangers (2021) regard these factors as first-order barriers to successful technology integration efforts in schools, while beliefs, attitudes, confidence, and skills are second-order. However, any of the first or second-order barriers, a piece of new knowledge, or a situation may influence teachers' use of technology for teaching (Christopoulos & Sprangers, 2021). As evidenced by the COVID-19 situation, many teachers, regardless of their beliefs, employed digital resources for online teaching and learning (Alabdulaziz, 2021). Their beliefs and attitudes changed as they experienced the "ease of use" and recognized the "perceived usefulness" of digital resources in mathematics teaching (Scully et al., 2021). Therefore, it is important to consider the influence of both "situations" and "factors" that bring changes in teachers' beliefs and, consequently, the teaching-learning process.

Methodology

As mentioned earlier, this paper presents COVID-19 related qualitative findings, which are part of an ongoing mixed-methods doctoral study. The main study collected qualitative data using semi-structured interviews. The participants were mathematics teachers who volunteered for individual interviews during an online survey. The aim was to recruit high school mathematics teachers with varied years of teaching experience from both government and private schools in Pakistan's rural and urban areas. Based on the criteria six teachers were selected and coded as T1 to T6. The final six included two (02) teachers in private schools and four (04) in government schools. Two (02) were female and four (04) were male with teaching experience ranging from four (04) to twenty (20) years. Due to COVID-19, the interviews were conducted and recorded online using the Zoom video-conferencing application. The data were collected using the following specific questions related to COVID-19 teaching and learning:

1. *What beliefs do you have about digital resources? What do you say about using digital resources in mathematics teaching?*
2. *To what extent, if at all, have your beliefs about the use of digital resources changed due to COVID-19?*

The data were analysed using themes and sub-themes that emerged from larger data collected during the mathematics teachers' online survey (Phase 1 of the study). The aim was to look for connections within data and identify thematic patterns.

Findings

This section presents the findings of the interviews conducted with mathematics teachers. The two main themes are explained using the interview codes. The first theme explains teachers' beliefs about digital resources in Pakistan. The interviews were conducted during the COVID-19 lockdown, their responses are, therefore, most likely to have been influenced by how they needed to cope with COVID-19 and may reflect some anxiety as they began using digital resources for teaching that they may not have used before.

Beliefs About Digital Resources

In response to the first question, the majority of teachers agreed that digital resources play an important role in mathematics teaching and learning. Most of the teachers indicated that they "never used" digital resources before the pandemic because they were not available at their school. The pandemic presented a unique opportunity to "self-learn" the use of digital resources with the "help of peers" and enhanced their "digital skills". For instance, T2, a male senior teacher, teaching in an urban government school for 20 years, mentioned:

> I had never used digital applications before COVID. We didn't have them at school, not even at home. Recently, I bought a ThinkPad in instalments. My friend showed me how to use it. Now, I can send links, digitally write and solve math problems, share pictures and notes. If a student asks a question, I can write, type, and share the answer on the screen. Now I see how useful and comfortable it is.

He further added:

> As COVID is all over these days, you can't survive without digital things. Students are at least connected with education .... If not 100%, then at least 50 % are learning… Today I took the Zoom class from home in which 50% of students were present. Digital resources are very useful for students who want to learn. Those who don't want, don't even study in the face-to-face physical environment ...

T3, a female teacher, facing issues of "lack of digital resources," explained:

> We can't use digital technology much. If we used audio-visual aid in our school, it would definitely help students and teachers, but we can't use them.

In Pakistan, the state of technology infrastructure and funding in private schools is better compared to that in public schools. However, T3's response was contradicting the general presumption. Therefore, the researcher asked her to elaborate further. She replied:

> Because there is no system, there is no such thing (digital resources) right now in the school. The real problem is that we can only teach here with a textbook and whiteboard. We don't have any other tools here …. At the same level at which we learned from our teachers, we are also teaching our students. Actually, digital learning is not so much appreciated here. I know I can use them if I want to .... But we have to follow what the headteacher tells us to do or use …

A teacher at a government school (T4), who was previously teaching in a private school, replied by comparing private and government school students' ability to "access technology."

> It is easy to teach online to the students in a private school. They can afford digital technologies, but not public-school students. The majority of them complain about not having Internet access at home.

T5, a male government schoolteacher from a rural area, mentioned that they do not have the "facilities" and their "teaching method or style" does not require digital resources:

> We should learn the use of digital technologies. Neither do we have the facilities, nor do we do the type of teaching that requires digital tools. Especially when we talk about the government sector, there are no such things as digital resources. We only use WhatsApp to communicate with each other.

T6, another male but fairly young teacher in a government school also mentioned "limited or no use" of digital resources. He explained:

> I think digital resources are very helpful, especially videos are very good. But unfortunately, we don't have much use for these resources, and we also don't have ideas about their uses.

Changes in Beliefs

In response to the second question, the majority of the teachers agreed that their beliefs about digital resources have changed after teaching online during COVID-19 lockdown. T2, experienced a significant shift in his beliefs about digital resources. The "affordances" of "video conferencing application" (Zoom) and "ease of use" impacted his beliefs, he explained:

> But since I started using the digital resource, it seems that the world is not over. Your home could be your classroom via Zoom .... Honestly, before COVID, I was not aware of the potential of digital resources… I used to travel a lot to give private tuition in the evening. Now I can give them using Zoom while sitting at home with my family. It's very easy …

T3, a female teacher at a private school not only observed changes in her beliefs but also in "peers" and the school leadership (principal). However, she thinks the senior teachers were resistant to change and were not motivated to learn new digital skills. She described:

> More than me, the beliefs of my colleagues and principal have changed. They have realised that if it were not for smartphones and the Internet, we would not have been able to teach students. Now they think that technology is good. Before COVID, some (senior) teachers did not even know how to use the Internet (Zoom) ... they were not skilled and did not want to learn, so we had to take their classes.

T5, a rural government schoolteacher recognised digital resources and online learning as an aid to "self-learn" skills that the government is unable to provide. He explained:

> My beliefs have changed quite a lot. Because of online education during COVID, now I know how I can improve my skills using online education, where to download resources and how to use YouTube channels. Now, I use online content for learning and teaching. I also share links with my students.

T6, another male teacher at a rural government school acknowledged that the use of technology has increased due to pandemic, he explained:

> Before COVID, nobody was talking about digital resources in Pakistan, but now everyone is talking about Zoom and WhatsApp. Recently, many students told me that during COVID they were taking classes online and learning from YouTube. The use of digital resources has increased quite a lot.

However, T1 and T4 did not evidence many changes in their beliefs. Both teachers considered online learning needs time to take effect. For T4, the role of "parents" was the key to the successful implementation of online learning, whereas T1 thought face-to-face teaching needs to be further enhanced with digital resources rather than focusing on online teaching and learning.

> I still believe that students should not be allowed to use digital devices such as smartphones. It is very hard to monitor the activities of (high school) students on smartphones. They are adults and know how to dodge their parents. Children from lower-middle-class families attend government schools. The parents have no understanding of how to use or monitor digital technologies.

T1 proposed:

> After teaching during the lockdown, I think online teaching and learning is too early for our students. We need to find ways to increase the use of digital resources in our face-to-face classrooms.

Discussion and Conclusion

This paper reports on an investigation into Pakistani mathematics teachers' beliefs about the use of digital resources and how the sudden transition to online education and the use of digital resources may have impacted their beliefs. The findings suggest most teachers hold positive beliefs about using digital resources for teaching and learning mathematics. In general, this is consistent with other studies related to mathematics teachers. In contrast to previous studies (Dundar et al., 2014) in which teachers find it complex and difficult to use technology, the majority of teachers in this study found it relatively easy to transit to online teaching and learning. This indicates that teachers already had knowledge and positive beliefs about the digital resources. Regardless of professional role (government or private), teaching experience, and gender, all teachers share a common belief that lack of resources, funding, and internet connectivity are the main barriers to the use of digital resources. These barriers exist for both students and teachers. Christopoulos and Sprangers (2021) called them the first order barrier, which impacts teachers' beliefs on the use of technology. For government teachers, the type or style of teaching could be another factor that influences their beliefs, as T5 mentioned that the "type of teaching that requires digital tools" is not practised at government schools. Nonetheless, this is mainly due to the unavailability of technology and professional support at government schools, along with teachers' financial ability to arrange digital resources for personal use (Dundar et al., 2014).

Teachers appear to be self-learning the use of various digital tools with the support of their peers. A male teacher (T2) explained, "my friend showed me how to use it," showing that teachers had insufficient (official) professional and technical support available during the pandemic (Christopoulos & Sprangers, 2021). With the rapid technological development around the globe, it would be difficult to predict how the teachers' self-learning efforts could ideally keep their professional development requirements up to date. Therefore, it would be useful to further investigate the effect of "professional development versus self-learning" related to the use of digital resources. The findings also show, despite having positive beliefs about digital technology, the teacher (T3) decided not to use digital technology and preferred to follow the headteacher's advice. This indicates that the teacher knew of the personal use of digital resources but lacked the confidence to make professional use of digital resources in school. The teachers believed initiating change might go against the pedagogical practices of senior teachers. T3, under the influence of the social context (peers and technological infrastructure), adopted the same teaching methods as peers in school (Ernest, 1989). Teachers are influenced by their social environment, which is shaped by students, their peers, parents, and seniors' expectations (Ernest, 1989). Therefore, it is not surprising especially for early-career teachers to be influenced by their peers and seniors. The support and the role of senior leadership (headteachers) did, however, play a critical part in establishing beliefs about using technology at school. Discouragement and lack of support from the leadership not only inhibited new ideas but also decreased the innovative use of technology among teachers (Scully et al., 2021). This pressure was released while working from home during the COVID-19 and teachers felt more autonomous about using technology and making changes in their practice.

With regards to the second question, COVID-19 played a major role in changing beliefs and increasing the use of digital technologies (Alabdulaziz, 2021; Scully et al., 2021). During the pandemic, teachers developed personal schemas, self-learned and created their own workarounds for teaching and learning mathematics (Carey et al., 2020). Beliefs do not need to be established in a physical classroom environment or within technology-related professional development programmes (Clark-Wilson et al., 2020); they can be established through the personal and professional use of technology (Jafri, 2020). As evident in the case of T2, teacher's beliefs originated after experiencing the ease of use and change in social context. The digital resources' affordances improved his agency and self-efficacy (Karatas, 2014). Further, the regular use during the pandemic improved their digital competencies and they were able to select the appropriate resources based on pedagogical and mathematical considerations and create documents to be used by the students during the online sessions.

In a particular social context (such as teaching students of low-income families), teachers' beliefs are hard to influence as exemplified in the T4's response, "I still believe ..." This finding suggests that the teacher's knowledge of students' extended social contexts (parents' financial capacity and digital skills) and students' online behaviour may play a role in shaping or retaining beliefs about students' learning with digital technologies. Unfortunately, in a post-COVID scenario, many middle-income families in developing countries could fall into low-income status due to worsening economic indicators. This could further dampen parents' financial capacity to afford digital technologies, which may drive teachers to hold the beliefs associated with these social contexts more strongly than other beliefs. Parents are an important part of the social context of the (online) teaching situation (Ernest, 1989). They conjointly work with teachers to organise, structure, and operationalise the online learning environment (McCarthy & Wolfe, 2020). Therefore, this finding suggests a relationship between parents' ability to create an effective home learning environment and teachers' beliefs about students' online learning. However, more information and further investigations are required to establish this result. Nonetheless, it appears that teachers holding positive beliefs about digital resources may create strategies enabling parents to become partners in adult students' online learning situations, such as COVID-19.

The recent transition to online education because of COVID-19 provided an opportunity to understand teachers' beliefs about digital resources in Pakistan. The study found that teachers with a higher level of thought and reflection may adapt and align practices as per the social context, situation, and perceived affordances of digital resources. Despite holding positive beliefs, the data may be insufficient to predict teachers' use of digital resources in future (post-COVID) Pakistani mathematics classrooms. Teachers may require training and resources to support and continue the online transformation. Teachers' beliefs and inclination towards digital resources may be subjected to the opportunities and constraints that exist in the social context (students, parents, schools, their peers, and seniors) and teaching situations (COVID-19). However, regular use of appropriate digital resources in different social contexts may facilitate teachers to construct new beliefs and find purposes that can serve their goals of teaching and learning mathematics.

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Playing the "Research Game" in Marginalised Fields

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In this paper we reflect on our combined work in some of the most marginalised educational contexts in the Southern Hemisphere. We draw on the work of Bourdieu to frame the paper. We propose the working in marginalised education settings requires a particular habitus or way of being to be able to play the research game. Underpinning our approach is the South African construct of Ubuntu, which is very much about collaboration—I am because we are—so that there is a move away from doing research "on" participants and contexts to one which is very much about doing research "with" participants and contexts. We find Bourdieu's notion of game as a powerful construct to theorise ways of thinking about the field of educational research.

At the outset of this paper, we seek to articulate two key points. First this is a position paper where we seek to articulate particular ways of being and acting in educational research when working in marginalised contexts. Our intent is to provoke researchers to rethink their ways of formulating research and enacting research when working in marginalised contexts. For this research, the paper does not sit well with traditional forms of research publication, which is part of our intent to disrupt traditional research paradigms with their stylised ways of reporting research. Our second key caveat is that while we use the term 'game' to frame the paper, in no way should this be trivialised as a metaphor. Rather, from Bourdieu's (1991) work with the notion of game, there is a strong sense of how the field of education in general, and mathematics education in particular have certain ways of being and acting within the field and, from this, certain rewards are bestowed on researchers. These rewards can be in the forms of capital, which again is a Bourdieuian construct to signify status. We will expand on these constructs in latter sections of the paper.

Background for the Paper

Both authors have worked extensively in our respective countries with some of the most marginalised communities within those countries. We draw on these experiences as they are quite different from our work in mainstream contexts. The practices that we have adopted have created ruptures in what would normally be seen to constitute "good" research practices. Our work, and the insights we have gained from more than a decade of research into our respective contexts form the basis of this paper. Our purpose in writing this paper is to challenge researchers who work in equity contexts to consider whether they are participating in hegemonic practices that support the reproduction of research paradigms (along with the production of deficit research narratives) that are unlikely to bring about change for our most marginalised learners. By reflecting on the work that we have undertaken, we propose to disrupt practices that may bring about status, in the form of capital for researchers working in this domain. We note that as with negative stories in the press receiving far more attention and viewership than positive stories, so too do deficit discourses about poor learner performance and poor teacher practices in marginalised communities receive high attention and citations. In order to tell different types of stories in contexts defined by marginalisation and deficit of economic and social capital, researchers need to partner with communities in ways that acknowledge our shared humanity and find a shared commitment to ways forward to the challenges faced in the communities we work with. In this respect, we propose several principles that have guided our research endeavours (sometimes intuitively and sometimes

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 322–329. Launceston: MERGA.

explicitly) in our attempts to effect change in "hard-to-teach" schools and educationally challenged communities.

Adopting a Bordieuan Games Framework

Not dissimilar to Wittgenstein's (1953) language games, Bourdieu (1991) proposed that games are part of social life except that one could argue, unlike a game of football or cards, the stakes are much higher in the game of life, or in the case of this paper, we argue, in the game of research. For Bourdieu (1991), the concept of games was a serious understanding of the field in which one is located. Whether this is a sport where one must have a feel for the game, or an aspect of social life, to amass power and status within that field, one must have a serious understanding of that field. Bourdieu (1991) argued that much like trumps in a game of cards, the forms of knowing and being within a social field act like forms of capital within that given field. Applying this games analogy to the field of research, when researchers amass certain forms of capital—such as publications, grants, consultancies, awards, positions on boards, citations, etc.—they become forms of capital that can be exchanged within the field for other forms of capital such as promotions, salary, larger more prestigious grants and so on. In order to gain the initial forms of capital, the researcher needs to read the game (with rules that are sometimes implicit and other times explicit) and engage with the game in order to succeed. These rules of the game are constructed in practices far removed from the grounded realities of the empirical fields where research occurs, and are often based on universal conceptualisations of the nature of educational empirical fields, dominated "by the North" (Valero & Vithal, 1998).

The game is located within a particular field, in this case, of mathematics education. In this field, different forms of knowledge and ways of being are seen to be more valued than others. For example, to publish in certain journals, or even conference proceedings, particular forms of research and styles of writing are more valued than others. Where the researcher conforms with those rules (of the game) they are more likely to be published. Similarly, when applying for grants, whether high or low stakes grants, there are different forms of knowledge, methodologies, foci, and targets that are more likely to receive interest than others. Although published 20 years ago, Lerman and colleagues (2002) analysed the papers published in a range of significant publications in mathematics education and illustrated this case in point. Some types of papers—either in terms of theory, paradigm and/or method, were published in different journals while others were absent. Lerman et al.'s (2002) analysis illustrated who and what gets published in the field of mathematics education. From a Bordieuan games perspective, this illustrates that different forms of research and styles of publication can convey status on researchers when they play the game of (mathematics education) research. For those who want to amass capital there is a sense of knowing how to play the research game if one wants to succeed. Bourdieu (1990) suggested that buying into the game is often acquired through an unconscious process so that the , vis a vis researcher, is unaware of the ways in which the game is played and how capital is amassed by some and not others. Often there is an assumption that the game is fair and there will be natural winners and losers. In this way, the research game is perpetuated in a relatively unproblematic manner, thereby "reproducing the conditions of its own perpetuation" (Bourdieu, 1990, p. 67).

When working in equity target contexts, there are markedly different challenges in the conduct of ethical research. As outsiders coming into a novel context and seeking to understand and/or change conditions of existence with the intention of improving mathematics learnings for marginalised learners, the rules of the game become foregrounded. Bourdieu (1990) argued that by standing outside the game, the observer can see the illusion that is created through the practices—the threats, the appeals, the steps that are taken by participants—to see how the game is enacted and the effect of that game.

Many of the assumptions we hold dear as researchers—such as objectivity, ethics, impartiality, truthfulness, impact—come under challenge when working in marginalised contexts. We provide a brief example. Intervention impact research often comes with an expectation of control groups. Control groups, however, are problematic to implement in the community contexts we have worked with. To establish such groups as separate from intervention groups (they do not receive the intervention support and are excluded from participation in the intervention), while expecting that they must agree to participate in being researched, goes against community values of access and fairness. Furthermore, we believe this practice would have detrimental consequences to the nature of researcher/community relationships. We further argue that promoting such research practices in schools and communities that go against the grain of community values and preferences, even if accepted might threaten researcher access to authenticity. In this respect, we have both spent the past decade resisting the ongoing pressure for our research to be contrasted with "control group" data.

We discuss several differences in the research that we have undertaken that we see as different from what is usually undertaken in the field. While beyond the scope of this paper, if one were to consider the types of successful high stakes grants awarded in Australia (for example), there would be notable trends in topics and methods that were embodied in those grants. Again, such an analysis would be indicative as to what was valued within the field, and what the researchers were doing that enabled their success, or capital amassing.

Ubuntu

Globally, there are many shared beliefs about the conduct and value of research. These shared values and beliefs are the foundations of the game of research. As noted earlier, often the game of research valorised by the university system is limited in scope and the practices that create symbolic power for those who observe the unspoken rules of the game. Those complicit with the rules, even at an unconscious level, can amass considerable capital. Bourdieu (1991) proposed that the game is able to perpetuate itself by and through the implicit buy in of the participants.

Researchers willingly, and often unwittingly, participate in the game through their actions. The research game is hegemonic and reflects the values of the dominant groups within the research community (Calhoun, 2003). There are some researchers who have articulated the divide between the north and south in reference to the hemispheres (Valero & Vithal, 1998). It is here we seek to offer our first challenge in the game of research.

We seek to challenge the hegemony of the northern viewpoint through the introduction of a term from Africa—Ubuntu. While Mandela brought some familiarity of this term to the world, noting, "In Africa there is a concept known as 'ubuntu'—the profound sense that we are human only through the humanity of others; that if we are to accomplish anything in this world it will in equal measure be due to the work and achievement of others"[4], it remains a little-known term in the general research community. We propose it has significant value when considering the research game. A general translation of Ubuntu is "I am because we are," which signifies the value of the collective in bringing about social change for the better and encompasses enacting humanity and humility. The term is derived from the phrase umuntu ngumuntu ngabantu, which translates as "a person is a person through other people." Increasingly, South African educators and researchers are exploring Ubuntu as a research and development paradigm and philosophy to guide meaningful and ethical work and doing

[4] https://quotefancy.com/quote/874394/Nelson-Mandela-In-Africa-there-is-a-concept-known-as-ubuntu-the-profound-sense-that-we

research ethics the African way (see for example, Seehawer, 2018; Seehawer et al., 2021; Mlondo, 2022).

There are many practices that have been foregrounded by organisations when working with marginalised groups. For example, the Indigenous Corporation Training (ICT, 2022) suggested a number of principles when working with Indigenous people. These included being trustworthy; transparent; respectful; invested; involved, and patient. While the principles have strong values and ways to work with Indigenous people, questions need to be raised as to how well the current game of educational research would enable the true incorporation of such values into a research project. To this end, we suggest while the principles have intrinsic value, their incorporation at a deep level may not be realised fully. Adopting an Ubuntu perspective of the research game would require researchers to be part of the solution of marginalisation and, in so doing, reconceptualise aspects of the game of research. Where the "I am" refers to the researcher, the "we are" suggests that through collaboration, the researcher and the participants (considered in a wide sense) become a collective we and the "we" collaboratively learn through the research and engagement process. This challenges the orthodoxy of contemporary research games.

In the following sections, we draw on the principles that we have adopted in our research programs that attempt to refigure the research game so to better work with marginalised learners and contexts, particularly given researchers' outsider status.

Rules of The Game in Marginalised Contexts

From our combined (see for example, Graven & Jorgensen, 2018) and separate work across our South African and Australian geographically and culturally diverse contexts, we have proposed a number of principles or rules, that regulate our work in marginalised contexts (Jorgensen & Graven, 2022).

Establish Trust and Mutual Understanding

Increasingly schools and systems are sceptical about the intentions of researchers. It has been common practice for considerable time that researchers would enter schools, conduct their research, and then publish findings. In some cases, particularly in marginalised contexts, research produced would paint a negative picture, based on deficit models of thinking of the learners and their contexts, and have little value or impact for the participants of the research (Graven, 2014). This is hardly surprising given that the research game rewards publications so the goal for the researcher is 'publish or perish'. Similarly, the granting agencies expect outcomes, one of which is publication. Indeed, in the Australian context, a large allocation of the prestigious Australian Research Council grant is based on track record, and reports on funded grants must list the publications arising from the grant. What is needed is for a sense of trust and respect to develop between the researcher/s and the participants. This may take time but standing outside the game of research, it becomes possible to see that time constraints, and the concomitant sense of urgency of grants and publications, is part of the doxa of the game. It is this sense of urgency to conduct the research, and subsequently publish from that collected data that reproduces the game of research that ultimately hinders building relationships of trust and mutual understanding.

Being Authentic and Abandoning Status

As researchers coming from the University, and at professorial level, there is not only a need to acknowledge the status of these positions and what that entails, but more importantly, find ways to minimise the imbalance of power. The importance of the "we are" becomes a salient point and the mutualistic relationship between the researcher and the participants needs to be established. By attempting to address and reduce the outsider status with its inherent power imbalance, the researchers need to shift from being in the context of the university (where academic demands dominate) to be in the context of the research. For us, this required us to (as far as possible) look at engagement and timelines from the perspective of the local communities. While grants may allow for charter flights to communities, this has the message of importance and status of the researcher. By taking a charter flight, even if well intended to reduce travel time for busy academics, it sets up an us/them divide from the very start of one's arrival. Playing the game of the local communities would involve to preferably travel as they do—usually by land in a 4WD. Similarly, where one stays and how one dresses and presents in community can either create a divide or foster inclusion and respect for the community.

Being Part of the Team: Pitch In!

Research processes require researchers to conduct their work, report on that work and generate outcomes. The game is quite clear. However, this game is more often than not incongruent with the games within communities and schools. While the expense of the conduct of research in many remote, hard to reach communities is very high, resulting in increased pressure that unexpected community events do not disrupt one's intervention or data collection plans, the realities and demands within the community and school are distanced from these needs. Many events happen in communities who are often resource poor and while events require urgent attention communities often struggle to manage with limited resources available. We have found ourselves in communities where there has been a funeral but with no people to erect tents for families who are grieving. The community needed support to cater for the people coming for the funeral so we spent our time erecting (and dissembling) tents for guests, supporting the preparation of catering to enable the smooth running of the funeral. We have been in sites over weekends where there are demands on the school and have found ourselves washing and painting walls, gardening, creating resources for classrooms. When teachers and communities have observed our willingness to participate in engaging with their needs (and parking our own), they have become more interested in our purpose for being in the school and community. Authentic relationships, partnerships and research with communities requires flexible navigation of the intersection of research and community practices and games.

Gaining Broader Community Trust

The communities in which we both work are ones where many foreign workers come into the community to support the local people. These include but are not limited to health and welfare workers, government agencies of all departments, not-for-profit organisations to offer various supports for the community. As researchers coming into the communities there is potential for an already visitor-weary school/community to see the researcher/s as yet another person coming into their community. The research game needs to be extended to think of schools as part of a much larger community. The communities are often inextricably linked with other organisations within the community so extending the reach of projects beyond the school gates assists non-school entities and people to be aware of the research projects and potentially become more involved in supporting research that they feel offers more than the fly-in-fly-out research projects. In our work in marginalised communities, we reach out to providers and agencies working in the broader community so that there is greater knowledge of the purpose of our presence in communities.

Being Identifiable

Researchers are usually outsiders to the school and coming into the school context can arouse suspicion and mistrust. Many strangers pass through communities and their purpose may be unknown to the community members. Part of being transparent and open is for community members to know the origins and purpose of the strangers in their midst. Having items to identify the researchers—such as shirts with the employing body's logo clearly visible—helps community members know the origins of the strangers. Also wearing name badges assists in the identification of researchers. While the status of "Doctor" or "Professor" may be status in the game of the University, it has little value (capital) in the field of the community. In our work, we do not use our titles so to reduce any potential power imbalances.

Gatekeeping and Authenticity

Gatekeeping is a term that suggests that there is a person or process that allows some researchers access, and potentially denies access to others. The gatekeeping process can have significant impact on the conduct of research (Poed et al., 2020). Gatekeeping intersects with other aspects of this paper, including trustworthiness of researchers and relevance of research projects to the school. Community and school gatekeepers may have a healthy scepticism of the intent of researchers who want to work *on* or *in* their schools. Given the time it takes to develop trust and rapport with teachers, students and community, the more expedient way to play the 'successful' game of research, the propensity to genuinely work *with* schools is quite an onerous task and can be beyond the usual parameters of research. While the hegemonic research game is for a researcher to have a project based on their expertise and implement this in schools, this process may be at loggerheads with the game of schools and may not genuinely meet their needs.

Beyond Formal Ethics: Responsiveness to Local Events and Customs

As researchers we are bound by the formal ethics of our employing bodies and the bodies/systems in which we conduct research. We are also bound by our own moral compass about the conduct of ethical research. The game of research may bind researchers to the objective structuring practices of ethics and ethical research. But there are also ethical considerations to be made in relation to the conduct of research in communities that are different from the hegemonic structures of University and School System ethics. Being respectful of the community norms is a very different game from the game of University Ethics. What are the rules of the game for communities in terms of gendered relations, hierarchical structures, or of cultural events such as deaths or births or other ceremonial occasions? Knowing the rules of these games is critical to the conduct of ethical research but these are quite different from the rules for the conduct of research within the University game.

Considering the Consequences of the Stories We Tell

As researchers, it is invaluable to consider the consequences of the stories that are told from the research process. As researchers, we need to consider our complicity in the reproduction of negative stories of teachers, communities, particular equity target groups and families. It is important to consider the role of deficit stories of the 'ability' of learners who live in poorer, more impoverished communities. These can lead to low expectations, self-fulfilling prophecies and a reproductionist agenda (Graven, 2014). Rather, than focus on what learners (and teachers, communities etc) can't do, more positive stories and foci could be developed on the strengths and willingness to challenge these deficit stories. Learners in impoverished communities bring a wealth of knowledge and strengths to mathematics classrooms but these may be different from the structuring practices of schooling (e.g., strong visual-spatial and navigation skills of Aboriginal children). By challenging the orthodoxies of entrenched and taken-for-granted practices, new forms of knowing and being can be foregrounded.

Conclusions

In this paper we have set out to highlight a need to embrace an openness to shifting the rules of the 'traditional' notions of the research game for those who work with highly marginalised communities. The demands and needs of learners, teachers and teacher aids cannot be known by researchers without an openness to navigating flexibly and through a process of building relationships new mutually acceptable rules of the game. This may not be necessary for university researchers working with teachers and learners in high performing and highly functional schools. These latter schools might face few disruptions to planning and might buy in whole heartedly to the research agenda of the researchers. Often participants hold aspirations that perhaps one day they might study further or conduct similar research. Such aspirations can support buy-in to the research and the learning process. This makes for a very different research context than those of marginalised communities, schools and learners where there the aspirations of participants have little alignment with the researchers and research activities and incentives for participation need careful navigation.

In navigating the rules of the research game we have found that it has been essential to our work and learning in various under-served and marginalised communities to reconsider how rules that apply to 'mainstream' research contexts may need to be adapted for use in remote and or marginalised communities. Much of this learning and experience cannot be published or shared in conference platforms but builds towards opportunities for engaged authentic research that builds towards powerful new knowledge in our field. This paper emphasises our need to remain open to flexibly adapting one's research goals, approaches and needs through engagement with participants and their school and broader communities. We express our deep gratitude for all those research participants and their communities who have allowed us to experience and understand ways to enact an ubuntu research perspective. The elaboration of an ubuntu research perspective, theoretically and methodologically, is emerging among various South African scholars across different fields of education. It is our hope that our reflections on our grappling with some of the rules of the research game from an ubuntu perspective contribute to this endeavour.

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Are Learners Referring to the General or the Particular? Discursive Markers of Generic Versus Empirical Example-use

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Supporting students of all levels to move beyond empirical arguments, which employ example-based reasoning to endorse universal truths and are thus mathematically invalid, remains a challenging goal in mathematics education. Arguments that make use of generic examples are both mathematically valid and accessible for even young learners. However, discerning whether students are viewing or using an example as a specific case, or a general case, is difficult. In this paper, we open the space between empirical and generic use of examples and establish categories of example-use regarding odd and even numbers. We reveal discursive markers pointing towards whether a learner is referring to particularity or generality in their example-based reasoning.

Background

A wealth of research corroborates what Stylianides and Stylianides (2017) described as "key and persistent problems" (p. 124)—students' reliance on *empirical arguments* to endorse universal statements. Empirical arguments are example-based arguments that provide inconclusive evidence for mathematical generalisations; by verifying the truth of a universal statement on only a subset of all possible cases, they fail to eliminate the possibility of the existence of a counterexample. Therefore, mathematically speaking, these arguments are invalid. In comparison, *deductive arguments* use definitions and theorems to produce "logically rigorous deductions of conclusions from hypotheses" (NCTM, 2000, p. 56) and are considered mathematically valid (*proofs*).

While formal, deductive proofs might be out of reach for young learners, the usefulness of *generic examples* have been widely acknowledged (e.g., Hanna, 2000; Mason & Pimm, 1984; Reid & Vallejo Vargas, 2018; Stylianides, 2007). The idea of a generic example can be traced back to Mason and Pimm (1984), who defined it as "an actual example, but one expressed in a way as to bring out its intended role as the carrier of the general" (p. 287). Hence, unlike empirical example-use, a generic example removes the need to produce endless specific examples by showing general (rather than particular) properties of the cases it exemplifies. To illustrate, Stylianides (2007) provided an excerpt of an 8-year-old student using a generic example for "odd + odd = even" where the student drew two sets of seven lines and proceeded to circle them in twos, saying, "[All] odd numbers if you circle them by twos, there's one left over. So, if you ... plus one, um, or if you plus another odd number, then the two ones left over will group together" (p. 7). Generic arguments, such as this, are more accessible to young learners and they have explanatory potential. They can help support students "not only to see that it [a theorem] is true but also why it is true" (Hanna, 2000, p. 8).

The issue remains though, "How is it possible to determine whether students are using or viewing an example generically?" For instance, in the above example, it is quite possible that some may view the same two sets of seven sticks used by the student in Stylianides (2007) illustration as a specific example rather than a general one. Determining whether learners are using examples generically and are therefore seeing the general rather than the particular in examples, is neither obvious nor straightforward (Mason & Pimm, 1984; Reid & Vallejo Vargas, 2018; Yopp & Ely, 2015). This is particularly difficult where students are not aware of what constitutes a valid mathematical argument or where the production of a written argument is not appropriate (e.g., with primary school students). In such situations, it might be

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 330–337. Launceston: MERGA.

necessary to look for more subtle signs that point to students implicitly recognising the genericity of an example and reasoning deductively. Accordingly, in this paper we explore the empirical-deductive space and use the commognitive framework (Sfard, 2008) to examine the ways in which students use examples in their reasoning.

The Commognitive Framework

Sfard (2008) defined mathematical *discourse* as a special form of communication, made distinguishable via four interrelated characteristics: keywords (e.g., number-words like “three”, “fourteen”); visual mediators (e.g., numerals, symbols, diagrams, pictures); narratives (e.g., definitions, proofs); and routines (repetitive ways of performing mathematical tasks). Learning is seen as a lasting transformation in a learner’s discourse, which is identifiable by changes in one or more of these four characteristics. According to Sfard (2008), learning occurs both at the *object-level* and the *meta-level*. Object-level learning is signalled by an expansion in the routines and endorsed narratives within one’s discourse. For example, when an individual who endorses even numbers as “numbers in the sequence 0, 2, 4, 6, 8 ...” learns that these are also “numbers ending in 0, 2, 4, 6, 8” or “divisible by two”. Meta-level learning occurs when learners transition into a discourse that is different from their familiar one, requiring a change in endorsing “propositions about the discourse rather than about its objects” (Sfard, 2007, p. 573).

Transitioning from endorsing empirical arguments for universal statements to endorsing deductive arguments requires a meta-level shift in learning. Whereas in empirical discourses learners converse about specific objects, in deductive discourses learners are required to converse about abstract entities. In commognitive terms, learners performing an empirical substantiation routine will use numeric keywords (specific numbers) or visual mediators signifying specific numbers to model the resulting sums they make, and they rely on the sums of such numeric examples (such as, 3 + 5 = 8) to substantiate a universal narrative (e.g., odd + odd = even). In contrast, a deductive routine for substantiating a universal narrative (e.g., odd + odd = even) relies on a series of propositions supported by definitions (e.g., odd is even + 1; a multiple of 2 plus 1; or $2n + 1$), theorems or axioms, whereby each proposition is logically deduced from the previous one in an organised way (e.g., $(2n + 1) + (2m + 1) = 2n + 2m + 2$).

Although examples are not necessarily part of a deductive routine, there is space for them to be so if they are used generically. Commognitively speaking, determining whether an example is being used empirically or generically should be visible via some change in a substantiation routine (i.e., changes in keywords, visual mediators or narratives). Hence, in this paper, we examine students’ verbal responses and their accompanying actions using the commognitive framework to characterise primary school students’ use of examples. We look for discursive indicators as subtle signs that examples are being used more empirically or generically. Specifically, we ask:

(i) *How can learners’ use of examples in their reasoning be categorised?*
(ii) *What commognitive indicators are present in learners’ reasoning that suggests that they are using examples of odd and even generically rather than empirically?*

Conduct of the Study

Data were collected from 28 Year 4 students (aged 8- and 9-years-old) from two New Zealand schools. As the unit of investigation in this study was discourse, teachers selected students to work in groups of four according to whom they considered would be willing and able to engage in a mathematical dialogue. The students were shown a cartoon dilemma which featured three characters, each with a speech bubble containing differing narratives on the sums of odds and evens (e.g., odd + odd = even; odd + odd = odd; odd + odd = sometimes even and sometimes odd) for students to reject or endorse and then substantiate their choices. The students had pens and paper, counters and Numicon tiles[5] available to work with. Each student group session was video-recorded and transcribed in its entirety, and the students' written work was added to the data corpus.

We used fine-grained discourse analysis that utilised Sfard's (2008) commognitive framework to examine episodes of students' dialogue for distinguishing features (words and their use; visual mediators and their use; narratives and routines) that marked their reasoning and example-use as being more specific or more general.

Findings

From the analysis of our data, we were able to classify all comprehensible instances of students' example-use into four categories: (1) Inductive use of numeric examples; (2) Inductive use of numeric-generic examples; (3) Deductive use of numeric-generic examples; and (4) Deductive use of nonspecific-generic examples. We observed a number of discursive markers within these categories that pointed towards generic, versus empiric, example-use when endorsing universal narratives about the sums of odds and evens:

(i) A change from numeric to nonspecific keywords and visual mediators;
(ii) A switch in the mathematical object of focus in substantiating narratives—from the number generated in sums to the structure of the addends and the sum;
(iii) A shift from present to future tense;
(iv) Changes in the use of determiners—from the use of specific nouns, definite articles and demonstrative pronouns to the use of indefinite articles and quantifiers that refer to the whole extent of the particular group or situation in focus; and
(v) The use of illustrative expressions such as "like" or "for example" to indicate the purpose of the example is to illustrate the general in the particular.

Inductive Use of Numeric Examples

Episode 1
Toby and Erin's Inductive Use of Numeric Examples to Endorse "odd + odd = even"

Speaker	What was said	What was done
Toby:	Yeah, [odd + odd = even] because five plus five equals ten. Seven plus seven equals fourteen …	
Teacher:	... Has anyone found an odd plus odd equals odd?	
Erin:	Yes! Two four... Oh yes, yes.	Takes 5 and 9 and Numicon tiles and forms a 7 x 2 array.  Starts counting her 7 x 2 array in twos.
Toby:	No. Because two, four, six, eight, ten, twelve. Fourteen.	Reaches over to count Erin's 9 + 5 array: counting in twos.
Erin:	So, it's even.	
Toby:	Yeah	

[5] Numicon tiles are tangible mediators of numbers 1-10 as dots within a frameless 2 x 5 rectangle.

In Episode 1, Toby initially recalled the sums of pairs of specific odd numbers [5 + 5; 7 + 7] to substantiate endorsing “odd + odd = even” and Erin visually mediated a specific instance of “odd + odd = even” [5 + 9]. The two students’ keywords were entirely numeric (e.g., “five”; “ten”; “six”) and they used the resulting number of the specific sums they have selected as the object in their substantiations for “odd + odd = even.” Furthermore, when the students physically formed symmetrical structures (a paired array) with their two odd (asymmetrical) tiles, it is the number rather than the symmetrical/asymmetrical shape that signified evenness. The students’ disregard of the symmetrical structure of the pairs of odd Numicon tiles they used was most evident when Erin briefly endorsed her selection of Numicon 9 and 5 tiles arranged in a *2 x 7* array as an example of “odd + odd = odd.” Erin and Toby then justified rejecting it as an instance of “odd + odd = odd” on the basis of confirming the *count* is “fourteen,” and therefore even (rather than referring to the symmetrical *shape* of the array). These discursive features suggest that these students are apprehending the particular, not the general, in their example-use.

Inductive Use of Numeric-generic Examples

Episode 2
Sadie’s Inductive Use of Numeric-generic Examples to Endorse “even + odd = odd”

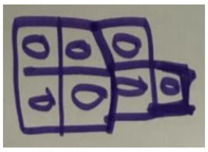
What Sadie said	What Sadie did
[Speaking to herself] Ok. It equals odd. Because this one here is hanging off.	Takes Numicon 9 and 2 and places them together. Points to the extra one from the two. 
So that one’s even. Ok. … because this bit here is hanging off.	Holding up Numicon 2. Places it with Numicon 9 (as before). Points to the extra one on top of the arrangement.
But can I have a experiment and see if Ruby’s [“even + odd = sometimes even and sometimes odd”] right?	Selects Numicon 10 and 3 tiles. Rotates Numicon 3 around to see if it will “fit” with Numicon 10.
Yeah, I think it’s Jed [“even + odd = even”]. This one’s not working. It’s still odd. Ok so I think it might be Jed that’s right. Yeah. Yeah.	

In Episode 2, even though Sadie selected pairs of Numicon tiles, she did not tend to refer to these addends with number-names (only once did she refer to the Numicon 2 tile as “my *two*”) nor did she use number-names for the sums. Instead, when Sadie selected Numicon tiles 2 and 9, she used the words, “one ... hanging off”, to substantiate the sum’s oddness, pointing to the “one” as she does so. Her use of these keywords and visual mediators indicate Sadie realised the generic mathematical (asymmetrical) structure of oddness. However, Sadie only used talk of mathematical structure to substantiate the odd *outcome* (“one ... hanging off”) for examples of “even + odd”; there were no dialogue substantiating why “even (with no “ones ... hanging off”) + odd (with “one ... hanging off”)” addends *together* make a shape with “one ... hanging off,” let alone why this would always be the case.

Sadie exhibited an ad hoc way of connecting the two Numicon pieces: she tried different rotations of Numicon 3 to see if there was a way to connect it with Numicon 10 to make a symmetrical (and therefore even) shape and commented, “it’s not working”, when she was unsuccessful. Accordingly, Sadie did not appear to be using the generic structure of the odd (asymmetrical) and even (symmetrical) addends to account for the resulting asymmetrical shape. Instead, her use of examples to endorse “even + odd = even” were: (i) related to more trial and check—she needed to “experiment” with more than one pair of “even + odd” to check for oddness; (ii) absent of deductive narratives and actions; and (iii) based on empirical observations of the two, confirming asymmetrical (odd) outcomes.

Deductive Use of Numeric-generic Examples

Episode 3
Jane and Zara’s Deductive Use of Numeric-generic Examples to Endorse “even + odd = odd”

Speaker	What was said	What was done
Jane:	[even + odd = odd] because there’s four and there’s gonna be three. It’s gonna be odd because there’s one more left. If that [extra one] wasn’t there, it’d be even because it would be three and three and that’s six. But if that’s there it’s seven so it’s not even.	Draws a Numicon-like 4 and 3 shapes, connected.  Pointing to the extra one on the bottom- right. Covers up the bottom- right square and then uncovers it again.
Zara:	Yeah, because if you have like two plus one, it will equal three. And there’s an even and there’s a one- and there’s an odd and it would still equal an odd. Cos there’d be that one extra.	Picks up Numicon tiles 2 and 1.
Jane:	For example, if there was six and one then we will have one still sticking out. Whatever odd number it is we’ll still have one more sticking out.	Takes Numicon tiles 6 and 1 and connects them. Changes Numicon 1 for Numicon 3 and connects it to Numicon 6. Then picks up Numicon 10 and 7.

At first glance, these utterances suggested that Zara and Jane were substantiating their endorsement of “even + odd = odd” with numeric examples and, as such, their example-use could be confused with that described in the first category (numeric talk with inductive substantiations). The two students used numeric keywords (e.g., three, six, seven), visually mediated numeric examples (e.g., Jane’s drawing of four plus three; Jane and Zara’s use of Numicon tiles) and used narratives that made use of those numeric examples—all of which were features consistent with the empirical example-use illustrated in the first category. However, there was several features that point towards generic example-use. First, in addition to using numeric keywords, both Jane and Zara also used generic keywords related to the mathematical structure of odd: “one more left”, “extra one”, and “one still/more sticking out”. Second, Jane visually mediated the resulting shape from adding different pairs of even and odd Numicon tiles and specifically pointed to the “extra one” by covering it (to indicate evenness) and uncovering it (to indicate oddness). Third, their narratives and actions indicated that they were using these examples for illustrative purposes: their narratives included the words “like” (Zara: “if you have *like* two plus one”), and “for example” (Jane: “*For example,* if there was six and one”), and Jane proceeded to interchange Numicon tiles to illustrate her substantiating narrative, “one sticking out”, holds for other examples.

Further, and even subtler, discursive markers, which also suggested that Zara and Jane were using numeric examples generically (rather than empirically), come from: (i) their use of determiners and (ii) a shift in tense. Note that in the previous category, Sadie used demonstrative pronouns "it" and "this" and she spoke only in the present tense (e.g., "*It* equal*s* odd. Because *this* one here *is* hanging off."). In contrast, Zara switched from using numeric words ("two," "one," "three") to using indefinite articles ("*an* odd", "*an* even"). Had Zara used more demonstrative determiners such as "this," "that," or the specific pronoun "it"—for example, "*this* odd," or "*that* even," or "*it* is even" and "*it* is odd"—her example usage would have suggested that the identity of the even and odd tiles she was referring to was known and they were specifically the ones (i.e., "two" and "one") she had chosen. Instead, Zara's use of the indefinite article "an" implies that the identity of the even and odd addends is neither known nor obvious and serves to make these nouns more general. In a similar way, Jane switched to using the determiner "whatever" when describing an odd addend with its generic "one sticking out", which implies her example holds for *any* odd number.

Furthermore, in this episode, both girls' shift in tense—from present to future—strengthens the sense one gets of them moving from the specific to the general in their example-use. Zara began in the present tense—"there'*s* an even and there'*s* an odd"—but then switched to future tense—"it *would* still equal an odd. Cos there*'d be* that one extra." This shift suggests that Zara may have used this particular numeric example to signify what *would* happen with *any* combination of "odd + even." Similarly, Jane switched to the future tense, when she presented the example of "six and one" and said, "... we *will have* one sticking out."

The object that both students used in their substantiations to endorse "even + odd = odd" was the structure of the sum (not the numeric result) and they showed how the generic structure of the even or odd addends combined to make the odd or even sum. Zara and Jane's use of "if ... then … because" narratives provide evidence of logical and conditional substantiations in hypothetical situations. For example, regarding her use of Numicon tiles for "four" and "three," Jane says: "*If* that [extra one] wasn't there [then] it'd be even *because* it would be 'three and three' and that's six. But *if* that's [the extra one] there [then] it's seven *so* it's [the sum] not even." In short, Zara and Jane's substantiating narratives made use of generic structural features to account for any hypothetical or potential case of "even + odd", which allow suggesting that Jane and Zara were indeed seeing the general in the particular examples they used.

Deductive Use of Nonspecific-generic Examples

Episode 4
Zara's Deductive Use of Nonspecific-generic Examples to Endorse "odd + odd = even"

Speaker	What was said	What was done
Zara:	Yes, so if you have something like a square. If you have something like this.	Draws an oblong rectangle.
Jane:	A rectangle.	
Zara:	Yes, it's an oblong. So, if you have like two circles on each, it will be even. And just keep on going down. But if you added on an extra one here, then it wouldn't be even. So, if you put like another one ["one" is taken here to mean another "odd"] there [referring to her drawing] then it would be even.	Draws two circles in the rectangle. Draws two lines going down from each of the circles. Draws the extra circle (bottom-right). 

In this episode, Zara drew a rectangle that signified her realisation of symmetry in a nonspecific even and her use of the words "square" and "oblong" referred to the generic symmetrical structure of "even" evident in her drawing. These keywords and visual mediators are consistent with generic example-use. Zara then drew a pair of dots inside the rectangle and lines from each of the dots that "just keep on going down." While her drawing resembled the Numicon tiles (which signify specific numbers) that she had worked with previously, it also implied that the even number continues indefinitely, signifies *any* even number, and highlights the "multiple of two" property in *any* even number. Zara then added "an extra one", which made her drawing asymmetrical and therefore prompted a realisation of "*not* even" (odd). This nonspecific odd embodied the same general properties as her former visually mediated nonspecific even had done; it implied that the odd number continues indefinitely, signifies *any* odd, and highlights the "multiple of two plus one" property in *any* odd number.

To endorse "odd + odd = even", Zara deductively used the generic structures of the two odd addends combined to make an even outcome. Note, as with the previous category, her use of "if ... then ... so" conditional statements and her use of "would" implies an imaginary or hypothetical situation. Note, also, Zara's use of the generic determiner "*another* one" implies the identity of the second odd addend (just like the first odd addend) is unknown—it is a generic odd. The combination of all these discursive markers indicate that Zara was seeing the general in nonspecific, and more abstract, examples.

Discussion

Supporting learners at all levels of mathematics education to develop valid arguments has been recognised as an obstacle in moving from inductive to deductive reasoning (e.g., Stylianides & Stylianides, 2017). Supporting learners to use generality in a particular example is part of the issue. Previous research pointed to the difficulty of knowing whether a student was aware of the general intent in an example (e.g., Reid & Vallejo Varga, 2018; Yopp & Ely, 2015). However, our study uncovered four different categories of example-use and several subtle discursive markers that *implicitly* pointed towards generic, versus empiric, example-use when endorsing universal narratives about the sums of odds and evens (Table 1).

Table 1
Categories of Example-use and Changes in Discursive Markers from Particularity to Generality

Categories of example-use	Inductive use of numeric examples	Inductive use of numeric-generic examples	Deductive use of numeric-generic examples	Deductive use of nonspecific-generic examples
Keywords and visual mediators	Numeric	↔		Generic
Object	Number	↔	Structure of the sum	↔ Structure of addends within the sum
Tense	Present	↔		Future
Determiners	Demonstrative pronouns	↔		Quantifiers expressing entirety
Articles	Definite articles	↔		Indefinite articles
Illustrative expressions	Absence of illustrative expressions	↔		Use of illustrative expressions

While example-use has previously been dichotomised as either empiric or generic, our findings show the occurrence of example-use to be more nuanced and multi-layered. The second category fell somewhere in between what has previously been considered *either* empirical *or* generic example-use. In this category, learners were better placed than those in the first category to potentially see the general in the particular because they recognised the generic (symmetric or asymmetric) structure of even and odd. Furthermore, where generic example-use has been viewed as a single category, our findings showed two distinct categories of generic example-use distinguishable by the level of abstraction: "numeric-generic example-use" and "nonspecific-generic example-use."

As is suggested by the overlapping of categories in Table 1, it is quite possible that students' routine ways of using examples may fall somewhere between two categories or be in flux between categories. It is also important to note that we are not claiming that all the discursive features described within these categories will necessarily be, or need to be, present in a learner's narrative to categorise the learner's example-use as fitting within one of the categories. Nor is it likely that one discursive feature alone will be sufficient to suggest that a learner is using examples generically. Hence, the intention behind observing discursive markers is not so that they may be used as an exclusive "must-have" tick list for distinguishing genericity from particularity in example-use, but for them to be used discerningly to strengthen researchers and teachers' conviction that a learner may be signalling generality in their example-use.

The present study is limited in its mathematical focus. The ways in which students use examples in this study might be different in other contexts. Equally, the four categories that emerged in this study are not necessarily transferrable to other tasks. However, by considering learning through a discursive lens, the study has brought otherwise unmentioned features of students' example use to the fore. Further work is required, and future studies would need to explore the way these discursive markers are present in learners' reasoning in different contexts and how they point to learners reasoning about and demonstrating generality.

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Impact of Listening Pedagogy on Mathematics Teacher Thinking During Lesson Study

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Listening is involved in daily school activities, but its importance is often overlooked and underestimated. Good teaching practices evolve from telling to listening in order to understand the needs of the student. In this paper, lesson study, a form of teacher professional development, is used as a contextual platform for developing the practice of listening pedagogy. The findings suggest that lesson study provided a conducive and favourable context for the learning of listening pedagogy. At the same time, it also allowed for rich teacher learning to take place which had an impact on teacher beliefs that contributed to the thinking of listening pedagogy.

Teaching is fundamentally built upon relationships. Listening encourages productive dialogue to occur between the teacher and students, when the teacher probes what the students know and understand as they share their ideas with each other and the class (Egan, 2009). The notion of listening is referred to throughout the Professional Standards for Teaching Mathematics document as part of the vision for mathematics teaching and learning (National Council of Teachers of Mathematics [NCTM], 1991). Specifically, listening appeared as one of the critical factors in both the teacher's and student's role in promoting classroom discourse (p. 35). However, listening is a more complex activity than described by these standards (Harkness & Wachenheim, 2008). Listening is more than hearing. While hearing is a physiological process, listening is a conscious process that requires one to be mentally attentive (Low & Sonntag, 2013). Listening pedagogy, centred on listening, has its roots in the Reggio Emilia approach in early childhood education and foregrounds the idea of respecting and understanding others (Rinaldi, 2006). It exemplifies listening to thought and what cannot be easily heard (Schultz, 2003). However, listening is often overlooked by teachers in the classrooms even though it is a necessary activity in schools and all learning environments (Fogelsong, 2016).

Research shows that teaching is still largely based on telling rather than listening (Duckworth, 1996; Hattie, 2012). To address these issues in the classroom, teachers should learn from one another, and possibly look at actual classroom scenarios, as part of their professional development engagements. Lesson study, a form of collaborative in-situ job-embedded professional development that focuses on the planning, implementation and reflection of lessons, provides a suitable context to cultivate listening pedagogy. In lesson study, teachers form a collaborative team and think deeply about teaching and learning that can change both the way they teach and the way they work with colleagues (Kusnick, 2008). Teachers are also able to share their expertise with each other while helping colleagues notice students' mathematical understandings (Stigler & Hiebert, 2016). Through this paper, I describe how listening pedagogy can serve as a critical enabler for effective teacher learning from lesson through a case study of Mdm. Dahlia, a primary school mathematics teacher.

Theoretical Background

In this study, teaching is viewed as a goal-directed activity. Schoenfeld (2011) referred to three factors that influence decision-making in his theory of goal-directed activity: Resources, orientations and goals (ROG). Resources largely refer to the knowledge that the individual

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 338–345. Launceston: MERGA.

possesses and utilizes for decision-making. Goals can be short, medium or long-term goals. There are also subgoals which help to attain the main goals. Orientations would encompass an individual's dispositions, beliefs, values, tastes and preferences. Specifically, orientations give rise to goals and goals draw on resources which is similar to the relationship reported by Thomas and Yoon (2014). Analysing what teachers think of and believe about listening pedagogy can thus be useful to understand its impact on their learning and classroom practices.

The role of ROG in influencing how a teacher listens in class should not be underestimated as it influences what he or she wants to achieve in the classroom. For example, if a teacher believes that listening is about listening for the "right" answers (Davis, 1997), then it is more likely for the teacher to engage in "funnelling" type of questioning. When funnelling, teachers ask a series of closed questions designed to progressively constrain students' responses until they offer a particular desired response (Wood, 1994). According to Schultz (2003), listening occurs at three levels: social and communal level, classroom level and student level. These levels encompass the idea of listening to silence. Listening to silence involves listening to what is said between and beyond words through a stance of questioning. This includes understanding how and when students might choose to remain silent and how they communicate through gestures and various media (Schultz, 2010). Listening to silence also requires teachers to listen to their students' thinking to identify gaps in their understanding. This refers to how teachers pay attention to students' mathematical ideas as they work on a problem (Doerr, 2006; Kazemi et al., 2016). To do so, teachers draw on their mathematical knowledge for teaching to make sense of their student thinking, which are often expressed in writing, speaking, and other representations such as mathematical language and diagrams. Furthermore, teachers need to orchestrate mathematically productive discussions around mathematically worthwhile tasks to make students' thinking visible (Stein et al., 2008). This is challenging work and it begs the question: How can teachers develop their expertise to listen to this silence?

Lesson study is one possible way to develop teachers' listening expertise. Lesson study requires long-term and cyclic teaching practices among the various professional development models and thus is effective in supporting the development of students' thinking because it is focused on students in each stage of instruction (Celik & Guzel, 2020). However, learning from lesson study is not a given (Lee & Choy, 2017) and experienced teachers need not necessarily learn from their participation in lesson study. For example, Akita and Sakamoto (2014) found that while novice teachers learnt from senior teachers, senior teachers did not learn from lesson study. The senior teachers only gave advice based on their knowledge. The focus of discussion during lesson study was mainly on test scores. So, in the case where the test scores of an experienced teacher's class remained high, he continued to teach using the same style with little input from the learning in lesson study. On the other hand, Bocala (2015) discovered that compared to novice teachers, experienced teachers participating in lesson study were less occupied with implementing the professional learning routine and more willing to experiment with the changes in their own thinking, their students' thinking, and their interactions with content that lesson study was designed to inculcate. They concentrated primarily on how they elicited and listened to students' thinking. Hence, the learning of experienced teachers from lesson study varies interestingly and is worth studying.

In this paper, I investigate an interesting case of Mdm. Dahlia who had developed more sophisticated notions of listening even though she was resistant to the ideas initially at the start of the study. By tracking the changes in her ROG, we can begin to unpack how adopting a listening pedagogy in the context of lesson study might offer a way to shift a teacher's instructional decisions from one that is more focused on listening for the right answers to one that is more focused on listening to students' thinking. The paper is framed by the following question: What are the changes in Mdm. Dahlia's resources, orientations, and goals about listening as she engaged in the processes of lesson study?

Methods

The data presented is part of a larger study that took place in a Singapore government primary school over two years. This study adopted a qualitative single-case study design. The case teacher, Mdm. Dahlia, an experienced teacher, was selected as an interesting case as she was initially resistant to ideas of listening pedagogy. Thus, by concentrating on a single-case study, the interaction of significant factors characteristic of listening pedagogy can be uncovered more effectively compared to a multiple-case study. It will help to elicit the extent of teacher learning of an experienced teacher during lesson study. Furthermore, a single-case study served to present the case longitudinally where changes in the teacher's thinking of listening pedagogy were analysed over a period of two years to reveal how her thinking changed over the duration of study.

The sources of data consisted of planning meetings, research lessons, post-research lesson discussions, teacher interviews and student focus-group discussions. All sessions were audio and video-recorded, except for the interviews and student focus-group discussions which were audio-recorded only. They were then transcribed including a running record of time. Descriptive and reflective fieldnotes were recorded during observation of all sessions (Creswell, 2014). The interviews were conducted by the co-Principal Investigator (co-PI) (C1) and a project collaborator (C2). Lesson artefacts collected included the lesson plan, photos of student work as well as photos taken during the lesson. The multiple sources of data were used to triangulate the themes that emerged in examining the changes in what and how Mdm. Dahlia thought of listening pedagogy over the two years.

Data analysis was carried out in five stages. The first stage involved reading the transcripts in conjunction with the fieldnotes to familiarize with the data. The second stage involved reading the transcripts of the lesson study sessions, interviews and student focus-group discussions to identify key moments and critical incidents (Goodell, 2006). They were then organized using chronology as an organizing variable into Excel. Critical incidents are classroom events which have significance for the teachers, make them question their practice and seem to provide an entry for their better understanding of teaching-learning situations (Hole & McEntee, 1999). Thirdly, using Schoenfeld's (2011) factors that influence decision-making, the critical incidents were coded and classified under the categories of ROG. In the fourth stage, Schoenfeld's (2011) form of representation using columns to characterize each lesson segment or incident (pp. 64-65) was used to track the changes in teacher's thinking. To depict the changes in Mdm. Dahlia over time, the critical incidents were colour-coded according to the codes generated in columns in a single continuous Excel sheet to make visible her ROG at each point of occurrence. The fifth stage involved connecting the codes and identifying themes through a systematic thematic content analysis (Fereday & Muir-Cochrane, 2006).

Findings

Here, a snapshot of Mdm. Dahlia is first presented by highlighting her background and inferring about her initial cluster of ROG. Her initial ROG about listening pedagogy is illustrated by describing a teaching episode in which she was the research teacher. Finally, the changes in Mdm. Dahlia's thinking about listening pedagogy after her participation in the lesson study at the end of the second year are highlighted.

Mdm. Dahlia's Initial ROG: "Not a Math Person"

Mdm. Dahlia is an experienced mathematics teacher with 22 years of teaching experience (R1) (see Table 1). She holds a Diploma in Education and is formally certified to teach English language, mathematics and science. She has been teaching students from the foundation stream for the past 17 years and had recently made the shift to teach standard stream students, as she did not want to "stagnate" herself in the foundation stream. At the same time, Mdm. Dahlia described herself as "not a math person" and "not very quick with numbers and concepts," which pointed to her conception of her own mathematical capability (R2). Mdm. Dahlia had the goal that all students should be happy in the classroom (G1) and look forward to learning. She also had the goal that students should learn the big idea from the activity (G2).

Table 1
Mdm. Dahlia's Initial Resources, Orientations and Goals

Resources	Orientations	Goals
R1: Rich teaching experience	O1: Beliefs about listening pedagogy – "Just a pedagogy"	G1: Students should be happy in the classroom
R2: Limited mathematical capability		G2: Students to learn the big idea

When asked about listening pedagogy in the first year, Mdm. Dahlia (D) viewed it as "just a pedagogy" and a "culture" (O1) and did not seem convinced of its effect in the classroom, as seen in the following excerpt:

> C2: Do you have anything else to share? Regarding your experiences so far? About listening pedagogy, about collaborative learning.
>
> D: I mean nothing much… it's just one pedagogy, it's not even a pedagogy I think it's just a culture- Yeah. It is a, you know, classroom culture, behavioural, it's a norm.

She thought that listening pedagogy does not bring about much changes in the classroom in terms of student performance, which she said would depend on "other factors" too. She also believed that listening routines were already in place in her classroom and did not view the idea of listening pedagogy as potentially resulting in a change in the way she would conduct lessons. She was thus seen resistant to ideas of listening pedagogy during the first year.

Mdm. Dahlia referred to listening as understanding what the person is trying to say, and then being able to echo it. She believed this will bring in fun to the learning as "it's fun to be able to understand, it's fun to be able to learn something new," relating to her G1. However, it was salient that Mdm. Dahlia's ideas of listening pedagogy were largely at the surface level, focusing mainly on students' responses, giving them opportunities to be heard by their peers as well as building onto students' responses. This would constitute of physical listening routines, similar to hearing.

Mdm. Dahlia's Teaching Practices: Listening for Answers and Telling

Although Mdm. Dahlia said that listening includes listening to students' thinking during the first interview, she was more concerned about listening for the right answers in her lesson. One of the things that she mentioned was that she "strives for accuracy" in her students' work. During the research lesson in Year 1, Mdm. Dahlia faced challenges to address student misconceptions. The topic for the research lesson was 'Area of Rectangles.' In her lesson plan, Mdm. Dahlia had considered her Year 4 (Primary 4) students' anticipated responses. The following task was planned for the lesson, which students were to discuss and present in groups:

> *Task: A rectangle has an area of 36 cm². What can be its length and breadth? Think of as many combinations of lengths and breadths, that will give the area 36 cm².*

During the lesson, Mdm. Dahlia reminded students to have their "eyes on the speaker" and "body facing the speaker" when their peers were speaking, referring to listening routines. During the group presentations, one group came up with 6 cm × 6 cm = 36 cm² as a possible solution. However, another student in the class thought that it was an incorrect solution. Mdm. Dahlia acknowledged it but continued and moved on to discuss the next group's solutions. It seemed that she did not expect to receive that solution although it was highlighted as a special case in the lesson plan. She struggled to address it immediately which could be attributed to her R2. Another group of students explicitly stated in their task sheet that 6 × 6 cannot be a combination as the question involves a rectangle:

> Describe the strategy that our group has chosen: *Find the factors of 36. Since it is a rectangle, 6 × 6 cannot be the correct answer.*

At the end of the group presentations, Mdm. Dahlia took the opportunity to address 6 × 6 as a possible solution by telling them it is a correct solution. However, most of the students disagreed that a square is a rectangle. When such confusion arose, Mdm. Dahlia found it difficult to convince them as she seemed rather unprepared in having anticipated the response. At this point, she faced difficulty in listening to the silence present in her class and thus her students' mathematical thinking.

> D: 6 times 6 cannot be the correct answer because 6 times 6 is a square. Is it? How many of you agree? Cannot be. That means a square is not a rectangle, cannot be an answer. It is not a rectangle. Square is not a rectangle. How many of you agree? How many of you disagree? Those of you who disagree stand up. You disagree, that means you say square is a rectangle. Eh, how come nobody stand up?... You have to have an opinion. Okay, let's refresh back. Square, what do you know about squares?

This was reflected in the way she turned the table around to question the students why they thought square is not a rectangle (see excerpt above). She adopted the approach of asking the students to convince her otherwise. After which, during the interview, Mdm. Dahlia expressed disappointment in being unable to convince the students during the lesson.

Changes in Mdm. Dahlia's ROG: Listening to Student Thinking

The impact of listening pedagogy was largely seen in Mdm. Dahlia when she was a lesson study participant in Year 2. Due to the structure of the lesson study in the school, Mdm. Dahlia was not selected as a research lesson teacher in the second year as other teachers were given a chance. At the beginning of the year, teachers were asked to share their thoughts on listening pedagogy thus far. Mdm. Dahlia stated that it was not the case that she has "never" used listening strategies before, but that she was clearer about listening pedagogy and more mindful about making it evident in the classroom now. One of the key changes in how Mdm. Dahlia listened was through her increased awareness of mathematical ideas shared during the planning meetings. For example, during one of the meetings, Mdm. Dahlia wondered about the use of the term 'units' or 'parts' for part-whole concept of fractions where there was a discussion of mathematical conventions. She seemed more sensitive to mathematical ideas shared by the teachers and the co-PI who was also the knowledgeable other. She also demonstrated increased confidence in sharing her own ideas about mathematics. Mdm. Dahlia displayed eagerness to learn, constantly asking questions and seeking clarifications. Consequently, there were several snippets of deep conversations pertaining to subject matter facilitated by the knowledgeable other. In another instance, she enthusiastically related from her teaching of Year 6 (Primary 6 or P6) students during one of the discussions, in relating the definition of a turn to the space around it. This was at a point when other teachers were struggling to provide a response to the definition even after multiple prompts by the knowledgeable other. This is seen in the excerpt that follows:

C1: How would you define a turn?

D: Prof, am I allowed to think in terms of angles now? I'm teaching P6 this year.

C1: Yes you can, you can add whatever you think. Ok yes so what's the missing part?

D: Because at P6, I mean if I'm a P6 kid this would translate to 360 degrees... Because they already learn angles, which is the space around.

Such instances helped shape her new Goal 3 (G3) of understanding and relating to students' thinking process. This change in Mdm. Dahlia could be attributed to the planning meetings which provided her a safe space where teachers discussed about their common problems faced in the classroom. This concurs with previous findings that lesson study offers a community for teachers to freely discuss their ideas without scrutiny (Cheng & Lee, 2011; Lee, 2008). Through her participation in the meetings, Mdm. Dahlia developed an improved conception of listening pedagogy. Since she had a strong belief in Orchestrating Productive Mathematical Discussions (OMD) (Stein et al., 2008) since the beginning as part of her G2, she began to assimilate ideas of listening into her orchestration which led her to appreciate listening pedagogy as part of OMD, rather than independently. In her second interview, she linked listening pedagogy to the five core conversational skills of restate, revoice, reason, questioning, and non-verbal communication. By aligning listening pedagogy to the five talk moves, she was more cognizant of the various aspects of listening. Through each move, Mdm. Dahlia took the opportunity to practice listening pedagogy. This pointed to a state of learning that was occurring within Mdm. Dahlia such that she was now listening more intently to her students rather than just hearing.

Moreover, she was more receptive to gathering students' responses in her classroom practice in order to achieve the planned learning outcomes, in line with her G3. This was reflected in her interview:

C1: So, it seems that you have kind of weaved some of these ideas into your own lessons. So, what are some changes that you see in your own teaching over the last one to two years?

D: So I think it (listening pedagogy) has helped me to be more receptive to… getting an array of responses. I think the takeaway is to get all their responses first and then worry about, you know getting them to see what we want them to see.

She also valued the different types of thinking that students have in the pursuit of achieving the big idea (G2).

D: Every child has got valuable thinking... that we can you know, benefit and learn from. So, I think that is the takeaway I want from my math lesson. And then at the end, having all the kids to… learn the big idea from the activity.

She believed that experienced teachers can tweak the structure of the lesson skilfully. Thus, she did not find implementing listening pedagogy challenging (R1). She was making her classroom more student-centred by paying attention to their thinking process via listening pedagogy. The changes in her ROG are depicted in Table 2:

Table 2
Changes in Mdm. Dahlia's Resources, Orientations and Goals

Resources	Orientations	Goals
R1: Rich teaching experience	O1: Beliefs about listening pedagogy – *Listening as part of OMD*	G1: Students should be happy in the classroom
R2: Increased awareness of mathematical ideas		G2: Students to learn the big idea
		G3: To understand students' thinking process

We can say that Mdm. Dahlia had become a better listener attentive not just to the voices and actions of her own students but also to her colleagues and the students participating in the lesson study. It was important that she realized that listening to one another is a collective action, not a technique or strategy, both in the classroom and in the professional development activity of planning meetings.

Discussion and Implications

Mdm. Dahlia predominantly held a product-oriented perspective towards listening where she displayed a disposition in the form of asking students various questions to check their comprehension of the responses shared in class and having them repeat the content. This initial understanding of Mdm. Dahlia was similar to that found in Nazari's (2020) study involving the impact of a listening instruction course on teachers' cognitions about listening. It took time for Mdm. Dahlia to understand the ideas of listening pedagogy in-depth and for them to be revealed in her thinking. Mdm. Dahlia's initial resistance to ideas of listening pedagogy gradually reduced as she took part in several cycles of lesson study which were collaborative in nature, providing a safe environment that aided in learning. While it was not possible to see large changes in teacher views of listening pedagogy within a period of two years, the study indicated the beginning of a positive change. Further research would be needed to determine the causes of teacher resistance to learning and how to address them effectively. Through her increased awareness of mathematical ideas, Mdm. Dahlia was displaying a hermeneutic listening approach to mathematics teaching (Davis, 1997) which enables teachers to bring the insights of constructivism into meaningful dialogue with the challenges of various critical accounts. The most promising change noted was reflected in her new goal of understanding students' thinking. This is consistent with findings from Suurtamm and Vezina (2010) which reported that changes in teachers' classroom practices, changes in teachers' understanding of mathematics, and changes in students' understanding of mathematics were mostly connected to listening to student thinking.

Through the findings, this paper has shown that lesson study provided an impetus for Mdm. Dahlia to change her ROG, particularly in her thinking of listening pedagogy. This change influenced her learning from lesson study in Year 2. Thus, there exists a dialogic relationship between listening pedagogy and lesson study which enhanced the teacher learning process. By adopting a listening pedagogy in the classroom, the relationship between the teacher and student can present new teaching and learning outcomes. The notion of teaching as listening rather than telling needs to be deeply embedded in mathematics classrooms, so that teachers can provide a more holistic support for students in their learning.

Acknowledgement. The data reported in this paper was gathered as part of a larger research study (Grant number: OER 19/17 LKE) funded by Office of Education Research, National Institute of Education, Nanyang Technological University, Singapore.

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RESEARCH PAPER

Preparing Job-embedded Primary Mathematics Specialists to Lead in Australian Schools

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The description *primary mathematics specialist* has become more frequently used in Australia in the past 10 years, suggesting sustained interest in deepening teacher expertise. Despite this, there is limited contextual research about how to prepare in-service teachers for such roles, and the organising structures needed to facilitate their progress. This paper describes background and emerging insights from a project offering sustained and supported learning experiences for in-service teachers to become primary mathematics specialist teachers in and across their school. Curriculum and contextual experiences designed around a theoretical framework of leadership development implemented over 2 years are described. Early insights include the *priming* influence of an extended focus on mathematical content and pedagogical knowledge on participants' confidence to lead others, and the benefits of being explicit about enabling structures that promote sustainable growth and change.

Literature and Research Problem

Increased mathematical content and pedagogical expectations of primary teachers and student mathematical performance challenges regularly appear in the national educational system narrative. A range of strategies have been trialled to address these issues, including the concept of specialisation or subject expertise in mathematics. For example, in 2010, participation in a new government-funded Primary Mathematics Science Specialist (PMSS) project was offered to Victorian schools with low NAPLAN results. This project has been sustained but with no publicly available research literature on how its implementation impacts individual teachers, their schools, and communities. In 2015, the Australian Institute of Teaching and School Leadership (AITSL) introduced primary teacher subject specialisation mandates for all initial teacher education courses. Priority was given to mathematics, science, and languages. In 2016, NSW government ministers proposed primary mathematics specialisation for in-service teachers to address declining student performance in this subject. Delays have meant that mathematics specialist preparation in NSW is in its early stages.

Developing individual teacher subject expertise across all Australian primary schools is a system challenge that borders on unrealistic. It may be helpful then, to look at the experiences of other countries who recognised the need decades ago to prepare in-service mathematics specialists to support their primary colleagues via "job-embedded professional development" (Nickerson, 2010, p. 54). More than 40 years ago, the National Council of Teachers of Mathematics (USA) board of directors recommended that state certification agencies offer teaching credentials for primary teachers that include mathematics specialist endorsements. The importance of mathematics teacher leadership and specialisation in the US was raised regularly in the following decades, echoed by calls for research into the impact of mathematics specialists on teacher practices and student achievement (Dossey, 1984; National Research Council, 1989; Reys & Fennel, 2003). The continued focus on the potential of this role led to the development of *Standards for Elementary Mathematics Specialists* in 2010, outlining standards for credentialing and degree programs (Association of Mathematics Teachers Educators, AMTE). These programs focus on content, pedagogical content, and leadership knowledge, and involve at least 24 semester hours and a supervised practicum. The US research agenda of primary mathematics specialisation and its impact on teacher and student learning

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 346–353. Launceston: MERGA.

continues to grow, with prompts for further international research on specialists as "hidden players in professional development" (Hjalmarson & Baker, 2020, p. 51).

Literature on effective professional learning emphasises a concurrent focus on content, duration, coherence, collective participation, and active learning to promote teacher growth and change (Desimone, 2009). In the mathematics education literature, this includes a focus on issues central to instruction, the promotion of "high-leverage practices" (Cobb et al., 2018, p. 71) and sustained support of two to three years (Sztajn et al., 2017). The intention to prepare primary mathematics specialists is present in Australia but there is limited education system and research literature on which to build this initiative. Recent research in Australia related to preparing teachers and leaders to generate whole school reform of mathematics teaching and learning has focused on the perceptions of mathematics leaders' successes and challenges (Sexton & Downton, 2014), principals' views about the preparation of primary pre-service teachers with a mathematics specialisation (McMaster et al., 2018), the nature of the School Mathematics Leader role (Driscoll, 2017), and changes in knowledge and beliefs of teachers attending short (6 days) and continuous professional learning (Roche & Gervasoni, 2017). These studies varied in their approach and emphasis, not yet providing a clear picture of how to prepare and support specialisation in Australian primary schools. The most recent research in Australia related to mathematics specialisation offers the concept of a "learning architecture" (Burrows et al., 2020, p. 1), the design and implementation of which can be effective or inhibitive in promoting deep and enduring mathematics professional learning in a school. As mentioned previously, however, there is no supporting research evidence to explicate its application.

It is acknowledged that no single strategy will support instructional improvement for large numbers of teachers, a broad perspective from the classroom to system coordinators is needed (Cobb et al., 2018). Embedded in this perspective are questions about how to prepare job-embedded change agents like mathematics specialists, and the possible structures through which they can learn to lead. If the proposal to prepare mathematics specialists is to progress as a national initiative to address the challenges of teaching and learning mathematics, it is reasonable to ask: are there innovative approaches that build the capacity of in-service teachers to fulfil such roles, and can the theoretical underpinning of these initiatives be used to generate a coherent model for change?

Methodology

The major research question for this study is:

> *How does implementing an innovative specialist expertise approach provide stimulus for teacher professional learning?*

The theoretical perspective rests within an interpretive design and case study methodology as it is concerned with gaining insight into the lived experiences of the participants, and their school context (Merriam, 1998). Case study methodology provided a systematic way of exploring themes through the collection and analysis of the multiple forms of data, including questionnaires, documents, observations, and semi-structured interviews. The tension of qualitative and quantitative data generation was managed, providing a clearer picture of how a teacher's mathematical and leadership knowledge had been impacted. The author was a curriculum designer, regular presenter as well as participating mathematics specialist. Being participant-observer and primary instrument for collection and analysis, opportunities for gathering meaningful data were maximised (Creswell, 2008). Systematic research procedures were maintained, and all participants were informed of the researcher's status.

The Primary Mathematics Specialist Initiative (PMSI)

An unrealised commitment by the NSW Department of Education in their 2016–2020 strategy to recruit primary mathematics specialist teachers was the stimulus for the author starting the Primary Mathematics Specialist Initiative (PMSI) in 2020. The first cohort (2020–2021) finished their 2-year project, and the next cohort (2021–2022) are in their second year. Initiated, developed, and implemented in 13 schools across Sydney, the long-term goal is to build the capacity of in-service teachers to lead the teaching and learning of mathematics across their school. The structure of PMSI is based on the "learning architecture" (Burrows et al., 2020, p. 1) of the PMSS run by the Victorian Department of Education and Training. This structure, appropriately adapted, was adopted as the theoretical framework of the project and the study presented (see Figure 1).

The three main areas of focus of the PMSI theoretical framework are *understanding and leading self*, *working with and influencing others*, and *being catalysts for change*. The first suggests mathematics specialist (MS) professional learning should involve building curricular and pedagogical capability. Increasing the confidence of the MS provides a basis for the development of collaborative practices, and finally whole-school change. Enabling structures in the original PMSS (Victoria) model included *slowing down to go deeper* and *cultivating professional discernment*, ensuring the deliberate scaffolding of time to reflect, analyse, discuss, and apply new learning.

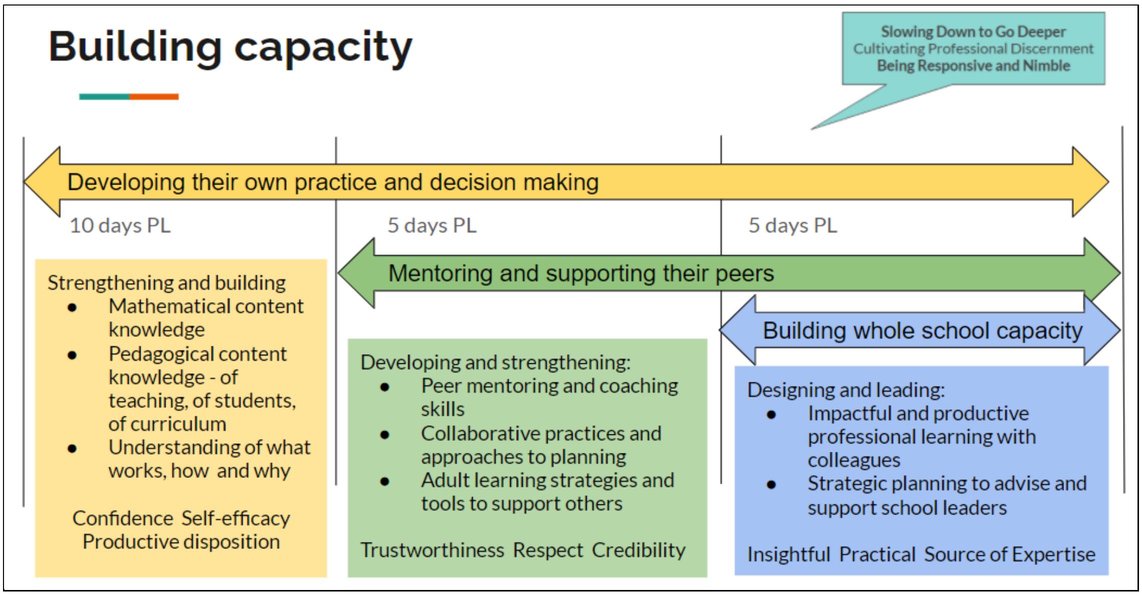

Figure 1. Primary Mathematics Specialists Initiative Learning Architecture (adapted from Burrows et al., 2020).

Adaptations of the theoretical framework for this project included an extension of time for the *understanding and leading self* phase, i.e., from February to December, rather than February to August. This change was informed by research related to the *intensity* of professional learning, including the number of hours and length of time of professional learning, as well as the volume of information addressed (Kennedy, 2016). Such intensity appears to be more effective in promoting teacher learning when the professional learning is aimed at developing teachers' insights into enacting new ideas. Following this line of thinking, it was conjectured that the longer the participating teachers had to trial and reflect on content and pedagogical content knowledge offered in PMSI's first phase, the greater the potential for growth and change. The second adaptation was the addition of a third enabling structure, *being* *responsive and nimble* to reflect the flexible, approachable, and receptive nature needed in such leadership positions.

Launched and facilitated by the author, PMSI started with a pilot group of six primary schools, and 12 teachers. The 2021 intake grew based on positive recommendations from the first group of schools, taking the full project to 14 (one withdrew owing to funding changes). For definitional clarity in this study, the intended role of a primary mathematics specialist is a generalist primary teacher who has specialised knowledge of teaching and learning mathematics, and leading colleagues. They work with students and teachers, team teach with other generalist teachers, and lead whole school reform in mathematics. In this, they may be considered a combination of the disciplinary expert teacher model terms such as Instructional Coach and Generalist Teacher with a Specialisation (Mills et al., 2020).

The PMSI curriculum reflects the overlapping emphasis of each phase of the theoretical framework. There were at least 20 days of professional learning in sequence blocks of two and three days over two school years. Between each of the learning sequences, school-based Applied Learning tasks were provided to deepen the mathematics specialists' contextual understanding of new knowledge and strategies gained, thereby increasing the opportunity to learn beyond the scheduled learning sequences. Applied learning was shared with the rest of the group with a view to generate opportunity for reflection with like-peers and provide opportunities to hone the professional discernment skills. Professional learning sessions were presented by leading primary mathematics researchers and educators, as well as the author who has been working in a primary mathematics specialist teacher role for more than 15 years. Opportunities were created for the cohorts to meet and create connections during Crossover days in the second year of PMSI, thereby strengthening the growing community of specialising teachers.

Method

At the end of 2019, the local community of schools were accessed with the support of the Director of Educational Leadership, and the permission of the central Director of Early Learning and Primary Education schools. A small funding base meant schools would not have to pay for administrative or professional learning costs. The 2-year commitment for two teachers to train as MS with a release time of 0.5 full time employee (FTE) load was outlined. The introductory document included information about the project's goals and the structure of the professional learning curriculum. Schools undertook a selection process, choosing a current teacher to remain with their class for a 0.5 FTE, and be released for the other 0.5 FTE to work with teachers, the other specialist, and leadership team.

Prior to the implementation of professional learning, a background survey was completed by all participants, including MS and principals in an allocated session. Among the data generated were participants' goals for PMSI, and the sub-strands they were confident to teach. These informed the structure of the first-year curriculum focus. Question prompts asked for confidence levels in relation to the teaching of mathematics, content knowledge of mathematics, knowledge of curriculum support resources and understanding of the Working Mathematically processes, addressing the needs of low attaining and high attaining students, planning a mathematics unit of work, assessing and interpreting students' learning needs, making connections across content areas in mathematics, and leading others in the planning and teaching of mathematics. These data were generated using a Likert scale from 0 (no confidence) to 10 (highly confident).

As the first year of professional learning focused on strengthening and building mathematical and pedagogical content knowledge and an understanding of effective teaching and learning strategies, the confidence scale was returned halfway through the 2-year project to MS and principals with their original responses. On it, they indicated any changes on the

Likert scale, qualifying the reasons for such change and identified changes in their Mathematical Knowledge for Teaching (MKT, Ball et al., 2008). At regular intervals participants responded to a variety of reflective surveys to describe growth and change. Making sense of the data in this study drew upon direct interpretation and categorical aggregation, whereby themes and codes were condensed and organised as data were generated (Stake, 1995). The theoretical framework of this study provided initial focus for this analysis.

School context was considered part of the curriculum design, including visits by the author at the beginning of the project, and with all MS during the second year as part of the scheduled professional learning. School demographics and observation of their 4-year plan were viewed as part of the planning process. As the first year's focus was on *understanding and leading self*, each session was planned for a central non-school based location to promote physical and conceptual focus on the MS themselves, and as a new learning community. In the following two phases, where the focus was on mentoring and supporting their peers and being catalysts for schoolwide change, the professional learning sessions were rotated across the participating schools.

The first COVID19 pandemic interruption to PMSI occurred after the first learning sequence block in February 2020 as teachers were no longer able to meet across schools. Cohort 1 had limited face to face meetings in their first year as a result. In order to continue the project, full day presentations were conducted via Zoom and practical materials sent to the participants.

Results

Reflections on early findings and analysis of this emerging research project have been organised into two sections. The first section reflects observations on the first phase of the theoretical framework. The second section analyses data from the second and third phases.

Implementation and Growth in the First Phase: Focus on Self

The purpose of analysing growth data generated by the entry survey was to gain an emerging picture of how the participants respond to the curriculum designed around the theoretical framework. Analysis of the (mean) growth data suggests the greatest changes in confidence for Cohort 1 were: an understanding of the Working Mathematically processes of the NSW syllabus, making connections across content areas in mathematics, and knowledge of resources. For Cohort 2 the greatest increase in confidence related to making connections across content areas and knowledge of resources, and the confidence to ask questions during class discussion. The specialists' narrative analysis of these changes in confidence provided important qualitative context to the quantitative data, revealing self-reported incidences of the Dunning-Kruger effect (Dunning, 2011). For example:

> Whilst many of the "ticks" on my reflection did not shift a huge amount, I still feel enormous growth in each area. I think this points to so much new knowledge and understanding that I did not have before. I think I could have easily moved my original crosses back 3 or 4 steps at the beginning of the year.

In this same reflection, participants described the subdomains of the MKT model (Ball et al., 2008) in which they had experienced the most change. Across Cohort 1 and 2, the greatest overall nominated growth in relation to MKT (percentage of responses) was Knowledge of Content and Students followed by Subject Matter Knowledge then Knowledge of Content and Teaching. Reflection on changes in beliefs and attitudes were evident at the end of the first year, as specialists reflected on misconceptions about what mathematics was and how to teach it. For example: "I have a much better understanding of multiple approaches or strategies when teaching a concept. There were subjects in mathematics (e.g., fractions) where I previously thought there really was only one way to teach it/approach it."

Phase one data demonstrates evidence of the enabling structures *slowing down to go deeper,* c*ultivating professional discernment*, and *being responsive and nimble*. Regular reflections referred to “allowing” themselves time to think. This included the benefits of “having the time and using enabling prompts has allowed me to anticipate student struggles so I am prepared to extend on ideas or provide enabling prompts.” At this stage, most of the references made by MS about change in confidence related to *professional discernment* referred to their interactions with students in class, for example: “I now feel more confident in my ability to make informed decisions about where to develop and lead students, based on their responses, understanding, and ideas.”

Of particular interest is the survey growth reported by all participants to “lead others in the planning and teaching of mathematics” as this was not an explicit focus of the theoretical framework phase of the first year. Growth in MKT seemed to have a *priming* effect on their confidence to lead others. This may have been related to the changes in student responses. Specialists reported changes in their practice. As a result, their students were “excited” and “more engaged” with an increased willingness to take risks and experiment with mathematical explanations. This change in student behaviour and attitude led to conversations with their school colleagues who were now taking an interest in how they were teaching mathematics, and the new resources they were trialling.

Overall, these data suggest an intensive one-year focus on strengthening and building mathematical and pedagogical content knowledge increased the confidence, self-efficacy, and productive disposition of participants to lead others.

Implementation and Growth in the Second and Third Phases: Focus on Others

The second and third phases of the theoretical framework focus on working with and influencing others and being catalysts for change. By this stage MS were working alongside colleagues to plan, implement, and reflect on lessons with others, creating scope and sequences of learning, learning trajectories, analysing data, and generating a whole school vision. Professional learning sessions focused on trialling lesson observation frameworks, programming and planning, developing connective practices to support the learning of colleagues, and leading others through change.

Data generated at the end of the initiative for Cohort 1 provided insight into the impact of the specialists on their peers. These included changes in lesson structure and the incorporation of rich tasks to differentiate learning experiences and provide assessment information. Many MS reported a greater emphasis on mathematical language, classroom dialogue, and reasoning strategies. Changes in attitudes of their peers were also documented, including an increase in “interest and desire” for information about rich tasks, and that “people are keen to come on board and learn” and are “more willing to discuss mathematics, the lessons they are trialling and how it went.” One MS was proud that their interactions with peers had built the confidence and capacity of their colleagues to “lead their own grade in implementing quality tasks.” Such distributive leadership is promising for principals looking to begin the process of whole school change.

Principal responses recorded in surveys across cohorts from 2020–2021 included reflections on the “renewed energy towards the teaching of mathematics” in their school, and the move to “open tasks and varying levels of questioning” to cater for a range of students. Two mentioned changes in whole school beliefs and practices related to ability grouping for mathematics as a result of collaborative practices by MS. Shifting such entrenched cultural practices suggests the positive leadership impact of the MS on teachers, and other leaders at their school. All but one of the MS from Cohort 1 indicated their position was being maintained and self-funded by their school at the end of the project, suggesting the potential vision each of these principals had for this role in future school plans.

Evident in the comments from specialists and their principals was the supportive nature of the specialist community created as a result of their involvement in PMSI. Regular reflections referred to their involvement in a “learning community” that had created a “network who are so supportive and incredible.” Due to the uniqueness of the role, preparing specialists were pleased to work with other teachers in this position.

These data suggest that the specialists’ potential to lead colleagues, and across their school was observed in the second and third phases of the theoretical framework. Reflections on difference in knowledge, beliefs, attitudes, and behaviour included inferences about the enabling structures of *slowing down to go deeper*, and *professional discernment*. Reflections on *being responsive and nimble* generally referred to the MS capacity to work around and through the challenges presented by COVID school interruptions, and their increased capacity to manage peers who were “resistant” to new ideas they were introducing.

Summary and Implications

Research on the impact of primary mathematics specialists highlights the promise it holds as a lever to support teaching and learning (Kutaka et al., 2017). Data generated during the 2-year PMSI aligns with this research. Insights to date suggest teacher professional learning was stimulated through curriculum and enabling structures that adhered to the underlying principles of the chosen theoretical framework. Growth and change were observed in three connected areas: individual knowledge of mathematics and teaching mathematics, strategies to mentor and support peers, and building whole school capacity.

An emerging insight from this research includes the *priming* influence of deepening and strengthening MKT to lead others in the same. This attends to the enabling structure of *slowing down to go deeper* and the positive benefits of an extended focus on the *understanding and leading self* phase of the framework. Such insights have positive implications for schools and systems providing continuous high-quality professional learning for teachers preparing to specialise and take leadership positions in their school. It does not suggest, however, that an equal investment in time to build on this leadership capacity should not be given; the year following this *priming* effect provides opportunity to embed and experiment with leadership practices that are beginning to take shape.

Another interesting observation is the frequent appreciation of an extended period of time to make long term and sustainable change in communities of teachers. Both MS and Principals referred to the association between “time” and “deep/er” learning, suggesting the 2-year period set out in the theoretical framework was positively influencing the change process. This frequency of reference may also have been related to the reinforcement in professional learning sessions of the enabling structure, *slowing down to go deeper*. If this is the case, being explicit about such conditions has the potential to make them more noticeable, and more effective.

The organising structures needed to manage the sustained and supportive professional learning of projects like PMSI included a near-vision view of participants, their individual growth and teaching context complemented by a wider perspective on the theoretical underpinnings and curriculum focus. This may have implications for those designing coherent systems and support structures that address the complexity of preparing in-service specialist teachers who lead others and be challenging for large education systems.

This paper aims to address the scarcity of Australian literature related to the fusion of disciplinary and leadership expertise in the primary school context. It aligns with effective professional learning research proposing intensive and lengthy concentration on issues central to instruction to promote MKT growth (Desimone, 2009; Cobb et al., 2018). It goes further by offering descriptive insight into the application of a theoretical framework and associated curriculum designed to promote sustained and supportive structures that prepare mathematics specialists to lead in and across their school. While no claim of generalisation is made about this case, it can be said that the theoretical framework has demonstrated potential and further research of its application would be constructive in building contextual literature.

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RESEARCH PAPER

Aligning Mathematical and Musical Linear Representations to Support Fractional Reasoning

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This paper describes the authors' journey in designing a linear representation that aligns mathematics and music to support fractional reasoning. The three authors, guided by the theoretical framing of realistic mathematics education, engaged in a task design process over a 12-month period to develop integrated resources. Data were collected in the form of Zoom meeting recordings, among other sources. This paper describes some key findings from a narrative analysis of this process. We posit that the contribution is twofold—methodological insight of a task design journey and findings around the obstacles and resolutions of aligning representations across mathematics and music, despite the obvious confluence.

The focus of this paper is the development of a key representation modelling the integration of music beats per bar and note values and fractions on a number line. Guided by principles of realistic mathematics education (RME), this linear representation was intended for use in supporting teachers' and their Grade 4–6 students' (ages 9–12 years) mathematical problem-solving across multiple constructs of fractions (a gap the authors identified in literature). This paper describes how the first author, a doctoral candidate, together with her two supervisors, embarked upon an exploratory research journey in search of a potential mathematics-music confluence. The journey involved 12 months of task design and materials development work. The tasks have been shared with ten mathematics teachers in two primary schools in the Eastern Cape province of South Africa.

Initially, identifying synergies between the mathematics and the music seemed straightforward. As the process unfolded, however, seeking alignment was not as simple as we had first anticipated. There are some obvious links, such as note values being described as fractions (for example, half notes and quarter notes). Such links do not readily translate, however, into an alignment of key representations such as the number line and the music bar. This prompted the research question: "How can mathematical and musical linear representations be aligned to support fractional reasoning?". We present a narrative analysis of the task design process as evidenced by a data set comprising journaling, e-mail communications, recordings of our Zoom meetings, trialled representations, and field notes on teacher feedback. Review of these data provide insight into the grappling that occurred during this methodological process, linking well to the MERGA44 conference theme: *Mathematical Confluences and Journeys*, and foregrounding our methodological journey. We share the rigour and structure through which we analysed the process of integrating fractional concepts in mathematics and music. The empirical contribution is, we believe, the identification of certain obstacles that may arise when designing tasks that integrate mathematics with another subject area (music, in the case of our paper).

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 354–361. Launceston: MERGA.

Literature Review

Multiple Constructs of Fractions

Fractions, an integral part of most mathematics curricula, are challenging to teach and learn. One of the reasons suggested for this challenge is the complex nature of multiple, and interrelated, constructs of fractions: fraction as measure, fraction as quotient, fraction as ratio, fraction as operator, and the part-whole fraction model (Siemon et al., 2015). In many primary-level mathematics classrooms, the part-whole construct is all too often the main, or only, focus. A single context or problem scenario is seldom considered from the perspective of multiple, interrelated constructs. Luneta (2013) argues that these multiple constructs of fractions, while derived from the ways in which they were used, should not be taught as discrete categories. Rather, students should be allowed opportunities to experience the multiple meanings of fractions so as to make sense of the same situation in various ways (Lamon, 1999). A further complexity is that, when working with fractions, the unit changes depending on the context in which the fraction is being used, creating challenges when trying to make sense of fraction representations (Lamon,1999; Siemon et al., 2015). The representations discussed in this paper draw principally on two constructs of fractions: fraction as measure (measures or distances from a given point (0) on the same scale, such as a number line) and fraction as ratio (the relationship between two quantities) (Siemon et al., 2015). We explore how the mathematics-music connection may provide rich opportunities for students to experience moving across these multiple constructs of fractions.

Task Design to Explore the Mathematics-music Connection

Literature on the connection between mathematics and music describes some of the ways in which the integration of the two can benefit education. Geist et al. (2012) note, for example, that musical elements such as rhythm, melody, pattern, and beat are known to aid mathematics learning. Benefits of such integration include increasing student motivation and engagement, as well as decreasing anxiety (Edelson & Johnson, 2003; Lovemore et al., 2021). Several studies have connected music note values with the part-whole construct of fractions. Both Courey et al. (2012) and Azaryahu et al. (2019), for example, explore ways to pedagogically use the connection between music note value names (e.g., half note) and their corresponding fraction (e.g., one half). In contrast, Cortina et al. (2015) suggest the design of tasks that instead encourage a fraction as measure construct. The current study seeks to find ways in which mathematics-music integration can support not only the part-whole construct, but also the fraction as measure and fraction as ratio constructs.

Graven and Coles (2017) describe task design as a process through which researchers and teachers develop tasks for a specific mathematics concept, learning trajectory, or group of students. Tasks are therefore designed to create particular opportunities for learning, and manipulatives are carefully selected (Jones & Pepin, 2016). Choy (2016), in studying teachers' design of fraction tasks, explains that task design is a deliberate, careful practice and should provide opportunities to work flexibly between multiple representations of a concept. Tasks for the current study have been designed to guide students in interpreting multiple representations and deepening conceptual understanding of a musical bar and fractional reasoning. Ainley et al. (2015) argue that the question, "Is this task purposeful for learners?" is important for designers to answer (purposeful here referring to an "engaging challenge for the learner within the classroom context" (p. 406) as opposed to a necessarily real-world application).

Theoretical Frame and Methodology

In grappling with the demands of integrating the mathematical and musical representations, the authors looked to RME for their theoretical framework. A primary principle of RME is the recognition of mathematics as a human activity (Freudenthal, 1991). Cobb et al. (2008) identify three key tenets of RME, the first being that the design of an instructional sequence should have an "experientially real" starting point for students in which they can actively engage in a meaningful way (p. 108). This, as van den Heuvel-Panhuizen (2003) explains, may be a mathematical activity which is realistic in daily life, or it may be a problem situation which students can "imagine", provided it is a context which is *real* in the students' experience (p. 10). This aligns with Ainley et al.'s (2015) considerations of task design. Cobb et al.'s second principle of RME is that students' informal reasoning, speaking and symbolising, established from the *real* context, should allow for progression to a more formal mathematical reasoning. And finally, Cobb et al.'s third principle is that mathematical activities should be designed in a way that supports the "process of vertical mathematisation" (2008, p. 109). This third principle was what particularly guided our end point: Students representing and working with formal mathematical modelling of fractions on a number line.

As noted, this paper reports on the initial phase of the first author's doctoral study: The task design phase of developing integrated strategies for teaching fractions and music ahead of sharing the strategies and related resources with the participating teachers. We, the three authors, are also participants in the task design process. Data were collected over a 12-month period, and included 15 hours of Zoom meeting recordings, 77 email threads, WhatsApp communications, reflective research journal entries, and field notes from meetings with the participating teachers. Ethical considerations were upheld via gatekeeper permission and informed consent.

Data analysis started as an inductive process. Data were reviewed and recurring patterns were noticed (McMillan, 2010). Segments from the transcriptions and recordings were identified as critical moments, pointing to a pattern of *obstacle—resolution—obstacle—resolution*. For each of these, the following set of three key questions came to the fore: (a) How does the task maintain the *fidelity* of the mathematics, the music, and the integration of the two? (b) How does the task *simplify* the complexity of the integration for subsequent implementation within a classroom setting? (c) What *key representation* would best support conceptual clarity? These align with the acknowledgement made by Courey et al. (2012) about the importance of designing lessons in which non-musician teachers can "deliver the intervention with fidelity" (p. 253). Categorising strategies were then used to group and compare the descriptive data. Aware of the limitation of categorising strategies (referred to by Maxwell (2013, p. 112) as "analytical blinders"), the authors also made use of connecting strategies to understand the grouped data in context and to recognise relationships between categories. This allowed for an in-depth narrative analysis and presentation of the task design grappling journey towards aligning the mathematical and musical representations.

Findings

We provide, in this section, a summary of the key obstacles and resolutions, plus describe in more detail one cycle illustrating the process of aligning the number line and music bar representations. We show how the questions of maintaining fidelity, simplification of complexity, and identification of key representations were addressed. Nine groupings of obstacle-resolution cycles were identified; the first being the selection of a starting problem scenario which, in line with RME principles, is *experientially real* for students. The chosen story-based scenario of African animals crossing a river allowed for children to jump across a constant distance, in a constant time, but with different rhythms. As is shown in Figure 1, below, this then led to a link between the river-crossing and a music bar.

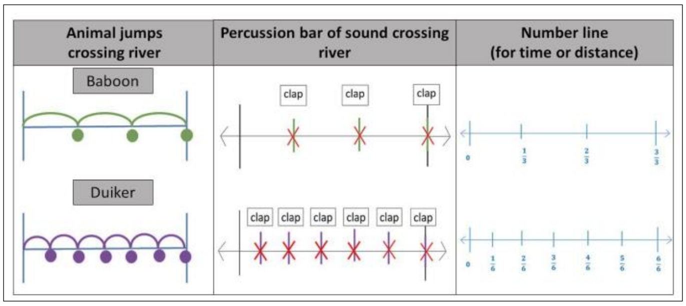

Figure 1. Integrated representations of animal river crossings, music percussion bars and number lines (adapted from Lovemore et al., 2022).

We made the decision that the unit in this scenario could be either the river-crossing (distance and time) or the music bar, so allowing for developing reasoning around both the fraction as measure (measure of time and distance) and the fraction as ratio (jumps per river-crossing or beats per bar) constructs. We saw the opportunity to align the visual representations of the animal river-crossing jumps, the music percussion line, and a number line (as is illustrated in Figure 1, above). Our grappling, informed as it was by the teachers' trialling and feedback, sought to address teachers' expressed concerns about their lack of prior musical knowledge and a lack of confidence to integrate music and fractions.

Obstacles of Aligning the Number Line and the Music Bars

We established that the whole (unit) should be clearly specified across contexts. The decision was that the river-crossing was one unit (of distance and time), and the music bar, similarly, would be one unit. The imagined distance over which the river-crossing stretched, or the time it took for an animal to jump across, could vary, just as the visual representation of distance of a musical bar or the time a bar takes to be played can vary. However, when aligning the music bar with a number line, the music notes are visually placed in the middle of the bar. This, we noted, could cause confusion as it did not align with the time and distance markers on the number line. There is a discrepancy between what one *hears* in music and how one traditionally *notates* the musical note values. In music, beats per bar are used to keep time. One would often hear musicians counting "1234, 1234," for example. In terms of a time progression (measurement of duration of time), for example, one would start at the zero seconds and hold a note for one second (0 to 1). However, this holding of the note would be counted as 1, with the 1 being said at the start of the note or the clap (the mathematical 0 point in time). A further complexity is that visually notated, the note count 1, time 0 to 1 second, is represented in the middle of the music bar, not at the starting line. So, for example, as shown in Figure 2, a whole note played for 4 beats is placed in the middle of the bar in music, yet the note is played (held) from the start of the bar to the end of the bar. Similarly, two half notes could be placed at one third and two thirds of the distance of a bar – yet these placements at thirds do not send the message that the first note needs to be played from the start of the bar to halfway (2 beats – half of 4 beats per bar) and the second is played from the middle to the end of the far (for another 2 beats).

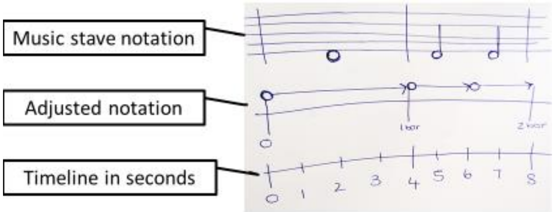

Figure 2. Representing the start and duration of a note value aligned with a timeline.

Merging of fractions on linear representations with musical representations could thus potentially create misconceptions or confusions. As is shared below, these sorts of nuanced differences resulted in much grappling for the authors.

> TL: What I'm seeing here, Mel, is where you are placing the crotchet [quarter note] or the note, it's fitting in between the lines on the timeline.
>
> MG: Sticking it between could be confusing. I think you have to actually do it on the seconds.
>
> TL: But in music we don't. In music we draw them [in the middle] …
>
> MG: Uhm, when we've got our second timeline … and when we play our music we play our crotchet [quarter note] at the start and it lasts one second …
>
> TL: So are you starting on the zero of the timeline? The first [note]?
>
> SR: You measure the second after its done. The note you measure as it starts.
>
> MG: So in relation to the timeline, we start at zero, instantly we start, 1234. And then it lasts that long.
>
> TL: So are we moving away from this idea where we place it in the middle?
>
> MG: Yes.
>
> SR: So you can't exactly align the two number lines. [Zoom meeting: 2021-04-12]

After much further deliberation, we concluded that the traditional notation of the music and the number (time) line could not exactly align. This would compromise the fidelity of both. We therefore decided to trial the adjusted notation shown in Figure 2, above, using a note value visually represented on the zero of the timeline that indicated the duration of that particular note would be four seconds. Due to concern expressed by participating teachers about their lack of confidence in working with western note value notation, we recognised that the representation in Figure 2, above, had to be reconsidered. In addition, the note values held some limitations of only working in circumstances where the music bar was written in a 4/4 time signature, meaning that each bar would consist of four beats per bar. This led us to a change of focus, looking now at the integration possibilities of fraction as ratio (beats per bar or jumps per river-crossing). After further task design grappling, we decided to use percussion beats (Xs) on a percussion line. This seemed a more obvious confluence with a number line (see Figure 1, above). This, however, led to fresh obstacles of fidelity in the musical and mathematical representations. Where, for example, would we clap together? In music, if two musicians clapped rhythms of different note values (e.g., if Musician A were to clap three beats per bar and Musician B, 6 beats per bar), their first clap together would be on count 1. However, in mathematics, the notion of equivalence would mean that Musician A's first clap (of three) would align with Musician B's second clap (of six), because $\frac{1}{3} = \frac{2}{6}$. Once again, we recognised the potential for misconceptions to occur if we were to overlay these representations. Almost a whole year after the first representations, through grappling with the confluences and contrasts, we eventually decided that we did not need to superimpose the mathematical number line representation and the music bar representation, a decision captured in the discussion below.

TL: Yes, and we're trying to represent what we hear, to make it look the same, but it's not. We don't actually have to put the note, the X, on the number line.

MG: So what we need to do here, is to draw some distinction between what we're going to do with the number line, when we're thinking distance, time, where there was a starting point of zero, whereas with the percussion we stick it in the middle of the line… We can see the similarity but we can see what's different as well… because we keep looking at this bar line and we see the similarities. But we're conflating two concepts. [Zoom Meeting: 2022-01-25]

Once this recognition of difference was established, we were able, as we discuss in our next section, to develop linked key representations, maintaining the fidelity of mathematics and music and simplifying the complexity to support moving between the constructs of fractions.

Resolution for Aligning the Number Line and the Music Bars

Having been freed from our self-imposed burden of superimposing the similar but different representations, we could focus on finding an alignment between the representations that built on the original animal river-crossing problem scenario. A key realisation was that it was not necessary to conflate the fraction as measure (measure of time and distance) and the fraction as ratio (beats per bar or jumps per river-crossing) concepts. Rather, the scenario allowed for moving flexibly *between* these interrelated concepts.

MG: I think a big AHA is here, when we dealing with percussion claps we're dealing *4 beats per bar*—that is a *rate (emphasis).* We don't normally put rate on a number line. This is a different concept of fraction as ratio to what we're going to do on the number line. When we say how long does it take to jump? That's a length of time, that is fraction as measure. If the river-crossing is 1 unit that's fraction as measure. [Zoom meeting: 2022-01-25]

RME principles encourage using students' informal models of problem situations and then guiding them through the process of reinventing to arrive at formal representations (van den Heuvel-Panhuizen, 2003). These principles guided us in our design of the key representation and task: Students represent animal river-crossing jumps on a mathematical number line, where the unit is a river-crossing. Due to complexities of musical notation, we noted that it would need to be specified to students that they could make use of certain animals jumps aligning with musical note values i.e., in this scenario kudu jumps (1 whole jump per river-crossing is equivalent to whole notes in music), ostrich jumps (2 jumps per river-crossing are equivalent to half notes in music), zebra jumps (4 jumps per river-crossing are equivalent to quarter notes in music) or monkey jumps (8 jumps per river-crossing are equivalent to eighth notes in music). To simplify the complexity, we explained to the participating teachers that this would work for a 4/4 time signature in music, but that other, more complex, options were also possible. The first author designed a set of resources where students would be given musical note values printed on transparent cards, cut to exact size where one whole note card will fit into one whole music bar, two half note cards would fill the measure of one whole bar, and so on. This was designed to indicate the similarity between the animal jumps and the music note values, where the measure of time is considered. Students would be able to choose where they place the pitch of the note values, as long as their notes fitted into the music bar.

The resource was then further developed to take the form of a triple number line, where the fraction as measure construct could be developed. Here, the number line with the river-crossing units would remain constant. Above it would be a number line indicating distance, and below it a timeline indicating the duration of the animal jumps or the music notes. The units of the distance number line and the time number line could signify different variables creating powerful opportunities for problem solving (e.g., If the river is 10 metres wide and it takes all animals 4 seconds to cross then … etc.). Figure 3, below, provides an example of the key representation being used to compose music. The resource has a musical stave represented above the number line, not to superimpose the two, but to indicate the alignment. The figure shows how we envisaged the representation being used to solve complex problems, moving between the fraction as ratio and fraction as measure constructs.

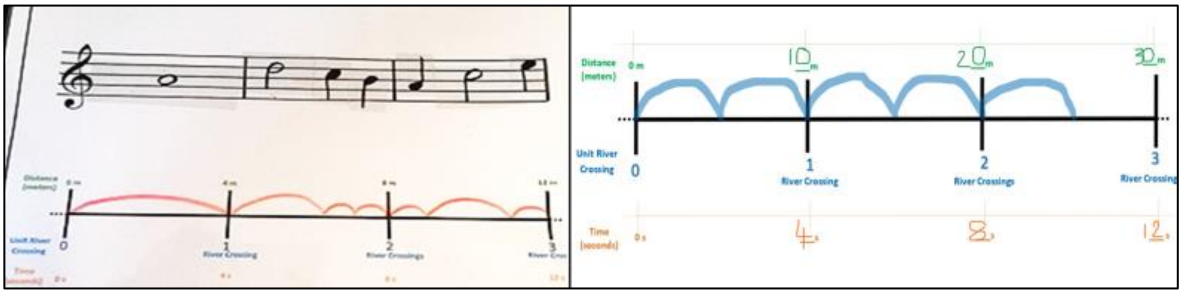

Figure 3. Example of the key representation being used to 'compose' music and the triple number line used to solve complex fractional problems.

Next, we discussed possible questions that could be posed to students: *Ostrich does two jumps per river-crossing. If one river-crossing is x metres, and it takes y seconds to cross, and ostrich jumped 5 jumps, how far would ostrich be jumping in metres? How long did it take Ostrich to get there? At what speed did Ostrich travel?* We recognised that students would be able to use the triple number line representation to support them in tackling such problems. In reflecting on the above representation, the third author commented,

> The power here is that you bring multiple aspects of fractional reasoning into play. The learning here is about shifting attention between different wholes and proportional ways of working. That's the power of this and integrating into music. [Zoom meeting: 2022-01-25]

Once we were satisfied that this key representation maintained fidelity (by aligning, but not overlaying, the linear mathematical and musical representations), we prepared sets of simplified resources to share with the participating teachers.

Conclusion

This paper has described the task design journey of searching for ways to align linear mathematical and musical representations to facilitate students' solving of problems which required moving flexibly between the constructs of fraction as measure and fraction as ratio. The process of grappling throughout this journey was meticulously documented and patterns of obstacle-resolution were identified. Three key elements guided the task design journey: Maintaining the fidelity of the mathematics and the music within the integration; simplifying the complexity of the integration for classroom implementation; selecting and designing key representations to best support conceptual understanding. In answer to our research question: "How can mathematical and musical linear representations be aligned to support fractional reasoning?", we conclude that rather than superimposing the mathematical and musical linear representations, a key presentation aligning a musical stave above a triple number line, can simplify the complexity while maintaining fidelity to support fractional reasoning. We anticipate that our findings will resonate with other researchers, task designers, or teachers integrating mathematics with other art forms or subject areas.

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RESEARCH PAPER

Linguistic Influences on Mathematics Learning: The Relations between Spacing/Spatial Relationship in Handwriting Legibility, Visual-Motor Integration (VMI), and Number Line Estimation

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Previous studies have uncovered the relationship between handwriting and mathematics performance, without articulating the underlying cognitive mechanism. With 197 Chinese fourth graders as participants, this study explored the mechanism by which the spacing/spatial relationship in handwriting legibility affects number-line estimation through VMI. The results indicate that (1) the spacing/spatial relationship in handwriting legibility had a direct and positive effect on students' number-line estimation performance; (2) VMI completely mediated the relationship between the spacing/spatial relationship in handwriting legibility and number-line estimation. Overall, the findings confirmed one specific mechanism of linguistic influences on students' numerical knowledge. Possible explanations for the results and implications for future research are discussed.

Handwriting, as one basic language skill (Kao, 1995), has been uncovered to be closely related to students' mathematics performance (e.g., Li & Nuttall, 2001; Rodic et al., 2015). However, little work has investigated the cognitive mechanism by which the relation between handwriting and mathematics performance emerges. Handwriting legibility, representing the construction, accuracy, or quality of the handwriting products (Maeland, 1992; Shen et al., 2012; Tseng, 1993), is typically operationalised as the spatial consistency of the handwriting products (Bo, 2014), including but not limited to indicators like the letter or character (and stroke/radical)'s spacing/spatial relationship, formation, size, and slope, etc. (e.g., Bumin & Kavak, 2008; Maeland, 1992). These spatial-oriented properties (Bo, 2014) imply that development in handwriting legibility may, to some extent, predict students' spatial advancement (e.g., Abou-El-Saad et al., 2017; Cheng-Lai et al., 2013; Li et al., 1999a, 1999b; Mealand, 1992; Kao, 1999, 2000). Combined with the identified promotion effect of spatial cognition on students' mathematics performance (see a meta-analysis by Xie et al., 2020), extant research comprehensively depicts an intermediary logic to construct the general relationship between handwriting and mathematics performance with the help of spatial cognition. Particularly, VMI, as one critical component of spatial skills (Xie et al., 2020), has been demonstrated to share a high coincidence in cognitive processing needs with handwriting legibility (Weil & Amundson,1994), necessitating the dynamic integration of perceptual, cognitive, and motor components (Kao, 1999, 200) and viewing copying geometric shapes as skill essentials. Previous studies on handwriting identified a significant link between the spacing/spatial relationship in handwriting legibility and VMI (e.g., Parush et al., 2010; Tseng & Murray, 1994). Furthermore, number line estimation was discovered to operate as a spatially meaningful numerical representation for mathematics (Gunderson et al., 2012). As Dowker and Nuerk (2017) suggested, "so far, much research about linguistic influences on numerical cognition has simply demonstrated that language influences number without investigating the level at which a particular language influence operates" (p. 3). In this context, focused on the spacing/spatial relationship in handwriting legibility, a specific cognitive pathway in which spacing/spatial relationship plays a crucial role in the development of number line estimation by helping youngsters to develop a superior VMI ability was hypothesised in this study. In specific with a focus on Chinese characters, the current research aims to (1) quantify the relationship between spacing/spatial relationship in handwriting legibility, VMI, and number

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 362–369. Launceston: MERGA.

line estimation; and (2) explore whether VMI plays a mediating role in the spacing/spatial relationship in handwriting legibility and students' number line estimation performance. These efforts may contribute to the field by deepening the understanding of the linguistic influences on numerical cognition, meanwhile providing empirical evidence about one specific mechanism in which how components in linguistic, spatial, and mathematics domains interact with each other.

Theoretical Framework

General intelligence theory (Spearman, 1904) demonstrates that there is a general intelligence factor out of human's different cognitive abilities (e.g., language, and mathematics) (Spearman, 1904). Sociocultural theory claims that language and thought evolve together, with linguistic experience driving the formation of concepts. Similarly, social constructivism (Ernest, 1991) views mathematics as a social construction, and linguistic knowledge acts as the basis of mathematical knowledge. From the broad concept of language, those theories underline the significance of language to mathematics and the inherent relationship between the two (Lu et al., 2022).

On this basis, focusing on the "visuospatial-orthographic" level of Dowker and Nuerk (2017)'s linguistic framework, narrowing to handwriting aspect, the psycho-geometric theory of Chinese-character writing postulated that the psycho-geometric pattern of the visual-spatial configuration of Chinese characters serves as a critical foundation for cognitive change (Kao, 2000). Chinese-character writing, according to Kao, is a dynamic integration of the writer' perception, cognition, and action, as well as a re-training and enhancement of the writer's visuospatial ability. This theory establishes the promotion effect of logographic character writing in students' spatial perception, representation, and imagination, thereby laying the groundwork for explaining the development of students' spatial cognition and, consequently, mathematics performances in the perspective of linguistics. In this sense, a general mediation framework involving spatial cognition is proposed here to claim the positive influence of handwriting on youngsters' mathematics performance. Further, combined with the conceptual basis clarified earlier, this study postulates a refined theoretical framework concerning spacing/spatial relationship in handwriting legibility, VMI, and number line estimation.

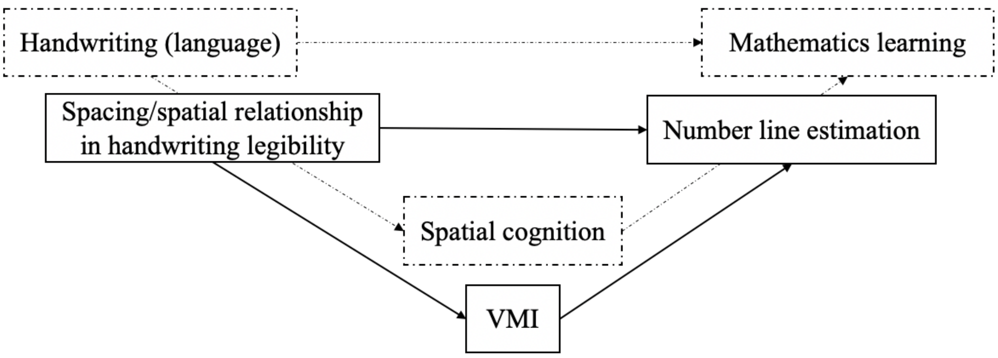

Figure 1. The theoretical framework of the present study.

Method

Participants and Procedures

Participants are 202 fourth graders from one mainstream primary school in central China. Five students did not participate in the handwriting task and were excluded from the analysis.

The final sample included 104 (52.79%) male students and 93 (47.21%) female students, with the mean age being 9.37 years.

Students were asked to complete a variety of paper-pencil tasks, including handwriting, VMI, and number-line estimation (See some sample tasks in Figure 2). Students were assigned tasks in their classrooms (one class at a time). The handwriting task required students to copy a 9*10-Chinese character template (All of the characters were chosen based on the Chinese characters' six main basic structural forms, and 25 out of the 30 basic stroke units, ensuring their representativeness; Li-Tsang et al., 2011) as accurately as possible in 4 minutes. The VMI required students to duplicate 24 line drawings of specified geometric designs (Beery & Beery, 2010). The number line estimation comprised 22 problems involving the numbers 0-1000, in which participants needed to determine the location of a target number on a blank number line, and the error between the target number's actual position and the participant's estimate reflected the participants' internal mental number line representation. The VMI and number line estimation tasks consisted of two phases, i.e., the practice and the experimental phases. In both phases, the invigilator (the first author) supplied detailed response instructions (see Booth & Siegler, 2006 for details). Two more teachers were assigned to manage class discipline and ensure that participants provided careful responses.

Figure 2. Sample characters/forms/items in the Chinese handwriting, VMI, and number line estimation tasks.

A five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree) was used to assess students' performance in spacing/spatial relationship in handwriting legibility (the four-item scale includes the following: 1. Correct spacing between strokes/radicals, the correct position of components; 2. Regular spaces between characters; 3. The alignment of the lines of writing; 4. Square configuration/baseline orientation: no characters are out of grid/line, nor do characters overshoot/undershoot the baseline; with appropriate margins). The first author conducted the handwriting evaluation with participants' identities and performances in other tasks blinded. The VMI evaluation was conducted by a group of undergraduates recruited from one college in XX province in China, with the test manual followed strictly (Beery& Beery, 2010). The number line estimate performance was graded using the criteria provided by Rittle-Johnson et al. (2001). Discrepancies during all the rating procedures were resolved by double reviews of original criteria. The reliability and validity of all instruments are good.

Data Analysis

According to the theoretical model, data analysis in this study included three steps. Firstly, descriptive and correlational statistics on variables of interest were computed. Then, confirmatory factor analysis (CFA) was utilised to validate the psychometric qualities of items within spacing/spatial relationship in handwriting legibility. Lastly, structural equation modelling was formulated to assess the effects of the independent latent variable on the dependent explicit variable, following the mediation effect examination to determine the influence of VMI on the effects of spacing/spatial relationship in handwriting legibility on number-line estimation.

Results

Descriptive and Correlational Statistics

Table 1 shows the correlation among variables. Results indicated that spacing/spatial relationship in handwriting legibility, VMI, and number line estimation were significantly and positively correlated with each other.

Table 1
Mean, Standard Deviations, and Correlations among Variables

	Correlation		
Variable	1	2	3
1 h	1		
2 vmi	.598**	1	
3 nl	.205**	.275**	1
Mean	4.050	17.210	14.360
Standard deviation	.667	2.958	3.768

Note. All correlation coefficients are statistically significant at the $p < 0.01$ level. *h* = spacing/spatial relationship in handwriting, vmi = VMI, nl = number-line estimation (the same as follows).

Structural Equation Modelling Analysis

CFA was first employed to estimate the measurement properties of the handwriting legibility subscale: $\chi^2 = 2.268$, $df = 2$, $p = 0.322$, RMSEA = 0.026, CFI = 0.999, TLI = 0.997, SRMR = 0.015, and the factor loadings were acceptable (Figure 3).

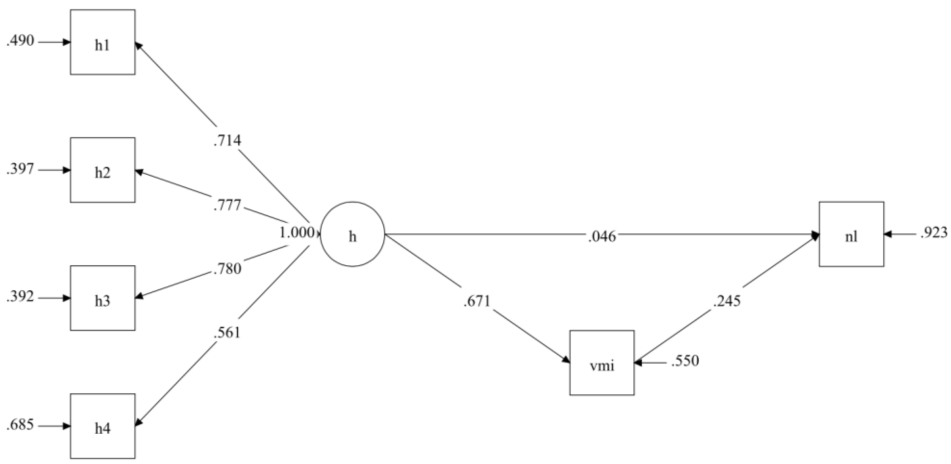

Figure 3. The structural equation model exploring the relationship between spacing/spatial relationship in handwriting legibility, VMI, and number-line estimation.

On this basis, structural equation modelling was further analysed, and the standardised path coefficients of the model were depicted in Figure 3. The model fitted the data adequately with $\chi^2 = 16.329$, $df = 8$, $p = 0.038$, CFI = 0.976, TLI = 0.956, RMSEA = 0.073, SRMR = 0.032. Besides, the model showed that spacing/spatial relationship in handwriting legibility positively predicted VMI and number-line estimation; VMI positively predicted number-line estimation. In further analysis of the mediation effect of VMI, bootstrapping processes were employed to obtain a 95% confidence interval with the original data resampled and replaced 1000 times. The results (Table 2) did not include zero, indicating a significant mediation effect ($p = 0.011$).

Table 2
Results of the Bias-Corrected Bootstrap Test

Paths	Standardised β	SE	Bias-corrected 95%CI	
			Low	High
h→vmi→nl	.164*	.065	.045	.300

*$p < 0.05$.

Discussion and Implications

Based on the general relationship between handwriting, spatial cognition, and mathematics performance, this study modelled one specific mechanism interweaving spacing/spatial relationship in handwriting legibility, VMI, and number line estimation. Meanwhile, our findings confirmed the positive and close relationship between involved variables, as well as the significant mediation effect of VMI in this relationship. Those findings fit well with the psycho-geometric theory of Chinese-character writing, and the basic ideas elaborated by general intelligence theory, sociocultural theory, and social constructivism.

Going beyond previous findings demonstrating the relationship between handwriting and spatial cognition, or handwriting and mathematics performance, the present study not only identified the more specific variables (i.e., spacing/spatial relationship in handwriting legibility, VMI, and number line estimation, respectively) out of these domains but further uncovered a coherent mechanism of spacing/spatial relationship in handwriting legibility on VMI, then in turn, on number line estimation. The positive and close relationship between spacing/spatial relationship in handwriting legibility, VMI, and number line estimation highlights that common cognitive processing needs—spatial thinking, underlies those different domains. This finding supports existing findings by Kao (1999, 2000), Li and Nuttall (2001), and others, by confirming the unique visuospatial properties embedded in Chinese characters and the sound spatial experience accumulated by legible handwriting practice; meanwhile, it emphasised the spatial-numerical association underpinning numerical knowledge (Schneider et al., 2009), fitting well with Dehaene et al.'s (1993) claim that there exists an analogical representation of magnitude using the left-to-right spatial arrangement in the human's mental world. Besides, the significant correlation between spatial ability and numerical ability identified in Xie et al. (2020) ($r = 0.22$, $p < 0.001$) also lends considerable support to the current study.

Moreover, this study demonstrates a substantial relationship between spacing/spatial relationship in handwriting legibility and VMI, which is stronger than earlier research concentrating on handwriting legibility and VMI (e.g., Bumin & Kavak, 2010; Hellinckx et al., 2013; Klein et al., 2011; Preminger, 2004; Wicki, 2014). Focusing on the more specific aspect of handwriting legibility may provide plausible explanations for this disparity. According to the characterisation of the present handwriting evaluation scale, as one critical indicator of handwriting legibility, spacing/spatial relationship encompasses highly spatial-orientated properties at both analytic (stroke/radical) and holistic (character) levels (Lam et al., 2011), such as the position/spacing of/between strokes/radicals/characters, alignment of characters, attention on the grid, etc. (e.g., Gilboa, 2014; Jameel, 2017; Klein et al., 2011; Parush, 2010; Linda et al., 2014). These profoundly spatial features are much in line with the visuospatial essence of VMI ability, suggesting the key role of spacing/spatial relationship in handwriting legibility. In addition, the script inconsistency relative to earlier studies might partially account for the present high correlation. As numerous studies (e.g., Kao, 1995, 2000; Lai, 2008) suggested, compared with the linear arrangement of alphabetic letters, Chinese characters are featured as planar structure; occupying two-dimensional space in height and width, Chinese characters are logographic in nature and has high visual-spatial properties. As such, Chinese handwriting processes, in comparison to alphabetic ones, would entail a more systematic integration of the writer's perception, cognition, and motor (Kao, 2000), which corresponds better to the cognitive processes of VMI. This finding furthers the literature by identifying a more nuanced link between Chinese handwriting legibility and VMI, so providing new evidence on their homogeneity proposed in previous studies (e.g., Li et al., 2018).

Moreover, the complete mediation effect of VMI obtained currently indicates that the effect of spacing/spatial relationship in handwriting legibility on number line estimation is indirect. In other words, Chinese handwriting practice emphasising appropriate spatial positions of each stroke/radical/character in reference to the square frame and its spatial relationships with others, would firstly near-transfer as students' sharpened VMI ability, then, in turn, far-transfer as development in number line estimation performance. This indirectness is expected for language (handwriting) and mathematics source from two relatively different fields, and they need some bridge to construct the linkage. This study verified that VMI performed this role appropriately. Based on the prior knowledge on the linkage between handwriting and mathematics, and the unique visual-spatial properties of Chinese characters, this study adds to the literature with the identification of spatial thinking underlying handwriting legibility and its linkage with VMI and number line estimation. These findings establish a theoretical and empirical foundation for future research on domain-specific cognitive processes associated with language and mathematics learning, as well as provide practical implications for teachers regarding how to enhance students' numerical knowledge through spatial-related handwriting practice.

Notably, while the current work focuses on the fundamental mathematics skill of number line estimation, it is constrained in its exploration of the mechanisms underlying more complicated mathematics learning processes. Indeed, this impact is possible. Previous work has indicated that geometric ability requires a variety of spatial factors, including identifying the characteristics and interconnections of lines, angles, and shapes (Ünlü & Ertekin, 2017), and geometric problems are frequently constructed by figures, all of which are strongly linked to visualisation analysis; logical reasoning problem solving also demands a high level of spatial ability, including the capacity to create visual representations, mental transformations, and visualization, etc. (e.g., Hegarty & Kozhevnikov, 1999; Oostermeijer et al., 2014). Thus, both geometric and logical reasoning problems might have a stronger link with spatial abilities (or VMI in this study) than numerical abilities (Xie et al., 2020). Alternately, according to Geary et al. (2017) and Lin (2011), numerical ability serves as a foundation for the development of arithmetic and facilitates the acquisition of more advanced mathematical competencies, hence it's reasonable to speculate that spatial thinking pervades the whole mathematics field. Future research should integrate more mathematics domains and explore their relative strengths with VMI and handwriting legibility. Moreover, utilising Chinese characters as the handwriting instrument makes this study limited in terms of logographic-alphabetic comparison. A robust body of research has found that compared with alphabetic handwriting, Chinese handwriting practices can contribute to the writers' superior VMI ability than their western counterparts (e.g., Cui et al., 2012; Lai & Frederick, 2012; Lim et al., 2015; Mao, 1995; Ng et al., 2015). In this regard, future research could advance the field by embracing this knowledge gap and conducting more empirical comparisons. Hopefully, this line of research can provide more opportunities for explaining the East-west disparities in mathematics success from the perspective of linguistics.

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RESEARCH PAPER

The Role of Mathematics Anxiety and Attitudes in Adolescents' Intentions to Study Senior Science

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In this study, we examined whether generalised mathematics anxiety, application of mathematics in science anxiety, and positive attitudes towards mathematics influenced adolescents' intentions to study biology, chemistry, and physics in Grades 11 and 12. Participants were 477 students in Grades 8–10 from two schools in Western Sydney. Girls reported higher levels of generalised mathematics anxiety and application of mathematics in science anxiety. Positive attitudes towards mathematics were a significant and positive predictor of students' intentions to study all science subjects, while application of mathematics in science anxiety was a negative predictor of students' intentions to study chemistry and physics.

The capacity to understand and apply mathematics is important for learning and comprehension of most science subjects and can act as a gatekeeper for student participation in science in the senior years of schooling and beyond (Douglas & Attewell, 2017; Shapka et al., 2006). Student attitudes, both positive and negative, towards mathematics and their prior achievement in mathematics can influence intentions to study science-based courses post-school (Lin et al., 2017; Sass & Kampa, 2019). For girls in Australia, there is evidence that their perceived ability in mathematics has an impact on their intentions to study physics, chemistry and biology in the senior years of schooling and beyond (Mackenzie et al., 2021; Watt et al., 2017). There is also evidence that positive attitudes to both mathematics and science is associated with higher achievement in each subject, further reinforcing the important relationship between student attitudes in both subjects (Berger et al., 2020).

Unlike other school subjects, mathematics has a long-held reputation as being a difficult subject to master and for inducing anxiety in some students (Hill et al., 2016). Mathematics anxiety has been defined as a negative reaction to mathematics and to mathematical situations (Ashcraft & Ridley, 2005). Mathematics anxiety is related to other types of anxiety, for example, general and test anxieties, but is also distinct and specific to learning and doing mathematics (Dowker et al., 2016). Numerous studies have documented the widespread prevalence of mathematics anxiety (Ashcraft, 2002; Hembree, 1990; Maloney & Beilock, 2012) and its negative correlation with mathematics performance (Barroso et al., 2020; Ma & Kishor, 1997; Miller & Bichsel, 2004). However, less is known about the impact of negative attitudes to mathematics on student interest in studying closely related subjects such as science. Mathematics plays a central role in the conduct of science by offering tools to help quantify, model, and represent scientific phenomena (Dierdorp et al., 2014). However, there is evidence that many students struggle to apply mathematics in science subjects (Rebello et al., 2007; Redish, 2017).

Higher levels of mathematics anxiety are related to poorer mathematics performance for both boys and girls, but because girls' mean mathematics anxiety is generally higher than boys, they are potentially more at risk as anxiety levels increase (Hyde et al., 1990; Stoet et al., 2016). There is also recent evidence that mathematics anxiety varies across mathematical tasks, with greater anxiety experienced by girls on mathematics tests but not during coursework (Geary et

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 370–377. Launceston: MERGA.

al., 2020). Therefore, the extent of mathematics anxiety felt at any particular time is highly dependent on the context within which mathematics is being experienced.

For this paper, we were interested in examining if domain-specific anxiety connected to 'applying mathematics in science', separately or simultaneously with generalised mathematics anxiety, was related to students' intentions to study senior science. Mathematics anxiety levels vary depending on the context within which the mathematics is being experienced, therefore, it is important to examine its impact on student subject choice across science subjects so that students can be better supported in pathways to further science study and careers. Participation in science subjects has persistent gendered patterns, with girls more likely to study biology, rather than physics (Yu & Warren, 2018). Given the higher rates of mathematics anxiety prevalent amongst girls, it was also important to examine how this anxiety was related to the intention to study different science subjects by gender.

While mathematics anxiety levels were of particular interest in this paper, we were also interested in how positive attitudes towards mathematics were related to intentions to study science. Positive attitudes towards mathematics encompasses students' enjoyment and valuing of mathematics, including their perceived relevance of mathematics. A significant body of research supports the notion that students who have positive attitudes towards a subject are more likely to continue studying it (Hsieh & Simpkins, 2022). We have previously found that, for most students, attitudes in mathematics and science were mirrored, with students demonstrating similar levels of confidence, liking, and valuing in both subjects (Berger et al, 2020). Also, there is evidence that students' attitudes to mathematics in early secondary school can subsequently impact on STEM course enrolments in the later years of schooling (Jiang et al., 2020). Indeed, students who are open to mathematics but not science have amongst the lowest STEM career expectations (Hsieh & Simpkins, 2022). However, it is less clear how positive attitudes towards mathematics influence students' science subject selections in the different science strands. Therefore, we examined the relationship between positive attitudes to mathematics and intentions to study science alongside our investigation of mathematics anxiety in relation to the study of science subjects.

Method

Participants

Participants were 477 students in Grades 8 (n = 193), 9 (n = 138), and 10 (n = 146) drawn from two single-sex, independent schools in Western Sydney. Female students were overrepresented in the sample (324 female, 153 male, 1 non-binary). 0.6% of the sample identified as being of Aboriginal and/or Torres Strait Islander descent. 60.8% spoke only English at home, while 34.4% spoke English at home in combination with another language and 4.8% did not speak English at home. 71.2% of the sample reported that their mother's highest level of education was a university degree, while 65.6% reported that their father's highest level of education was a university degree. Of the sample that began the survey (n = 477), 36 did not complete the measures of interest for this paper, leaving a final sample of 441 students.

Measures

Mathematics anxiety. Generalised mathematics anxiety was measured using the eight-item Negative Affect subscale of the *Mathematics Anxiety Scale–Revised* (MAS-R; Bai et al., 2009). The MAS-R has been validated for use with adolescent populations, demonstrating adequate validity and reliability (Bai, 2011). A sample item was: *I worry about my ability to solve math problems.* Participants were asked to identify how much they agreed with each statement on a scale from 1 (strongly disagree) to 5 (strongly agree), and scores were added to determine a total score for the subscale. The scale demonstrated excellent reliability in this sample, α = 0.90.

Application of mathematics in science (AMS) anxiety. Three items, designed for this study, were used to measure participants' domain-specific anxiety surrounding the application of mathematical skills and concepts in science. The three items were: *How anxious do you feel when you have to use mathematical concepts during lab activities in biology* (e.g., working out the magnification when using a microscope)?, *How anxious do you feel when you have to use mathematical concepts during lab activities in chemistry* (e.g., drawing a graph of time vs. temperature for a chemical reaction)?, and *How anxious do you feel when you have to use mathematical concepts during lab activities in physics* (e.g., using a formula to work out an unknown value)? Students were asked to respond to each item on a scale from 1 (not at all anxious) to 4 (very anxious), and responses were averaged to form a scale score. The scale demonstrated good reliability, α = 0.89.

Positive attitudes towards mathematics. Positive attitudes towards mathematics were measured using the Positive Affect subscale of the MAS-R (Bai et al., 2009; Bai, 2011). This six-item subscale includes items that reflect positive attitudes towards mathematics, such as: *I enjoy learning with mathematics* and *Math relates to my life*. Participants were asked to identify how much they agreed with each statement on a scale from 1 (strongly disagree) to 5 (strongly agree), and scores were added to determine a total score for the subscale. The scale demonstrated excellent reliability in this sample, α = 0.90.

Intentions to study senior science subjects. Three items measured participants' intentions to study biology, chemistry, and physics in Years 11 and 12. Participants were asked to rate how likely they were to study these subjects using a scale from 1 (very unlikely) to 5 (very likely).

Procedure

This study received institutional ethics approval and principal approval from each school prior to implementation. All students in Grades 8, 9, and 10 from each school were invited to participate. Standing parental consent was used in this study. Parents provided permission at the beginning of the year for their child to participate in research projects approved by the school throughout the year. Parents were then notified about the study and were given a two-week period to indicate that they did not consent for their child to participate in the study. Parents could also "opt in" to the study if they had not previously given permission for their child to participate in school research projects. Students were also asked to indicate whether they wanted to participate in the study, with a question included in the online survey that allowed them to withdraw from the study if desired. Study participation involved students completing a 25-minute online survey in class time. The response rates for the two schools were substantially different: 57.3% at the girls' school and 17.7% at the boys' school.

Results

Analyses were conducted using IBM SPSS Version 27. Except where otherwise noted, statistical significance was set at $p < 0.05$.

Descriptive Statistics and Correlations

Descriptive statistics for all study variables are shown in Table 1. Girls had significantly higher mathematics anxiety than boys with a small effect size, $t(438) = -2.65$, $p = 0.008$, $d = 0.28$. Girls also had significantly higher AMS anxiety than boys with a small effect size, $t(287.56) = -3.55$, $p < 0.001$, $d = 0.35$. However, there was no significant difference in positive attitudes towards mathematics between the genders, $t(438) = 1.07$, $p = 0.29$, $d = 0.11$. Girls were less likely to report they intended to study physics in the senior years, $t(436) = 5.23$, $p < .001$, $d = 0.55$, but there were no gender differences in students' intentions to study chemistry, $t(436) = 0.76$, $p = 0.45$, $d = 0.08$, or biology, $t(436) = -1.90$, $p = 0.06$, $d = 0.20$.

Table 1
Descriptive Statistics for Entire Sample and by Gender

Variable	Range	Entire sample			Girls			Boys		
		N	*M*	*SD*	*n*	*M*	*SD*	*n*	*M*	*SD*
Mathematics anxiety	5–40	441	23.52	7.37	307	24.12	7.28	133	22.11	7.42
AMS anxiety	1–5	440	2.00	0.90	307	2.09	0.93	132	1.78	0.80
Positive attitudes towards mathematics	5–30	441	20.39	6.04	307	20.17	6.05	133	20.84	6.02
Intentions to study biology	1–5	439	3.30	1.45	308	3.39	1.46	130	3.10	1.40
Intentions to study chemistry	1–5	439	3.10	1.44	308	3.06	1.47	130	3.18	1.38
Intentions to study physics	1–5	439	2.64	1.40	308	2.43	1.35	130	3.17	1.39

Correlations for all study variables are shown in Table 2. There was a large positive correlation between mathematics anxiety and AMS anxiety ($r = 0.53$, $p < 0.001$). Mathematics anxiety was moderately negatively correlated with positive mathematics attitudes ($r = −0.44$, $p < 0.001$). There was also a moderately negative association between positive mathematics attitudes and AMS anxiety ($r = −0.45$, $p < 0.001$). Positive attitudes towards mathematics were moderately positively correlated with intentions to study chemistry ($r = 0.44$, $p < 0.001$) and physics ($r = 0.41$, $p < 0.001$) but the association with intentions to study biology was small ($r = 0.13$, $p = 0.005$). Mathematics anxiety had a *small* negative association with chemistry intentions ($r = −0.19$, $p < 0.001$) and physics intentions ($r = −0.20$, $p <0 .001$); but the association between AMS anxiety was *moderately* negative with chemistry intentions ($r = −0.32$, $p < 0.001$) and physics intentions ($r = −0.34$, $p < 0.001$). While there was a large positive correlation between physics intentions and chemistry intentions ($r = 0.49$, $p < 0.001$), the correlation between physics intentions and biology intentions was small ($r = 0.11$, $p = 0.03$).

Table 2
Correlations Between Study Variables

Variable	1	2	3	4	5	6
1. Mathematics anxiety	—	.53**	−.44**	.01	−.19**	−.20**
2. AMS anxiety	.53**	—	-.45**	−.08	−.32**	−.34**
3. Positive attitudes towards mathematics	−.44**	−.45**	—	.13**	.44**	.41**
4. Intentions to study biology	.01	−.08	.13**	—	.44**	.11*
5. Intentions to study chemistry	−.19**	−.32**	.44**	.44**	—	.49**
6. Intentions to study physics	−.20**	−.34**	.41**	.11*	.49**	—

Note. ** = Correlation is significant at the 0.01 level (2-tailed). * = Correlation is significant at the 0.05 level (2-tailed).

Regressions

We conducted three regressions to examine the relative contribution of gender, mathematics anxiety, AMS anxiety, and positive attitudes towards mathematics to students' intentions to study biology, chemistry, and physics in the senior years of high school. All predictors were entered simultaneously, and results are shown in Table 3.

Table 3
Regressions Predicting Intentions to Study Biology, Chemistry, and Physics in the Senior Years of High School

Predictor	Biology intention			Chemistry intention			Physics intention		
	B	*SE*	β	*B*	*SE*	β	*B*	*SE*	β
Gender	.29	.15	.09	.01	.14	.00	-.64	.13	-.21***
Mathematics anxiety	.02	.01	.11	.02	.01	.09	.02	.01	.08
AMS anxiety	-.15	.09	-.10	-.31	.08	-.19***	-.31	.08	-.20***
Positive attitudes towards mathematics	.03	.01	.13*	.09	.01	.39***	.08	.01	.34***
R^2		.03			.22			.24	
F for change in R^2		3.70**			29.37***			34.13***	

Note. * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$

The regression predicting intentions to study biology was significant but accounted for a very small proportion (3.3%) of the variance, $R^2 = 0.03$, $F(4, 430) = 3.70$, $p = 0.006$. The only significant predictor was positive attitudes towards mathematics (β = 0.13, $p = 0.02$). The regressions predicting intentions to study chemistry ($R^2 = 0.22$, $F(4, 430) = 29.37$, $p < 0.001$) and physics ($R^2 = 0.24$, $F(4, 430) = 34.13$, $p < 0.001$) were also significant, and accounted for a larger proportion of the variance (21.5% and 24.1% respectively). AMS anxiety was a negative predictor of students' intentions to study chemistry (β = -.19, p < 0 .001) and physics ($\beta = -0.20$, $p < 0.001$), while positive attitudes towards mathematics was a positive predictor of students' intentions to study chemistry (β = 0.39, $p < 0.001$) and physics (β = 0.34, $p < 0.001$). General mathematics anxiety was not a significant predictor of students' intentions to study any science subjects, and gender was a significant predictor in the case of intentions to study physics only, such that boys were more likely to intend to study physics than girls (β = -0.21, $p < 0.001$).

Summary

In this study, girls had higher mathematics anxiety and AMS anxiety than boys. While girls were less likely to indicate an intention to study physics in senior high school, there were no gender differences for chemistry or biology. Mathematics anxiety and AMS anxiety were related constructs but had different relationships to other study variables. AMS anxiety appeared to be more predictive of intentions to study chemistry and physics than mathematics anxiety. Positive attitudes towards mathematics were a notable positive predictor of students' intentions to study all science subjects.

Discussion

Mathematics anxiety is a well-documented phenomenon in educational research with widespread incidence and negative impact on performance in the subject itself (Dowker et al., 2016; Miller & Bichsel, 2004). In our previous work, we have established various links between mathematics and science attitudes. For instance, liking, valuing, and confidence in mathematics was associated with similar attitudes in science (Berger et al., 2020). We have also shown that attitudinal constructs influence senior science subject choices (Mackenzie et al., 2021). In this study, we extend our understanding of the interrelationships between mathematics and science to the phenomenon of mathematics anxiety. In particular, because mathematics anxiety depends on the context within which mathematics is experienced, we explored whether generalised or domain-specific forms of mathematics anxiety were more predictive of intentions to undertake chemistry, physics, and biology in the senior years of high school.

Girls frequently have higher levels of mathematics anxiety than boys (Stoet et al., 2016). The findings of our study extend that observation to a domain-specific form of mathematics anxiety. In addition to higher levels of generalised mathematics anxiety, the girls in this study also had higher AMS anxiety than boys. That is, girls were more likely than boys to report anxiety about scientific tasks that required the application of mathematical concepts, like using formulas and drawing graphs. This finding is concerning given that mathematics forms an integral part of most science disciplines (Douglas & Attewell, 2017) and science provides an applied avenue for the development of many mathematical concepts and skills (Berger et al., 2020).

An interesting, but somewhat surprising, finding was that generalised mathematics anxiety was not a significant predictor of students' intentions to study any of the science subjects in senior high school. While we observed a significant and negative correlation between generalised mathematics anxiety and intentions to study physics and chemistry, in the presence of other variables (positive attitudes towards mathematics and AMS anxiety), this relationship was no longer significant. This suggests that general mathematics anxiety minimally influences adolescents' attitudes and feelings towards learning in science. However, given the strong and positive relationship between generalised mathematics anxiety and AMS anxiety, we argue that the role of mathematics anxiety in adolescents' science subject selections warrants further investigation. For example, it is possible that generalised mathematics anxiety precedes more domain-specific mathematics anxieties. While outside the scope of this study, future research could investigate such temporal relationships via longitudinal research designs.

In this study, we observed that positive attitudes towards mathematics was a notable positive predictor of students' intentions to study all science subjects. This suggests that positive attitudes towards mathematics are more influential than mathematics anxiety in guiding students' intentions to study science. As previous research has found that students with higher mathematics than science motivation have greater STEM achievement and course taking in high school than students with higher science than mathematics motivation (Snodgrass Rangel et al., 2020), our finding supports an increased focus by teachers and parents on supporting adolescents' positive attitudes towards mathematics. Positive attitudes towards mathematics were more strongly related to intentions to study chemistry and physics in comparison to biology, which further reflects the greater congruence between mathematics and physics and chemistry (Jansen et al., 2015).

Practical Significance

There has been much focus in recent years on addressing gender inequities in participation in the STEM disciplines (Hsieh & Simpkins, 2022). Girls may need different activities to support their attitudes and intentions for mathematics and science, particularly where mathematics underpins the content being taught (Berger et al., 2020). Factors like domain-specific forms of mathematics anxiety present a possible avenue to improve the proportion and experience of girls in science. Mathematics and science teachers share the responsibility for addressing domain-specific forms of mathematics anxiety. Another avenue for improving domain-specific mathematics anxiety involves identifying the mathematical skills required in science to help students to make more explicit connections between the disciplines. This will require closer collaboration between mathematics and science faculties than traditionally has been the case in schools (Furner & Kumar, 2007). Finally, positive attitudes towards mathematics were the strongest predictor of students indicating an intention to study chemistry and physics. This reinforces the importance of supporting students to develop positive attitudes towards mathematics. Pedagogical interventions that emphasise the relevance and value of mathematics, particularly in science, are a way to strengthen students' positive attitudes and motivation (Furner & Kumar, 2007).

Limitations and Conclusion

While this study extends our understanding of the role of mathematics anxiety and attitudes in adolescents' science subject selections, there are several limitations to note. First, our study focused on adolescents' *intentions* to study science, rather than their actual subject selections. It is possible that adolescents' subject selection intentions change over time, but we argue that there is value in considering their intentions as these indicate current interest and valuing of each subject. The second limitation to note is that our research design was cross-sectional, which does not allow for determination of the direction of effects. As stated previously, we recommend that future longitudinal designs are implemented to disentangle the relationships between variables considered in this study. A third limitation of our study was the relatively small response rate from the boys' school. Further research with different populations is needed to determine whether the relationships between mathematics anxieties and attitudes and science subject selections observed in this study are also present in adolescents from different school contexts.

This study makes an important contribution to the field by investigating how mathematics anxieties and attitudes influence adolescents' science subject selections. While generalised mathematics anxiety did not appear to influence intended subject selections, AMS anxiety and positive attitudes towards mathematics appear to be particularly influential in adolescents' intentions to study chemistry and physics. These findings have important practical implications for mathematics and science educators to support more students (particularly girls) in continuing their study of science in the senior years of high school.

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Identities of Mathematics Teacher Educators in a "Hybrid" Mathematics and Mathematics Education Department

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Prospective secondary mathematics teachers in Australia are typically taught by mathematics educators and mathematicians who work in different faculties and seldom collaborate—a situation that can lead to conflicting views about how to teach mathematics. This paper reports on findings from semi-structured interviews with three mathematics teacher educators in a hybrid mathematics/mathematics education department. Valsiner's zone theory is used to analyse how their beliefs, institutional context and professional learning opportunities shape their identities as MTEs. Findings reveal that the MTEs had developed similar beliefs within supportive institutional context but drew on different sources of professional learning.

There is increasing scrutiny and regulatory pressures on initial teacher education (ITE) in Australia, evidenced by two recent reviews (Australian Government, 2022; TEMAG, 2015). Additionally, the declining performance of Australian students in mathematical literacy in the Programme for International Student Assessment (PISA) has raised concerns about the apparent failure of education systems to produce mathematically capable students (Thomson et al., 2019). Rather than taking a deficit view of mathematics teacher educators (MTEs) who prepare future teachers of mathematics as being part of the *problem* (Dinham, 2013), we regard these developments as acknowledging the vital role of teacher educators in preparing high quality teachers.

In Australian secondary ITE programs, MTEs include both mathematics educators and mathematicians who typically work in different faculties or departments and seldom collaborate. Within mathematics departments, all Australian students learn mathematics together with no acknowledgement of the differing needs of future teachers, mathematicians, or engineers. Leikin et al. (2017) found that in this situation mathematicians focussed on preparing professional mathematicians rather than teachers. Pre-service teachers (PSTs) have described the tensions between how they were taught mathematics at university and how they were taught to teach mathematics (Marshman, 2021), saying that mathematicians tended to present them with the mathematics, and it was up to them how they engaged with it. This presents the challenge of how to facilitate integration of mathematical content knowledge and pedagogical content knowledge when preparing future secondary mathematics teachers.

Innovative models of ITE exist in Australia that not only foster collaborations between mathematicians and mathematics educators but also productively integrate the content and pedagogical knowledge needed by PSTs (Anderson & Tully, 2020; Goos & Bennison, 2018). These collaborations can also lead to the formation of hybrid, *in-between* identities in some MTEs who cross the boundary between these disciplines. Hybrid MTEs might possess qualifications and experience in both disciplines, or change careers to move between disciplinary communities. Thus, hybrid MTEs can feel like they belong to both communities, or neither (Goos & Marshman, 2021). Analysing MTEs' identity trajectories will increase our understanding of how crossing disciplinary boundaries supports or hinders their learning.

This study is part of our larger research program investigating the professional formation of MTEs in terms of identity development (Goos & Bennison, 2019). We view identity as "a

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 378–385. Launceston: MERGA.

performative process of *becoming* that addresses social interactions and institutional contexts, while acknowledging that an individual's knowledge, beliefs, and attitudes can influence identity enactment" (p. 405). The present study responds to an opportunity that originated following our presentation at the 14th International Congress on Mathematics Education (ICME-14) on hybrid MTEs and explored models of ITE in terms of collaboration between MTEs who are mathematics educators or mathematicians (Goos & Marshman, 2021). In universities in some countries, including Bohemia University (pseudonym) in the Czech Republic, prospective mathematics teachers are taught both mathematics and pedagogy within a *hybrid* mathematics/mathematics education department. The Head of this department invited us to extend our research to this new international context thereby providing an opportunity to explore the identities of MTEs in a very different international context from Australia. This paper reports on preliminary findings from interviews conducted with MTEs at Bohemia University. The research question addressed in this paper is: How do beliefs, institutional contexts and professional learning opportunities interact to shape the identities of MTEs in a *hybrid* mathematics/mathematics education department? MTE names are pseudonyms.

Theoretical Perspectives

Wenger (1998) described identity as the "pivot between the social and the individual" (p. 145). He claimed that identity development involves a temporal learning trajectory that connects the past, present, and future. Lately, identity has been found to be a useful concept to understand the learning and change of school mathematics teachers (Lutovac & Kaasila, 2018). Yet this kind of research is limited when it comes to the learning and development of MTEs – research that is needed if we are to improve ITE for future secondary mathematics teachers.

In our previous research we have used Valsiner's (1997) zone theory to analyse teacher and MTE identity (Goos & Bennison, 2019). Valsiner extended Vygotsky's concept of the Zone of Proximal Development (ZPD) to incorporate the social setting and goals and actions of participants. He redefined the ZPD and introduced two additional zones – Zone of Free Movement (ZFM), and Zone of Promoted Action (ZPA). The ZFM represents what the environment allows. The ZPA, on the other hand, represents what is promoted. According to Valsiner, the ZFM and ZPA are dynamic and work together as a ZFM/ZPA complex to direct development along a set of possible pathways. Individuals, however, are active participants in this process and are free to change their environment or interactions with the people in it. These features of Valsiner's zone theory make it suitable for capturing the complexity and dynamic nature of identity.

The analytical approach we have developed (Goos & Bennison, 2019) interprets Valsiner's (1997) ZPD, ZFM, and ZPA in nuanced ways that are aligned with the influences on teacher learning and development. This approach has enabled us to trace identity trajectories of mathematics teachers, teachers who were embedding numeracy into subjects other than mathematics and, more recently, Australian MTEs participating in the Inspiring Mathematics and Science in Teacher Education (IMSITE) project (http://www.imsite.edu.au). This approach has allowed us to foreground the temporal character of identity-as-becoming. For MTEs, the three zones are interpreted in a way that is aligned with the factors that influence MTEs' learning and development. According to our adaptation of Valsiner's zone theory, the ZPD represents the possibilities for development of new knowledge, goals, beliefs, and practices. For MTEs, this zone includes their mathematical knowledge, pedagogical content knowledge, knowledge of how new teaching practices are learned, and beliefs about mathematics teaching and learning. The ZFM represents environmental constraints that structure access to particular areas or resources or ways of acting with resources. For MTEs, this zone includes characteristics of teacher education students, structural characteristics of teacher education programs, organisational structures, and university cultures. The ZPA includes activities, objects or areas in the environment where actions are promoted. For MTEs, this zone includes reflections on practice, research with teachers, professional development, and informal interactions with colleagues.

Research Design

The study is being conducted in 2021/2022 and is funded by a School of Education and Tertiary Access Pilot Research Grant. Ethics approval was granted for the study by the university's Ethics Committee (A211656) and all participants gave informed consent. Data collection involved an online survey and interviews. Some preliminary findings from interview data are reported in this paper.

Context

The hybrid mathematics/mathematics education department at Bohemia University has 18 academic staff and sits within the Faculty of Education. The department is only involved in ITE, preparing PSTs to teach in primary and secondary schools. Prospective secondary mathematics teachers initially study a three-year Bachelor of Mathematics for Education which includes 34% pure mathematics, 4% mathematics education and 2% of credit is assigned to mathematics time in schools. To become fully qualified mathematics teachers, they then study a two-year Masters degree, with 12% mathematics content, 13% is mathematics education and 12% of credit is assigned to mathematics time in schools. A shortage of mathematics teachers means that approximately 60% of PSTs are teaching in schools whilst studying their Masters.

Methodology

Participants. All MTEs in the hybrid department at Bohemia University in the Czech Republic were invited to participate in the study by completing an online survey about their beliefs about teaching and learning mathematics and their teaching practices. The final question invited respondents to indicate their willingness to participate in a semi-structured interview by providing their name and contact details. Of the 18 MTEs who commenced the survey, 15 provided responses to all items and five consented to be interviewed. Interviews were conducted in December 2021 and January 2022.

Data collection. Semi-structured interviews were conducted via Zoom. Participants were asked about their backgrounds (e.g., qualifications, teaching experience and their pathway to becoming a MTE), beliefs about teaching and learning mathematics (e.g., beliefs about the way mathematics should be taught and their role as a MTE), their professional context (nature of students, delivery methods, forms of collaboration with colleagues, accreditation requirements) and their opportunities for professional learning (e.g., the influence of colleagues and their research on their teaching). Interviews lasted between 25 and 49 minutes, were recorded and transcribed using Otter.ai. Transcripts were checked manually and edited to ensure accuracy.

Data analysis. Interview data were analysed thematically using our adaptation of Valsiner's (1997) zone theory described earlier. Transcripts were annotated to identify responses that provided information about a participant's ZPD, ZFM and ZPA. For example, a question about a participant's beliefs about their role as a MTE might provide insights into their beliefs about mathematics teaching and learning (ZPD); a question about accreditation requirements could uncover constraints on their teaching practice (ZFM) and a question about the influence of their research on their teaching might reveal an avenue of professional learning (ZPA). For each participant, relevant excerpts of text were then electronically copied and pasted into table with columns for the ZPD, ZPA and ZFM to represent the MTE's zone configuration. The final step in the analysis was to develop a case study for each participant.

Findings

Case studies of Josef (a mathematician) Eva (whom we have classified as a hybrid) and Marie (a mathematics educator) are presented in this paper. These MTEs illustrate the identities of three types of MTEs working within a hybrid mathematics/mathematics education department.

Josef—a Mathematician

Josef is a mathematician and mid-career academic. He has a PhD in mathematics, and predominantly researches in the field of mathematics. He mainly teaches geometry "from elementary to like really high school and university geometry" to future teachers.

Since Josef began teaching in the hybrid department, his beliefs about how to teach mathematics have changed. He described how he had begun teaching in a mathematics department using a very traditional approach: "because I started from mathematical faculty, I also had some other views on how it should be taught. I was more concentrated on many exercises. … we couldn't like miss anything". Now, however, he is more concerned that his students understand the mathematics they are learning because of their future role as teachers:

> But I guess they should do much more for mathematicians while they need these methods in their head as they need to learn this for their future research …. But mathematical teachers, they don't need these results. They don't need so many methods - they don't need so many techniques. But they need to understand them really well. So, it must be taught in different way.

As a result of this change in beliefs, he promotes mathematical understanding through question driven discussions:

> I usually lead discussions with them. I ask them too many questions, why and why and why until they know why is why. I simply always ask them, and I don't believe anything. I said they don't believe me if I say anything until they have proof, which comes from mathematics.

Josef felt that his approach was aligned with a desire within the hybrid department to "teach them more abstract competencies, like how to think, how to discuss, how to work". He had found support for developing his pedagogy within the department:

> I went the department with so many people who do research in education … From my perspective, what I know about the didactics - I'm sitting here with so many people who discuss how they teach and how should, you should do it. So, I attended many conferences. … I learn from these things. … I also read sometimes something that is interesting in geometry. In geometry, I think the education or in didactics, there are not so many materials for what I do actually.

While Josef had support within the department in which he worked, constantly changing accreditation requirements and high administrative load impacted on him:

> Also, the problem should not be, but it is, is the government here has changed accreditation so many times while I'm teaching. I'm teaching maybe six or seven years here, at Faculty of Education, but I'm in the fourth accreditation system here … So, this is really what makes it hard - bureaucracy here makes it hard. Very much administrative also.

More recently, the COVID-19 pandemic with the resultant change to online teaching had placed constraints around his use of discussion when teaching mathematics:

> No one, from what I know, likes totally online teaching for mathematics. It's hard to discuss with people and also my way of teaching doesn't fit too much with only online teaching. So, if it is possible, we do hybrid or face to face teaching.

Josef's identity has changed since he joined the hybrid department. Changes to his beliefs about how mathematics should be taught (ZPD) were prompted by the needs of his students who were preparing to become mathematics teachers ("they need to understand"). He now uses questioning and discussions, focusing on mathematical thinking and understanding. This shift in practice, reflective of a change in identity, has been guided by a ZFM/ZPA complex that includes his interactions with colleagues which outweigh constraints imposed by university systems and the COVID-19 pandemic.

Eva—a Hybrid

Eva is a very experienced hybrid MTE and senior academic. She has a PhD in pure mathematics and originally worked as a pure mathematician. When she commenced in the hybrid department, she made a career choice to move into mathematics education: “I did not manage to do both [research in pure mathematics and mathematics education] at a high level. Therefore, I had to decide what I will continue with as a main research area”. She teaches mathematics content (algebra) and mathematics education courses.

For Eva, becoming a MTE meant changing from using mathematics to promoting understanding of mathematics:

> I think that when I worked as mathematician, I used a lot of formulas and mathematically precise expressions and ideas. Now I am much more constructivist than I was before … when students develop something. And then we are working on it together to find out if it's correct how to use it, but what to do with it, to be able to use it, and so on. So, I think that this is the main difference.

She described her teaching as “very interactive. I do not like to present something without interaction from students”. This is evident in how she uses discussions so that her students can build their own understandings:

> I am moderator of the discussions. I am somebody who opens the door for them to see what they can do. … Sometimes I'm in the judge - I'm evaluating their discussions - their ideas in the discussions, and when they do not have the same opinion, I have to say sometimes as somebody who says, who helps them to see the you know, for example, sometimes they accept something which is not correct, which is wrong, the very same mistake and they do not see it, because their knowledge is not developed enough. You know, I have to take care of these difficulties and help them to see. So, I am I am more guide through the topic, than the real presenter.

There have been many people who have influenced the way Eva teaches, including a teacher from her final years of secondary schooling, her colleagues and two internationally renowned mathematics educators whom she named (David Clarke and Guy Brousseau):

> I was partly influenced by one of my professors at the upper secondary level, who taught me in the last year of upper secondary and he was excellent. And he used the method that I am copying very often, even now, and then the practice and my colleagues whom I could follow.

She also draws on her research to inform her teaching: “I am using it mainly in the courses in maths education ... I do not do something in research and something else in teaching. I’m linking it together.” Eva works collaboratively with other MTEs in the hybrid department in a mentoring role:

> We work together and we are preparing all this together. And during my professorial life and at the faculty I already worked with similar young colleagues who start, either my former PhD students or those coming from other paths who are very often pure mathematicians, who for example, take some seminars to my lecture, and they learn from what I show them, and I give them and ask them to do, how to work with students later. So, I hope when I finish there will be enough young colleagues to take over.

The collegial atmosphere within the department works to alleviate constraints imposed by accreditation requirements: “For accreditation there are very, very many decisions, which are taken by the faculty and at the level of the university, which must be followed”.

Eva’s identity has changed since she joined the hybrid department from being a mathematician to becoming a hybrid MTE. Her beliefs about mathematics (ZPD) were initially associated with practicing mathematics (“formulas, mathematically precise expressions and ideas”) and are now centred on promoting her students’ understanding of mathematics (“help them to see"). The ZFM/ZPA complex that has guided this change is dominated by influential people, including her colleagues, and her research. Constraints imposed within her professional context (e.g., accreditation requirements) seem to have limited influence on her practice.

Marie—a Mathematics Educator

Marie is a mathematics educator and Head of the hybrid department. She originally studied teaching mathematics, geography and English and taught in a primary school before commencing her PhD in mathematics education. She initially taught mathematics, analytical geometry, and problem-solving courses but now teaches only mathematics education courses.

Marie's beliefs about teaching mathematics were formed through her initial experiences in the hybrid department:

> When I was starting teaching and, in the faculty, Professor H came to the university, and he completely redesigned some courses for future teachers and analytic geometry was one of them. And it was organised in such a way that there are hardly any lectures, but students got some problems to solve, and by solving the problems, they can, they should discover some something. So, it was like a discovery teaching. And during the seminars and lectures, we discussed it and put it together. So, it was really, for me, it was like a miracle.

According to Marie, "there is a consensus that, of course, definitions and theorems are still important and must be covered, but I believe that in the department nowadays there is a consensus that it could be done in a more inductive way". Now her focus is not on teaching mathematics but on teaching her students how to teach mathematics for understanding: "My main role, I believe, is to get my students to think hard about how to teach because they know what to teach". She does this by getting her students to analyse and discuss textbooks, readings and videos of lessons: "It's usually observing a part of the lesson, for example, a video lesson, or it's the analysis of a textbook from some point of view, or some reading. And then during the seminar, we speak about it". Marie wants her students to adopt constructivist approaches and teach for understanding:

> We [the hybrid department] advocate constructivist approaches to teaching. When they go to school, I always tell them, you can use any method, but all the methods should be aimed at their understanding. So, if you if your pupils learn by heart, then you failed. So, you must do it differently. And so that's the message I try to tell them during all my courses.

Mathematics is culturally valued in the Czech Republic and this is evident in the way Marie feels the hybrid department is viewed within the Faculty of Education:

> A big advantage of my place is that we are in the Faculty of Education. ... So, we have both mathematicians, and math educators. But the mathematicians who come to our department are the ones who are also active mathematicians, but their approach to math education is positive. … mathematics is regarded as like, I don't know, the Queen. We are important. So, we feel important.

Within the hybrid department, there is a culture of collaboration, evidenced by the way in which Marie described how programs were developed collaboratively for accreditation:

> So, when we were preparing the content of this study, so we got together as a department, and we spoke about individual courses. So, we agreed on the content as a group, … And then they teach, and it's their responsibility that they teach to the syllabus.

Marie's research is intimately connected with her teaching. Her students are her research participants, and her data analysis informs her teaching:

> My research is very much informing my teaching. … my second stream of research is in future teachers and teachers' noticing, and knowledge-based reasoning. So, this is really connected, because I use my students also as research subjects. So, they analyse their work and then they use it to inform my teaching. For example, if I give them some video recordings to see, and then when I analyse the reflections I can see well, they didn't notice what I wanted them to notice. So, this very basic thing next time I am I am

> preparing a task for them, which would focus their attention on it. So, I think I am lucky, because my research is really very tightly connected to my math education courses.

In addition to her research, Marie's teaching is informed by her reading: "I try to read a lot. So, I tried to incorporate some research results" and, prior to the COVID-19 pandemic, visitors to the hybrid department: "when we had some visitors from abroad coming, I always invited them to my courses with a view to get some inspiration from their teaching".

Marie's identity as a MTE began developing while she was teaching with Professor H. Her beliefs about how to teach mathematics (ZPD) were shaped by the "miraculous" constructivist methods he introduced her to that have led her to focus on pedagogy that promotes mathematical understanding ("get my students to think hard about how to teach"). The ZPA/ZFM complex that guides her development includes the privileged position of mathematics, the collaborative nature of the department, and her opportunities to link her research and teaching.

Identities of MTEs in the Hybrid Mathematics/Mathematics Education Department

The case studies of Josef (a mathematician), Eva (a hybrid MTE) and Marie (a mathematics educator) illustrate how the identities of these MTEs have been shaped by their environment and the people in it (i.e., the hybrid mathematics department.). The hybrid department has a privileged position in a faculty that values mathematics and brings together three types of MTEs who have a common goal – preparing future mathematics teachers. This common goal has contributed to shaping the *beliefs* (ZPD) about how mathematics should be taught of Josef, Eva, and Marie. Their beliefs have developed under the influence of their *institutional context* (ZFM) and *professional learning opportunities* (ZPA) that canalise their identity development.

Within the hybrid department, the MTEs have developed similar beliefs about the value of constructivist teaching methods and the importance of discussions to build mathematical understanding. Unlike the mathematicians in Leikin et al.'s (2018) study, Josef believed that mathematics needs to be taught differently to prospective teachers so that they developed a deep understanding of the mathematics ("it must be taught in a different way"). Eva, who taught both content and pedagogy courses, described herself as a "moderator of discussions" who helped her students "to see". As a mathematics educator, Marie impressed on her students the importance of thinking about how they will teach for understanding ("if your pupils learn by heart, then you failed"). While beliefs are only one factor that contributes to an individual's ZPD, it seems that this component of the ZPDs of all three MTEs is aligned when it comes to their goal what is required to prepare future mathematics teachers.

The institutional context provided by the hybrid department is one where mathematics is valued and there is a professional culture of collaboration. All three MTEs described instances of collegiality. Josef described regular conversations about teaching ("people who discuss how they teach and how you should do it"), Eva described how she mentored less experienced colleagues ("they learn from what I show them") and Marie described how courses were planned collaboratively for accreditation ("we agreed on the content as a group"). None of the MTEs mentioned insurmountable constraints imposed by their professional context. Thus, the ZPA offered affordances rather than constraints. While there seem to be no formalised professional learning opportunities for the MTEs, there is a culture of learning from colleagues within the hybrid department and beyond. Josef's teaching, for example, is informed by his colleagues within the hybrid department whereas Eva and Marie seem to draw mainly on their research and international mathematics educators. This difference is perhaps due to Josef's research being in mathematics rather than mathematics education. Nevertheless, the ZPA promotes interdisciplinary collaboration. Thus, the ZFM/ZPA complex experienced by the three MTEs is similar but differences exist in the professional learning opportunities each MTE draws on.

Concluding Remarks

The MTEs who teach prospective secondary mathematics teachers in Australia are usually mathematics educators and mathematicians who work in different faculties and seldom collaborate – a situation that can lead to conflicting views about how to teach mathematics. Interdisciplinary collaboration between mathematicians and mathematics educators in ITE in Australia sometimes leads to *hybrid* MTEs who cross the boundaries between the disciplines (Goos & Marshman, 2021). This study extends our earlier research on the identities of MTEs (e.g., Goos & Bennison, 2019) involving interdisciplinary collaboration *across* the boundaries that separate disciplines by providing insights into interdisciplinary collaboration *within* an organisational unit (the hybrid mathematics/mathematics education department). Although we cannot make strong claims because we are drawing on the preliminary analysis of interviews with only three MTEs, our findings suggest possible insights that can be drawn from investigating the identities of MTEs in an ITE context that is different from Australia. We identified three types of MTE—mathematician, hybrid, and mathematics educator—in a hybrid department that seems to afford collaboration between mathematicians and mathematics educators. This collaboration appears to have resulted in similar beliefs that guide the MTEs' teaching practices. We hope to gain further insights following analysis of data from the online surveys and remaining interviews.

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RESEARCH PAPER

Diagrams in Mathematics: What Do They Represent and What Are They Used For?

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Diagrams are succinct, visual representations of real-world or mathematical objects that serve to communicate properties of the objects and facilitate problem solving. This study explored perceptions of mathematics teachers in Singapore, who are heads of mathematics departments in their respective schools, related to diagrams and their use in the teaching of mathematics. An open-ended survey was adopted to illicit responses to i) what is a diagram, and ii) when do you use diagrams in your mathematics instruction. The findings of the study show that participants generally viewed diagrams as visual representations of real life or mathematical objects and they used them mainly as visual aids when illustrating mathematical concepts and relationships.

The revised school mathematics curriculum for secondary schools in Singapore places emphasis on Big Ideas in School Mathematics (BISM) (Ministry of Education, 2018). One BISM is diagrams for representation and communication of mathematical objects. Though in the 'eyes' of mathematicians and curriculum developers the role of diagrams may be apparent the same may not be for mathematics teachers in general. To facilitate development of mathematics teachers enacting the revised curriculum in Singapore secondary schools a study was carried out amongst school leaders, specifically the heads of mathematics departments from twenty secondary schools to ascertain their understanding of diagrams specific to the teaching and learning of mathematics.

The Study

Diagrams in Mathematics

> Diagrams are succinct, visual representations of real-world or mathematical objects that serve to communicate properties of the objects and facilitate problem solving. For example, graphs in coordinate geometry are used to represent the relationships between two sets of values, geometrical diagrams are used to represent physical objects, and statistical diagrams are used to summarize and highlight important characteristics of a set of data. Understanding what different diagrams represent, their features and conventions, and how they are constructed helps to facilitate the study and communication of important mathematical results. (Ministry of Education, 2018, p. S2-5)

For the purpose of this study, we adopt Winn's (1987) definition that a diagram is a 2-dimensional, visual representation, that exploits spatial layout in a meaningful way, enabling complex processes and structures to be represented comprehensively. In the teaching and learning of geometry, diagrams are simultaneously concepts and spatial representations of abstract ideas (Gagatsis et al., 2010). This facilitates the construction, argumentation, and understanding of geometrical ideas (Dimmel & Herbst, 2015). Diagrams have also been the oldest form of communication to convey formally or informally mathematical concepts and proofs (Cellucci, 2019; Shin et al., 2018). In mathematical proofs, diagrams may illuminate key inference steps, but they do not replace the rigour of axiomatic presentations (Hilbert, 2004; Shin et al., 2018). However, the heuristic conception of diagrams (Cellucci, 2019), stemming from the analytic method of mathematics involving both deductive and non-deductive rules resonates with diagrams as a heuristic tool in mathematical problem solving (Polya, 1945). Drawing diagrams is a significant problem-solving heuristic, and research shows

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that a visual representation through a model or diagram is the most effective amongst others for problem-solving (Hembree, 1992; Uesaka & Manalo, 2012; Wong, 1999).

Larkin and Simon (1987) noted a substantial connection between diagram use and problem solving, specifically, when the diagrammatic forms are representative of a cognitive process or are schematic in nature. Stylianou (2011) examined the functions of representations in mathematical problem-solving. In doing so, she summarised roles of such representations in students' problem-solving activities. Representations act as tools that support understanding where different aspects of the problem can be combined to observe how they interact. Representations also serve as a means to record information and reduce the cognitive load imposed on one's working memory. Regarding visual representations, Stylianou (2011) noted that diagrams are flexible exploration devices that allow a solver to generate new information about a problem or reveal information that is not immediately obvious. Additionally, representations may also be used to monitor and assess progress in problem-solving. The findings from Stylianou's study evidently show how representations take on different utility value as the objective of a problem-solving activity changes. They illustrate the different roles they play and highlight the benefits of employing them.

De Toffoli (2018) distinguished between different types of representations and purported that choosing the right one is essential. One benefit of visual representations such as diagrams can be credited to its capability to transform a mathematical idea from one form to another. They are useful in providing a complementary language to sentential representations of the same knowledge. Nelson (1993) cited in Small (2012), states that "diagrams, either with or without accompanying words, can be extremely powerful tools for reasoning and explaining" (p.21). Moreover, they make apparent those quantifiable relationships that define a problem (Sunzuma et al., 2020). This alternate form of representing the same knowledge facilitates the process of sense-making and knowledge construction (Mudaly, 2012) which illuminates why diagrams have received much attention from many research areas.

A cognitive advantage of diagrams is that they act as external sketches where interconnected pieces of information can be put together and therefore relieve students' working memory load (van Essen & Hamaker, 1990). A diagram facilitates conceptualisation of the problem structure. It serves as an intermediate step between a mental representation and a physical representation of a concept. Diagrams highlight important relations between quantities and operations in a given problem and assist students to extract pertinent information (Larkin & Simon, 1987). Particularly, some cognitive scientists have concentrated on the role diagrams play in various cognitive activities such as memory, perception, inference and problem-solving (Hamami & Mumma, 2013; Shin, 2015). This brings about the discussion about working memory and the cognitive load imposed on a learner when faced with a problem. Diagrams are able to reduce cognitive demands and alleviate working memory (Murata, 2008; Ngu et al., 2014). Furthermore, they facilitate connections between concrete and abstract representations which support students' problem-solving activities. They allow both teachers and students to view a problem in its entirety as all parts are displayed on the diagram at the same time (Sunzuma et al., 2020).

The Research Questions

The study reported in this paper is part of a larger study (Manoharan, 2021) that investigated teachers' perceptions of diagrams for the teaching and learning of mathematics in Singapore secondary schools. The research questions that guide the study reported in this paper are:

1. *What are teachers' perceptions about what diagrams in mathematics represent?*
2. *When do teachers use diagrams in their mathematics instruction?*

The Instrument

The instrument, a pen-and-paper survey comprising an open-ended questionnaire, was used to probe the understanding of mathematics teacher leaders about diagrams and their role in the teaching and learning of mathematics. Details about the survey are reported in Manoharan (2021). To answer the research questions, responses to prompts in the first section of the survey were coded. The questions were:

- What do diagrams in mathematics represent? [Item 1]
- In your teaching of mathematics when do you use diagrams and why? [Item 2a]
  Do you use diagrams to make connections between mathematical ideas? [Item 2b]

Subjects

A total of 20 heads of mathematics departments whom we refer to as mathematics teacher leaders (MTL) from secondary schools in Singapore participated in the study. These MTLs had taught mathematics across grades 7 to 10 for at least 3 years prior to their participation in the study. The participants were in-service teachers, attending higher degree courses or professional development sessions at the National Institute of Education when they completed the survey. The collection of data was governed by the IRB of the university.

Data and Analysis

A hybrid process of deductive and inductive thematic analysis was used to facilitate the coding process. A thematic analysis, as noted by Braun and Clarke (2006) is a useful and flexible method that can potentially provide a rich and detailed account of data for qualitative research. A hybrid approach employed for this study incorporated both the 'bottom-up' inductive approach of Boyatzis (1998) and the "top-down" deductive approach outlined by Crabtree and Miller (1999). The process of data extraction, coding and categorisation was divided into two stages:

Stage 1: A set of *priori* codes were created drawing from the literature reviewed. This allowed for the coding process to be structured and grounded in existing theories as an initial coding cycle (Linneberg & Korsgaard, 2019).

Stage 2: During the process of coding, it became apparent that not all the responses could be captured by the set of *priori* codes. Through iterative rounds of revising the codes and coding the data, a final set of *posteriori* codes was derived. Swain (2018) notes that this approach of encoding the data results in theory being a precursor to, and an outcome of, data analysis. The set of codes derived from both literature and the actual data, serves as a conceptual framework which guided the process of analysis. Table 1 shows the sets of *priori* and *posteriori* codes. Table 2 shows samples of responses and corresponding codes.

Table 1
Priori and Posteriori Codes

Priori Category Code Description		Posteriori Category Code Description	
[Item 1] Category Code: Representations (R)			
R1	Source of information (Stylianou, 2011)	R1	External representations (Winn, 1987)
R2	Means to record information (Stylianou, 2011)	R2	Source of information (Stylianou, 2011)
R3	A heuristic used in any part of problem-solving (Cellucci, 2019; Polya, 1945; Uesaka & Manalo, 2012)	R3	A heuristic used in any part of problem-solving (Cellucci, 2019; Polya, 1945; Uesaka & Manalo, 2012)

R4	Simplifying tool – known and unknown quantities can be represented. (Stylianou, 2011)		
[Item 2a] Category Code: Affordances (A)			
A1	Facilitate understanding (Cellucci, 2019; Stylianou, 2011)	A1	Facilitate understanding (Cellucci, 2019; Stylianou, 2011)
A2	Diagrams act as a means to connect between existing knowledge and skills (Mudaly, 2012)	A2	Reduce cognitive demand (Murata, 2008; Ngu et al., 2014; Stylianou, 2011)
A3	Reduce cognitive demand (Murata, 2008; Ngu et al., 2014; Stylianou, 2011)	A3	Assist with visualisation (Polya, 1945)
A4	Exploration device (Stylianou, 2011)	A4	Exploration device (Stylianou, 2011)
A5	Monitoring and assessing students' understanding (Stylianou, 2011)	A5	Monitoring and assessing students' understanding (Stylianou, 2011)
[Item 2b] Category Code: Making Connections (MC)			
MC1	Connect between existing and new knowledge (Mudaly, 2012)	MC1	Connect between existing and new knowledge (Mudaly, 2012)
MC2	Boost germane cognitive load (Ngu et al., 2014)	MC2	Boost germane cognitive load (Ngu et al., 2014)
MC3	Connect between different representations (De Toffoli, 2018)	MC3	Connect between different representations (De Toffoli, 2018)

Table 2
Sample Responses and Codes

Teacher	Response to Item 1: What do diagrams in mathematics represent?	Codes
MTL7	Diagrams in Mathematics are **visual representations** of mathematics objects, their properties and relationships with each other. Diagrams are often used as a **problem-solving tool.**	R1 R3
MTL12	Diagrams are a means to **consolidate data and information** for the ease of analysis and pattern finding. Diagrams serve to **convey** and **explicate concepts and information**. Diagrams aid in **problem solving** as information can be represented in a more **comprehensive manner to enable the problem solver to gain better perspective of the problem and form connections**	R2 R3
	Response to Item 2a: When do you use diagrams and why?	
MTL3	I will use a diagram when I can. I believe that multiple-representations and multimodal representations will help students to **make connections** between the representations and **deepen their understanding** of the concept. A diagram also **explicates the relationships** between aspects of the concepts. Different diagram can be used to **highlight certain features for pattern recognition** or for study of trends as in the case of statistical diagrams. Also, I believe a diagram helps in problem solving. When we download the information given and make sense of it in a diagram, we **reduce cognitive load** so as to **free up our mind to focus on ways to solve the problem.** The diagram may aid in **exploring various ways to solve a problem.**	A1 A3 A2 A4

Findings and Discussion

What Do Diagrams in Mathematics Represent?

Table 3 shows the main perceptions held by MTLs about what diagrams represent.

Table 3
Frequency by Category of Responses to Item 1

Survey Item	Category codes for representation	Frequency (n=20)
What do diagrams in mathematics represent?	External representation (R1)	13
	Sources of information (R2)	12
	Heuristic—problem-solving tool (R3)	6

The responses revealed three key perceptions, all of which have been identified in other theoretical or empirical work on diagrams as in the review of literature of this study. Consistent with the definition of diagrams of this study, 13 of the MTLs identified the powerful ability of diagrams to provide a visual representation of real world or mathematical objects. MTL1 stated that diagrams may "represent statistical information" and are also able to "summarise the problem" given. They make quantifiable relationships and structural features of a problem apparent (Sunzuma et al., 2020). Furthermore, 12 also recognised that diagrams provide a concise method of presenting information. Many respondents indicated that diagrams "communicate information" such as mathematical "concepts and properties" (MTL2, MTL3, MTL4, MTL20). However, only 6 perceived diagrams as a heuristic to be used in any part of problem-solving to facilitate understanding and consequently guide problem-solving activities. This finding is of concern as diagrams are a significant heuristic for problem-solving (Hembree, 1992; Polya, 1945; Uesaka & Manalo, 2012; Wong, 1999). Furthermore, mathematical problem-solving is the primary goal of mathematics instruction in Singapore schools.

When and How Diagrams are Used

Perceptions of the MTLs related to when do they use diagrams and why are shown in Table 4. The responses were coded in two categories, namely affordances of diagrams and making connections.

Table 4
Frequency by Category of Responses to Items

Item 2	Category Codes	Frequency (n = 20)
(2a) In the teaching of mathematics when do you use diagrams and why?	Affordances	
	Assist with visualisation (A3)	16
	Facilitate understanding (A1)	10
	Reduce cognitive demand (A2)	6
	Monitoring and assessing students' understanding (A5)	4
	Exploration device (A4)	2
(2b) In your teaching of mathematics, do you use diagrams to make connections between mathematical ideas?	Making Connections	
	Connect between existing and new knowledge (MC1)	7
	Connect between different representations (MC3)	7
	Boost germane cognitive load (MC2)	2

From Table 4, it is apparent that all the MTLs used diagrams to gain from the multitude of benefits that diagrams afford when engaging students in mathematics instruction. This finding concurs with that of earlier works established by many others (see Cellucci, 2019; Mudaly, 2012; Stylianou, 2011). Sixteen of the MTLs used diagrams to assist with visualisation - when "explaining concepts, as it helped students visualise" (MTL20) and "guided them when solving problems" (MTL17). Half of the MTLs noted that diagrams facilitate understanding by serving as external representations of interrelated concepts and properties. Transforming sentential representations to diagrammatic representations allows one to make connection between concepts. It supports schema acquisition which enables problems-solving. MTL11 mentioned that students having difficulty understanding the problem will benefit from a diagram as it "will provide a clearer picture."

Less than a third of the MTLs, placed emphasis on the following during their mathematics instruction: (i) diagrams reduce the cognitive load of oneself when working on a mathematical task, (ii) diagrams may be used to monitor and assess one's understanding of mathematical ideas, and (iii) diagrams as an exploration device—tool for representing, visualising, etc. of mathematical ideas.

When asked if they made connections using diagrams, all the MTLs responded positively. However as shown in Table 4, less than half of them were cognisant that diagrams facilitate connections between existing and new knowledge and does the same between different representations. MTL 6 presented Figure 1, as an example of how she engages students in reasoning with diagrams whilst connecting their knowledge about congruent figures and properties of circles.

AB is a diameter of the large circle, centre *O*.
CD is a diameter of the small circle, centre *O*.
AC and *BD* are tangents to the small circle.

Show that triangle *OAC* is congruent to triangle *OBD*.
Give a reason for each statement you make.

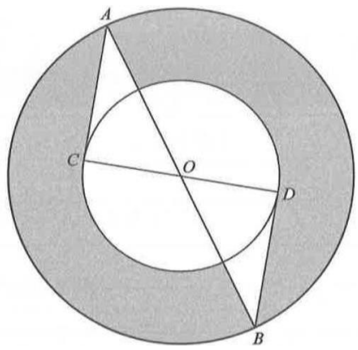

Figure 1. Reasoning with diagrams.

MTL11 was the only respondent who mentioned that diagrams can facilitate big ideas such as Proportionality. She illustrated, see Figure 2, how diagrams may be used to teach the concept of arc length of a circle. Using the diagrams and comparing some calculations, students would be guided to comprehend the proportion of arc length over the circumference of circle. In addition, students' knowledge of fractions and circle properties are drawn on to teach the new concept of arc and sector length.

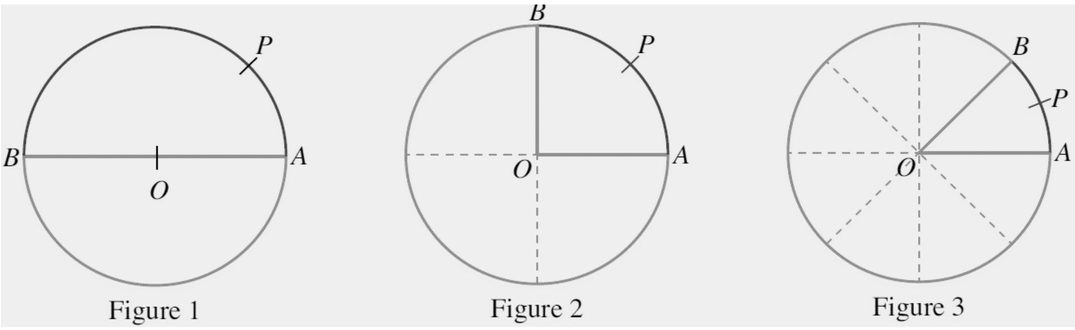

Figure 2. Arc length of a circle.

Conclusions

This study involved secondary school mathematics teacher leaders. As the revised school mathematics curriculum for implementation in schools from 2020 onwards has placed heightened emphasis on big ideas in mathematics with diagrams being one of them, the findings of this study are timely.

Diagrams as external representations and as sources of information, emerged as the dominant perceptions. MTLs acknowledged the capacity of diagrams to externalise concepts and relationships (van Essen & Hamaker, 1990) which facilitates problem-solving for learners. They also noted that critical information about a problem can be recorded in a single, coherent diagram (Stylianou, 2011). However, very few teachers were able to identify diagrams as an effective heuristic that can be used in any part of problem-solving (Polya, 1945). Indeed, most associated diagrams with primarily being external sketches that represent information but did not consider the more specific capacity of diagrams. Some distinctive functions of diagrams include, but not limited to, serving as simplifying tools which can elucidate unknown quantities (Chu et al., 2017), as exploration tools which allow for manipulation of concepts, and as monitoring tools to assess learning (Stylianou, 2011). These findings suggest that teachers may be more accustomed to using diagrams for the most recognized use – as external representations. There was also a lack of affirmation about the use of diagrams in making connections between existing and new knowledge, and also between different representations. Although aware of the connected nature of mathematics, the findings suggest that teachers lack the understanding of how to utilise diagrams as a Big Idea in making these connections thus not fully realising the potential of this vision.

Conversations about teaching towards Big Ideas and ways to enhance pedagogy will greatly impact the extent to which teaching mathematics can progress towards a larger conceptual understanding (Woodbury, 2000). Teaching towards Big Ideas affords opportunities for teachers and curriculum specialist to rethink, refine, and possibly reinvent how they communicate with the use of diagrams. To prepare teachers for the challenge ahead of illuminating diagrams as a big idea in the teaching and learning of mathematics, professional development of mathematics teacher leaders in this area is critical. Therefore, it is recommended that mathematics teacher leaders engage in professional development to deepen their knowledge on diagrams and their potential for the teaching and learning of mathematics.

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RESEARCH PAPER

A Comparison of Classroom Pedagogical Practice Named by Middle School Mathematics Teachers in Australia and Chile

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The professional lexicons of middle school mathematics teachers in communities around the world were documented as part of The International Classroom Lexicon Project. This vocabulary captured teachers' naming of classroom pedagogical practice. Reported in this paper are attributes of the lexicons of teachers from Australia and Chile. A comparison of the lexical items revealed commonalities and differences in the named phenomena and in the schema employed for their organisational structure. The analysis revealed differences attributable to cultural and contextual factors. A categorisation system was developed to classify the complexity of similarity of lexical items of one lexicon with another.

Background

The International Classroom Lexicon Project began with the recognition that classroom practices named by one educational community are not necessarily named by others (Mesiti et al., 2021a). Some communities have had their named activities translated into English in ways which misrepresent their true meaning, whilst other named activities have been omitted from the *lingua franca* of research (Clarke, 2006). Research undertaken in the field of mathematics education and language has generally focussed on the language of the learner; the language brought by the teacher to the classroom; and the language arising from the practice of mathematics (Planas et al., 2018; Austin & Howson, 1979). These domains appear to exclude studies of teachers' professional language about phenomena of the mathematics classroom. In response, the primary goal of the International Classroom Lexicon Project was to document the professional vocabulary of teachers to describe middle school mathematics classrooms. These are the words that teachers use to talk *about* the classroom when in conversation with colleagues. At the core of the research project was a recognition of the importance of teacher knowledge, a commitment to document teacher knowledge, and a commitment to share this knowledge with the wider community to improve teachers' reflective practice.

The professional lexicons of Australian and Chilean mathematics teachers are expressed in English and Spanish. English is spoken by 1.348 billion people (370 million native speakers) whilst Spanish (the official language of Chile) is spoken by 543 million people (471 million native speakers). English is the most spoken language and Spanish is the fourth most spoken language around the world (Lane, 2021). The professional lexicons are a collection of terms by which teachers name the objects and events that constitute their professional activity. In this paper we compare the lexicons of the Australian and Chilean middle school mathematics teachers to build our understanding of the variation possible within two teaching communities that speak two of the most common languages in the world by asking the following research questions:

1. *When comparing the national lexicons from Australia and Chile in what ways are they the same and in what ways do they differ?*
2. *How might these differences be categorised?*

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 394–401. Launceston: MERGA.

Teachers' Professional Vocabulary

The identification of a technical or professional language in English, to support the description and analysis of classroom practice is underdeveloped (Grossman et al., 2009; Lampert, 2000; Lortie, 1975). Few opportunities are present in the school environment to engage with peers about the problems and challenges of practice (Connell, 2009). This absence of informal learning opportunities results in an English "language of practice [that] remains flat or nonexistent" (Lampert, 2000, p. 90). Similarly, in two case studies reporting the development of communities of practice using videotapes in Chile, participants reported that they were entirely unfamiliar with the idea of creating communities to engage with discussions about various aspects of practice (Grau et al., 2017). Successful learning communities are characterised by professional experiences, opportunities and the sharing of a common language (Grossman et al., 2001).

Theoretical Framework

Researchers have argued that differences amongst languages, both linguistic and semantic, influence our lived experience (Boroditsky, 2001; Levinson, 2003). This position, a weaker interpretation of the much-debated Sapir-Whorf hypothesis that "language shapes thought" (Sapir, 1949), has been characterised as *linguistic relativity*. Others have similarly argued that "categories set up, and hence the distinctions made by language, not only express the social structure but also create the need for people to conform to the behavior associated with these categories" (Marton, Runesson & Tsui, 2004, p. 28). The theoretical position adopted for this project is in line with the notion of *linguistic relativity;* that differences and absences in vocabulary matter and these may indicate a diversity of teachers' view of the classroom.

Research Methodology

The International Classroom Lexicon Project: The Research Design

The International Classroom Lexicon Project involved research teams from Australia, Chile, China, Czech Republic, Finland, France, Germany, Japan, Korea and the USA. In each participating country, experienced teachers of middle school mathematics participated as legitimate members of both the research teams and their wider practitioner community, representing insiders, informers as well as collaborators. The research teams enacted a "negotiative" methodology (Mesiti et al., 2021a) by participating in collaborative consultations with teacher partners regarding the identification and inclusion of terms in the lexicon. This approach ensured authority was accorded to teacher voice in the generation of each national lexicon. Each resulting lexicon was then negotiated with its wider community through a thorough, structured, iterative process of validation to refine and ratify the structure and content of this national lexicon. The main phases for the documentation of national lexicons and each community's validation processes have been detailed elsewhere (Mesiti et al., 2021b).

The Australian and Chilean Lexicons

The lexicons consist of terms (or short phrases) that are familiar, have an agreed meaning, and are in use by middle school mathematics teachers. These terms are illustrated with a description from the classroom, an example and sometimes a non-example and this detail has been translated into English by the country team of origin (see Table 1 for a selection from the Australian and Chilean Lexicons). All items included in the lexicons have been validated locally and nationally.

Table 1
Example of Lexical Items from the Complete Australian and Chilean Lexicons (Mesiti, Hollingsworth, Clarke et al., 2021; Grau et al., 2021)

Lexical Item	Australian Lexicon	Chilean Lexicon
Term name	prompting	puesta en común (putting in common)
Description	The teacher guides the student (usually with a verbal comment) towards a more appropriate or effective response of solution method.	The teacher orchestrates the joint review of a completed task without giving out the right answer. This implies that knowledge is distributed among students and the teacher.
Examples	A teacher comments: “Check your working.” Or “Try multiplying.”	The class is reviewing the results of a worksheet. The teacher asks: “Did anyone get a different result? Why?”
Non-examples	The teacher provides the next step in the solution.	

In the table above, the operational definition of each of the terms *prompting* and *puesta en común* are operationalised with a description, examples and non-examples. A non-example, something that might be thought of as indicative of the practice but is not, was sometimes included to assist with the provision of a fuller definition.

Comparison of the Australian and Chilean Lexicons: The Research Design

Phase 1: Independent Reviewing

The members of the cross-national team included an Australian researcher (speaker of English and Italian) and three Chilean researchers (speakers of both English and Spanish). Each member independently reviewed both lexicons to identify terms that were identical, similar with regards to pedagogical intention, as well as entirely absent from the other lexicon.

Phase 2: Group Negotiation

Phase 1 was followed by a whole group negotiation. Each candidate pairing of terms was examined and members of the team discussed possible lexical similarity. This involved considering the items in terms of:

- agency (whose action);
- observable form;
- inferred function;
- breadth (range); and
- relation to mathematical content.

Discussion centred around revealing the ‘fullness’ of the definition – deciding which characteristics were crucial in understanding the meaning of the term and what action, activity or cognitive activity was being represented. This was supported with readings and re-readings of the operational definitions (definitions, examples and non-examples). The members of the cross-national team approached this challenging task mindful of respecting the origins of the lexicons as representing the teachers’ vocabulary and their definitions. Once terms were identified as “similar” or related in some way, various types of associations were detailed. These included:

- *Exact* (match in name and intention);
- *Containment* (where one term was a subset of another term);
- *Inter-Related* (terms with properties in common); and,
- *No correspondence* (unique to its lexicon).

These relationships, as well as critical distinctions, were revisited at a following meeting to validate initial pairings.

Results and Discussion

The Organisational Structure

The Australian Lexicon contains 61 terms as representative of a professional vocabulary of middle school mathematics teachers (see Table 2). The terms are generic in nature, feature a significant number of gerunds. Students' voice, action and participation are significant characteristics (Mesiti, Hollingsworth, Clarke et al., 2021). The Chilean Lexicon features 74 terms in total mostly constituted of short phrases as opposed to single-word terms (see Table 2). These phrases refer to generalist concepts of pedagogy not specific to mathematics, and feature terms that focus on lesson structure. Most of the terms that involve agency are actions performed by teachers (Calcagni et al., 2021).

Table 2
Lexicon Characteristics of the Australian and Chilean Lexicons

Lexicon Characteristics	Australian Lexicon	Chilean Lexicon
Number of terms	61	74
Number of organisational categories	5	5
Terms belonging to more than one category	37	5

Table 3
Organisational Category Names of the Australian and Chilean Lexicons

Lexicon	Organisational Category Names (number of terms)
Australian	Administration (8) Assessment (11) Classroom Management (7) Learning Strategies (27) Teaching Strategies (49)
Chilean	*Didáctica de la disciplina* - subject-matter didactics (22) *Metodologías generales* - general pedagogies (18) *Interacción pedagógica* - teaching interactions (16) *Estructura/rutina* - structure/routine (12) *Clima de aula* - classroom climate (9)

The number of terms identified in the lexicons are similar in number and are communicated with an organisational system encompassing five categories (see Tables 2 and 3). Both systems were inspired by classroom teachers' groupings of lexical items and the names they gave to these groupings. When comparing these organisational structures, category names and terms grouped within were examined. The following findings were identified:

- only the Chilean category *Didáctica de la disciplina* (subject-matter didactics) includes terms with a mathematical focus, this category also includes a number of practices that are classified within the Australian *Teaching Strategies* category;
- the Australian category *Assessment* includes terms that can be found within the Chilean category *Metodologias generals* (general categories);
- the Australian category *Classroom Management* appears similar to intention to the Chilean category *Clima de aula* (classroom climate); and,
- the Chilean *Interacción pedagógica* (teaching interactions) category includes terms similar to those within the Teaching Strategies category.

Whilst most of the terms in the Chilean Lexicon belong only to one category, five terms have been assigned to two (see Table 2). For example, the Chilean term argumentar (arguing) is found in both the *Didáctica de la disciplina* (subject-matter didactics) and the *Interacción pedagogica* (teaching interactions) categories. In contrast, the Australian Lexicon includes 37 terms that belong to more than one category and significant overlap of 24 terms is found between the *Learning* and *Teaching Strategies* categories (see Table 2). One possible explanation is that the Australian terms indicate a flexibility in teacher pedagogical expertise as many practices can be seen as providing both teaching and learning opportunities. This includes terms such as: *answering questions*, *applying*, *defining* and *justifying* as well as *reasoning* and *summarising*. On the other hand, the Chilean categorisation system indicates a nuanced understanding of *didactics* (locally defined as "the art of teaching"): the consideration of content knowledge, student cognitive characteristics and pedagogy within the scope of learning and teaching.

The Named Phenomena

A summary of the results of the analysis of exploring commonality and difference between the lexicons of Australia and Chile is given in Table 4. (Note that a term may be involved in more than one pairing, for example, consider the Australian term *questioning* in Figure 1.)

Table 4
Comparison of the Australian and Chilean Lexicons by Term Correspondence

Type of term correspondence	Australian Lexicon	Chilean Lexicon
Exact	10 pairs	
Containment (by Australian term)	12 pairs	
Containment (by Chilean term)	4 pairs	
Inter-Related *(terms with properties in common)*	12 terms	16 terms
No correspondence *(unique to the lexicon)*	28 terms	33 terms

About half the terms (33 Australian terms; 41 Chilean terms) corresponded in some way (whether exactly, containing or contained, or inter-related). Many more Australian terms contained the Chilean terms and indicated a difference with level of detail. The Australian Lexicon is distinctive in its identification of general pedagogical practices and this comparison highlights the level of specification. Whereby the Australian Lexicon identifies the phenomena of *questioning* the Chilean Lexicon has indicated four questioning-related terms (see Figure 1). Another form of inter-relatedness includes the pairing with respect to peer support. In this case the construct differed by actor. The Australian term refers to the student activity of peers assisting each other; the Chilean term refers to the teacher promoting such student activity.

Exact

Australian Term	Chilean Term
group work	trabajo en grupo (group work)
justifying	fundamentar (providing justifications)
monitoring	monitoreo del aprendizaje (monitoring learning)
positive reinforcement	refuerzo positive (positive reinforcement)

Inter-Related

Australian Term	Chilean Term
peer support	promover apoyo entre pares (promoting peer support)
questioning	pregunta per informacion (information question) contra-preguntar (counter-asking) pregunta metacognitiva (metacognitive question) pregunta por elaboración (elaboration question)

Containment by Australian Term

Australian Term	Chilean Term
summarising	sistematización (systematisation)
worked example	resolución en conjunto de tareas matemáticas (solving a mathematical task together)

Containment by Chilean Term

Australian Term	Chilean Term
clarifying	facilitar el aprendizaje (facilitating learning)
questioning	secuencia pregunta-respuesta (question-and-answer sequence)

Figure 1. Examples from the lexicons of type of term correspondence.

No correspondence

Australian Terms (absent from the Chilean Lexicon)	
engaging (engagement)	on task
motivating	reflecting
elicit understanding	guiding
giving praise	modelling

Chilean Terms (absent from the Australian Lexicon)	
enunciar el plan de la clase (stating the lesson plan)	desarrollo de la clase (lesson development)
curso normalizado (normalised class)	petición de ayuda (asking for help)
uso del humor (use of humour)	valorar procedimientos (valuing procedures)
presentar contenidos con contexto de la vida real (presenting contents in real-life contexts)	

Figure 2. Examples from the lexicons of ‘No Correspondence’ (absent terms).

Many of the terms in this comparison were determined as unique to their constituent lexicon: 28 Australian terms (46%) and 33 Chilean terms (45%) (see both Table 4 and Figure 2). The Chilean terms absent from the Australian Lexicon include terms related to structure of the lesson (*conectar con plan curricular*; connecting with the curricular plan), stages of the lesson (*inicio de la clase*; beginning of the lesson), resource use (*uso de material concreto*; use of concrete materials), and problem-solving approaches (*simplificación de un problema*; simplification of a problem). The Australian terms absent from the Chilean lexicon relate to areas of affect (*encouraging*), management (*collecting work*), assessment (*feedback*), student work (*student responses*) and thinking processes (*prompting, applying*) [additional examples have been given in brackets]. Of particular interest to the Chilean researchers was the emphasis of terms related to thinking processes, both dialogical (*guiding*) and imitative (*demonstrating*), in the Australian Lexicon. Affect-related terms are almost entirely absent from the Chilean Lexicon (except for the term *uso del humor;* use of humour).

Research studies in Chile focusing on mathematics teaching have consistently reported that lessons are characterised by teacher-led instruction and question-and-answer sequences and the Chilean Lexicon appears to reflect this finding. The lesson begins with the presentation of concepts and procedures and is followed by individual student work to practise skills (Preiss et al., 2016). One of the enduring difficulties with teachers' continuous education is the lack of coherence between the needs of professional development and the courses and methodologies currently offered by the programs. They tend to be based in direct instruction, instead of focusing on teachers' reflections about their own practice (Grau et al., 2017). Australia continues to endure a stubborn shortage of qualified mathematics teachers in schools (Weldon, 2015), which results in teachers without specific training in mathematics education teaching "out-of-field" (Weldon, 2016). This situation would likely affect teachers' naming of practices about the mathematics classroom, thus professional development involving comparing lexicons might well be useful to promote teachers' nuanced understanding about classroom practice.

Conclusion

In this paper we compared the lexicons of the Australian and Chilean middle school mathematics teachers to indicate the variation possible within two teaching communities that speak two of the most common languages in the world. These lexicons capture the words familiar and in use by teachers when identifying the phenomena of their mathematics classroom. The lexicon documented is in the teachers' native language (and translated into English) and is supplemented with a description and illustrative examples from the classroom. Together these elements work together to provide a full and rich operational definition for each of the lexical items.

In contrasting the Australian Lexicon with the Chilean Lexicon, we were able to identify differences and commonalities in structure, phrasing, specification, and context. More than half the terms in the lexicons were related. The methodological approach detailed in this paper allowed for the development of a five-type correspondence categorisation that improved the identification and naming of complexity around the notion of similarity. This was made possible with collaborative detailed discussions to characterise the complexity of correspondence by the members of the cross-national team. The analysis outlined in this paper confirms the cross-national researchers' commitment to document and share teacher knowledge as an initial step towards improving teachers' reflective practice.

Acknowledgements. This project has been funded by a Discovery Grant from the Research Council of the Australian Government (ARC-DP140101361) and partially funded by Measurement Centre (MIDE UC), Pontificia Universidad Católica de Chile and Proyecto REDES 170141 de CONICYT—Chile and an Australian Government Research Training Scholarship.

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Teacher Questioning to Support Young Students to Interpret and Explain Their Critical Mathematical Thinking

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This paper examines the types of teacher questions that assist young students to interpret and explain their critical mathematical thinking (CMT). Focusing on students who enter their first year of formal schooling (aged 5–6 years), this paper draws on data from a one-on-one task based clinical interview conducted with 16 students. Teacher questioning data were analysed for question type (probing, factual, guiding) and further analysed against the conceptualized critical mathematical thinking framework for young students. Findings indicated that when teachers used clarifying, noting relationships, and offering opinions style probing questions, young students were supported to interpret and explain their critical mathematical thinking.

The ability to apply mathematical thinking in a critical way is essential to our students' future success in education and employability (Heard et al., 2020; National Council of Teachers of Mathematics (NCTM), 2000). Research, policy, and curricula have indicated the importance of including critical thinking in the discipline of mathematics starting in the early years of schooling (NCTM, 2000; Wood et al., 2006). However, to date, little research has been undertaken to better understand how critical thinking and mathematical thinking amalgamate, which has left this area poorly defined and under researched. Consequently, this has impacted on classroom practices, and recent reports indicate that typically educators teach generalised critical thinking skills without a theoretical underpinning and without examples for domain specific areas (Sweller, 2022). This eventuated in the emergence of a new term, Critical Mathematical Thinking (CMT) that brings together critical thinking and mathematical thinking in a domain specific way (Monteleone, 2021). A recent systematic review of the literature, drawing on both mathematical thinking and critical thinking, led to the development of a theoretically informed conceptual framework, the Critical Mathematical Thinking Framework for Young Students (CMTFYS) (Monteleone, 2021) to better understand the key features of CMT for young learners.

The question remains as to how teachers can support young students to interpret and explain their critical mathematical thinking. It is well established in the literature, that the role of teacher questioning is central to supporting young learners to elicit their thinking (Franke et al., 2009; Sahin & Kulm, 2008), however, it is not clear what examples of teacher questions posed best support young students to exhibit domain specific thinking such as CMT. Thus, the focus of this paper is to begin to identify the particular types of teacher questions that help young students to interpret and explain their CMT as they participate in mathematics learning experiences.

Critical Mathematical Thinking For Young Students' Conceptual Framework

While the literature surrounding critical thinking (e.g., Facione, 2011) and mathematical thinking (e.g., Clements & Sarama, 2014) are diverse, there are also many commonalities. In the development of the conceptual framework, seminal work were reviewed and five critical mathematical thinking themes with supporting sub-themes emerged from the literature (Monteleone, 2021). The themes, *Interpreting* (e.g., Facione & Facione, 2008), *Analysing* (e.g., Lai, 2011), and *Creating* (e.g., Facione, 2011) emerged from the critical thinking literature while *Evaluating* (e.g., Wood et al., 2006) and *Explaining* (e.g., Diezmann et al., 2001)

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 402–409. Launceston: MERGA.

emerged from both critical thinking and mathematical thinking literature. Findings of a larger study identified that the CMTFYS provides the definition of CMT capabilities in young students (Monteleone, 2021), and offers a unique contribution to Critical Mathematical Thinking as a term. Indicating that CMT is more diverse from mathematical thinking alone. Figure 1 presents the CMTFYS which includes the five themes and 14 sub-themes. In this figure the themes are in bold and the sub-themes are italicized. The bolded themes and italicized sub-themes offer a unique contribution of critical thinking literature to this framework.

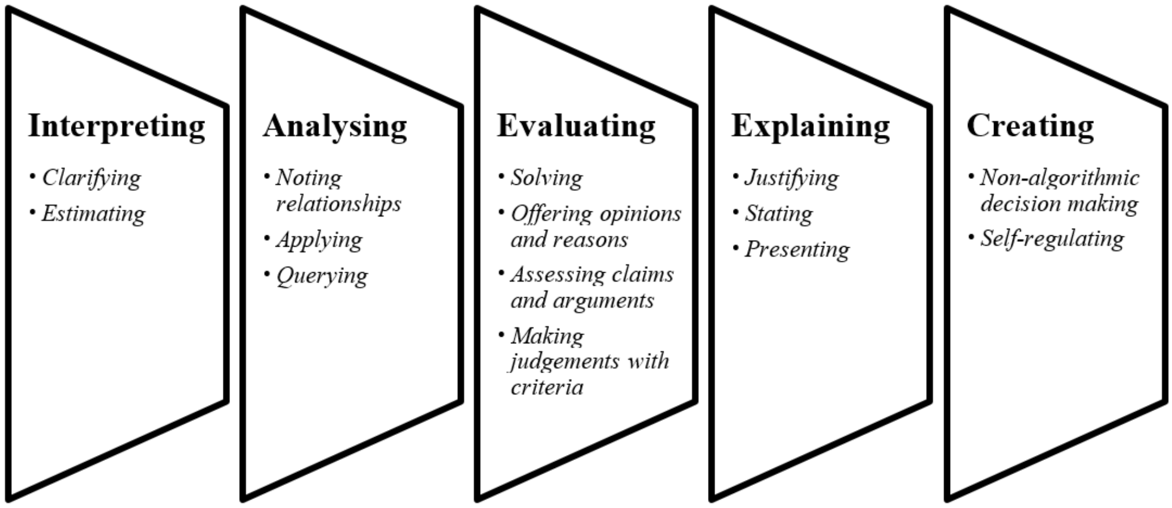

Figure 1. Critical mathematical thinking for young students' conceptual framework (CMTFYS) (Monteleone, 2021).

Further investigation is needed to understand the role of the teacher to support students to elicit their CMT. Therefore, for the purpose of this paper, the CMTFYS conceptual framework serves as a lens to help classify teacher questions and provided a platform to exemplify the types of questions that can be used to support young learners.

The Role of the Teacher Questioning in Mathematics

It is known that within mathematics learning in the early years, the role of the teacher is central, especially in supporting young students to elicit their thinking (Franke et al., 2009). Literature informed approaches to teaching mathematics that appear to be the most beneficial to young learners require teachers to: (i) engage young students by integrating mathematical concepts; (ii) embed problem solving; and (iii) allow for construction of ideas (Lessani et al., 2017). Adopting the mentioned teacher approaches has been found to provide a platform for students to engage in reasoning, creativity and allow for communication of their mathematical ideas (Wood et al., 2006). In order to enact this in classrooms with early years learners, teachers must engage young students in mathematical talk through the use of questioning. Encouraging student mathematical talk has been found to assist learners to engage in thinking processes, including justifying and reasoning about their approaches and solutions in class (Hunter & Anthony, 2011).

Teacher questioning plays an important role in promoting students' thinking in mathematics classrooms (Franke et al., 2009; Mata-Pereira & da Ponte, 2017; Sahin & Kulm, 2008). A range of teacher questioning used in the classroom supports student engagement in building mathematical understandings (Martino & Maher, 1999). Yet, research undertaken with grade two and three students identified that over 76% of teacher questions in mathematics lessons were at best surface level, requiring a yes or no answer (Di Teodoro et al., 2011). It is evident that there continues to be an ongoing challenge with the types of and diversification of teacher questions used in mathematics classrooms.

In mathematics education, teacher questioning can be categorised in a myriad of ways; for example, *factual*, *probing* and *guiding* questions (Sahin & Kulm, 2008). *Factual questions* tend to provide very little information about a student's understanding of a concept or content and, are often lower order with little opportunity to discuss strategies with others (Sahin & Kulm, 2008). *Probing questions* have been found to extend student's understanding, knowledge, and mathematical thinking, moving students from low level to higher order thinking (Sahin & Kulm, 2008). Franke et al. (2009) recognises the need for probing questions as a way for teachers to gain further clarity about a student's explanation. *Guiding questions* are considered questions that direct students to derive concepts or procedures to solve problems (Mata-Pereira & da Ponte, 2017). Both probing and guiding questions have been found to best support students to display higher levels of mathematical thinking. It is important to note that a critique of the teaching question literature is that at times the focus is too narrow, only focusing on teacher questions and not fully understanding the impacts of this across an entire teacher-student conversation. Hence, it is important to understand the impact of the teacher question on the student learning.

Thus, to ascertain the role of teacher questioning in supporting young learners to elicit their CMT, the study was underpinned by the following research question:

What types of teacher questions help young students interpret and explain their CMT?

Research Design

The findings presented in this paper are drawn from a larger study that employed an explanatory mixed methods design (Creswell, 2013) to examine how young students elicit their critical mathematical thinking. The focus of this paper is on the qualitative data which was collected to better understand the statistical analysis in more depth, particularly with regards to the CMT capabilities of young students and the role of teacher questioning.

Participants and Context of the Study

In total, 161 Kindergarten students participated in the larger study (aged: 5 years 1 month to 6 years 8 months). These Kindergarten students were from three urban primary schools in NSW Australia and were in their first six months of formal schooling. The participating schools had similar demographical features, with the Index of Community Socio-educational Advantage levels between 1092–1112 (a score above 1000 indicates a high-level of socio-economic status). These schools had similar above average Australian results in National Numeracy assessments (NAPLAN).

There were different data collection stages within the larger study to narrow student participants. The reason for narrowing the student group was to identify young students who exhibited some CMT capabilities in mathematics lessons which was observed by the researcher and evidenced across quantitative measures. This smaller group would provide an opportunity to explore more deeply the types of CMT students display and the role of teacher questioning to support students to explain and interpret their CMT. In total, 16 students were identified to participate in interviews. This included nine male and seven female students. The 16 students were represented from each of the schools of the larger study (25% School A; 32% School B; 43% School C).

Data Collection Methods

To investigate the types of teacher questions that assist young students' to interpret and explain their CMT, the data collection methods included a task based one-on-one clinical interview. The interview consisted of eight learning experiences. The clinical interview process followed *Piaget's methode Clinique* (Hunting & Doig, 1997) that identifies the cognitive capabilities of a child within the learning social context. Previous studies in mathematics education have adopted this method to assess young children's mathematical learning (Clements & Sarama, 2014; Hunting & Doig, 1997; Warren et al., 2012). Of particular importance, is the balance of the researcher encouraging students to elaborate on their responses while refraining from steering students towards a desired answer (Miller, 2014; Warren et al., 2012). The dialogue between the student and the researcher supported the researcher to clarify, ask questions and pose problems during the interview.

The interview included learning experiences which were designed to: (i) begin with an open-ended question (Nicol & Bragg, 2009); (ii) provide multiple entry points for students (Jorgensen et al., 2010); (iii) use physical manipulatives (MacDonald & Lowrie, 2011); and (iv) cover a range of mathematical content appropriate for the age group. Table 1 presents examples of learning experiences from the clinical interview where the students more commonly drew on CMT pertaining to explaining and interpreting as delineated in Figure 1.

Table 1
Example Learning Experiences from the Clinical Interview

LE	Description of the learning experience
LE1	*Framed photo finding the middle* This is a framed photograph of Joey (*hold up frame*). I would like to hang this frame in the middle of a wall. Now, imagine this piece of paper is a blank wall *(hold up A3 paper)* and this is the picture frame I need to hang (*hold up smaller frame*). How can I hang this frame in the middle of the wall?
LE2	*Counting unseen items* This is a mini bean bag *(show mini bean bag).* It is filled with little beans like these (*show zip lock bag with some beans*). It's too tricky to count them one by one. Can you think of another way to find out how many beans are in this mini bean bag?
LE6	*Cubby house – identifying number of tiles required* I have just finished building a cubby house for my children at home *(show picture of the cubby house)*. I would like to put these tiles down on the floor of the cubby house *(show square tile)*. How can I work out how many tiles I need?

The interviews were administered in a context that was familiar to the student (e.g., in a breakout room). Each student interview took between 25–35 minutes to administer. All interviews were video recorded to capture both the student responses and the researcher questioning. These videos were downloaded and were later transcribed for data analysis.

Data Analysis

Analysis of the transcripts occurred in a three-step process. First, all student responses were coded using the CMTFYS to determine the types of CMT young students exhibited. Second, all teacher questions were coded as either: factual (Sahin & Kulm, 2008), probing (Franke et al., 2009; Sahin & Kulm, 2008), or guiding questions (Mata-Pereira & da Ponte, 2017). Third, the coded teacher questions were re-analysed and deductively coded using the 14 sub-themes of the CMTFYS. It was important to code the student responses first to ensure these young learners were exhibiting CMT, before determining the teacher question (TQ) that supported this. Table 2 provides an example of the final coding of the researcher (R) and student (S23) conversation for Learning Experience One.

Table 2
Example of Coding for the Transcript of Learning Experience One

	Transcription	Alignment with the CMTFYS
R	Is there another way you can check?	*TQ: Probing – Clarifying*
S23	That there on the sides. There, that's the long way of them. And that is the length.	*Student CMT: Explaining-Justifying*
R	The length, I heard that very special word length. How can we check it*?*	*TQ: Probing – Clarifying*
S23	... If you put it with a measuring tape on the even number, you know it's the middle because the odd number that doesn't add up, it would totally be in the middle. It's the same length as the other, so that's the middle.	*Student CMT: Explaining - Justifying*
R	You could use the same strategy?	*TQ: Probing- Noting Relationships*
S23	… you could draw a line like that, two lines, above one line. Then make another one to show where your pictures standing, and then put the other line here ... So, you can do those lines. And then you put the photo on.	*Student CMT: Explaining- Stating*
R	Can you show me*?*	*TQ: Guiding - Assessing*
S23	And then what you do is you hang it up like that. Then you'll know which is the middle.	*Student CMT: Explaining - Stating*

Results and Discussion

In total, 333 probing, guiding and factual questions were posed to the 16 students for each learning experience and across all interviews. Table 3 displays the percentage of the types of researcher questions used in the interview across the eight learning experiences.

Table 3
Percentage of Types of Researcher Questions used in Interview

Questions Type	Example question	Percentage
Probing	What do you think? Can you explain that to me?	59% (196/333)
Guiding	Can you show me? Is that the best way?	25% (84/333)
Factual	Which one do you think has more? Why are both sides halves?	16% (53/333)

The data evidenced that all types of teacher questions (factual, probing, guiding) were used across the interviews. This aligns with past research (Franke et al., 2009; Mata-Pereira & da

Ponte, 2017; Sahin & Kulm, 2008). In addition to this, the analysis revealed that probing questions were the most used to support young students to elicit their CMT. While guiding and factual questions were also used by the researcher in the interview, it appeared that these types of questions were not as common. Thus, the following section provides further understanding of probing questions, specifically, how this type of questioning aligned with the CMTFYS and how these questions support young students to interpret and explain their CMT.

Using Probing Teacher Questioning to Support Students to Interpret and Explain their CMT

Further analysis of the probing questions utilised, aligned to the CMTFYS, revealed that the probing questions aligned with three types of CMT. These included: clarifying, which emerges from the critical thinking literature (CT); and noting relationships and offering opinions, which align with both critical thinking (CT); and mathematical thinking (MT) literature. Table 4 provides the: (i) frequency percentage and type of the probing questions used by the researcher; (ii) a description of the type of probing question used during the conversation; (iii) an example of the researcher questions drawn from the transcripts; and (iv) an example of the student response showing student's interpreting and explaining their CMT.

Table 4

Probing Researcher Questions to Support Young Students to elicit their CMT.

Type of question	Description of Question Type	Example of Researcher Question	Example of student CMT
Clarifying (53%) [CT]	The researcher rephrased or re-used the students' terms to gain further insight into their CMT.	*How do you know they're the same?*	"You need to do this [S using arm spans to gesture equality] so you can see it is the same." *(Student CMT Interpreting – Clarifying)*
Noting Relationships (37%) [CT and MT]	The researcher asked questions to gain further understanding of the relationship's young students were seeing across mathematical concepts.	*How do you know it is the middle?*	"How about I measure it, we bring the pencil here [moves pencil to measure to show the location of the middle]." *(Student CMT: Interpreting – Estimating)*
Offering Opinions (10%) [CT and MT]	To redirect student thinking, the researcher included questions to support young students to provide opinions about their thinking.	*Can you tell me what that would look like?*	"I am pretending this is a measuring tape …. Then you get five tiles … then you lay them out in a row of five." *(Student CMT Explaining - presenting)*

The analysis of data revealed that probing clarifying teacher questions were the most commonly used in the interviews, followed by probing noting relationships and then probing offering opinions. It is noted in previous studies that probing questions can support students to move from lower level to higher levels of thinking (Franke et al., 2009; Sahin & Kulm, 2008), however, the use of the CMTFYS provides new insight into understanding the role of teacher questions, in particular, how specific questions during conversations can help students further interpret and explain their CMT. The descriptions offered for each type of teacher probing question, in Table 4, extends on the research by Franke et al. (2009) and Sahin and Kulm, (2008) by providing specific teacher questions to be used during a conversation. For example, teachers can pose *probing clarifying questions* to young students.

Research indicates that supporting young students to communicate their ideas (Wood et al., 2006) and engage in mathematical talk to articulate their thinking (Hunter & Anthony, 2011) is deemed important. More so, in order to prepare students to engage in CMT, support for teachers is required to assist in moving beyond generic approaches adopted in their mathematics classroom (Sweller, 2022). If young learners need to display CMT, then teachers need to pose specific questions that align with themes and sub-themes from both the critical thinking and mathematical thinking literature. The findings of this study begin to address this gap by providing a domain specific theoretical framework (CMTFYS) and begins to align teacher practices, in particular the questions they ask to this framework. By beginning to understand the types of teacher questions that support students to interpret and explain their CMT, both the talk used by the teacher and students in mathematics interviews has the potential to provide opportunities in broader classroom discussions (Monteleone, 2021).

Conclusion

The CMTFYS is a platform for teachers to understand how young students can engage with CMT in early years classrooms. The findings of this study suggest there are particular types of questions that can be presented to young students to assist them to interpret and explain their CMT while engaging with mathematical learning experiences. It appeared that teachers could ask probing questions that aligned more closely to support students in *clarifying, noting relationships* and *offering opinions*. This adds to the field by demonstrating that it appears there is a nuanced difference in how probing questions can be asked for young students to elicit their CMT. Thus, the CMTFYS is a new framework that can assist teachers to identify student's critical mathematical thinking and ascertain the types of questions teachers can use to help young students exhibit CMT.

Critical Mathematical Thinking is a new term in mathematics education and an emerging field of research. The findings from this study, informed by the CMTFYS conceptual framework, provide a contribution to the literature in two ways. First, how CMT is conceptualised in early years mathematics and second, the types of teacher questions that can support young learners to interpret and explain their CMT. It is evident that more work is to occur to apply this framework across the school setting (e.g., middle and upper primary classrooms; secondary classrooms) and with larger samples of students. The findings have potential to support teachers to reflect on their own teacher questioning techniques by considering how their questions align with the CMTFYS. In addition to this, teachers can consider how their questioning can shape the conversations with young students to elicit CMT. It is apparent that ongoing support is needed to assist teachers to understand CMT in a domain specific way, which should include the provision of evidence informed teacher practices to support CMT in their classrooms.

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RESEARCH PAPER

The Role of Mathematics Learning in the Interdisciplinary Mathematics and Science (IMS) Project

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While interdisciplinary approaches in the STEM subjects are widely advocated there are concerns that disciplinary learning can be compromised, especially in mathematics. The Interdisciplinary Mathematics and Science (IMS) project is a three- year longitudinal study in four Victorian primary schools that has developed a pedagogical approach to mathematics and science learning where data modelling and representation are common to each. Investigations include astronomy, ecology, chemistry, fast plant growth, force and motion, water use, heat and temperature, body height, light and microorganisms. The paper describes the role of mathematics in the IMS pedagogical model and design of learning sequences.

The promotion of integrated or interdisciplinary approaches to mathematics and science learning has been characterised by enquiry-based processes that reflect the way these disciplines contribute to problem solving in real-world contexts. There is also increasing interest in the potential of interdisciplinary approaches that integrate mathematics learning with learning in other disciplines and in particular in the context of Science, Technology, Engineering and Mathematics (STEM) (Doig et al., 2019; Maass et al., 2019). One of the concerns of such approaches is preserving the integrity of disciplinary knowledge, especially when opportunities for rich mathematics learning are not realised. Nevertheless, Lehrer (2021) argues that such interdisciplinary work a) opens up possibilities of knowledge transfer between disciplines as science and mathematics constructs interact, b) emphasises disciplinary knowledge as relevant to solving important problems, and c) can build the sort of connected and structured knowledge systems that expert STEM practitioners display. By focusing on the development of mathematics and science disciplinary representational practices students come to understand the distinctions between these practices which in turn supports their reasoning and knowledge-building, and develops their representational competence (diSessa, 2004; English, 2012).

Background Literature

Enquiry-based approaches to mathematics learning have been promoted in a range of studies, for example in problem-solving challenges (Sullivan et al., 2016) and in teacher orchestration of student work (Dorier & Maass, 2020; Pinto & Koichu, 2021). Student-led representations have been central to the problem-solving process as a tool for mathematical thinking. A number of recent studies have focused explicitly on young students' meta-representational competence in solving problems in real-life contexts often integrated with scientific concepts and investigations (English, 2012; Makar, 2016). For example, in a study of third graders' predictive reasoning students interpreted the aggregate properties and

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 410–417. Launceston: MERGA.

variability of a "real-world" data set comprising monthly maximum temperatures over time (Oslington et al., 2020). Other studies have described the development of mathematical thinking in constructing and interpreting graphs from data collected in their investigations of ice melting, growth of plants and measures of change in the growth pattern of chickens (Mulligan, 2015). While studies of meta-representational competence often focus on the students' mathematical understanding portrayed through representations, there needs to be complementary studies of how teachers use pedagogical strategies to support students' interpretation of data.

The *Interdisciplinary Mathematics and Science (IMS)* project was conceptualised and designed to explore the principles and possibilities of interdisciplinary alignments in a variety of topics, across a range of grade levels in the primary school (https://imslearning.org/). The principle underpinning the project is that robust learning involves the invention, evaluation, refinement and coordination of representational systems in both science and mathematics, and by focusing on how these science and mathematics systems interrelate, more robust learning of foundational concepts will occur. The key challenge was to generate tasks and learning sequences where science opened up productive possibilities for new mathematics learning, and vice versa. The project operated under several design constraints: a) the challenge of productively aligning scientific and mathematical concepts and practices at the appropriate developmental level, so that they are mutually reinforcing, b) the expectations of teachers regarding appropriate content at that grade level, and c) teachers' disciplinary knowledge and pedagogical capabilities needed to support students. In this paper mathematics learning is described in 12 learning sequences which were implemented through a pedagogical model that promotes the development of mathematical concepts and meta-representational competence.

The IMS Pedagogical Model

The IMS pedagogical model focused on students' investigations across a range of scientific problems with an emphasis on constructing and refining representations (Prain & Tytler, 2021; Tytler et al., 2022). This process enabled students to develop connections between everyday ideas and mathematics and science representational systems. We also drew on the work of Lehrer and Schauble (2020) who describe their approach as establishing the need to create/invent representations, explore what they reveal, make decisions about appropriate representations, and engage with an expanded set of representational tools. In the IMS project the disciplinary focus shifted back and forth between mathematics and science, with each iteration involving new questions and idea refinement (Tytler et al., 2021), and productive knowledge-building in each subject. From these perspectives we developed and refined a pedagogical model which consists of four stages, each with a disciplinary purpose, shown in Figure 1.

Orienting: Teachers pose questions, explore student ideas and guide students to focus their attention on what is worth noticing, asking for predictions, questioning what they have noticed, asking for ideas about what could be measured, and introducing resources for the later stages of the enquiry.

Posing representational challenges: Students are challenged, individually or in groups, to invent/construct representations that reflect a process of claim-making and predictive and causal reasoning or justification. The process involves students in meaningful material exploration, organised by and feeding into the representational practices.

Building consensus: This stage entails teacher guided sharing/display and comparison/evaluation of the comprehensiveness and clarity of the representations. The teacher guides comparative review, feedback, and refinement/revision, drawing strategically on the variation in students' representations to guide an emerging consensus.

Applying and extending conceptual understanding: Students are given new representational challenges to extend their new knowledge and practices in related situations, or further concepts are introduced through representational tasks, to repeat the cycle.
n some sequences an iterative process involved more than one cycle of stages focused on the refinement of the same concept (e.g., motion and force or variability), or developing a sequence of concepts (shadow patterns leading to modelling of earth's rotation).

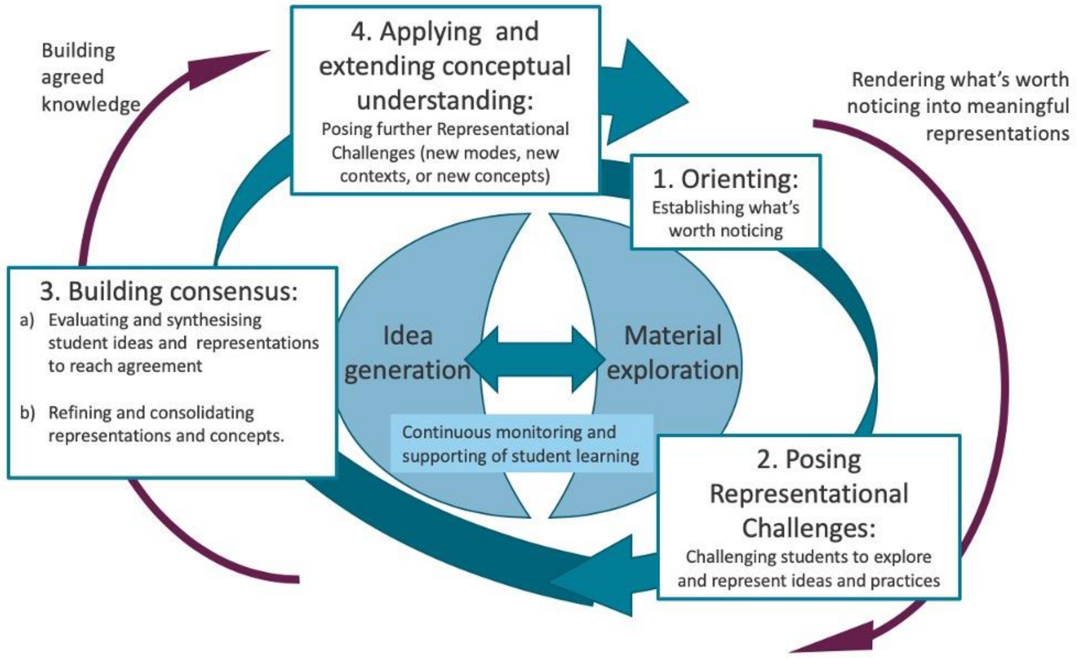

Figure 1: The Interdisciplinary Mathematics and Science (IMS) pedagogy model (from Tytler et al., 2021).

Methodology

The IMS project study was conducted in two metropolitan and two regional primary schools in Victoria, Australia and one regional school in Wisconsin in the US. For each of six grade levels the sample comprised between one and six classes each with 25-30 students, a case-study teacher, and up to 21 case study students representing a range of abilities.

The study adopted a design experiment methodology (Cobb et al., 2003) based on a cycle of planning, trialling, data generation/evaluation, and refinement of 12 learning sequences over a three-year period for Grades 1 through 3 and Grades 4 through 6 (see Tytler et al., 2021). Each cycle involved modifications to teacher support, including changes in framing of the pedagogy and the sequences, workshop design, development of ways of effectively integrating the science and mathematics, and methods of assessing student learning. Professional planning and review meetings were conducted where the learning sequences were refined in consultation with teachers. The workshops afforded opportunities for teachers to engage in the learning experiences and raise questions about the mathematical and scientific content knowledge and statistical ideas inherent in the investigations.

Learning sequences were implemented in each of four school terms for the first two years and one term in the third year, comprising between three and eight weekly lessons of 1-2 hours duration, depending on school timetables and the demands of the topic. At least two members of the research team provided support to the teachers and students during the lessons as well as liaising with the research assistant in collecting data. Data sources comprised records of teacher planning, reflective workshops and review meetings, classroom observations and video capture of lessons conducted by case study teachers, student work samples and class displays, pre- and post-student assessments, and individual and focus group interviews with case study teachers and students.

Analysis of these data focussed on micro-ethnographic discursive analysis of video capture to analyse teacher and student interactions and reasoning during lessons, reflecting the stages of the IMS pedagogical model. The analysis process required independent multiple views of these data from two perspectives: mathematics and science learning respectively, prior to the research team identifying similarities and differences between these perspectives and establishing how data modelling was common to establishing interrelationships.

Development of Learning Sequences

In planning interdisciplinary mathematics and science sequences we focused on concepts that were common to both disciplines and where the mathematics and science contexts productively interacted, and mutually reinforced. Some sequences drew upon the enquiry-based modules of Primary Connections (Australian Academy of Science, 2012). Table 1 provides an overview of the mathematics and science concepts across 12 learning sequences. In all but one, the sequence is identified by the science context, but the mathematics and data modelling is developed strategically and synergistically within the science exploration, and feeds back into representational work.

Table 1:
Interrelationships between science and mathematics concepts in learning sequences (adapted from Tytler et al., 2021).

Grade	Topic	Science	Mathematics
1, 4	Astronomy	Shadows, sun movement, day and night, earth and space perspectives.	Angle as rotation, estimating and measuring, recording and graphing shadow length (formal and informal measures), recording and interpreting time, perspective taking and spatial reasoning.
1, 4	Ecology	Living things, diversity, distribution and adaptive features related to habitat.	Data modelling of living things in sample plots, constructing tables and graphs, variability and sampling, spatial reasoning, mapping, area, coordinates, directionality, constructing and using a scale.
1, 4	Motion	Dynamic concept of motion, Measuring and representing distance/time/speed relations, constant speed, acceleration.	Measuring and representing distance/time/speed relations, using informal and formal units, noticing variation, graphing, and slope of ramp. Using, ordering and recording decimals.
2	Chemistry	Dissolving and mixing, physical change, particle ideas, chemical reactions, change to substances experimental methods.	Measuring and representing time sequences, recording and interpreting timing, measuring substances using formal and informal units, common fractions, and proportional reasoning.

2/4	Fast Plant Growth	Plant growth, structure and function, growth needs and patterns, plant life cycles: germination, flower structures, and fertilisation.	Measures of height, width, leaf and root size, shape and pattern, informal and formal units (cm and mm), time (days, 24-hour time), constructing a scale and graphs.
2	Force and Motion: Helicopters	Flight and air flow, gravity, representing forces, modelling and design, and variable control.	Estimating and describing helicopter parameters, measuring time and timelines, variation and its sources, graphing, and spatial properties.
2	Water Use	Personal use and conservation of water, impact of water use and conservation on the environment.	Estimation and measure of water use, informal and formal measures of volume/capacity, 24-hour time, time line, data collection, organisation and representation.
3	Heat and Temperature	Heat sources and heat flow, temperature measurement, conduction, insulation, and material/design.	Attributes of time and temperature, count and interval, units of time and temperature, relation between informal and formal measures of time and temperature, constructing a scale, and representing data.
4/5/6	Light	Properties of light: vision, reflection, refraction, and image creation.	Angle type and measure, rotation, reflection, symmetry and directionality.
5/6	Measurement: Body Heights	Anatomy, relation between growth and age, estimation and measure of heights, interpreting variability and differences between populations.	Establish need to measure, identifying attributes, comparing, estimating and measuring (informal and formal) ordering, clustering, comparing samples, measures of central tendency and variation, dot plots, timeline, aggregation and predictive reasoning
5/6	Astronomy	Solar system, day and night, planetary features, moon movement and phases.	Ratio of planetary size and distance, angle, compass points, tracking position over time, perspective taking, cosmological distances.
6	Magic Microorganisms	Structure and function of microorganisms, magnification, and growth patterns	Spatial patterns, multiplication, ratio, and proportion, measuring area, sampling and distribution and measurement tools and units.

In each of the learning sequences (except Body Heights), the science context contextualised the mathematical enquiry by creating a need to explore and represent underlying patterns (spatial, numerical) in ways that fed back into questions that directed data representation.

Illustrations of Learning Sequences

Each of the 12 learning sequences were implemented with particular grade levels across a number of cohorts and refined through multiple iterations. Data analyses from each learning sequence enabled different insights into the mathematics–science interrelationships. Three examples described below provide pertinent illustrations of the role of mathematics in the learning sequences: Ecology, Body Heights, and Force/Motion.

At a fundamental level, the Ecology sequence engaged Grade 1 and 2 students in developing effective counting and tallying skills as well as representing data as a simple table, picture or bar graph. Students, in groups, investigated and individually represented the number and location of different living things in different habitats represented by sample plots. But the exploration extended to a wider range and depth of mathematical ideas. Students then evaluated and refined their data representations, tabulated class data, represented the distribution of particular animals across the different plots, and proposed reasons for this. Spatial skills were also involved by visualising and drawing a map of the area and including a key and invented icons (or simple coordinates). Following the groups' refinement of representational work to produce documented counts of living things in the different plots, teachers raised questions about variation in populations across the different sites: Where are particular living things found? Why are the numbers of particular animals different across the plots? Moreover, the mathematical representations and data interpretation supported the students' reasoning about the distribution of living things in various locations leading to the scientific concepts of diversity, habitat and adaptation.

The Body Heights sequence involved a series of investigations conducted in Grades 5 and 6 across six classes in one school (Mulligan et al., 2022). Students were initially challenged to consider whether students in their Grade 5/6 class would meet the 1.4m height requirement of a theme park ride. Students estimated their own height, compared their height to their peers in an iterative process in which they clustered, displayed, graphed and interpreted class data. Comparison of estimates with actual measures then initiated ideas about measures of central tendency. They compared height data from Prep/Grade 1 classes with their own to make inferences about the sampling and to draw conclusions about student growth patterns over six years. These findings indicated that students were more than capable of measuring height—they were able to organise and interpret measures to support the data-modelling process and development of their statistical reasoning.

The Force and Motion sequence provided a coherent example of the interrelationships between the mathematics and science concepts and processes. The investigation involved concepts of forces due to air flow, uplift and gravity in the design of helicopters including variations in wing length, shape and weight, controlled by the number and size of paper clips attached to the body or the wings. The students utilised their measurement skills in making estimates of height of the drop, understood the need for a fine-grained measure of time in seconds and parts of seconds (one or two-place decimals), ordered and interpreted times, related time to the speed of the helicopter, and recorded, organised and made inferences from the data.

The science and mathematics, while they had commonalities, were somewhat distinct, but mutually reinforcing. Students were able to explain the relative affordances of different ways of representing their data sets and recognised and responded to the influence of variability. They discussed different ways of constructing data tables to display multiple trials with different conditions, and displayed these on a timeline with an interval scale. Students were able to recognise and use the median value of a set of numbers as being a fair representation of the data set, and to articulate the inevitability of variation on repeat measures, and suggest sensible reasons for this. That they could do this, through gentle guidance, was surprising to the teachers. The science provided the setting that drove authentic mathematical problem solving and meta-representational competence, and the mathematics in turn contributed to the science in raising questions about, for instance, the need to control for variation, and the effect of weight or wing design on the helicopter flight.

Implications and Conclusions

Further research, that evaluates the efficacy and impact of the IMS approach on teaching and learning is necessary to validate and upscale the approach in a variety of contexts and with a more diverse populations. Attention to professional learning and support might be prioritised: enabling sustained research with teams of mathematics and science education researchers in collaboration with teachers and school systems; and the development and implementation of programs to support professional learning about the synergistic nature of mathematics and science learning.

The generative nature of the mathematics and science interdisciplinary model has significant implications for curriculum review and practice. The IMS learning sequences demonstrated a range of possibilities for mathematics learning supporting the Australian Curriculum–Mathematics, particularly the reasoning and problem-solving aspects of the Proficiencies (Australian Curriculum, Assessment & Reporting Authority [ACARA], 2018). Student learning through purposeful invention, comparison and refinement processes was evidenced by students' explanations and representations of data supported by explicit teacher scaffolding and the building-consensus process. Through situating mathematics in meaningful investigations, the purpose of measurement and data-modelling processes were realised.

The IMS project provided multiple opportunities to develop students' conceptual knowledge: number and pattern, spatial reasoning, measurement, and data-modelling simultaneously—as well as developing statistical concepts and representational processes that are central to mathematics and science investigations. These outcomes were achieved at varying levels of depth and competence for all students, often well beyond curriculum expectations, when appropriate problem contexts were explored systematically over a series of iterations.

Moving forward, one way of prioritising mathematics could be to integrate scientific problems during the mathematics learning space. The science curriculum provides this flexibility in ways that the tightly prescribed practices in mathematics content strands do not often allow. Blending these disciplines in the mathematics learning space might enable teachers to focus more on the mathematical proficiencies of problem solving and reasoning (ACARA, 2018). More flexible learning structures and more time within school curricula would support such as approach as was necessary in the IMS project. The depth of practice achieved by this interdisciplinary approach would compensate for the time invested. A re-conceptualisation of the role of data modelling in mathematics curricula would necessitate a shift in emphasis from a traditional "siloed" approach to one that is more flexible and interrelated. This approach would encompass forms of interdisciplinarity that honour the epistemic processes of both subjects in ways that lead to rich learning in each.

Acknowledgments. Research Team: Russell Tytler (Deakin University), Joanne Mulligan (Macquarie University), Peta White, Lihua Xu, Vaughan Prain, Chris Nielsen, Melinda Kirk, Chris Speldewinde (Deakin University), Richard Lehrer, Leona Schauble (Vanderbilt University). Our thanks extend to the Victorian Department of Education and Training, schools, teachers and students.

The research reported in this paper was funded by the Australian Research Council Discovery Grant No. DP180102333 *Enriching Mathematics and Science Learning: An Interdisciplinary Approach.*

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RESEARCH PAPER

Exploring the Alignment Between Pre-service Mathematics Teachers' Beliefs and Espoused Practice

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Researchers in mathematics education have focused on teacher beliefs as an important area of study because of their influence on teaching practices. In this study, we focused on two aspects of beliefs, the nature of mathematics and the teaching and learning of mathematics, with eight pre-service mathematics teachers (PMTs) to explore alignment between these beliefs and their espoused teaching practice. Data were collected through questionnaire prompts and semi-structured interviews. Analysis revealed that the PMTs generally held mixed beliefs about both the nature of mathematics and about its teaching and learning, indicating little alignment within and between the beliefs expressed and their espoused practice.

In trying to understand what contributes to the quality of mathematics teaching, some researchers have attributed the predominant influence of teacher beliefs. For example, studies have shown the influence of teacher beliefs for decision-making and in underpinning teacher actions in mathematics classes (Beswick, 2006). As such, beliefs are increasingly seen as a key component of pedagogical content knowledge (PCK) (e.g., Hashweh, 2005), and even Shulman, the original conceiver of PCK, later recognised the lack of consideration of non-cognitive attributes (including beliefs) in his initial theorisation (Shulman, 2015).

> The affective aspects of teacher understanding are important both because a lot of what teachers 'know and do' is connected to their own affective and motivation states, as well as the ability to influence the feelings, motives, persistence, and identity formation of their students (p. 9).

Whilst some researchers have explored teacher beliefs and teaching practices together (e.g., Beswick, 2012), and others have focused on the consistencies and inconsistencies of teacher beliefs to teaching practices (Roehrig et al., 2009), little is known about the alignment for pre-service mathematics teachers (PMTs). When PMTs enrol in mathematics education courses, they have already developed beliefs about the nature of mathematics, and its teaching and learning from their own experiences of learning mathematics (Beswick, 2019). In this article we aim to better understand how beliefs might account for teaching practices of pre-service mathematics teachers (PMTs) in the early stages of becoming mathematics teachers. We propose that studying the alignment between PMTs' beliefs and their espoused practice can help to understand why they perceive teaching of mathematics in the way they do. Furthermore, the study illuminates the need for teacher educators to be aware of PMTs' beliefs at the beginning of their courses so that they can account for and help to strengthen or change those beliefs and inform the design of teacher education programmes. Our main research question is thus: How do PMTs' beliefs about the nature of mathematics align with their beliefs about the teaching and learning of mathematics?

Literature Review

Several beliefs relating to mathematics, have been identified by researchers, including beliefs about the nature of mathematics, and beliefs about the teaching and learning of mathematics (e.g., Beswick, 2007). These two aspects of beliefs unveil mathematics teachers' views regarding the role of teachers and teaching, and the nature of mathematics activities (Weldeana & Abraham, 2014). For instance, two teachers holding the same knowledge about

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teaching mathematics, may teach differently due to the differences in beliefs they hold, because they use their existing beliefs to interpret whatever comes into their mind (Stipek et al., 2001).

Beliefs about the Teaching and Learning of Mathematics

Beliefs about teaching and learning of mathematics are related to the choices teachers make with respect to their teaching roles, the nature of instruction, and the activities and resources used in their mathematics classroom teaching practice (Ernest, 1989). Yang et al. (2020) related these beliefs to teachers' mathematical understanding and their preferred mathematical activities, teaching approaches, and their conception of how mathematics is learned. The most frequently considered beliefs about the teaching and learning of mathematics are transmissive and social constructivist (Meschede et al., 2017). In the transmissive view, mathematics teachers see effective teaching and learning of mathematics as teacher-centred, where the students' role is to follow their teacher's instructions (Meschede et al., 2017). The emphasis is on memorisation of rules, procedures, and facts.

From a social constructivist view, mathematics teaching challenges students' thoughts and guides them towards a complete understanding of mathematical concepts (Weldeana & Abraham, 2014). Students are involved in doing mathematics and developing different ways to solve mathematical problems or tasks, as opposed to being passive recipients of knowledge (Ernest, 1989). Students take an active role by individually processing and constructing knowledge (Meschede et al., 2017) as opposed to merely following procedures. This view is evidenced in the way a mathematics teacher uses teaching and learning resources, provides autonomy to students, and considers varied ways to arrive at the correct answer. In this article we consider social constructivist teaching as an effective teaching practice for mathematics.

Beliefs about the Nature of Mathematics

Beliefs about the nature of mathematics are closely related to the question, "What is mathematics?' and these beliefs are considered to have more impact on mathematics teachers' practices than do beliefs about the teaching and learning of mathematics (Beswick, 2012). Beliefs about the nature of mathematics are generally conceptualised as either static or dynamic (Weldeana & Abraham, 2014). The static position is that mathematics is a body of formulas and mathematical facts that are procedure-driven (Yang et al., 2020), and suggests learning mathematics means an accumulation of facts, rules, procedures, and skills for the fulfilment of some external end, or producing one correct answer (Ernest, 1989). Such beliefs might be seen to produce teachers who teach in a traditional teacher-centred way (Stipek et al., 2001). In contrast, those holding a dynamic view of nature of mathematics understand the nature of mathematics as a process of inquiry (Yang et al., 2020), similar to a problem-solving view (Ernest, 1989) in which mathematics is seen as an active sphere of human invention and creation that is always growing. Likewise, mathematics is regarded as a tool for thought (Stipek et al., 2001). Mathematics teachers who hold this view are likely to employ student-centred teaching often associated with improved students' learning (Baeten et al., 2016).

PMTs' Beliefs and the Issue of Alignment

If we want to help PMTs develop effective teaching practices, then we need to address their beliefs (Beswick, 2006). Research has focused on how PMTs' beliefs about the nature of mathematics and its teaching and learning relate to their teaching practice (e.g., Yang et al., 2020), but little is known about the alignment between these beliefs (Penn, 2012) and how this alignment or misalignment influences their practice. We propose that it is important to investigate these alignments at this early stage of the PMTs' careers to better understand how their beliefs inform their decisions in planning to teach. The expectation is that when a PMT holds static beliefs about nature of mathematics and transmissive beliefs about its teaching, then their beliefs are aligned suggesting a teacher-centred approach. Whereas a PMT holding dynamic and social constructivist beliefs would be expected to align with a learner-centred approach (Baeten et al., 2016). PMTs holding static and social constructivist beliefs, and vice versa, are seen to have misaligned beliefs and we have no expectation about their teaching.

Methodology

A qualitative research method was employed to explore the alignment of eight PMTs' beliefs about the nature of mathematics, its teaching and learning, and their espoused practice. Qualitative research enables analysis of the how and why (Yin, 2009), and here, the how and why of alignment of each of the PMTs' beliefs. In this study, the PMTs were in a teacher education college setting. Detailed information was collected using a variety of qualitative data collection instruments such as questionnaire prompts, and semi-structured interviews.

Participants

Eight PMTs in their second year, second semester (their last year) were purposively selected from one teacher education college in Tanzania. Second year PMTs have participated in their first block teaching practice (BTP) and therefore have some experience of teaching mathematics in classrooms. Twenty PMTs were invited and eight consented to participate. Ethical approval was granted under the guidelines of the Social Sciences Human Research Ethics Committee (SSHEC) in Australia (project number 22979).

Data Collection

Data were collected through questionnaire prompts and semi-structured interviews. Two questionnaire prompts were provided to each PMT, one on beliefs about the nature of mathematics and the other on the beliefs about the teaching and learning of mathematics. PMTs were to agree with the prompts they thought were correct. Semi-structured interviews were conducted with each PMT to gain more information on their beliefs and their perceived teaching of mathematics. The duration of the interviews ranged from 30 to 45 minutes. Questions were based on the beliefs identified and some guiding questions such as:

> In your opinion, what is mathematics? How would you teach your students operations on fractions? (Choose one operation and describe how you would teach it).

Analysis

The prompts endorsed by the PMTs in the questionnaire reflected two beliefs about the nature of mathematics, and the teaching and learning of mathematics respectively. We employed a deductive thematic analysis process for the data from the semi-structured interviews (Braun & Clarke, 2006), which was managed using NVivo software. The data were coded deductively using elements drawn from the literature as static and dynamic (beliefs about the nature of mathematics, e.g., Ernest, 1989; Stipek et al., 2001), and as transmissive and social constructivist (beliefs about teaching and learning mathematics, e.g., Tatto et. al., 2008). The first author carried out the initial coding, and the other two research team members reviewed sample coding from each participant. There was no disagreement between authors regarding the codes, but there was a considerable discussion concerning the representations and explanations of the examples for teaching. For example, for Francis we discussed the seeming contradiction between a constructivist position of his fraction images and the transmissive position of his interview statements.

Findings: Questionnaires

In this paper, we present data from three PMTs, Francis, Liam, and Shaibu. These three were chosen because they represented a range of different beliefs. The belief prompts that were endorsed by each of the three PMTs regarding the teaching and learning of mathematics are shown in Table 1, and about the nature of mathematics in Table 2.

Table 1
Prompts on Teaching and Learning of Mathematics that were Agreed by Each PMT

Transmissive Prompts	Names*			Social Constructivist Prompts	Names*		
	FR	LI	SH		FR	LI	SH
The best way to do well in mathematics is to memorise all formulas	A**		A	In addition to getting right answer in mathematics, it is important to understand why the answer is correct	A		
Students need to be taught the exact procedures for solving mathematical problems	A		A	Teachers should allow students to figure out their own ways to solve mathematical problems	A	A	
It doesn't really matter if you understand a mathematical problem, if you can get the right answer	A			Time used to investigate why a solution to mathematical problem works is time well spent	A		
Students learn mathematics best by attending to teachers' explanations	A		A	Students can figure out a way to solve mathematical problem without teacher's help			
When students are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed	A			Teachers should encourage students to find their own solutions to mathematical problems even if they are inefficient	A	A	
Hands-on mathematics experience is not worth the time and expense	A			It is helpful for students to discuss different ways to solve problems	A		

*FR: Francis, LI: Liam, SH: Shaibu (note same in Table 2)
** means the participant agreed with the statement (note same in Table 2)
A = Agree

Summarising from Table 1 and Table 2, we see Francis agreeing to all prompts in both tables except for the prompt stating, "Students figure out ways to solve mathematical problems without teachers' help" (Table 1), which suggests he holds mixed beliefs. Liam agreed to only dynamic prompts in Table 2, and only social constructivist prompts in Table 1 (though he chose few prompts in each case). Whereas it was difficult to tell which side Shaibu was based on with respect to the nature of mathematics, as he agreed to two prompts in the static scale and only one in the dynamic scale (Table 2). However, as shown in Table 1, he agreed to only transmissive prompts, which suggests he leant towards this view of teaching mathematics.

Table 2
Prompts on Nature of Mathematics Agreed by Each PMT

Static Prompts	Names*			Dynamic Prompts	Names*		
	FR	LI	SH		FR	LI	SH
To do mathematics requires much and correct application of routines	A		A	Many aspects of mathematics have practical relevance	A		
Fundamental to mathematics is its logical rigor and precision	A			Mathematical problems can be solved correctly in many ways	A		A
Mathematics involves the remembering and applying definitions, formulas, mathematical facts, and procedures	A		A	In mathematics many things can be discovered and tried out by oneself	A		
Mathematics is the collection of rules and procedures that prescribe how to solve mathematical problems	A			Mathematics involves creativity and new ideas	A	A	
Mathematics means learning, remembering, and applying	A			Mathematics helps to solve everyday problems and tasks	A		
When solving mathematical tasks, you need to know the correct procedure, else you would be lost	A			If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts)	A	A	

A = Agree

Findings: Semi Structured Interviews

The beliefs expressed in the questionnaire prompts about teaching and learning mathematics were reflected in the PMTs' interview accounts of how they would teach their students operations on fractions. The three examples are set out below.

Liam started by emphasising the procedures to be followed when dividing fractions. When asked if there is any other way or representation, he still emphasised the procedures:

> Students will have to follow the division procedures as they are in the textbook such as for $\frac{a}{b} \div \frac{c}{d}$, the first procedure is to invert the $\frac{c}{d}$ to be $\frac{d}{c}$ and multiply it by $\frac{a}{b}$. Thus, $\frac{a}{b} \times \frac{d}{c}$.
>
> For example, $\frac{1}{3} \div \frac{2}{6} = \frac{1}{3} \times \frac{6}{2} = \frac{6}{6} = 1$

When asked if there is any other representation for this, his answer was:

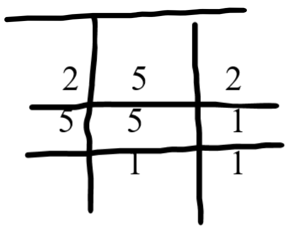
> No, there is no other way, they are supposed to just follow the procedures I explain.

Further, when asked why we invert the second part of fraction when doing division, his response was:

> We invert to change the division sign into multiplication sign, it is the procedure.
>
> The first procedure here is to change those mixed numbers to improper fractions, this will be;
>
> $(2 \times 3) + \frac{1}{2} \div (5 \times 6) + \frac{2}{5} = \frac{7}{2} \div \frac{32}{5} =$, next step is to invert the right-hand side and then multiply by the left-hand side. Thus, $\frac{7}{2} \times \frac{5}{32} = \frac{35}{64}$. The important thing for students is to make sure they follow the procedures as they are.

In his interview, Shaibu started by telling how he would guide his students in learning how to add fractions with the same denominators and those with different denominators, explaining it as follows

> I would like to teach using examples. For example, evaluate the following fractions
>
> $1.\frac{3}{5}+\frac{1}{2}$ and $2.\frac{2}{8}+\frac{1}{8}$. …… Here the first thing I will teach my students is to find LCM (lowest common multiples) for denominators 5 and 2 by using prime factorisation procedure because denominators are different. But if the denominators were the same no need for LCM.



$LCM = 2 \times 5 = 10$

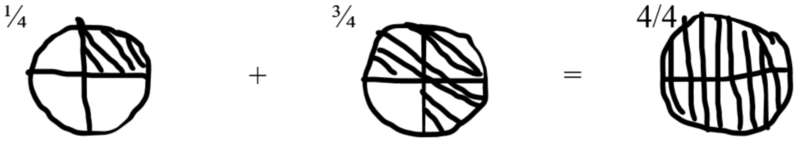
> Therefore, $\frac{3}{5}+\frac{1}{2} = \frac{((10 \div 5)\times 3)+((10 \div 2)+1)}{10} = \frac{(2 \times 3)+(5 \times 1)}{10} = \frac{6+5}{10} = \frac{11}{10} = 1\frac{1}{10}$, since the numerator is greater that denominator, I will tell them to change it to mixed number.

When we asked Shaibu about why he was controlling everything in the class, his response was:

> Because some students are slow learners, so as a teacher you need to make sure you have covered all that is required.

Francis, however, proceeded as follows:

> I will go with 2 oranges and cut them into four pieces, to teach that $\frac{1}{4}+\frac{3}{4} = \frac{4}{4} = 1$, I will take one piece from the first orange and let them know that's a ¼ and then take 3 pieces from the other and that's ¾, I will then put those pieces together and they will see that it goes back to 1 orange. (Francis).



When asked if there is any other way to arrive at the solution or teach the concept, Francis acknowledged varied representations for arriving at the correct answer. For example, he said:

> … because I will be in the class teaching, I may use students as my real objects let say the class has 60 students where 20 are girls and 40 are boys. Then I may tell them that they are a fraction of:
>
> $\frac{20}{60}+\frac{40}{60}=\frac{60}{60}=1.$

In general, Francis held misaligned beliefs between the questionnaire prompts, when compared to his espoused practice. Liam held a contradicting belief between those identified in the questionnaire prompts, and the semi structured interviews. While this confirmed his leaning towards a dynamic view of mathematics, he shifted from a social constructivist to a transmissive view of the teaching of mathematics. Moreover, looking closely at the example two which he provided, we noticed some mathematical error or procedural/conceptual mix-up. Instead of,

$$\frac{(2 \times 3)+1}{2} \div \frac{(5 \times 6)+2}{5}, \text{ he wrote } (2 \times 3)+\frac{1}{2} \div (5 \times 6)+\frac{2}{5}$$

These two representations have two different answers; however, we are not sure if this was just a typo error. Shaibu's interviews confirmed he holds a largely static view of mathematics, and a transmissive view of teaching, as shown in his teacher-centred approach in his espoused teaching.

In summary, the espoused teaching practices of the three PMTs were largely procedural and transmissive in nature, even when there were teaching and learning resources for students to manipulate (e.g., Francis). They acted as a knowledge giver, viewing the students as recipients of knowledge.

Discussion

The purpose of this study was to explore the alignment of PMTs' beliefs about the nature of mathematics and the teaching and learning of mathematics. Through qualitative research, the PMTs were asked to complete open-ended questionnaires and participate in semi-structured interviews. Whilst there is evidence that mathematics teachers' beliefs about the nature of mathematics do align with their espoused teaching and learning beliefs (Ernest, 1993), the findings from the three PMTs presented in this report suggested both alignment and misalignment between the nature and teaching of mathematics. For instance, Francis showed misalignment in both beliefs. His beliefs about nature of mathematics and its teaching and learning through prompts were mixed within and between them, which makes him misaligned. Shaibu, who leaned towards the nature of mathematics as static and the teaching and learning of mathematics as transmissive in approach, suggested alignment in his beliefs. Liam suggested alignment in the beliefs identified via prompts as dynamic and social constructivist, however strongly learned towards transmissive approach in interview. The potential explanation for the misalignment of beliefs may be attributed to misunderstanding of the meaning brought by the prompts, or that the perspective that there are only two (incompatible) ways of understanding mathematics is incorrect. Further, the misalignment might imply that the PMTs were guided by both views in their teaching practices. The other reason might be attributed to the fact that, these PMTs were beginning teachers and therefore had shifting beliefs based on their previous learning experiences as well as still developing pedagogical practices.

The misalignment for Francis and Liam was also evident in their espoused teaching practices where they demonstrated a procedural way of teaching. Francis showed some elements of conceptually teaching or representations. He tried to teach using teaching and learning resources to help students understand the conceptual part of fractions. The teaching and learning materials he used during his espoused practice were figures like circles (mentioned about using oranges). The possible explanations for misalignment, might be they were trying to bring onboard what their tutors taught them (the experiences they went through or saw), but for some reasons they could not apply (if the experiences were student-centred). Maybe it was because they did not understand what they were taught or there was minimal practice on their BTPs. These findings confirm the findings by Penn (2012), who found that the majority of PMTs' beliefs about the nature of mathematics and the teaching and learning of mathematics did not align. Hence, Liam's contradictions in the prompts and Francis's shift from conceptual to procedural in espoused teaching might have resulted from the course expectation to teach in a constructivist way, even though the PMT held a transmissive approach.

These findings collectively suggest that it is possible for a PMT to hold beliefs that are contrary to the expectations we might have for their teaching practice provided by their beliefs (e.g., Liam and Francis). A further question to ask is whether these PMTs were aware of the contradictions in the beliefs that they held, and this remains as a suggestion for further studies.

Conclusion

Beswick (2006) suggested if we want to see changes in teacher' practice, we first need to study their beliefs and help them actualise their fullest potentials. In this regard, this study has endeavoured to study PMTs' beliefs and our findings suggested that PMT beliefs are often misaligned. Ernest (1989) suggested two key reasons that can cause misalignment between beliefs and practice as: 1) powerful influence of the social context, and 2) level of consciousness of beliefs and the extent to which the teacher reflects on their mathematics teaching practice. We therefore argue that these reasons may have caused misalignment between and within PMTs beliefs and these misalignments suggest a more nuanced understanding of PMT beliefs and how they relate to nascent views of effective practice.

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A Snapshot of Gender and Mathematics Anxiety in Years 5 to 8

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Studies such as the 2012 Programme for International Student Assessment (PISA) indicate that there are gender differences among students in measures of mathematics anxiety. In this paper we explore students' mathematics anxiety levels and intentions to choose mathematics in Year 11 and to choose a career that includes an emphasis on mathematics. The data are examined to identify any differences across a sample of students in Years 5 to 8, with a focus on gender and year level.

In 2009, the G20 leaders acknowledged one of the key priorities for global recovery was improved standards in mathematics and numeracy (International Labour Organization [ILO], 2010). Failure to improve mathematics and numeracy standards was identified as a threat to the strength of economies (Organisation for Economic Cooperation and Development (OECD), 2012). Low achievement in mathematics has been linked to lower school completion rates, unemployment, poor financial decisions, and poor health (English & Gainsburg, 2016). However, studies such as Mack and Wilson (2015) continue to highlight how the numbers of students opting for STEM subjects continues to decline. In particular, they identified that the number of students opting for mathematics continues to decline and also that, of those who do opt for a mathematics subject in Year 11 or 12, the preference is for elementary mathematics rather than intermediate or advanced mathematics. Jaremus et al. (2019) confirmed that female representation and participation in Year 12 STEM subjects was declining and that female enrolments in mathematics, as well as in digital technologies, was of particular concern. There are many reasons cited for such changes as discussed by O'Keeffe et al. (2018). Of concern and of relevance to this paper is the persistence of issues around mathematics anxiety and gender (Sax et al., 2015).

Catholic Education South Australia (CESA) continues to work with academic partners to be informed through data to find ways to counter these issues. In this paper we discuss mathematics anxiety, with a lens on gender and year level. This stems from a larger project that was implemented over three years (2018 to 2020) as part of CESA's STEM Learning Initiative (O'Keeffe et al., 2021). The first two years of the project involved working with teachers and students in schools to develop and implement an inquiry approach to integrated mathematics, science, and technology teaching and learning while also collecting data from students, teachers, and school leaders to gauge the impact of the project. The third year involved data collection only so as to evaluate the sustainability of the project. In this paper, we focus on one aspect of the student survey data collected in both pre- and post-survey modes in 2019. Of particular interest is students' mathematics anxiety levels and the ways in which these were influenced by their engagement in integrated/inquiry approaches to mathematics.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 426–433. Launceston: MERGA.

Background

Mathematics anxiety is defined by Ashcraft (2002, p. 181) as a "feeling of tension, apprehension, or fear that interferes with math performance." Uusimaki and Nason (2004) discuss how mathematics anxiety manifests itself as intense frustration or powerlessness about one's capacity to do mathematics, and how it can be depicted as a learned emotional response. Elevated mathematics anxiety levels are most prevalent in situations where a person needs to communicate their mathematical knowledge, such as in a test situation or working through mathematical problems. For example, Tooke and Lindstrom (1998) suggested that mathematics anxiety surfaces most dramatically when the person is perceived to be under evaluation. The outcome of this is that low achievement is being reported for students when in many cases their low achievement levels may have a greater correlation to their mathematics anxiety rather than to their knowledge and understanding of the mathematical content. This aligns with a reciprocal theory perspective that mathematics anxiety contributes to poor performance which in turn contributes to higher mathematics anxiety (Gabriel, 2022).

It is well documented that the transition between school sectors is a time of upheaval and distress for many students (Hanewald, 2013) and, as a result, is a common time for negative perceptions of mathematics and anxiety to develop or deepen (Attard, 2012; Hanewald, 2013). The 2012 PISA survey looked specifically at mathematics self-efficacy and mathematics anxiety (along with mathematics self-concept among students and student engagement). Some of the key findings of relevance to this paper are that "almost 30% of students reported that they feel helpless when doing mathematics problems" (OECD, 2013, p.80). Of this 30% it was clear that girls and socio-economically disadvantaged students were more likely to have lower self-efficacy levels. The 2012 PISA study also highlighted that 43% of students believed they were not good at mathematics, despite 59% reporting that they get good grades.

In relation to gender, the 2012 PISA data indicated that more boys believe they are better at mathematics than girls, and girls recorded higher levels of mathematics anxiety than their male counterparts in 56 of the 65 OECD countries. Mathematics anxiety also increased, with students in 2012 more likely to be anxious about mathematics than those in the 2003 survey. Thirteen countries, including Australia, showed a statistically significant increase in the mathematics anxiety recorded by their students.

Looking to the 2018 PISA data, girls outperformed boys in reading but remained behind their male counterparts in mathematics. The 2018 data also indicated that girls, across the majority of OECD countries, are more likely to express fear of failure than boys and this gender gap is even more pronounced among the top performing female and male students. Amongst the students who were doing well in mathematics, one in three boys considered working as an engineer or science professional, in comparison to one in five of the higher performing girls (OECD, 2019). Holmes et al. (2018) also pointed to this gender imbalance around career expectation beginning in the middle years of schooling, and suggested that lack of female role models, self-beliefs and dispositions all play key roles. The criticality of the middle years is further echoed by Steinke (2017) who found that, at around age 12, many girls who were considered highly confident and capable female students tended to lose interest in STEM subjects such as science and mathematics.

In Australia, despite considerable contribution in the form of policy activity and programs female participation in STEM careers, uptake of STEM subjects has not altered substantially over two decades (Marginson et al., 2013, Jaremus et al., 2019). Explanations for this gender disparity have changed over time and across disciplines. Previously this gender disparity was attributed to girls/females having less aptitude and interest for STEM careers and subjects and a lower mathematics ability (Panizzon et al., 2018). However, as discussed by Panizzon et al. (2018), substantial research has found little empirical support for these claims. Bøe et al. (2011)

and Archer et al. (2012) have posed that socio-cultural factors and constraints, rather than student ability, have constituted the most powerful explanatory factor behind gender disparity in STEM. Ganley and Lubienski (2016) noted that the gender disparity, though small, was persistent and warrants further exploration as the gaps cannot be explained and increase over time (whereas literacy gender gaps narrow over time). Hence, the initial small gaps lead to *stark disparities* in mathematics-related career pathways, adding to the issue of the gender pay gap.

Research Design

CESA's key aim in supporting teachers and schools to engage in this project was to increase student engagement across science, technology, and mathematics. The intention was to create opportunities for schools to build leadership and teacher capability to transform STEM learning in a manner that privileged and integrated intentional curriculum and capabilities aligned to CESA's learning framework. The project team, in collaboration with CESA's Learning and Technologies Consultants, supported schools to develop a school-specific approach to integrated mathematics, science and/or technology, with a focus on driving pedagogical change through inquiry-based projects. The move towards inquiry-based projects involved professional learning to build teacher confidence and expertise in facilitating, assessing, and teaching discipline knowledge through inquiry. A total of 29 primary and junior secondary schools participated in the project with at least three teachers involved at each site.

As part of the wider CESA project, data were collected from principals, participating teachers, and their students. Principals and teachers were invited to "opt-in" at various stages of the data collection process, including interviews, focus groups and surveys. There was some turnover in staff throughout the project, but each year of data collection is stand-alone and was not contingent on those teachers being involved in previous years. Students whose teachers had opted to participate in the project were invited to "opt-in" to digital pre- and post-surveys and in-person focus groups. This paper reports on one aspect of the student survey data.

Student surveys were carried out at the beginning and end of each school year and had a particular focus on eliciting students' understandings of and dispositions towards mathematics and science. The survey sought to identify students' levels of mathematics anxiety as well as their intentions to choose mathematics in Year 11 and to choose a career that includes an emphasis on mathematics. To establish students' levels of mathematics anxiety, we used the following questions from the PISA 2012 study (OECD, 2012):

Q1. I often worry that it will be difficult for me in mathematics classes
Q2. I get very tense when I have to do mathematics homework
Q3. I get very nervous doing mathematics problems
Q4. I feel helpless when doing a mathematics problem
Q5. I worry that I will get poor grades in mathematics.

A five-point Likert scale was used with response categories: strongly agree (5), agree (4), neither agree nor disagree (3), disagree (2), and strongly disagree (1). A higher score for each question corresponds to a higher level of mathematics anxiety. The five responses were summed to give an overall indication of mathematics anxiety. The minimum score that a student could obtain for mathematics anxiety was 5 (no anxiety) and the maximum score was 25 (high anxiety). All surveys were coded to enable data matching while maintaining student and school confidentiality.

A total of 644 students in Years 5 to 10 consented to take part in the various elements of data collection in 2019 and completed the pre-survey; 455 students completed the post-survey. Of the post-survey data, only 179 students (in Years 5 to 8) had completed both the pre- and post-survey and could be matched for comparison. The sample of paired data included 49 students in Year 5, 38 in Year 6, 64 in Year 7, and 28 in Year 8. Note that in 2019, Year 7 was moved to secondary for most CESA schools. All project schools were part of this transition.

Findings

This section presents the student data for levels of mathematics anxiety before and after a two- or three-term focus on integrated science, mathematics, and technology, as well as the ways anxiety correlates with students' intended subject and career choices. Any variances in sample size in the presented tables is because of incomplete surveys, for example, a student who gave year level data but not gender was included in year-level data and not in gender data.

Mathematics Anxiety

Table 1 presents summary data for male and female student mean anxiety levels aggregated across Years 5 to 8, for both the pre-survey and the post-survey. The pre-survey data, collected at the beginning of 2019, indicated statistically significant differences (determined by independent t-tests) between male and female anxiety scores (without separating by Year level). Examining the pre-survey data more closely by year level (not shown in the table) shows that the Year 5 male students were more likely to be mathematically anxious than the Year 5 female students (not statistically significant), but for all other year levels (Year 6 through to Year 8) the data indicates that female students are more likely to have higher mathematics anxiety than male students.

Table 1
Student Mean Mathematics Anxiety Scores by Gender (min score is 5; max score is 25)

	Gender	*n*	mean	standard deviation
Pre-survey	Male	168	12.61	5.70
(p <0.001)	Female	266	15.23	5.46
Post-survey	Male	118	13.47	5.88
(p = 0.014)	Female	186	14.73	5.69

When grouped by school sector only, the mean mathematics anxiety score for primary students in the pre-data was 13.73 (n = 250) in comparison to 14.72 (n = 240) for secondary students. The gap between the cohorts increased at the end of the year; the primary students' mathematics anxiety decreased (mean score 12.70, n = 165 in post-data) while the secondary mathematics anxiety scores increased (mean score 15.71, n = 172). This reflects a statistically significant difference ($p < 0.05$, using independent t-tests) between the mean mathematics anxiety scores for primary and secondary students. Table 2 shows the data by year level and highlights the increased anxiety across each year level at the end of the year with larger increases evident in Years 7 and 8.

Table 2
Student Mean Mathematics Anxiety Scores by Year Level (min score is 5; max score is 25)

	Year 5	Year 6	Year 7	Year 8
Pre-survey	13.49	13.86	13.96	16.54
	(n = 100)	*(n = 150)*	*(n = 173)*	*(n = 67)*
Post-survey	12.23	13.08	15.00	17.04
	(n = 68)	*(n = 97)*	*(n = 110)*	*(n = 62)*

Table 3 presents the data summarised by school sector and by gender. Primary students, both male and female, recorded a decrease in the mean mathematics anxiety over the year while secondary students' mean mathematics anxiety recorded an increase.

Table 3
Student Mean Mathematics Anxiety Scores by School Sector (Primary or Secondary) and Gender (min score is 5; max score is 25)

	Primary		Secondary	
	Male	Female	Male	Female
Pre-survey	12.74 *(n=114)*	14.52 *(n=136)*	12.41 *(n=81)*	15.81 *(n=159)*
Post-survey	12.63 *(n=86)*	12.78 *(n=79)*	14.88 *(n=48)*	16.01 *(n=124)*

There were 179 students who completed all relevant pre- and post- data questions enabling paired t-tests to compare the means. It is evident from grouping by gender and year level (Table 4) that the Year 5 girls exhibited lower mean mathematics anxiety than their male counterparts but that from Year 6 this pattern reversed and the girls consistently recorded higher mean mathematics anxiety.

Table 4
Student Mean Mathematics Anxiety Scores by Year Level and Year Gender (min score is 5; max score is 25; n = 179)

	Year 5		Year 6		Year 7		Year 8	
	Male	Female	Male	Female	Male	Female	Male	Female
Pre-survey	12.38	11.24	11.85	15.00	11.20	14.74	-	16.15
Post-survey	12.48	10.76	11.43	14.33	13.00	15.14	-	16.85

Mathematics Anxiety, Subject Choice and Career Intentions

To facilitate cross-tabulation of the mathematics anxiety scores with intention to choose a mathematics subject in Year 11, five groups were constructed with scores grouped as 5–8, 9–12, 13–17, 18–21, 22–25. Five groups were chosen as the original set of questions had five ratings. The relationship between anxiety and likelihood of choosing mathematics at Year 11 in the pre survey is presented in Table 5. There was a correlation (p < 0.001) between these, with students who were mathematically anxious (Groups 4 and 5) being the least likely to choose mathematics in Year 11. It is worth noting, however, that there was still a high percentage within these groups who intended to choose mathematics.

Table 5
Intention to Choose a Mathematics Subject in Year 11, by Group (group 1 is least mathematically anxious; group 5 is most mathematically anxious)

Percentage	Group 1 *n = 77*	Group 2 *n = 102*	Group 3 *n = 121*	Group 4 *n = 83*	Group 5 *n = 52*	Total *n = 435*
Strongly disagree	1.3	2.0	5.0	4.9	21.6	5.6
Disagree	2.6	8.8	8.3	16.0	11.8	9.3
Neither agree or disagree	10.5	17.6	33.1	38.3	15.7	24.4
Agree	38.2	38.2	32.2	22.2	37.3	33.4
Strongly agree	47.4	33.3	21.5	18.5	13.7	27.4

In the pre-survey, Table 6 shows that 29.1% of the students were not planning on pursuing a career involving mathematics (combining strongly disagree and disagree). Although the largest percentage of these students were in Groups 4 and 5, there was no significant correlation. It is worth noting that students may not have been exposed to careers involving mathematics as career education and pathways were not traditionally explored in Years 5 to 8.

Table 6

Intention to Pursue a Career Involving Mathematics, by Group (Group 1 is least mathematically anxious; Group 5 is most mathematically anxious)

Percentage	Group 1 *n = 77*	Group 2 *n = 102*	Group 3 *n = 121*	Group 4 *n = 83*	Group 5 *n = 52*	Total *n = 435*
Strongly Disagree	2.7	6.9	12.4	18.5	13.7	10.9
Disagree	17.3	13.7	19.8	19.8	25.5	18.2
Neither agree or disagree	36.0	40.2	35.5	33.3	35.3	35.8
Agree	20.0	24.5	20.7	22.2	21.6	21.4
Strongly agree	24.0	14.7	11.6	6.2	3.9	13.6

Tables 7 and 8 present the post-survey data showing how student intentions changed over the project in relation to choosing mathematics in Year 11 or choosing a career which involves mathematics. As is evident in Table 7, students with the highest levels of anxiety are the ones who are less likely to choose mathematics in Year 11 ($p=0.007$), but again it should be noted that there was still a large percentage of these groups who indicated that they were more likely to choose mathematics at the end of the year. The students in Group 3 are interesting as they did not rate themselves as being very anxious and yet 25.5% indicated that they were less likely to choose mathematics in Year 11.

Table 7

Student Change, From Beginning of 2019, in Likelihood to Choose Mathematics in Year 11, by Group (Group 1 is least mathematically anxious; Group 5 is most mathematically anxious)

Percentage	Group 1 *n = 52*	Group 2 *n = 70*	Group 3 *n = 96*	Group 4 *n = 44*	Group 5 *n = 42*	Total *n = 304*
More likely to choose	75.0	68.6	63.8	38.6	56.1	59.9
No change	11.5	18.6	10.6	22.7	9.8	19.5
Less likely to choose	13.5	12.9	25.5	38.6	34.1	20.6

Table 8

Student Change, From Beginning of 2019, in Likelihood to Pursue a Career Involving Mathematics, by Group (Group 1 is least mathematically anxious; Group 5 is most anxious)

Percentage	Group 1 *n = 52*	Group 2 *n = 70*	Group 3 *n = 96*	Group 4 *n = 44*	Group 5 *n = 42*	Total *n = 304*
More likely to choose	44.2	38.6	38.3	29.5	24.4	34.9
No change	21.2	21.4	23.4	36.4	24.4	37.4
Less likely to choose	34.6	40.0	38.3	34.1	51.2	27.7

From Table 8 it is evident that the relationship between pursuing a career in mathematics and levels of mathematics anxiety is not as clear. While there is a similar pattern with the highest percentage of students less likely to pursue a career in mathematics coming from the students in Groups 4 and 5, there was no significant correlation. It is interesting that there are almost 100 students who are more likely to study mathematics in Year 11 than pursue a career involving mathematics (59.9% compared with 34.9%).

Summary and Conclusion

In 2018, O'Keeffe et al. reported that mathematics anxiety was prevalent in South Australian schools and the data presented in this paper would suggest that little has changed in recent years. This is not unique to South Australia and follows the global pattern reported through PISA data by Gabriel (2022). Of particular focus in this paper is the way in which gender appeared as a factor. The data shows that, overall, female students still indicate that they have higher levels of mathematics anxiety than male students. This finding supports other studies in this area, including PISA (OECD, 2012, 2019). However, this gender imbalance is not the case for all year levels, with Year 5 female students in this study presenting as less anxious than their male counterparts. This is reflective of the findings of Holmes et al. (2018) and Steinke (2017) who pointed to the criticality of the middle years. This warrants further investigation, especially in light of the connection between mathematics anxiety and choosing mathematics in senior levels of schooling.

The connection between higher mathematics anxiety and choosing mathematics in the senior levels of schooling, though not surprising, was clear and statistically significant. Students who are more anxious about mathematics are less likely to choose mathematics subjects. This has previously been flagged in studies such as Panizzon et al. (2018) and Jaremus et al. (2019) and is of concern as the number of good female role models in senior mathematics subjects will continue to remain low until more female students opt for mathematics in the senior years.

The connection between mathematics anxiety and choice of career involving mathematics was not evident in the data. However the highest percentage of students more likely to choose a career involving mathematics were from the least anxious grouping of students, indicating lowering mathematics anxiety should help increase the number of students being willing to consider a career involving mathematics.

As a final comment, it is worth noting that in a previous study O'Keeffe et al. (2018) warned of a potential for mathematics anxiety of Year 7 students to increase once they transitioned to high-school settings. In their 2018 work, O'Keeffe et al. noted that the average mean mathematics anxiety score for males in Year 7 (at the time in primary school) was 12.66 (n = 293, SD = 4.94) and females was 14.46 (n = 325, SD = 4.9). While the data cannot be directly compared as it was not the same cohort of students, it is worth highlighting that the mean score for male Year 7 students in this study (in a high school context) was 15.03 (n = 46) and the mean score for females was 14.98 (n = 63). While the female scores in both studies are relatively similar, the secondary cohort of Year 7 females exhibit a slightly higher mean mathematics anxiety score. The figure for male students is notably higher, and further highlights the need to understand the transition to high school for Year 7 students. This aligns with the work of Attard (2012) and Hanewald (2013) who reminded us that school transitions are a time of upheaval and distress for many students and during these transitions students are more likely to develop negative perceptions of mathematics and increased anxiety in mathematics.

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Numeracy Across the Curriculum in Initial Teacher Education

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In Australia and Ireland, the school curriculum requires numeracy to be developed in all subjects. Initial teacher education standards similarly require that all graduates know and understand numeracy teaching strategies. This paper reports on a study that investigated pre-service secondary teachers' preparedness to teach for numeracy across the curriculum in Ireland. Analysis of questionnaire and interview data from participants in three universities showed they had largely superficial understanding of how to teach for numeracy in their subject specialisms and had experienced varied opportunities to learn about this aspect of teaching. The findings have implications for the design of courses in numeracy pedagogy for future teachers.

The school curriculum in many countries identifies numeracy as a cross-curricular competence that should be developed in all subjects rather than only in mathematics (Alberta Education, 2019; Australian Curriculum, Assessment and Reporting Authority [ACARA], n.d.; National Council for Curriculum and Assessment [NCCA], 2021; Norwegian Directorate for Education and Training, 2012). To date, however, only limited research has investigated how to prepare and support teachers to recognise the numeracy demands and opportunities within the subjects they teach. Most of the available studies involving practising teachers (e.g., Bolstad, 2020; Goos et al., 2014; Liljedahl, 2015), and much less is known about how initial teacher education can help future teachers understand and implement strategies for developing their students' numeracy capabilities. This paper reports on a study that explored the perceptions and experiences of Irish pre-service secondary school teachers in relation to how well prepared they feel to teach for numeracy in subjects across the school curriculum. The findings of the study may have broader implications for the design of courses in numeracy pedagogy in initial teacher education programs.

Background and Context

More than twenty years ago, the Quantitative Literacy Design Team was formed in the US to look into the meaning of numeracy in contemporary society. Led by Lynn Arthur Steen, this group of eminent scholars argued that numeracy development for students must be situated in an interdisciplinary educational context, permeating the whole school curriculum (Steen, 2001). This challenging goal has been taken up by curriculum authorities in some countries. In Australia, for example, the national curriculum was developed with cross-cutting *general capabilities* that intersect with learning areas representing disciplinary knowledge, skills, and understanding (ACARA, n.d.). Numeracy is one of these general capabilities. The Australian curriculum website explains that "students become numerate as they develop the knowledge and skills to use mathematics confidently across other learning areas at school and in their lives more broadly" (ACARA, n.d., Numeracy section). A similar situation exists in Ireland, where a national strategy was launched to improve literacy and numeracy in children and young people (Department of Education and Skills, 2011). A revised curriculum framework for the junior secondary years introduced a set of eight *key skills* that act as cross-cutting competences for teachers to embed in the learning outcomes of every subject. Numeracy is identified as one of these key skills (NCCA, 2021) and is described in terms of expressing ideas mathematically; estimating, predicting, and calculating; developing a positive disposition towards investigating, reasoning, and problem-solving; seeing patterns, trends, and relationships; gathering and interpreting data; and using digital technology to develop numeracy skills and understanding.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 434–441. Launceston: MERGA.

Yet there is no guidance for teachers, in any of the newly developed Irish subject syllabuses, as to how numeracy or the other key skills can be recognised and developed in specific curriculum contexts.

When the school curriculum includes numeracy as a cross-curricular competence, as is the case in Australia and Ireland, initial teacher education policies and standards likewise require that programs address pre-service teachers' personal numeracy competence and their ability to promote numeracy in the subjects they teach. The Australian Institute for Teaching and School Leadership (AITSL) has developed a professional standards framework that specifies what teachers at all career stages need to know and be able to do. The graduate standard relating to teachers' numeracy capabilities states that graduate teachers should "know and understand literacy and numeracy teaching strategies and their application in teaching areas (AITSL, 2011, p. 13). Similarly, in Ireland, teacher education graduates are required to demonstrate knowledge and understanding of numeracy as it relates to school curriculum requirements (The Teaching Council, 2020). However, Irish universities may take diverse approaches to program design incorporating numeracy into the initial teacher education curriculum. These approaches can range from semester-long courses on numeracy, sometimes in combination with literacy, to providing only one or two lectures on numeracy as part of a general pedagogy course, or instead expecting curriculum methods courses to address subject-specific numeracy demands. Thus, pre-service teachers in Ireland experience great variation in opportunities to learn about numeracy pedagogy. To gain insight into this emergent feature of initial teacher education, we analyse responses to a questionnaire and interview survey of future teachers to address the following research question: What are pre-service secondary teachers' perceptions of their preparedness to teach for numeracy in subjects across the school curriculum in Ireland?

Research on Numeracy in Initial Teacher Education

The brief account of policy initiatives outlined out above points to the need for more serious research attention to be given to design of initial teacher education in countries where numeracy is a cross-curricular responsibility for all teachers. Accounts and evaluations of some past and current Australian initiatives can provide some insight into course design and associated challenges. In an early example, Watson and Moritz (2002) explained how they created a Quantitative Literacy component as part of the mathematics course within a postgraduate teacher education program at their university. This component was based on a website they had developed previously, which compiled more than 200 newspaper articles containing significant mathematical content, together with teaching notes and student questions to support discussion of aspects of quantitative literacy in their daily lives. Forty pre-service teachers completing primary, middle years, and secondary specialisations completed four assessment tasks requiring them to plan and teach lessons based on the website materials. While the quality of pre-service teacher responses varied, Watson and Moritz considered that the tasks were effective in introducing quantitative literacy into the teacher education curriculum.

Another early initiative, described by Groves (2001), involved developing a semester-long course in response to local teacher education guidelines before national professional standards were introduced in Australia. The course, titled *Numeracy Across the Curriculum*, targeted future secondary school teachers and addressed the nature and scope of numeracy in everyday life, the inherent demands and opportunities for numeracy in different subject specialisms, and teaching strategies to respond to school students' numeracy learning needs. The course was first offered in 1999 and enrolled 146 pre-service teachers who were preparing to specialise in 24 different secondary school subject areas. It was designed and taught by mathematics education academics at a time when there was little awareness, amongst teachers in schools and university staff teaching methods courses, of the role of numeracy within their subject areas. There were many obstacles encountered in delivering the course, such as the limited oncampus time available due to the scheduling school placement and pre-service teachers' apprehension about their personal numeracy abilities. Feedback from pre-service teachers was mixed, with some finding great value in the course and others clearly indicating their dislike for it. Perhaps the greatest challenge to engaging pre-service teachers was the lack of recognition, at that time, of the need for all secondary school teachers to take responsibility for development of students' numeracy capabilities.

A more recent example of implementing a numeracy course for pre-service teachers is reported by Forgasz and Hall (2019). The impetus for designing the course arose from the inclusion of numeracy as a general capability in the Australian curriculum and the introduction of numeracy standards for graduate teachers. The course, titled *Numeracy for learners and teachers*, is compulsory for all primary and secondary pre-service teachers. It provides an introduction to the meaning of numeracy, drawing on the numeracy model developed by Goos et al. (2014), to show pre-service teachers how to plan and implement numeracy-enriched tasks and lessons in areas such as science, financial literacy, persuasive writing, geography, history, the arts, technology, and health and physical education. The numeracy model consists of four core elements: attention to real-life and curricular *contexts*; application of *mathematical knowledge*; use of physical, representational and digital *tools*; and promotion of positive *dispositions* towards the use of mathematics to solve real-world problems. A fifth overarching element—a *critical orientation*—involves selecting and applying mathematics to a real-world problem as well as interpreting and critiquing of results. The numeracy course was designed and coordinated by Forgasz, a mathematics educator, and taught in collaboration with teacher educators who specialised in other curriculum areas. It was evaluated for three consecutive years, using online surveys before and after its delivery and semi-structured interviews held after the course had finished. Analysis of pre-service teachers' responses revealed that their understanding of the differences between numeracy and mathematics improved after participating in the course, as did their confidence in incorporating numeracy into their teaching. It may be that the introduction of numeracy standards for graduate teachers increased the motivation of pre-service teachers and teacher educators alike to engage with numeracy across the curriculum as part of their respective professional obligations. In Ireland, however, standards for initial teacher education programs are less stringently monitored, which does little to encourage universities to prioritise numeracy pedagogy.

Research Design and Methods

Participants in our Irish study were undertaking a Professional Master of Education (PME) degree, which is a two-year, postgraduate program for students who already have an undergraduate degree in a subject area designed to qualify secondary school teachers. In Ireland, the PME is the most common pathway into secondary school teaching, and it offers the widest range of teaching subject specialisms. A secondary PME program was offered in all seven universities that existed in Ireland at the time of the study (the 2018/2019 academic year). All pre-service teachers who were enrolled in secondary PME degrees in three of these universities were invited to participate. These universities were selected on the basis of their proximity to the location of the researcher (first author), thus facilitating travel to each site for data collection. University A and University B introduced numeracy as part of other courses within their initial teacher education programs, while University C had a semester-long course dedicated to literacy and numeracy for teaching.

The 204 pre-service teachers who agreed to participate represented 62% of the overall PME cohort in the three universities. Nineteen of these pre-service teachers attended University A, 82 attended University B, and 103 attended University C. Almost half (95, 47%) were specialising in English or other languages; nearly one-quarter (49, 24%) in social science subjects (history, geography, business); with the remainder preparing to teach STEM subjects (science, mathematics, technology; 35, 17%) or arts and practical subjects (music, art, physical education; 24, 12%).

Between late September and early October 2018, pre-service teachers in each university were invited to complete a paper-based questionnaire during, or immediately after, a general pedagogy lecture that was attended by all PME students in their respective cohorts. The questionnaire consisted of 41 closed and open items. Its first section collected demographic data, while the second section assessed pre-service teachers' interpretations of numeracy and how they would teach for numeracy. A third section presented seven numeracy tasks for the pre-service teachers to solve. This paper draws on participant demographic data and analysis of responses to the following two open-ended items from the second questionnaire section:

1. What is your understanding of the term numeracy?
2. Explain with the aid of a specific example how you would incorporate numeracy teaching and learning into a lesson.

Twelve questionnaire respondents volunteered to participate in follow-up interviews: five from University A, two from University B, and five from University C. They were preparing to specialise in a wide range of subjects, including mathematics, music, English, languages, geography, and business studies. Two rounds of focus group interviews were planned in the first half of 2019, by which time all pre-service teachers had been introduced to some aspects of numeracy in their PME programs. The first focus group explored their understanding of numeracy and the second aimed to investigate their experiences in the initial teacher education program and preparedness to teach for numeracy within their subject specialism. However, due to scheduling difficulties in Universities A and B, it was not possible to arrange the second round of focus group interviews and so individual telephone interviews were conducted instead with these participants. Interviews were audio-recorded and transcribed in full. Transcript excerpts quoted in this paper identify the participant's university, gender, and subject specialism.

Thematic analysis (Braun & Clarke, 2006) was used to generate initial codes and then identify overarching themes from the open-ended questionnaire responses and interview transcripts. In this paper, interest centres on pre-service teachers' understanding of numeracy, their descriptions of how they would incorporate numeracy into a lesson in their subject specialism, and the opportunities they experienced within their initial teacher education programs to learn about teaching strategies for numeracy across the curriculum. "Opportunity to learn" is a key indicator used in international comparative studies such as PISA and TIMSS to show whether students have access to high quality education. Here, "opportunity to learn" is used in the same sense as in the Teacher Education and Development Study in Mathematics (TEDS-M) to refer to whether or not pre-service teachers have studied a particular topic in their teacher education program (Tatto et al., 2012).

Research Findings

Pre-service Teachers' Understanding of the Meaning of Numeracy

Thematic analysis of responses to the first questionnaire item yielded three distinct themes for describing the meaning of numeracy. A little more than half the pre-service teachers (113, 55.4%) described numeracy as a form of *mathematical knowledge*, for example, by referring to "numbers", "the ability to count", "maths skills", or "using figures, sequences or graphs". Just under one-third (64, 31.4%) interpreted numeracy as *using mathematical knowledge in different contexts*, mentioning that numeracy was "the study of maths in real life", or "using numbers in other subjects". A smaller proportion of respondents (20, 9.8%) explained numeracy in terms that referred to *thinking processes*, such as "decision making", "problem solving", and "logical". Seven participants left this question blank.

Further insights into pre-service teachers' understanding of numeracy were gained in the later focus group and individual interviews conducted with twelve volunteer participants from the three universities. By this time, all had been exposed to some aspects of numeracy for teaching, but only those from University C had experienced a formal course that addressed this topic. Pre-service teachers from University A claimed to understand what numeracy meant in their teaching specialisms; however, their understanding was limited to seeing numeracy as dates or numbers. For example, one female participant who specialised in teaching Irish language referred to:

> … dates, years, and how to count … there's a variety of ways of counting things in Irish. (A3; F; Languages)

Those from University B said they did not understand what numeracy meant—a situation some also said they had noticed with practising teachers:

> Well, I don't really know if the truth be told, I'm trying to understand it myself. We had one lecture on it and it was literacy and numeracy about this time last year. But, other than that, it was glossed over. I really don't think there's a lot of understanding in numeracy in established teachers. (B1; M; Mathematics)

Participants from University C indicated that their understanding of numeracy had changed as a result of completing the semester-long course in literacy and numeracy for teaching. One male pre-service teacher, specialising in Business Studies and Geography, explained that before completing the course

> It was just figures, the numeracy section in my [lesson] plans always had the date, just so that the students would know what date it is. (C4; M; Business Studies & Geography)

He went on to explain that the literacy and numeracy course

> … opened up my eyes to what numeracy actually is and the importance of context in numeracy as well. Let's say the percentages on loans or whatever, but then using that in context in the real world or real life settings". (C4; M; Business Studies & Geography)

Pre-service Teachers' Perceptions of Incorporating Numeracy into a Lesson

Responses to this questionnaire item described lesson ideas that were consistent with the pre-service teachers' interpretation of numeracy as involving either mathematical knowledge such as dates, time, percentages, units of measure; use of mathematical knowledge in real-world contexts such as finance; or thinking processes related to problem solving. A second layer of analysis classified the responses in terms of the level curricular awareness displayed by each respondent; in other words, how well they could recognise the inherent numeracy demands of the subjects they were preparing to teach. The most common responses (82 pre-service teachers, or 40.2%) were classified as demonstrating *no curricular awareness* and included superficial answers such as "write the date on the board" or "listing the three tasks that students must undertake". The remaining responses were nearly evenly divided between those displaying *some curricular awareness* (55 responses, or 27.0%) and *relevant curricular awareness* (59 responses, or 28.9%). The former category included examples such as using statistics, reading and interpreting graphs, and calculating percentages, but without specific explanation of how these activities would help students learn the target subject. Examples of relevant curriculum awareness, which linked numeracy with subject-specific content and goals, included using ratio and proportion when introducing perspective in art, interpreting patterns in musical notation and scales, and investigating time-distance relationships in movement activities in physical education.

Interview responses largely confirmed these low levels of curricular awareness of the inherent numeracy demands within secondary school subjects, with pre-service teachers from

Universities A and B usually mentioning trivial uses of numbers and dates in lessons. For example, a female pre-service language teacher from University B said:

> There were a few dates and I said, "How long?" and "What's the difference between the dates?" And I will consider it to be numeracy. (B2; F; Languages)

Those from University C seemed to have somewhat more curricular awareness of numeracy demands in the subjects they were preparing to teach; for example, a male pre-service teacher specialising in Geography and English explained:

> Overall, in terms of Geography, I do use a lot of graphs. Or even just speech writing in English, I do think using the statistics and discussing them is great. (C3; M; Geography & English).

Pre-service Teachers' Opportunities to Learn about Numeracy for Teaching

Pre-service teachers can experience opportunity to learn about numeracy for teaching by studying this topic in their initial teacher education programs. In the Teacher Education and Development Study in Mathematics, Tatto et al. (2012) designed a questionnaire that asked future teachers who were about to graduate whether they had experienced opportunity to learn in several broad areas hypothesised to influence knowledge for teaching mathematics: tertiary-level mathematics, school-level mathematics, mathematics education pedagogy, general pedagogy, teaching diverse students, learning through school-based experiences, and the coherence of their teacher education program. In the present study, opportunity to learn about the meaning of numeracy and to develop subject-specific numeracy pedagogy was investigated by comparing the courses that pre-service teachers were required to study and their perceptions of the adequacy of these courses. As this issue was explored only in focus group and individual interviews involving twelve participants, any findings are illustrative rather than conclusive. Table 1 presents a description of what is offered in each university initial teacher education program in relation to numeracy learning opportunities.

Table 1
Description of Initial Teacher Education Courses that Incorporate Numeracy

University	Course Name and Description
University A	Becoming a teacher: Identity and agency Helps student teachers to identify and critically analyse influences which shape the individual in becoming a teacher and to also focus on the key concepts of communication and reflection as significant processes in professional identity formation.
University B	Professional studies Examines the role of assessment, curriculum, literacy and numeracy, and ICT in education.
University C	Literacy and numeracy development in the post-primary classroom Examines and reflects on the role, function, development and pedagogy of the core curriculum skills of literacy and numeracy in the post-primary classroom.

Pre-service teachers from University A and University B referred to their experiences in their initial teacher education programmes as "tick the box" exercises and "slightly confusing", as they felt they did not know enough about numeracy. For example, a female pre-service teacher at University A who was specialising in music and Irish language teaching described her experience in her subject specialism pedagogy lectures in terms that convey a superficial interpretation of numeracy:

> Yeah, we just had two hours. Now, in fairness, our Irish lecturer did say, she was just teaching us how to tick a box like, which is fair, we're here to get marks at the end of the day. But she did say, these are examples of ways you can include numeracy in your lesson planning: the date, the birthdays in your classes, what age are the students. I was using a date every single day, it didn't matter that they were giving me the exact same month, at least they were giving me the date. (A4; F; Music & Irish)

On the other hand, a male pre-service mathematics teacher from University B described receiving little or no guidance at all as to how to incorporate numeracy into his lessons, even though he was aware that he should be doing this:

> It's mentioned, like even in the lesson plans that they give us, there is a section for literacy and numeracy but like, we've never, never been really told what we should put in there. (B1; M; Mathematics)

In contrast, pre-service teachers from University C said they found the literacy and numeracy course they undertook to be helpful. Nevertheless, they identified a missing connection as numeracy was never addressed in their subject specialism pedagogy courses where they were learning how to plan their lessons. For example, one interviewee explained:

> They could even mention it in the pedagogy classes. I suppose they don't mention it when we are in English pedagogy or learning to teach geography, you know. They don't mention numeracy at all, it's more literacy really. (C5; M; English & Geography)

While this is only a very small interview sample, it is possible to hypothesise that a dedicated course on numeracy across the curriculum, even if taught in conjunction with literacy across the curriculum, is more effective for raising pre-service teachers' awareness of the numeracy demands of the subjects they teach than attempts to address numeracy within general pedagogy lectures or subject specialism pedagogy courses. This is an issue that needs to be investigated in further research.

Conclusion and Implications for Initial Teacher Education

This paper investigated the preparation of secondary pre-service teachers in three Irish universities for their role in teaching for numeracy across the school curriculum. Its contribution to numeracy research and development in initial teacher education complements previous studies that have described and evaluated a single course or course component offered by one university in the Australian context. The pre-service teachers in our study demonstrated largely superficial understanding of the meaning of numeracy in real-world and curriculum contexts and described mostly trivial examples of how they would incorporate numeracy into a lesson. Thus, there seems to be a lack of alignment between initial teacher education program requirements regarding numeracy and the Irish pre-service teachers' lived experiences in their respective universities' programs.

Nevertheless, there were noticeable differences between the perspectives of pre-service teachers in Universities A, B, and C, which suggests that the concept of *opportunity to learn* might be a useful way of exploring these differences. Only in University C was there a dedicated course on literacy and numeracy in the school curriculum; in Universities A and B, numeracy was meant to be addressed as a topic within other courses focusing on pedagogy. Clearly, the greater allocation of content and time for numeracy in University C would have provided greater opportunity to learn than in Universities A and B.

Four approaches to addressing numeracy in initial teacher education emerged from the literature and empirical evidence discussed in this paper:

1. A dedicated, semester-long course on numeracy across the curriculum;
2. A semester-long course that combines literacy and numeracy across the curriculum;

3. Some lectures on numeracy across the curriculum as a topic within a general pedagogy course;
4. Incidental attention to numeracy within subject-specific pedagogy courses.

Arguably, none of these options, on its own, is likely to be sufficient, and an ideal approach might combine options 1 or 2 with option 4 so that pre-service teachers experience a thorough, research-informed grounding in numeracy alongside opportunities to apply their understanding of numeracy to develop resources and lesson plans in their subject specialisms.

Yet this recommendation raises another important question: Who should be responsible for providing pre-service teachers with opportunities to learn about numeracy across the curriculum? If numeracy is every teacher's responsibility, can the same kind of responsibility be placed on teacher educators? A collaborative model of course design may be beneficial, led by mathematics education staff with expertise in numeracy across the curriculum but also involving teacher educators in other areas of the school curriculum. Such an approach would not only strengthen the content of a numeracy across the curriculum course, but also provide non-mathematics teacher educators with valuable insights into the inherent numeracy demands of their own subject specialisms that could then be shared with their teacher education students. In this way, teacher educators may experience opportunities to learn about numeracy across the curriculum that are just as worthwhile as the opportunities they offer to their students.

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Teacher Actions to Progress Mathematical Reasoning of Five-year-old Students

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Opportunities for five students to access higher mathematics and critical thinking is often restricted until they have developed sufficient knowledge. This case study focuses on two reception class teachers and their teacher actions used to develop mathematical reasoning with their students. The findings illustrate the impact these teachers have on their students in the mathematics classroom by developing mathematical practices with five-year-old students that show that these students can critically reason and engage in mathematical practices.

Students in Aotearoa/New Zealand enter and attend school on or near their fifth birthday and are termed *New Entrant* students. They are assessed for mathematical knowledge within days of their entry to the schooling system. This should hold potential for them to then be given opportunities to learn mathematics from where they are on entry. However, Young-Loveridge (1998) contended that teachers then spend eighty percent of these students' first year teaching and practising existing skills. This means that a considerable proportion of New Zealand New Entrant students are provided with limited opportunities to succeed beyond the skills they enter school with as five-year-old students. Also, because of the repetitive form of instruction, they have limited opportunities to develop rich reasoning in mathematics (Hunter & Hunter, 2018). In contrast, if teachers scaffolded students by explicitly praising their engagement in mathematical practices and creating *talk rooms,* students would then be provided with opportunities to develop rich mathematical reasoning. However, we know that developing such classrooms is challenging, more so because there appears to be limited models for teachers to draw on to support their practice. The aim of this paper is to provide a possible model for New Entrant (reception) teachers, that could be representative of classrooms premised within such forms of inquiry and deep reasoning. The focus of this paper is to explore and examine two New Entrant teachers as they construct rich mathematical understandings with their five-year-old students. The question we ask is:

> *What actions do teachers take to provide all students with the opportunities to learn and communicate their mathematical reasoning in sense-making ways?*

Literature Review

Readiness is an important concept in mathematics teaching, but it can also be misinterpreted and used as a reason to delay allowing students opportunities to progress mathematically. Decades ago, Young-Loveridge (1987) warned that for many New Zealand students, mathematics was a subject that teachers used readiness mistakenly, to hold back on allowing their students to explore number concepts and key ideas until it has been achieved. This idea of readiness was developed from Piaget's (1952) view of cognitive development. Young-Loveridge explained how educators have taken from Piaget the idea that students are not ready to learn numerical mathematics until they have a base of cognitive abilities of conservation and classification. She says that there is no clear evidence to show that teachers should delay teaching rich mathematics until specific skills or knowledge have been acquired. MacGinitie (1969) also argued that to hold off exposing mathematical ideas or big ideas to New Entrant students was not an effective use of time. Nevertheless, this thinking has prevailed and in Aotearoa/New Zealand with a large amount of time placed on these young students

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 442–449. Launceston: MERGA.

gaining number knowledge to succeed in mathematics. This was exemplified in Aotearoa/New Zealand with mathematics teaching revolving around the New Zealand Numeracy Project (Ministry of Education, 2006). The Numeracy Project followed a linear trajectory of learning with students placed in ability groups. The initial focus for young children prescribed knowledge-based activities, which many teachers enacted in procedural and rote-type learning. A large focus of learning was around students *mentally* solving or using *imaging* to solve number problems. If unable to use a specific strategy students would remain at that level until the knowledge was learnt. This is despite many studies (e.g., Clements & Sarama, 2018; Steffe et al., 1976; Smith et al., 1981), which showed that students without what is considered necessary knowledge still benefit from deep mathematical understanding to the same degree as those who have the necessary knowledge.

In more recent times in Aotearoa/New Zealand, the focus of mathematics has been around students constructing reasoning embedded within a body of knowledge. As part of this, access to a range of materials and ways of representing reasoning was considered important for young children to construct rich forms of conceptual reasoning. Selling (2016) described how teachers should “attend to precision” (p. 547) as students use different representations. She states that it is important for teachers to name the representation being used as students show and explain their mathematical ideas. Mathematical practices are a set of skills or practices students use to become effective mathematicians. These practices include students justifying their ideas, making claims, the use of mathematical explanations, using representations and making generalisations – among others with and without materials (Selling, 2006). Selling also discussed the impact of teachers explicitly making connections across different representations by comparing solutions. Mueller and colleagues (2014) also showed how using materials supports students to clarify their ideas, think critically, and make connections across big mathematical ideas, and deepen their mathematical knowledge as connections and translations are made across representations. Hunter and Hunter (2017) added that for deepening students’ mathematical knowledge they needed opportunities to develop mathematical practices and reasoned conceptual knowledge throughout all mathematical areas.

Mathematical practices are an important tool which affords students opportunities to develop deep conceptual understandings. Through mathematical practices Hunter and Hunter (2018) outlined how students can develop their mathematical identities and strengthen positive mathematical dispositions from as early as five years old. They explained that students need to be provided with the opportunities to productively engage in mathematical practices for the practices to become real and meaningful. Selling (2016) argued the importance of teachers making mathematical practices explicit to facilitate “equitable learning opportunities” (p. 511). She described how it promoted the students as doers of mathematics as their peers are exposed to the work or reasoning they are doing. Hunter (2009) also discussed the impact of teachers responding in the moment to students engaging in mathematical practices. In doing so, teachers supported the development of these mathematical practices as tools that supported mathematical learning.

Mathematical practices work in tandem with classrooms that are discourse heavy. Researchers (e.g., Chapin & O’Connor, 2007; Franke et al., 2009; Hunter & Hunter, 2018) described the importance of talk in the mathematics classroom. Mathematical talk provides opportunities for students to not only justify and explain their ideas but, through this process, clarify and make sense of their reasoning and that of their peers (Chapin & O’Connor, 2007). Chapin and O’Connor (2007) detailed a set of what they term talk-moves, which teachers can employ to position their students to sense-make in mathematics classrooms. Nevertheless, for students to participate in such learning environments a safe and secure classroom environment needs to be developed. Mutual respect and high expectations are crucial for students to contribute their reasoning and respond to their peers’ questions (Hunter & Hunter, 2017). Many researchers (e.g., Boaler, 2008; Hunter, 2009; Hunter & Hunter, 2017) discussed the importance of teachers taking the time to teach and explicitly set up the norms expected in their classroom. Relating these norms and expectations to cultural experiences or values enhances the students' ability to connect to big mathematical ideas and experience success in real life mathematics (Bishop et al., 2009).

We began this paper by outlining the paucity of papers that provide teachers of very young children with models of talk-rooms to promote engagement in rich conceptual reasoning. The aim of this paper is to provide one possible exemplar of actions two teachers took to induct their young students into rich mathematical discourse environments.

Research Methods

This small-scale study was conducted at two New Zealand, primary schools with a diverse community of learners. These included students of Māori, Pāsifika, Indian, Korean, and New Zealand/ European ethnicities. The school was sited in a low-socioeconomic community. The teachers were experienced and of New Zealand/European ethnicity. Eight of Teacher A's students had attended school for two terms, two had attended school for six weeks, and three had attended school for less than two weeks. Teacher B's students had all attended school for less than ten weeks (n = 14).

Data consisted of video and audio recorded lessons observations wholly transcribed, and field notes. Analysis involved developing codes and notes and noticing the emerging common themes, which related to teacher actions to facilitate and progress mathematical reasoning amongst the new entrant students.

Findings and Discussion

Teachers structuring mathematics with ability grouping or streaming has been predominant in many New Zealand classrooms. This mode of grouping has traditionally begun almost as soon as students enter school as five-year-old students. In both classrooms described in this study, the teachers chose to structure their mathematical activity around the use of heterogenous grouping patterns, rather than taking a streamed approach. The classrooms also drew on pedagogical practices promoted within culturally sustaining (Paris, 2012), ambitious mathematics (Kazemi et al., 2009), which promoted engagement in inquiry and mathematical practices (Hunter, 2009). Lesson structures conformed to those outlined by Smith and Stein (2011). Problems were launched then students were given a short time to work on the problem with a buddy before returning to talk about their solutions in a larger group.

Providing Equal Opportunities for all New Entrant Students

From the data it is clear to see that both teachers considered that the benefits of having students in mixed ability groups outweighed possible deficits. Teacher A described and justified why she used multi-ability groups rather than streaming her students when she stated, "some of these students can add and write number sentences whilst some students can not recognise their numbers when counting to twenty – but they are all in the one group succeeding in the maths." Teacher B added to the discussion in describing how "there are some students in this group who struggle to form their numbers, so we provided materials, such as a hundreds boards to support them writing their numbers." Clearly, these teachers considered that it was their responsibility to ensure access to the mathematical reasoning through supportive means. This was evidenced in the substitution of the hundreds board rather than holding a child back because of the inability to record numbers.

Materials encourage students to solve mathematical tasks conceptually and a hands-on approach is needed for five-year-old students to develop mathematical representation. In the two classrooms the students were provided with many opportunities to use multiple resources to support them to develop their mathematical ideas. A range of materials was available in both classrooms and students were pressed to show their reasoning in whatever way they chose. For example, in the observed lesson Teacher A had a box of mixed materials the students could access at any stage of the task to support them with its solving and explaining. The teachers also supported the students' representations by modelling and highlighting representations and scaffolding them to connect to mathematical concepts. Their actions matched those promoted by both Mueller et al., (2014) and Selling (2016). These researchers demonstrated the power of teachers building on student representations, as either recorded or hands on explanations. In their research they showed how such actions supported the students to explain and clarify their reasoning. This was evident in this research.

Beyond having multiple forms of concrete and other representations the teachers also carefully launched the tasks to ensure all students had access to the context and the mathematics. This is essential with such young learners. For example, Teacher A used the repeat talk move multiple times as she developed what the task required them to do mathematically. This repetition provided students who had difficulties identifying specific numbers when reading the task the opportunity to understand the numbers they were working with. Making the repeat talk move public and explicit encouraged all students to be aware that the discussion was happening with the whole class, not just between the teacher and the individual student. Chapin and O'Connor (2007) described how teacher action of using repetition by different students highlights the importance of a contribution or essential point made.

Setting Expectations for Individual and Collective Engagement

In order for five-year-old students to develop effective mathematical practices teachers must provide opportunities for the students to engage and struggle with the mathematics. The tasks must be relevant and responsive to the cultural and social needs of the students. In doing so the students will see the mathematics as relevant and real to life (Bishop et al., 2009; Hunter & Hunter, 2017). Teacher A's task was written about a particular student's news item shared the day previously. The student reported it was her brother's birthday and they celebrated with homemade Indian naan. When launching the mathematical task, the teacher was able to bring this student into the conversation, and she shared her experience with her peers. The teacher then facilitated a discussion where the students compared their likes and dislike for naan. With the students able to relate easily to the context, the teacher was then able to form a challenging task around naan. As Teacher A described, "Fractions are huge concepts for New Entrant students to develop as they are so used to the bigger the number the bigger the value it holds, so we are flipping their thinking." This was shown when she generalised and made a connection to her thinking by asking, "If you were to have a piece of naan, would you want a half or a quarter?" One student said: "half because the piece is bigger." Listening carefully another student stated, "I agree, half a naan is bigger than a quarter of the naan." Illustrated in this exchange was the importance of the task and ensuring all students had access to the context. The students were clearly using the context to make sense of the mathematics. Although Smith and Stein (2011) previously described the importance of the launch and practical ways to implement it, their work was with much older students. In this classroom with young students, it remained both an important and effective tool, which served to progress the sense-making of them all and kept them engaged.

Establishing an expectation for engagement of all students includes maintaining the high cognitive demand in the task. Teacher B directly addressed the potential for students to opt out if they became daunted by the task. She stated, "Todays task is going to be a challenge, but I know you can do it—otherwise I wouldn't have written it." Through her explicitly voiced support the students responded positively. She maintained the pressure on them to persevere but at the same time she indicated to them that they were not alone in working with the mathematical task: “I can hear lots of people talking and collaborating with their thinking that means you are sharing your maths ideas and that is really important.” Other researchers (e.g., Boaler, 2008; Hunter, 2009) have illustrated the powerful role such positive praise and support holds in maintaining students persevering in mathematics. Although both these research studies were in classrooms with older students, the benefits remained the same for these five-year-old students and their perseverance with the task at hand.

The egocentric behaviour of five-year-old students has the possibility to interfere with a teacher’s drive for collective mathematical reasoning. Both teachers in this study attended to building norms, which included students’ accountability, not only to themselves but to each other. For example, Teacher B emphasised the ways in which collective action enhanced the outcomes in saying: “We are going to learn and notice the maths together—let’s use everyone’s brains to succeed when solving this problem—just like when we use the parachute everyone needs to work together to make it succeed.” These students had just come back into the classroom from using a parachute outside and the teacher directly supported the students to relate to this experience as shared success. This expectation that all students were to join in and problem solve together encouraged the five-year-old students to persevere with the challenging task at hand and show collective responsibility within each group. Teacher A also emphasised the role they needed to consider when working as a pair or a group and the importance of their contributions to the collective understandings when she stated, “When you come back together as a class to take it to the next step.” On interview, Teacher A expanded her thinking about the role of the collective in gaining deeper and richer reasoning when she said,

> When the pairs of students solving the task together got as far as they could on their own and had as much success with their problem. Then as a whole group we take the problem as far as we can and have further success together as a whole.

Through this statement it is clear the teacher did not expect one person in the group to solve the problem and teach the others, the expectation was for all students to engage equally and contribute collectively. As Boaler (2008) showed, it is important for students to learn that working and reasoning together means that as a collective they are all more likely to reach higher mathematical understandings.

Deepening Students’ Use of Mathematical Practices

Mathematical practices have received considerable attention in the past decade. Many researchers (e.g., Boaler, 2008; Hunter & Hunter, 2017, 2018; Selling 2016) have noted their importance in student engagement in successful problem solving within reasoned mathematical discourse. Most often studies related to mathematical practices have described their implementation with older students, but what makes this study different is that it took place successfully in classrooms with five-year-old students.

Expectations were explicitly placed on students making explanations that were mathematically based but within the contexts of the problems. For example, Teacher B modelled and supported her younger students to make an explanation. She used what Selling (2016) described as naming and highlighting to make the students aware when a mathematical practice was used. She was also frequently heard prompting and questioning the students to draw out the mathematical explanation. This is exemplified below:

> Student: This is a half.
>
> Teacher: What makes it a half?
>
> Student: There are two pieces and a line in the middle.

Teacher: A line in the middle?

Student: Yes, a line in the middle so both bits are the same size.

Teacher: So, you are saying: This is a half because there are two pieces, and each piece is the same size?

Student: Yes

Teacher: that is an excellent mathematical explanation that you have just made.

Clearly, two things happened in this exchange; the teacher drew out the information from the student so that a full mathematical explanation was constructed; at the same time the student retained authorship of the mathematical idea and explanation. Selling promotes naming such actions as central for all students to learn what makes an appropriate mathematical explanation. The mathematical status of the student was also raised in the teacher actions in supporting the student ownership of their mathematical reasoning (Boaler, 2008).

Likewise, when students made a complete mathematical explanation Teacher A commented audibly with statements like, “Wow, what a great mathematical explanation, as you have explained what it means to cut an item in half.” This teacher action highlighted publicly to the students the importance of what made this explanation a valid one. Selling (2016) explained the importance of such actions and the use of positive language when publicly naming mathematical practices such as “great” and “good” in developing a positive disposition in students and contributing to them constructing positive mathematical identities.

Without justification of mathematical explanations, generalising mathematical concepts often does not occur. In both these classroom lessons a natural focus was placed on requiring students to extend their mathematical arguments to providing reasons or justification. Teacher B consistently told her students to “Talk to each other about what you think and why.” Even with such young students yes or no answers were not accepted during lessons, with both teachers consistently prompting their students to explain why. As a result, the students in these lessons would make a mathematical claim and then almost naturally back it up with their justification. This can be heard in a student explanation, “We are folding this in half because then we will have two pieces, that means it is in half. If we are folding into quarters, we would have four pieces because quarters are something broken into four.” As Teacher A commented after the lesson, “The justification is really important. Even at five, they can do it. So, for this lesson, the keyword was ‘because’.” They know that when they give an answer they have to explain why, so ‘because’ and then their reasoning.” Analysis of student talk in the lessons showed a pattern in which the older students endeavoured to provide full and detailed mathematical explanations and justification. As a result, the younger children were being inducted into a mathematical environment that modelled reasoned mathematical discourse.

Teacher questioning had a key role in establishing a mathematical environment founded within reasoned discourse. Students were positioned to question the mathematics presented to them. The use of asking students to agree or disagree with a mathematical idea but expecting them to do so in a mathematical manner and using the word because was evident in both lessons. For example, when asked about their reasoning a student responded with, “I disagree because there are two dots on that side but there should be 3.” This statement was followed by the teacher then asking all the listeners to consider whether they agreed or disagreed with the statement. This encouraged the students to be active members of the lesson, and to not merely accept a rebuttal without considering all the mathematical reasoning being offered. Chapin and O’Connor (2007) described how such actions create a habit in students of reasoning and developing critical thinking. Such actions provide space for students to practise mathematical analysis and critical thinking.

Conclusion and Implications

We have illustrated in this article the importance of teacher actions for providing all students with opportunities to learn and communicate their mathematical reasoning in sense-making ways. Importantly, we showed the value of teachers believing in the potential of their New Entrant students to learn rich mathematics in sense-making ways on school entry and onwards. It was evident that these young students, when provided with opportunities to engage in mathematical practices, used solutions to develop productive mathematical discourse and conceptual understandings. These findings correspond with those described by previous researchers (e.g., Hunter & Hunter, 2018; Chapin & O'Connor, 2007; Selling 2016). However, in this research the students were of a considerably younger age than those in previous research.

Teacher actions are of critical importance in inducting students into such talk environments. As Hunter and Hunter (2017) described previously, learning to provide mathematical explanations and respond to questions take many students time and practice to learn. Within these two classrooms this was apparent but through careful scaffolding this occurred. The teachers used many actions described by Boaler (2009) and Selling (2016) to establish their safe supportive classrooms. Also evident was the model provided by older students in inducting their younger peers into an environment, which actively required interactive participation. Many New Entrant classrooms in Aotearoa/New Zealand hold a range of different aged children because they all enter school on their fifth birthday. Therefore, teachers need to consider how a constant flow of new students can be catered for. This research suggests that teachers should scaffold the older students their peers so that the mathematics community continues to develop.

Materials also hold an important place as scaffolding tools. It was clear that the two teachers recognised their importance in both supporting the students to learn to construct mathematical understandings and also to provide rich explanations and justification. This parallels what Mueller and colleagues (2014) illustrated in their research when they showed the importance of materials for students in reasoning critically and making connections across big mathematical ideas. Selling (2016) in her research also showed the importance of teachers making connections across making representations. Although she did this with senior students in this research it was shown as also valid and important to progress mathematical reasoning with significantly younger students.

Mathematical practices have received considerable attention in recent times but most often with older students. What was illustrated in this research was their place and importance in New Entrant classrooms. They appeared to provide these students with many opportunities to access mathematics with deep understandings. This was because they were in classrooms that promoted a talk-rich environment as promoted by other researchers including Chapin and O-Connor (2007), Franke and colleagues (2009) and Hunter and Hunter (2018, 2019). Teacher actions also matched those described previously by Selling (2016) and Hunter (2009). Like both these researchers showed, this research illustrates the importance of teachers responding in the moment to student engagement in the talk to support development of mathematical practices.

Although this research reports on only two teachers and their two lessons the results have potential to provide other teachers with a model of the types of action and talk they can promote in their classrooms with very young students. More research is needed to examine the outcomes of other classrooms on the mathematical achievement of young students.

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RESEARCH PAPER

Wicked Problems as a Context for Probability Education

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This study focuses on pre-service teachers' experimentation with a game-modding process in a constructionist setting whilst they experimented with randomness embedded in wider socio-scientific issues that call for decision making under uncertainty. In this process, participants created 39 different game mods. Our observations of the participants while they worked on the mods suggest that grappling with wicked problems while using digital socio-scientific games can offer new contexts for harnessing causality to facilitate students' meaning-making for randomness embedded in such contexts. In order to bridge the deterministic and the stochastic in wicked problems, the students transfer agency to specially designed numerical consequences of choices, by inserting proportional thinking, game theory, and probability.

Introduction

Human brains revolt at the idea of randomness. Just learning the concept of randomness seems to be difficult (Prodromou, 2008; Prodromou & Pratt, 2013; Pratt & Noss, 2002). Piaget's seminal work (Piaget & Inhelder, 1975) reported how the organism initially fail to apply operational thinking to the task of constructing meaning for random phenomena. In this sense, Piaget offers us a first hint that we only begin to gain some mastery over the stochastic after we learn how to exploit our well-established appreciation of the deterministic. Prodromou (2008) studied students aged 14–15 who made sense of distribution, adopting a range of causal meanings for the variation observed in the basketball computer simulation and in the graphs generated by the simulation. The carefully designed computer simulations offered ways for harnessing causality to facilitate students' meaning making for variation in distributions of data, much as, according to Piaget, the organism invents, much later, probability as a means of operationalising the stochastic. The basketball simulation developed helped shed light on how students at one level let go of the deterministic whilst at the same time re-applying those ideas in new ways to construct meanings about probability distribution. In this study, instead of using digital media for experimentation, we used a constructionist setting where experimentation is part of a modding process for a game.

The game-modding environment provokes deterministic meanings that we might be able to harness in a productive way while a) students experiment safely with randomness, that b) is situated in wider socio-scientific issues and our everyday lives, making it more interesting and relevant. We study participants' meanings of randomness that are both embedded and generated in a real context, illustrating explicitly the way they made decisions under uncertainty when grappling with complex and wicked problems. We conjecture that grappling with wicked problems while using digital socio-scientific games might offer new contexts for harnessing causality to facilitate students' meaning-making for randomness embedded and generated in such contexts and enhance the development of language that is used when making decisions about wicked problems. Such understandings and language pervade almost everything we do and directly inform the way we conduct our everyday lives.

Theoretical Background

The use of a constructionist setting for experimentation in the modding process fosters: 1) the learning of concepts related to randomness, and 2) meaning making of the difficult concepts of randomness that are embedded in a wider socio-scientific issues. In each case, the role of ideas about randomness and probability in the thinking process was that they would be the

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 450–457. Launceston: MERGA.

ticket to find a solution to a problem or to investigate an open problem thoroughly and comprehensively, even if it has no solution. Mathematics is the mechanism to pose or solve problems. There has been very little study, however, of understanding probabilistic ideas and ideas about randomness and putting them to use in the context of grappling with wicked problems and in the frame of a post-normal science epistemology. Increasingly, the real problems in our society are complex and there is an increased awareness that such complexity, like that observed in environmental and sustainability problems, may lead to powerlessness, paralysis, and apathy if learning is not scaffolded to generate a sense of agency (Kollmuss & Agyeman 2002). How can we infuse such an element in education? How can we help students reduce confusion and stress about issues surrounding them and appreciate that mathematical thinking can be useful and relevant in such contexts?

In the study reported in this paper, we engaged in design research aiming to engage students (or prospective teachers in the role of students) with such issues by using a digital socio-scientific game, an online environment that was used for participants to approach randomness anew, with a disposition to re-structure (Wilensky, 2010). The aim was to explore the ways in which randomness is conceptualised in education in the quest to make it more attractive and afford meaning making to students and aid in grappling with complex problems. We approach meaning making of randomness from a social and situated perspective, whilst studying the dynamic interactions of participants, though participants' relations, their interactions with the game, mathematical concepts and everyday knowledge or everyday events that involve problems including "wicked problems" (Rittel & Webber, 1973), which are described as dysfunctionalities within a complex system (Conklin, 2005). The term "wicked problems" (WPs) was coined by Rittel (1972) to describe design problems in the domain of social planning and a particularly challenging type of ill-structured problem (Rittel & Webber, 1973). WPs involve a high degree of uncertainty, lack definite right or wrong solutions, are highly contextualised, involve political considerations, and are characterised by a high level of inherent ambiguity and normative conflict.

Wicked Problems

Wicked problems (WPs) are characterised by the lack of clarity in both their aims and solutions, and they deal with the complex, fuzzy, multifaceted, contentious issues in both big projects (e.g., urban development) and small (e.g., a healthy personal diet). Wicked problems are difficult to contain and structure, are interconnected and interdependent, and are ill-defined and dynamic, as their parameters are continually in flux (Rittel & Webber, 1973; Coyne, 2005). Unlike with mathematical problems, whose consequences are minimal, when dealing with wicked problems, the effects of a planner's actions can matter a great deal to the people who are affected by those actions, and planners can be held liable for the consequences of the solutions they generate. Faced with such problems, individuals often feel overwhelmed, develop denial and resignation, followed by inertia due to a sense of determinism, which permeates societies (Hulme, 2009; Lazarus, 2009). Yet wicked problems need action at many levels, which requires taking risks and acknowledging the randomness inherent in the problem. For the individual, it is important to engage in becoming knowledgeable on the problem and to also engage in actions. In the context of personal diet, which is used by the games in this study, individuals' actions can include planning their own individual diet; maintaining their individual diet; and challenging their own beliefs and eating habits. Additional complexity is added by interest not only in a personally healthy diet but also in contributing to community decisions—whether a classroom (as with the participants of this study) or some larger community of interest—about diet that engage with the wickedness of planning a diet, as well as the wickedness in designing and developing a game that educates people about diet.

The design of digital educational resources for fostering new ideas in collective design and meaning making in a mathematically rich area of study like probability and randomness, brings the study of students' grappling with a WP in the design of a diet to the fore. Similarly, the modding brings the study of WPs in the mod to the fore. In this study, we looked at the process of modding collaboratively, focusing on their attempt to grapple with WPs and develop personal agency, action competence, and meaning making about concepts of randomness. In this context randomness (mathematics) and thinking about randomness (mathematical thinking) will not resolve the problem nor will it help define the problem clearly, but it may help engagement with the problem, development of meaning, and understanding of subproblems and issues.

A paradigm shift is needed to move from solving well-defined siloed problems to a post-normal science approach that grapples with complex real-world problems (Lehtonen et al., 2019). So, consider a transformational stance to schooling in an attempt to integrate such a post-normal science approach in teaching and learning, addressing and perceiving students as young citizens (McLaren, 2013). Consider the challenge of harnessing wicked-problem education to become syntonic and integrated with the innovative educational push towards cultivating the eight key competences for lifelong learning (European Commission, Directorate-General for Education, Youth, Sport and Culture, 2019).

Method

In this study, our 10 participants worked on joint design of games in the ChoiCo online digital environment, forming different *communities of interest* (CoI)—a group of practitioners from diverse disciplinary and/or professional backgrounds who engage in that activity collaboratively within their own communities of practice to design digital educational resources (Fischer, 2001)—in different contexts. The ten participating CoI members were practitioners in different levels of education (from primary to tertiary education) who specialised in primary education, physical education, and/or English or German education. This diversity in knowledge domains, perspectives, and cultures was meant to enhance the CoI's creative potential. We engaged in design research aiming to engage students (or prospective teachers in the role of students) with such issues by using a digital socio-scientific game. Using the ChoiCo game-authoring system, we designed a game for the students to work with. The game was concerned with healthy diet, which we perceived to be a WP in a domain that was already familiar to the students.

The ChoiCo environment consists of Play Mode and Design Mode, so the users could alternate between playing and designing, changing the content as well as the rules of the game. In play mode, the player made selections from among many options. Each choice had consequences for a number of game attributes and none of the choices had only positive or only negative consequences. For example, Food choices (Figure 1), the game discussed in this article, offers different food choices such as "hamburger", "vegetables", "chicken", etc. The player makes a food selection and observes the effect of the choice on four attributes (nutritional_Value, Kilos, Pleasure, and Health). The player tries to keep these variables within set limits for as long as possible. In design mode, the user can either design a new game from scratch or modify an existing game. We embedded mathematics in the game by carefully designing numerical consequences, and by inserting proportional thinking, game theory and probability.

Data and Analytical Approach

Our data were the online discussions of the participants and their written reflections when they used ChoiCo to reason about randomness and implement randomness in the design of a novel game that simulated a realistic everyday situation in which people make decisions about their food preferences. We developed two similar games in ChoiCo that provided several different options of food to eat (Figure 1). One was fully deterministic; the other included use of a random number generator. The aim of both games is to make food choices and keep the game attributes “Nutritional Value”, “Kilos”, “Pleasure” and “Health” within specific numerical values. In play mode, the participants played the two games and wrote down their food choices, the game values, and their justifications for the strategies they used to stay in game as long as possible, referring to the numerical values of the game attributes “Nutritional Value”, “Kilos”, “Pleasure” and “Health”.

Figure 1. ChoiCo game.

The participants reasoned about whether the notion of randomness was embedded in the game and compared the two games in terms of their results (food choices and game values) and the notion of randomness involved in the game. We expected that the absence of realism in the game would trigger participants to modify the game or design new games that simulate a more realistic situation where people make decisions about their daily diet. The modding process was divided into three main phases: “Play-Modify-Create”. In the modding process of the ChoiCo game, creativity manifested as the result of using the idea of randomness to either create a realistic context of something new (a novel game related to diet) from something that did not exist before or modify the game in the ChoiCo environment, which supports students’ autonomy, risk taking, freedom, flexibility, and playful learning.

Results

This study shows that in participants’ engagement in the game modding process, they used the idea of randomness in the process of modding, which in turn acted as a scaffolding for the development of some important understandings about randomness. Four episodes were selected to illustrate the implemented alterations through four distinctive phases, which are used to structure the story of how the relationship between causality and variation shifted as users moved through these phases: A) Playing, experimenting; B) Changes related to the elements of the game, such as the rules, codes, and values of the game; C) Application of algorithmic building and modifying the code of the logical expressions; D) Creating a new game.

The first episode involved two CoI members: Amanda and Sophie, both master of primary education students. In this episode, there was a mod created that introduced changes to the elements of the original game. Amanda and Sophie played both games and initially engaged with observing the implementation of randomness in the second:

> Sophie: Game one provides no randomness. This is because the values of each food choice remain the same and can be replicated with each turn.”

> Amanda intervenes: I believe the algorithm provides messages based on set game/food choice values, for example, if the pleasure value is too high or kilo value is too low. You can stay longer in the game by checking the game values of each food choice before selecting to ensure it contributes positively to your score…

Their attention focused on game two and Amanda continued:

> Amanda: The concepts of chance included in game two is randomness, or pseudo randomness as it is generated by the software. This can be seen in Table 4, where the pleasure value for each food item has a range and the software will select a number at random within that range. This means the results cannot be predicted or replicated.
>
> Sophie: If the focus was simply on comparing the food choices and their values (not cumulative values), then this game could be used to demonstrate randomness to students. By playing the game and recording their food choices and values, they could see that in game 2 the same choices do not replicate the same scores and therefore are random. Alternatives to this, that may be clearer for students to comprehend include rolling the dice, random draws, and the random number function in Excel. It should be noted that randomness is also produced by each player's preference and food selection, however, as the algorithm prefers healthy choices to keep you in the game. This is somewhat limited and not such a clear example of randomness.
>
> Amanda: The updated game, I uploaded increases the range of pleasure values to include both positive and negative for each food choice. This will enable students to see a wider range of variables amongst the pleasure scores, making it clearer to Year 5 students to investigate this concept of randomness.

The next episode, Episode 2, lasted four days and the participants were four CoI members: Sue and Sarah, preservice primary teachers, and Charles and Tess, in-service secondary teachers. All of them are studying for the Master of Education degree. They discussed the concepts of chance embedded in game one:

> Charles: In game one, on my first attempt I clicked [on] hamburger and died immediately. The concept of randomness was displayed here as I made a random choice … students can see the values for each food, and after playing several times, discover that alternating between *always food* and *sometimes food* keeps them in the game longer.

As Charles began to explore the effect of making different food choices causing variation directly on the food values, it appeared that the students felt that they were not able to make their choices randomly because of the pop-out messages. The discussion continued considering other features of the game.

> Sue: Although it is true that values had been allocated by the programmer with certain amounts of randomness, if students are able to see the values, chance does not come into play. Discussions surrounding bias could be developed when analyzing this activity. During play, pop-outs say things like "You might have health issues" and "Start eating healthier meals."
>
> Tara: I also believe the game can be more realistic by removing all the nutritional information from the Point Information box. In real life, we cannot analyse the factors of nutritional value, health, kilos, and pleasure for every single item that we eat. We can make predictions about the outcomes based on what we know is healthy versus unhealthy and what we enjoy, but the actual outcome is out of our control to an extent. The game coder can implement pseudo randomness in the game by hiding the information in the Point Information table so that it is not visible to the player.
>
> Sue: In general, eating food in real life often does not come with detailed nutrition information so the game could also hide parts of the Point Information table so that the user has to make their selection using their prior knowledge rather than rely on the data in the table.

In their attempt to make the game more realistic, the participants removed the pop-out messages and hid the game values. Gradually, the discussion became more focused on Game 2 and was oriented towards making decisions about the embedded randomness and cross-curriculum issues such as nutrition. Participants used different strategies to discern randomness. For example, when this CoI group discussed randomness embedded in pleasure, Charles made the same food choices in Game 1 and Game 2 to observe the different game values when randomness was introduced in Game 2. It appeared that Charles explicitly recognized that pleasure from food might be chosen randomly from the values within a predefined range. This insight was accompanied by the acknowledgment that the repetition of the same choices did in fact show variability:

> Sarah: In Game 2, a truer element of randomness is evident in the points allocated for pleasure which are randomly generated with a particular pre-defined range. Making a choice contains an element of chance, but students maintain relative control as they can look at the range and make appropriate choices. We ran the same pattern of 10 foods through both Games 1 and 2 (vegetables, legumes, kebab, eggs, pasta, diary, fish, chicken, ice cream, fruit).
>
> Charles: I played Game 2 three times. Each time the pleasure values were different despite making exactly the same food choices, yet all other values remained the same. Making these comparisons is a good way of demonstrating randomness to students.

A special feature of this stage is the CoI members attempt to make the game more realistic by changing the elements of the original game. They discussed the attributes' values that had been allocated by the programmer with certain random ranges.

> Sue: It is not however, particularly realistic. To make the game more realistic we made some changes to the game. Firstly, it was important to allocate randomness to health as well as pleasure. While fruit is obviously a healthy food, it would not be healthy for someone to make almost all their diet fruit, so it is appropriate to introduce randomness into health values. Rather than having a set number such as a 10 for diary, I allocated a random range between 5 and 15. And have repeated this range of 10 for each item. Another to which I assigned random vales is nutrition. Using vegetables as an example, it is more realistic to have a random value assigned to a vegetable because in reality, not all vegetables are nutritionally alike. Starchy potatoes are a lot less nutritious than kale. Also, I have raised the starting level of health to 40. Starting the game by eating a hamburger should not cause immediate death.

Students applied algorithmic concepts both in the database (variables, types) and in the block-based editors creating conditional statements, logical expressions, etc. The modified games' rules appeared to be created by logical statements that progressively were expressed by logarithmic building. For instance, in episode three, another group, Yeeka and Engel, both special education master's students, modified the game and wrote:

> The game was modified to make gameplay more engaging for diverse students, with each game being different due to random variation without being overly unpredictable. Pseudo-randomness was added to the Nutritional Value to simulate real-life variations from different food preparation methods and freshness and choice of ingredients. Some of the food variables were altered to balance out the positive/negative effects more realistically. The additional Hydration Value—based on the water content of foods—and choices of Water and Juice also add realism and complexity to the game. The difficulty of the game increases as play advances due to higher minimum limits set for Health, Hydration and Pleasure values.

As we evidenced above, due to the random variation inherent in the game, randomness was implemented to other variables (Nutritional Value, Kilos, and Health) and new variables also created (Figure 2) to illustrate real-life variations when making a food selection. They mapped these expressions to the relevant block-based code.

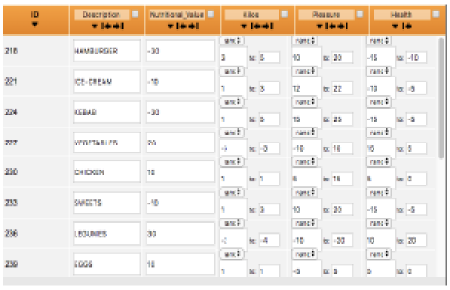

Figure 2. Nutritional Value, Kilos and Health Variables.

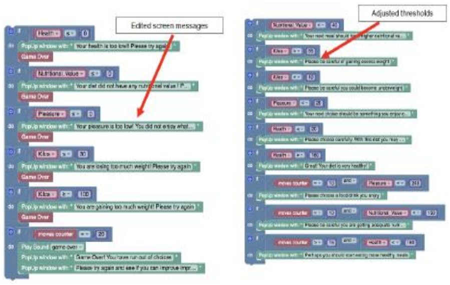

Figure 3. Conditional expressions and new rules.

The participants studied and edited the logical expressions in the code to create new rules and also edited the screen messages to make the behaviour of the game more realistic (Figure 3). Eight of the 39 different mods created had new attributes. In two of them, the participants also added new layers and designed a new setting for the story. For instance, the fourth episode, involves a CoI of three, Anna, Judy, and Angus, bachelor of secondary mathematics education students, who designed a new game that included recommend servings of food items per day versus its calories and nutritional value. As Anna wrote,

> We include in our game recommended serving per day of any food item in view of its total energy count and nutritive value to measure a well-balanced ratio of the essential nutrients: carbohydrates, fat, protein, minerals, and vitamins in food or diet. Users' choices provide students with feedback about making healthier choices that can affect their body and overall functioning.

Another CoI group (Mike and John, Bachelor of Primary Education students) created a different map-based game scene in which they included restaurants with their food menus, and the varieties of food they eat at home.

> John: While the knowledge of each food can give rise to certain likelihoods over others within each variable, nothing is certain, and thus the values of each food within the game for each variable need to reflect this notion of variation and randomness to food and nutrition in real life … hamburgers are unhealthy but only from fast food restaurants. They can be very healthy if made at home. Thus, the health and nutrition value scores need to reflect this variation.

Conclusions

Randomness was embedded in the variables of the ChoiCo game through a mechanism that can be seen as an example of what Papert refers to as a quasi-concrete object (Turkle & Papert, 1991), in the sense that the virtual objects can be manipulated in ways akin to how we learn about material objects through their use. We were exploiting here an affordance of computational objects to facilitate an intuitive connection to abstract formalisms (e.g., risk, handling randomness) in ways that are not available through conventional approaches. In our game, formal mathematical ideas of socio-scientific issues are instantiated as on-screen (Papert, 1996) quasi-concrete (Turkle & Papert, 1991) objects, virtual artefacts that can be manipulated on screen like material objects and experienced as concrete and tangible (Turkle & Papert, 1991). Those quasi-concrete materials foster participants' risk taking and experimentation (Calder, 2011), allowing space for students to explore the essential nutrients, carbohydrates, fat, protein, minerals, and vitamins in food or diet, make decisions about their food choices, whilst taking into consideration the impact of food on their health, weight, and pleasure. Participants experienced randomness through game modding (instead of stochastic experiments, e.g., rolling dice, etc.) and they seemed to harness causality (Prodromou, 2008) in order to re-construct a game that embodied realistic elements, based on testable conjectures, which sensitised the CoI's appreciation of meanings as they shifted between randomness, concepts of chance, and knowledge about nutrition and healthy eating. The aforementioned shift amongst those different domains, indicates the immediate need for a paradigm shift, from solving well-defined siloed problems to solving wicked problems in an attempt to integrate a transformational stance to teaching and learning.

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RESEARCH PAPER

Embodied Task to Promote Spatial Reasoning and Early Understanding of Multiplication

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This study enquires into the embodied processes of children in solving multiplication tasks, considering how such processes can expand access to spatial reasoning skills and simultaneously develop students' understanding of multiplication. The analysis focused on four Year 2 students as they completed two embodied tasks. The aim was to understand how embodied tasks could stimulate students to use spatial reasoning to explore and understand multiplication as equal groups in array forms. The findings suggest that engaging students with embodied tasks stimulate them to think about mathematics spatially and reflect on their thinking about mathematical ideas.

Introduction

Although numerous research findings strongly confirm the intense link between spatial reasoning and mathematics competency (e.g., Barmby et al., 2009; Gunderson et al., 2012), less study has been conducted focusing on how spatial reasoning can be used to facilitate mathematical understanding (Lowrie et al., 2020; Newcombe, 2018; Newcombe et al., 2019). As spatial reasoning is naturally embodied (Thom et al., 2015; Thom & Hallenbeck, 2021), one way to promote spatial reasoning in mathematics learning is through engaging students with embodied tasks (i.e., the tasks that stimulate physical and sensory systems). The link between spatial reasoning and embodied experiences is described by Thom et al. (2015) in the following assertion, "A child who engages in the act of mentally rotating a shape is not just performing an act of spatial reasoning. She is demonstrating embodied mathematics. She is recursively enacting her embedded and embodied knowing (p. 81)." Here, spatial reasoning is about how bodies regularly sense and make sense of the situations of the physical world.

Therefore, the potential of the embodied task as a learning strategy is grounded on the theory of embodied cognition. The theory claims that our knowledge or cognition is shaped by the experience of our physical and sensory systems within our environment (Shapiro, 2019; Shvarts et al., 2021; Varela et al., 2016). The embodied theory highlights two important points; first, cognition is contingent on the types of experiences gained by having a body with different sensorimotor abilities; and, second, these individual sensorimotor abilities are themselves inherent in the larger biological, psychological and cultural contexts (Varela et al., 2016).

Regarding the relationship between spatial reasoning and embodied activities and their potential to promote mathematical understanding, this study aimed to understand how embodied tasks stimulate students' spatial reasoning and encourage the students to use this reasoning to explore and understand mathematical concepts. To reach the goal, learning tasks involving embodied experiences were designed to develop students' awareness of the idea of multiplication as equal groups (e.g., 2×3 as two groups of 3) through array structuring tasks. The concept of multiplication was chosen for three reasons. First, the idea is foundational in early mathematics since it is the groundwork for proportional thinking used in real-world applications (Fosnot & Dolk, 2001). Second, although it is foundational, many students retain a poor understanding of the concept as they are mostly taught multiplication as a list of mathematical facts to be memorised without understanding the underlying mathematical concepts (Hendriana et al., 2019). Third, the underlying concepts of multiplication can be discussed in terms of space as the concepts can be effectively represented and communicated

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 458–465. Launceston: MERGA.

through several spatial representations, such as number lines, bars or arrays (Kosko, 2019). Thus, this study aimed to address the question,

> *How could embodied tasks stimulate students to use spatial reasoning to explore and understand multiplication as equal groups*?

Embodiment, Spatial Reasoning and Understanding of Multiplication

The fundamental claim of embodied cognition is that the features of our cognition are shaped by the aspects and the experiences of our body with the environment (Shapiro, 2019; Varela et al., 2016). In this theory, knowledge is developed as the result of the sensory and motoric experiences of our body with the external world, such as learning tools, through a mechanism called perception-action loops (Shvarts et al., 2021). As students interact with a learning tool, the perception-action loops can be described as a simultaneous interaction between body and mind in producing knowledge. Here, the initial perceptions toward the learning tool guide users' actions on the learning tool and, at the same time, the actions generate verified or extended perceptions toward the learning tool (Shvarts et al., 2021).

Moreover, in the embodied cognition theory, an embodied action can be viewed as the extended visible or tangible form or a concrete example of conceptual understanding (de Freitas & Sinclair, 2013; Thom et al., 2015; Thom & Hallenbeck, 2021). Thom et al. (2015) coined this action as an observable knowing where students' understanding is reflected by or represented through their physical actions. For example, when an individual student arranges unit cubes into several equal rows, this student might be described as embodying the concept of the equal group of multiplication. When a student shows that $a \times b = b \times a$ by rotating $a \times b$ array to form $b \times a$ array, this student may be representing the embodiment of the commutative nature of multiplication. Thus, the embodied cognition theory suggests that students' experiences need to be situated in designed embodied tasks that support them to develop the intended understanding. It follows that student understanding can be assessed by investigating their physical actions in situated conditions. Therefore, in this study, we define an embodied task as a task that requires physical actions to perform purposive actions under a situated condition.

Embodiment and spatial reasoning are strongly related and even significantly overlap. Spatial reasoning is embodied since spatial reasoning arises from making sense of the embodied sensorimotor experiences (Thom et al., 2015; Thom & Hallenbeck, 2021). For example, in the context of mathematics, a child who is mentally rotating a shape is not only performing an act of spatial reasoning but also demonstrating embodied mathematics (Thom et al., 2015). An individual (whether a child or an adult) mentally manipulating spatial properties of an object using gesture, movement, drawing, modelling, and so on, with or without signed/spoken/written language, is simultaneously demonstrating embodied mathematics (Thom & Hallenbeck, 2021). Therefore, embodying mathematical ideas in physical movements will foster the use of spatial reasoning as such reasoning is stimulated once a student navigates his/her body in space during the movements.

The array is recognised as a powerful spatial representation that allows access to several big ideas of multiplication, such as equal groups and the binary nature of multiplications (Barmby et al., 2009; Battista et al., 1998; Kosko, 2019). The relationships among embodiment, spatial reasoning and the understanding of multiplication can be demonstrated by using spatial tools, such as arrays, to explore, practice and communicate mathematical ideas underlying multiplication. For example, a student who is structuring arrays to reason about multiplication is considering embodied events as the spatial information embedded in the array (e.g., shape, size, location, and distance) is oriented, moved, or managed by the potentials of our body (sensorimotor capacities). Here, in relation to the concept of the body in/of mathematics (de

Freitas & Sinclair, 2013), the ideas of multiplication are animated and practised through and by body experiences of structuring arrays. Such an embodied experience intensely fosters spatial reasoning as it involves spatial structuring thinking that facilitates students to develop a meaningful understanding (Battista et al., 1998).

Method

This study aims to develop embodied learning tasks that stimulate students' spatial reasoning and promote students' awareness of multiplication concepts as equal groups of objects. Therefore, design research was employed consisting of three phases, namely design preparation, teaching experiments, and retrospective analysis (Gravemeijer & Cobb, 2013).

During the design preparation, a literature review was conducted to form the basis for designing the embodied tasks. The embodied tasks were designed to help students develop early awareness of multiplication as equal groups. Two related embodied tasks were formulated. Task 1 invited students to explore different ways of counting 12-unit cubes and used the cubes to represent their counting strategies. Here, it is expected that the students will develop several counting strategies, such as counting by twos or threes and represent the strategies in arrays. By arranging cubes in arrays, it is expected they will see the group structure of the cubes in arrays. As the follow-up, Task 2 asked students to imagine several arrays consisting of 12-unit cubes and then draw the imagined arrays on a grid paper. It is expected that the students will be aware of the group structure of multiplication while imagining and drawing the arrays.

In the next phase, the teaching experiments, the designed embodied tasks were tested in the classroom setting involving eight students (four Year 2 students, two Year 3 students, and two Year 4 students). However, for the current paper, the analysis focused on the findings from the Year 2 students (50% girls) as the data produced by those students best exemplify how embodied learning tasks could stimulate students' spatial reasoning and promote students' awareness of the concepts of the equal groups of multiplication. During the experiments, the teaching-learning activities were observed directly, and video recorded. As well, students' written work on display boards and worksheets were collected.

In the final stage, the retrospective analysis, students' embodied responses toward the tasks together with the relevant written works were analysed for two purposes, namely (1) to gain an understanding of the relationship between the embodied tasks and students' responses, and (2) to understand how the embodied tasks promote the intended understanding. The retrospective analysis was conducted task by task chronologically to see how each task stimulated students' responses. For this paper, the analysis focused on how the two embodied tasks stimulated students to think of and reflect on the array structure about equal groups in multiplication.

Results and Discussion

Students' Responses on Task 1

The first task asked the students to explore different ways of counting 12-unit cubes and used the cubes to represent their counting strategies. The data showed that three out of four students initially counted the unit cubes one by one. But, after they were asked to count them differently, they developed various counting strategies, such as counting by twos or threes, and represented their counting by arranging the cubes in arrays (see Figure 1). Student A, for example, initially counted the cubes by threes before she decided to count them by twos, where she made six groups of two in an array. Student B's first attempt was counting by threes and arranged the cubes in an array model representing four groups of three.

Meanwhile, Student C, at the first attempt, counted the cubes by twos, connected their cubes, and then arranged the unit cubes to form three groups of four. Student D initially counted the unit cubes by fours and then by twos. In contrast with Student B, who arranged their unit cubes horizontally or in rows, Student D arranged them vertically in columns.

Moreover, it is identified that the students glanced over at other students' array constructions to get inspiration of the array structure from others. For example, as Student D was attempting to make another unique array, he quickly constructed a 2×6 array for his second attempt after seeing Student A, who made a 6×2 array. He recognised that six groups of 2 can be represented as two groups of 6. As the students arranged the unit cubes in arrays, the teacher used this opportunity to examine whether the students could see the structure of the groups in an array. During the discussion, the students conveyed that 12 cubes could be represented in various grouping forms, such as two groups of 6 or three groups of 4. The teacher used the students' production of group structure as the context to introduce the notion of multiplication as equal groups. For example, six groups of 2 can be written multiplicatively as 6 × 2.

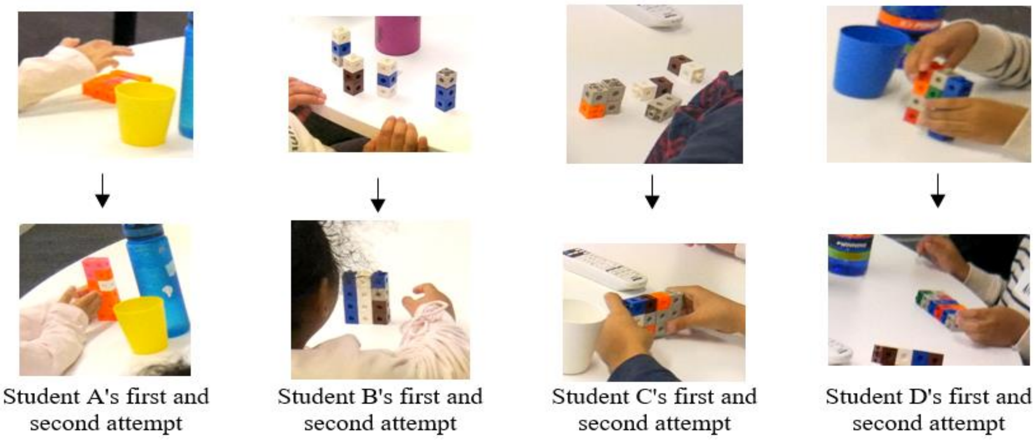

Figure 1. Several students' self-constructed arrays to track their counting.

Students' Responses on Task 2

In Task 2, without cubes, the students were asked to imagine several arrays consisting of 12-unit cubes and then draw the imagined arrays on paper. Through the task, it is expected that the students will be aware of the group structure of multiplication while imagining and drawing the arrays. The data showed that the embodied task of drawing the imagined arrays stimulated the students to validate their conjecture of the expected array structure. For example, Student A intended to draw the array for two groups of six cubes (see Figure 2). She initially drew a big rectangle and split the rectangle vertically by drawing six vertical partition lines. She then drew a horizontal partition line splitting the rectangle horizontally into two sections and counted the number of unit cubes on the first row, where she got seven-unit cubes instead of six. Upon realising this mistake, Student A erased one vertical partition line to make six columns.

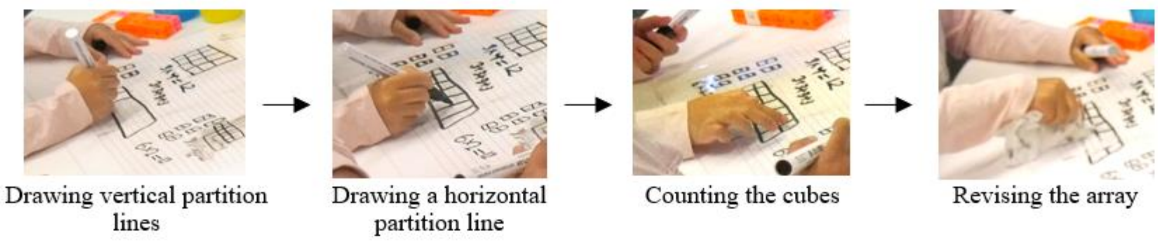

Figure 2. Student A's experience of reconstructing a 2×7 array into a 2×6 array to represent two groups of 6.

Similar to Student A's experience, Figure 3 shows that Student C intended to draw six groups of two. He modified a 6×3 array into a 6×2 array because he realised that the 6×3 array does not represent six groups of two as he expected. Initially, he drew a big rectangle and split the rectangle into two sections vertically, then split the rectangle horizontally by drawing horizontal lines forming six rows. Next, to draw two columns, Student B drew two vertical partition lines, although he instead attained three columns. He suddenly became aware of his mistake after counting the generated unit cubes (the square cells) by twos (row by row), where he found 18 cubes instead of 12. Immediately, he removed the last column creating a 6×2 array.

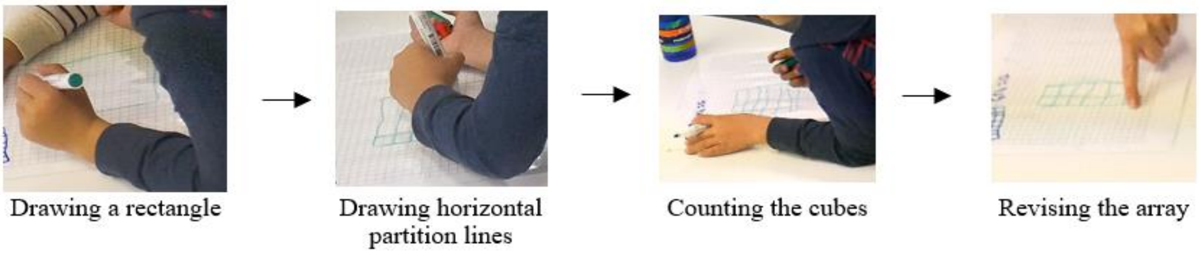

Figure 3. Student C's experience of reconstructing a 6 × 3 array into a 6 × 2 array to represent six groups of 2.

It was identified that Students A and C's mistakes were similar. They drew one more column than what they were intended to have. As they drew the number of vertical partition lines equal to the intended number of columns, they may have thought they had already drawn the correct number of columns. In fact, drawing vertical partition lines generate n+1 columns. For example, drawing two vertical partition lines generate three instead of two columns. As the students could do self-assessment and correction simultaneously, they utilised their spatial reasoning on the spatial visualisation of the group structure. Such an experience contributed to their conceptual understanding of the structures.

Discussion

Regarding the intimate link between spatial reasoning and embodied activities and their potential to promote mathematical understanding, this study aimed to understand how embodied tasks stimulate students' spatial reasoning and promote mathematical understanding. The impact of the embodied tasks was sought through discussing two key findings. First, the embodied tasks in the learning stimulated students to think of the equal-group structure of multiplication. Second, the tasks stimulated them to reflect on their thinking of the mathematical concept.

Embodied Tasks of Structuring Array Stimulating Students to Think of the Equal-group Structure of Multiplication

The findings from students' responses on the first task show that the students' physical actions of arranging and rearranging the unit cubes in several different group structures express what they have in their minds about the spatial structure of the array. Considering the idea of the body in/of mathematics (de Freitas & Sinclair, 2013), the actions serve as the extension of their thinking of the array structure as they acted purposively to express their thinking of the structure. Therefore, the actions themselves can be regarded as the representation of their understanding of the structure. Their ability to construct 12-unit cubes in several arrays shows that they understood the underlying array structures where 12 can be represented in various ways of grouping, such as three groups of four, two groups of six, or six groups of two.

Furthermore, the array is used as a spatial tool to represent the group structure. Students' actions through the spatial tool consequently stimulate the use of spatial reasoning where, for instance, the students need to consider the changes in the group structure of the array as the result of moving or modifying unit cubes. Thom et al. (2015) consider this kind of action as observable knowing of which and by which the students' spatial reasoning grows. For example, as each student was asked to construct a unique array from the 12 cubes, Student D looked and imitated the 6×2 array made by Student A to construct a 2×6 array by mentally twisting the 6×2 array. Student D could differentiate the group structure between the 6×2 and 2×6 array due to the rotation.

The embodied actions of structuring arrays and their reasoning of the spatial structure of the arrays interplay simultaneously during the process of constructing the meaning of the group structure. On one side, the embodied actions stimulated the students to activate their spatial reasoning and serve as the way to clarify their mathematical reasoning. For example, once the students develop their conjecture that the 12-unit cubes can be represented in four groups of three, the actions of arranging the unit cubes into four rows of three will verify the conjecture. On the other side, and at the same time, spatial reasoning navigates students' embodied actions of arranging and rearranging the array structure. For example, bodily reconstructing a 6×2 array into a 2×6 array requires the students to envision the spatial changes on the 6×2 array; as a result, the transformation and how the changes generate the new spatial structure. Here, the simultaneous interactions between the perceptions of the spatial changes on the array (as the result of the embodied actions) and the body actions on the array (as the result of the spatial perceptions) create perception-action loops (Shvarts et al., 2021). In this context, the perception is generated by the spatial information of the array, and the embodied actions are triggered by the perception. By considering Shvarts et al. (2021) theory of embodied instrumentation, the perception-action loops are the generator of knowledge and understanding where the progressive and simultaneous interaction between spatial perceptions and the embodied actions on the array generate understanding of the group structures underlying the array.

Embodied Tasks of Drawing Arrays Stimulate Students to Reflect on Their Thinking About the Group Structure

Although understanding the concept of equal groups of multiplication is known to be challenging for many students (Battista et al., 1998), Students A and C's responses on Task 2 suggested that the embodied experience of structuring arrays by drawing the array has the potential to promote students' awareness of the group structures. As the students were engaged in the embodied tasks, they could develop an awareness of the concept of the equal group and how the concept can be expressed on arrays. They understood that they had to modify their previous array to get the intended array and realised that parts of the array needed to be revised to attain the correct one. Students also recognised that, as they modified the arrays, they must preserve the same number of unit cubes in each row or column. Such awareness reflected their understanding of the array representation of multiplication.

Moreover, their embodied spatial experience of structuring the array and the array itself (the representation of the groups in rows and columns) allowed them to reflect their thinking, for example, thinking whether they created the array for 12-unit cubes or not. By drawing the array, they animated the virtual ideas of multiplication (i.e., equal grouping) through the embodied experience of drawing. Looking at the notion body in/of mathematics (de Freitas & Sinclair, 2013), their experiences of recognising incorrectness through the visualisation of the array and bodily modification of the array reflected their conceptual understanding of how arrays can be used to express the abstract ideas of equal groups of multiplication.

At the end of the lesson, the students' conceptual understanding of equal groups of multiplication could be asserted as they represented the concept of equal groups in various ways, such as through arrays and repeated additions, which helped them define the product of multiplication (see Figure 4). Student A, for example, defined 3×6 as the sum of 3 groups of 6, which can be represented as an array having three rows of 6.

The analysis of the tasks suggested that the mechanism underlying the mutual connections among the three constructs (i.e., embodied tasks, spatial reasoning, and multiplications) can be explained through the framework of embodied instrumentation proposed by Shvarts et al. (2021). In the embodied instrumentation, the interaction between body capacities (sensory and motoric skills) and the situated learning condition (e.g., learning multiplication through physically structuring and drawing arrays) generate knowledge or understanding resulting from progressive perception-action loops. The perception-action loops are simultaneous interactions between body and mind in producing knowledge (Shvarts et al., 2021). In the context of the current study, perception was generated by the ability of the body's capacities (sensory and motoric skills) to see, hear, or become aware of the spatial structure of arrays to represent the equal groups as the result of bodily modifying the array. Meanwhile, the action was the act of bodily modifying the array, which is stimulated or guided by the perception. As the perception-action loops were continuously verified and refined through time simultaneously, the perception-action loops generated the intended verified knowledge and understanding (Shvarts et al., 2021). For example, it was identified that the students initially did not recognise the notion of equal groups of multiplication. However, as the students intensively interacted with the spatial tools situated by the embodied tasks, they began to develop that awareness. Throughout the process, spatial reasoning played significant roles both as the mediator between the perceptions and the actions and as the catalyst that stimulated progressive perception-action loops. As the students worked with the spatial tool to represent mathematical ideas, spatial reasoning facilitated them in constructing appropriate perceptions about the mathematical ideas spatially. At the same time, their body actions were guided by their spatial reasoning as they are dealing with a spatial environment (e.g., spatial tools) to communicate mathematical ideas. In the context of the current study, where spatial tools are predominantly used to express mathematical ideas, having a good sense of space may foster the development of perception-action loops.

Student A's group structure of 3×6 | Student B's group structure of 3×6 | Student C's group structure of 5×7 | Student D's group structure of 5×3

Figure 4. Students' self-constructed representations for several multiplications.

Conclusion

The students in this study demonstrated the connections among embodied actions, spatial reasoning, and mathematical understanding. Their embodied actions were portrayed through the embodied experience of structuring and drawing arrays. Meanwhile, the use of spatial reasoning was observed through the use of a spatial tool (the array) to communicate and express mathematical ideas of multiplication. Finally, the progression of their mathematical understanding was reflected by their ability to define multiplication as equal groups in the form of array structure and repeated addition.

The findings of the current study highlight two important points. First, engaging students with embodied tasks potentially stimulates them to think of and reflect on mathematical ideas spatially. The embodied tasks promote spatial reasoning to explore, communicate, and make sense of mathematical concepts spatially. Here, the embodied experiences and the spatial experiences provide the contextual meaning for the explored mathematical ideas. Second, the use of embodiment theory should be considered in task design. The embodied-based tasks foster the formulation of knowledge and understanding through the mechanism called perception-action loops. In this mechanism, actions stimulate an understanding and, at the same time, understanding guides action. Progressive perception-action loops allow students to develop their thinking and reflect on their thinking of the situation and ideas being explored.

Acknowledgments. Thanks go to the Indonesian Government for their support through the Mora Scholarship and to Tracy Logan and other colleagues at SERC (University of Canberra) for advice and feedback. Special thanks to the teacher, the students and the parents involved in this study.

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Evaluating Factors that Influence Young Children's Attitudes Towards Mathematics: The Use of Mathematical Manipulatives

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The use of manipulatives to develop conceptual understanding appears to be a prevalent practice in many mathematical learning experiences, particularly in the early years of schooling. This study evaluates the impact of mathematical manipulatives on young children's attitudes towards mathematics (YCATM). The modified three-dimensional model of attitude (MTMA) and Bruner's experiential stages were used to investigate how manipulatives influence YCATM. The findings suggest that young children enjoyed using manipulatives, contributing to their Vision of Mathematics and Perceived Competence. However, the transition between enactive, iconic, and symbolic experiences can contribute to the formation of negative attitudes.

Children's attitudes towards mathematics are strongly related to their receptiveness to learning and understanding mathematics, their achievement, the value of the subject, self-confidence, and enjoyment (Stiles et al., 2008). Underlining the necessity for mathematics and how children develop mathematical understanding is the need to understand the factors that influence attitudes towards mathematics. Attitude is a multi-dimensional construct with affective, cognitive, and behavioural dimensions (Walker et al., 2020). Investigating attitudes towards mathematics as a multi-dimensional construct provides an erudite view between attitudes and mathematics achievement (Walker, 2020). Connected to this is the phenomenon of negative attitudes and mathematics anxiety. Before these terms, Gough (1954) used "Mathemaphobia" and believed that many failures in mathematics are attributed to a phobia of mathematics. Gough (1954) claims "victims" suffering the phobia avoid studying mathematics and proposes addressing the issue by identifying causes and influences. For this reason, it is crucial that attitudes towards mathematics and the factors be understood so that positive attitudes can be fostered and nurtured. While there is a wealth of knowledge about older students' attitudes towards mathematics, in the case of young children's attitudes towards mathematics (YCATM), there is a dearth of research (Ingram et al., 2020).

The dearth of research extends to investigating how the use of manipulatives by children influences their attitudes towards mathematics. Manipulatives are an established mathematics education resource that can be a "positive tool to improve student learning" (Liggett, 2017, p. 90) and a tool to develop conceptual understanding in mathematics (Quane & Brown, 2022). Further, manipulatives are an established form of mathematical representation (Moyer, 2001). Goldin and Shteingold (2001) remarked: "that a mathematical representation cannot be understood in isolation" (p. 1). Rather, a representation of mathematics is part of a more comprehensive system of mathematical conventions and meaning. Mathematics representations can be a process and a product and are broadly classified as external or internal representations (Goldin & Shteingold, 2001), with manipulatives being an example of a product and an external representation. However, research indicates that manipulatives are more than physical, external representations. Bruner (1996) suggests that our world can be represented and translated into experience in three stages: enactive (action), iconic (perceptual organisation), and symbolic (words and symbols). Research has focused on how teachers effectively use manipulatives to facilitate mathematical learning (Quane & Brown, 2022; West, 2018) and the challenges of using manipulatives (Moch, 2002) and warn that manipulatives do not necessarily lead to success and can even be detrimental to learning (McNeil & Jarvin, 2007). Few studies, however, have explored the use of manipulatives from a child's perspective

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7) pp. 466–473. Launceston: MERGA.

and how they influence attitudes towards mathematics. Given the dearth of research, this study investigated the range and nature of YCATM and, in doing so, identified a range of factors that were found to influence attitudes. Numerous factors were identified, including, but not limited to, the use of technology, game-based pedagogies, tests and assessments and manipulatives. The focus of this paper is on the use of manipulatives and how they influence attitudes towards mathematics. The guiding research question is:

> *How do manipulatives used during mathematical learning experiences influence young children's attitudes towards mathematics?*

Theoretical Framework

To explore the confluence of YCATM and the use of manipulatives, the Modified Three-dimensional Model of Attitude (MTMA) was used to define the construct of attitudes towards mathematics (Quane et al., 2021). Bruner's (1966) experiential stages of learning were used to analyse how children used manipulatives and their attitudinal response to using the manipulatives. To further categorise and develop a more nuanced understanding, the mapping mathematical materials framework by Larkin (2016) was applied to describe how children used manipulatives (Table 1).

Table 1
The Modified Three-dimensional Model of Attitude (MTMA) with Reference to Bruner's Experiential Stages of Learning

MTMA Dimension	Bruner's Experiential Stages of Learning
ED: Emotional Tendency	Children's initial emotional response and reaction during enactive learning experiences using manipulatives and emotions towards iconic and symbolic representations of manipulatives.
ED: Overall Sentiment	Children's general reactions and emotional beliefs regarding mathematics, including non-verbal cues (posture, gestures and body language) and verbal cues to the use of manipulatives (enactive, iconic and symbolic representations of experience)
VM: Topics, Tasks and Processes	Types of mathematical learning experiences and processes identified by children; the number of mathematical topics and how children communicate their mathematical understanding and learning. For example, children's use of manipulatives during mathematical learning experiences (enactive); children's drawings of manipulatives to represent mathematical concepts, ideas, and thinking (iconic); children using or discussing the use of manipulatives to then represent mathematics in written form using words and symbols (symbolic).
VM: Value and Appreciation	How and what children view as important and acknowledge as worthwhile about mathematics. For example, what worth or importance do children place on using manipulatives in mathematics as a direct sensory experience (enactive), using pictorial representations of manipulatives (iconic), using manipulatives to aid in the representation of mathematics abstractly (symbolic).
PC: Mathematical Mindset	Children's mathematical mindset and perceptions of themselves related to their ability to do mathematics. Children's mathematical mindset when using manipulatives, drawing, or making iconic representations of their use of manipulatives or recording their use in symbolic form.
PC: Self-concept	Children's beliefs in their mathematical ability and their expectancy for success when using and representing manipulatives.

The MTMA moves beyond the dichotomies of "liking" versus "disliking" (Capps & Cox, 1969) and "positive" versus "negative" (Lipnevich et al., 2013) to capture the complexity of attitudes in three broad dimensions. These three broad and interconnected dimensions were conceptualised by Di Martino and Zan (2010) and encompassed the Emotional Disposition (ED), Vision of Mathematics (VM) and Perceived Competence (PC) dimensions. The MTMA provides six explicit sub-dimensions that can be used to classify and describe YCATM, placing a premium on the *developmental* aspects of children (Quane et al., 2021). Further, moving the definition of attitude away from a dual classification system to include a more extensive and nuanced description of attitude affords the opportunity to also identify factors that influence their attitudes. Each original dimension of the original TMA was modified to include two sub-dimensions, as shown in Table 1. The six sub-dimensions of MTMA were used to identify how the use of manipulatives influence YCATM. A manipulative is "an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered" (Swan & Marshall, 2010, p. 14). This paper reports on the use of physical manipulatives.

Method

A mixed-methods methodology was used to investigate the influence of manipulative use on YCATM. Qualitative research techniques involved using children's drawings, written descriptions, interviews, and observations. These methods include the use of children's drawing, and the affordances and limitations (see Quane et al., 2021) have been previously presented. The use of child-centric research techniques afforded children to express what is important to them regarding their mathematical learning. Quantitative analyses were conducted first to identify the range of attitudes, followed by qualitative analyses to describe the nature of attitudes. The frequency and distribution of scores provide the *range* of children's attitudes. The *nature* of children's attitudes is a narrative that has been developed based on acts, actions, artefacts, actors, and significant events that children discussed during the interview when talking about their drawing and during the mathematical learning experiences observations. Three phases of data collection and analyses were enacted: an exploratory study (n = 25); main study (n = 81) and overall data analysis (N = 106) involving systematic, numerical, thematic, and comparative analyses. Children's drawings were coded using and inductive, deductive, and anticipatory responses (Quane et al., 2021). The 106 Year 2 and 3 children attended three South Australian schools and were from ten classes. The data generated has been re-analysed using open coding and TMA and Bruner's experiential stages to explicate how manipulatives influence YCATM. After the generated data were coded, indicators were developed aligning to the framework outlined in Table 1.

This research examined YCATM and the factors that influence attitudes in both lesson and non-lesson contexts, bridging the gap in researching attitudes towards mathematics. Children's drawings, written descriptions, and interview responses (N =106) were collected in a non-lesson context. That is, data collection occurred with children in a non-classroom environment. All children were assigned an alpha-numerical code to ensure anonymity. The letter indicates which school the child attended, and the number indicates the order in which the drawing and interview was conducted. Observations of mathematical learning experiences (n = 27) were conducted after the non-lesson data collection. In the lesson context, three children from each class were purposefully selected based on their attitude classification from the non-lesson context and observed during mathematical learning experiences. Children's attitudes were classified for both the lesson and non-lesson contexts. In reporting the findings, the context where the data were generated is indicated providing transparency regarding the source of data. The observations focused on overt behaviours that were observable by an onlooker. It is not that concealed or non-observed actions, reactions or behaviours are considered inconsequential. Rather, they are hard to detect in a classroom situation. Unknown circumstances may have impacted what the children were experiencing on the day they completed their drawing and the days of the observed mathematical learning experiences.

Findings

Mathematics representations emerged from the non-lesson context as a theme of interest, with 41 (39%) children discussing the use of manipulatives. During the non-lesson context, children discussed using manipulatives such as unifix cubes, dice and counters to represent and solve mathematical questions and problems. Figure 1 includes selected children's drawings that depict the use of manipulatives. Additionally, children went beyond identifying different manipulatives to describe how they used manipulatives in the context of mathematical topics, their emotions towards using particular manipulatives, and their perceived competence in using the manipulative. During mathematical learning experiences, children's use of manipulatives was observed to see how their attitudes towards manipulatives were enacted. A more extensive range of manipulatives was noted in the lesson context, including unifix cubes, dice, counters, geoboards, Polydrons, measuring instruments, attribute blocks, paddle pop sticks, dice, clocks, number charts, and tens frames, were associated with doing mathematics.

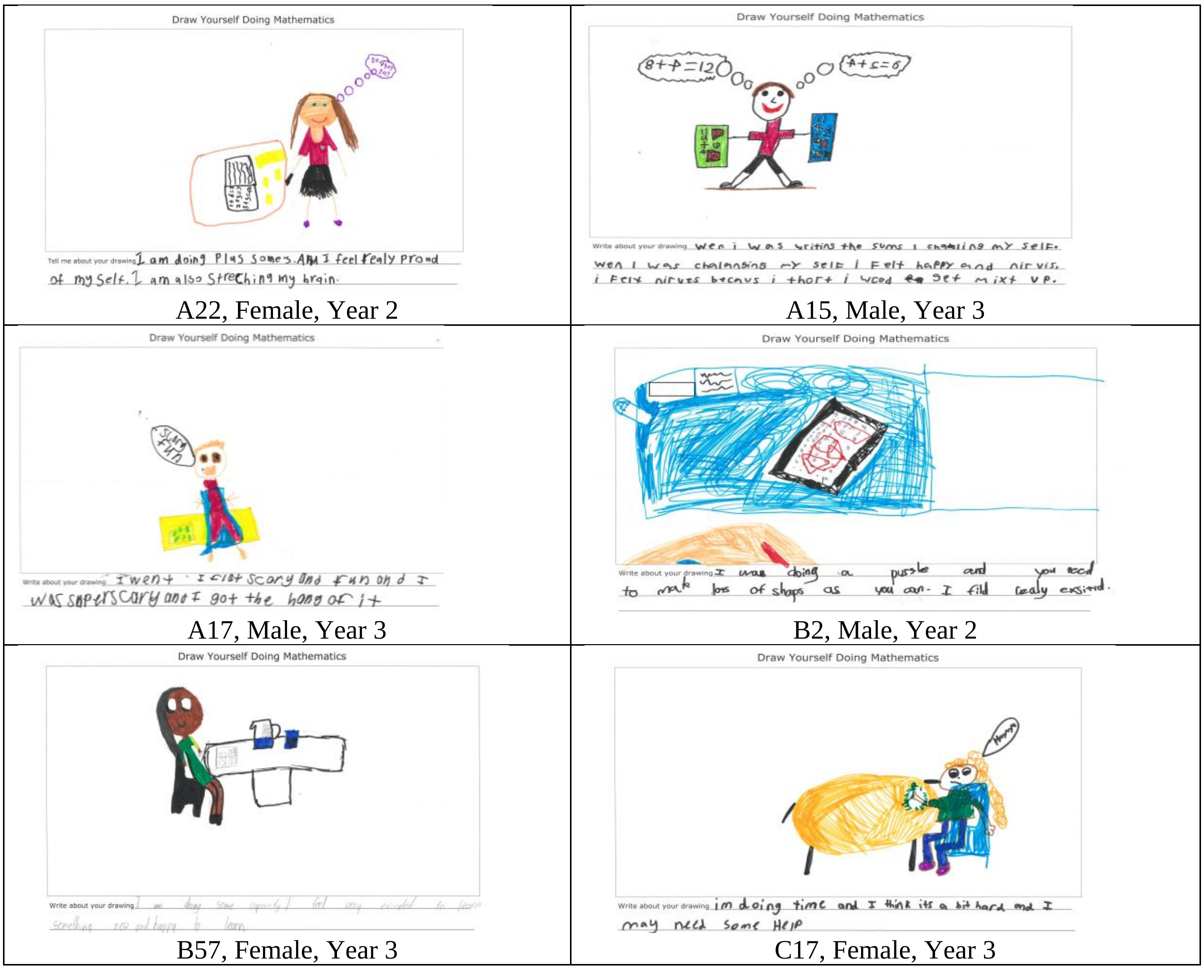

Figure 1. Children drawing themselves doing mathematics.

In reporting these results, it is easy to resort to the traditional dichotomy definition of attitudes. However, this overlooks the complexity in describing YCATM and how young children view manipulatives, their emotional response towards their use and how combined with their perceived competence. The findings are reported and discussed using MTMA and Bruner's experiential learning stages to describe children's attitudes.

Attitudes Towards Mathematics and Manipulative Use: Enactive Stage

In the non-lesson context, children connected the use of manipulatives to a variety of mathematical topics, predominately number (operations and place value), followed by 2D shapes, time, reading analogue clocks, and money. Children's Emotional Tendency towards using manipulatives varied greatly depending on the topic and manipulative used, which contributed to children's overall Emotional Sentiment towards mathematics. The majority of children, even children who were classified as having a Negative Attitude towards mathematics, appeared to exhibit Positive Attitudes towards manipulatives during enactive learning experiences. For example, when using manipulatives, A10 showed perseverance and interest in what she was doing, and this was a stark contrast to the other observed learning experiences when A10 exuded disdain and negativity.

In contrast, not all children appeared to appreciate using manipulatives in the same way as A10, leading to negative emotions and views of manipulatives. For example, C9 found the enactive phase frustrating and prohibitive, and this was seen in both the non-lesson and lesson context. During the non-lesson context, C9 did not draw any iconic representations of manipulatives. However, he did speak extensively about using manipulatives. C9 described his 'hate-love-hate' relationship with mathematics, attributing number concepts, particularly multiplication, as the cause of his disdain. C9 provided several examples in a non-lesson context of what he thought was annoying "cause [sic] you need to make like one hundred groups of sixty-five," referring to using counters to represent multiplication as an array. In a lesson context, C9 used manipulatives as an opportunity to disengage by wandering the room, collecting a single counter from several locations before returning to his desk. Another child, C17 (see Figure 1), revealed in the non-lesson context that she "sometimes feel a little bit anxious, anxious where I just want to give up" and related these feelings to using analogue clocks to tell the time. C17 has drawn herself using an analogue clock, stating, "I feel like I need some help, and I need to get my brain thinking more."

Attitudes Towards Mathematics and Manipulative Representation: Iconic Stage

Children's drawings were noteworthy sources to examine the confluence of manipulative use, representation, and attitude formation. The use of iconic imagery in the non-lesson context was documented by children in their written descriptions of their drawings. Figure 1 shows a range of iconic representations of manipulatives representing topics from addition and skip counting, subitising, 2D shapes, capacity and clocks. A range of emotions was depicted in the drawings, ranging from happiness and enjoyment (A22, B2, and B57) to feeling nervous and scared (A15, A17, and C17). These sources provided opportunities for further inquiry. For example, A22 liked to use manipulatives, such as the blocks that she has drawn, to help her find a solution, "well, why I chose to do plus sums is because I really like umm solving them with different things and I especially like using the blocks." A22 was able to represent the manipulatives iconically, seeing value in representing her mathematical thinking in multiple ways, contributing to her Vision of Mathematics and her Perceived Competence.

However, during the lesson context, many children appeared to struggle to create iconic representations of the representations used in the enactive stage. Further, they were yet to understand the notion of a productive struggle. For example, B8, when using Polydrons to model a cube, quickly realised that it was not possible to draw multiple 1:1 scale iconic representations of the physical representations that he was creating on an A4 piece of paper. This impediment led to outward signs of frustration, anger and ultimately not pursuing more than two possible solutions. Similarly, B52 struggled to create iconic representations of familiar objects during a fraction learning experience. While other children in B52's group acted as enablers and shared their strategies on how they substituted the familiar object with a diagram, B52 was adamant that it was not achievable. It appears that transitioning from enactive to iconic caused some children to outwardly exhibit negative emotions resulting in disengaging, especially in tasks with higher levels of cognitive demand.

The transition between enactive and iconic experiences was exacerbated by introducing iconic experiences before children were ready or had developed the necessary conceptual understanding, as seen in the following vignette. C9, C11 and C21 could identify 3D solids when engaging in a learning experience with 3D wooden attribute blocks and use these blocks to identify and describe related objects in their environment. The children used the attribute blocks to determine the number of faces, corners, or edges by rotating the block to assist in identifying features. However, all struggled with visualising iconic representations of 3D solids as a 2D representation in the form of a net. This struggle is contextualised in the discussion.

Attitudes Towards Mathematics and Manipulative Representation: Symbolic Stage

Children created symbolic representations of manipulative use in both the lesson and non-lesson context. As seen in Figure 1, children wrote number sentences to accompany the representation on manipulatives. For example, A15 writing the number sentence 4 + 2 = 6 to match the numbers shown on the dice that formed part of the enactive stage. In doing so, we see the number formation, including number reversal and how he feels about the symbolic stage, where he admits to feeling nervous about getting "mixed up."

In a non-lesson context, children with Positive Attitudes depicted more complex number sentences and were able to describe mathematical processes to perform the operations depicted. Children with Positive Attitudes were more likely to draw iconic representations of manipulatives and showed how they enacted their use in their drawings. For example, B1 (Year 2) wrote 636 + 636 and depicted the process of partitioning ("chunking") to work out the answer. Other children used number lines to show repeated addition of two-digit numbers, while others drew MAB (longs and units) to show the processes they used to add numbers. These children were confident in using multiple representations (an indicator of the TTP) to answer the questions (an indicator of their SC). Further, children with a Positive or Extremely Positive Attitude classification embraced the notion of creating and developing their own symbolic representations, confidently showing their mathematical thinking and working. Children with Neutral, Negative and Extremely Negative Attitude classifications are yet to develop this confidence. Children with a Negative and Neutral Attitude classification struggled in transitioning from physical representations (manipulatives) to visual and symbolic representations.

The lesson context provided further examples of how children used symbolic representations of manipulatives and how creating symbolic representations influenced their attitude towards mathematics. The following vignette is from two children in the same class attempting the same task that required multiple ways of adding numbers to 12. A22 worked independently on the task, regularly making statements about what she was doing. Her self-talk was audible to others but appeared to be directed at no one in particular. The child worked on the task, continuing to self-talk when the teacher clarified the instruction about the task, stating that a number c only be used once. A22 stood up to get an eraser from a different desk and returned to her seat, uttering a mild expletive before erasing some of her work. A22 resumed the task independently and soon resumes the self-talk uttering "9 + 2 + …" and "I've got two done," quickly followed by "I've got four questions done." The child continued to work on the task. In contrast, A18 was reluctant to make a start and appears to be avoiding and delaying work. A18 pushed her chair backwards, away from the table, physically distancing herself from her work, finding other unrelated reasons for not completing the task. Even with teacher prompting, A18 was hesitant and exhibited signs of discomfort and distress. She stood in the doorway, arms folded on her chest, frowning and huffing. She moved further into the corridor so that she could not be seen, occasionally glancing back into the room, remaining there for approximately two minutes. A18 returned to her desk, stating that she had only got one number sentence (3 + 4 + 5) with two children suggesting two solutions. A18 ignored their assistance and began counting out some pop sticks and proceeded to write a second number sentence in her workbook, opting not to share her solutions with other children.

Discussion

The use of manipulatives was not done in isolation; as previously discussed, enactive manipulative use accompanied by symbolic and iconic representations. Attitudes towards mathematics varied in all three stages of Bruner's experiential stages of learning. The largest variation in attitudes were noted in transitioning between the iconic and symbolic stages with several factors identified that contributed to this variation.

The transition from physical to internal representations (Goldin & Shteingold, 2001) via the enactive, iconic and symbolic stages (Bruner, 1996) influenced children's attitudes. The transition between representations is a vital development in the learning and acquisition of mathematics, especially as one of the goals of mathematics education is for "children to create and think critically about mathematics" (Perry & Atkins, 2002, p. 201). Children need time to develop confidence in using physical representations before introducing "conventional notation" (Perry & Atkins, 2002, p. 201). Further, children need the connection between the enactive, iconic and symbolic representations or informal and formal representations to be made explicit (McNeil & Jarvin, 2007). Conversely, spending too long on a particular representation or method can result in frustration and boredom in children, causing Negative or Neutral Attitudes towards mathematics.

During the enactive phase, manipulatives made the intended learning accessible and enjoyable, thereby fostering Positive Attitudes towards mathematics. Moch (2002) found similar results, reporting that children who were previously reluctant were more eager and enthusiastic.). The eagerness of the children in this study manifested in many ways, where we see children such as A10 and A22 wanting to complete tasks that involve the use of manipulatives where previously, there were reluctant to engage. In contrast, some children in the study appeared reluctant to use manipulatives to work through cognitively demanding tasks, even after the teacher prompted the use of specific manipulatives. It seemed for these children, that the use of manipulatives was viewed as a last resort and not a useful mathematical tool. For a minority of children who described discomfort with mathematics, manipulatives were a tool that was used to actively or passively disengage from mathematical learning experiences. McNeil and Jarvin (2007) found similar results, reporting that manipulatives, in some cases, can be detrimental to learning. Further research is recommended to identify the relationship between disengagement and manipulatives, as it is possible that students are unfamiliar with the manipulative and do not know how to use it to support their conceptual development.

The transition between enactive and iconic was further inhibited by introducing the iconic representation of 3D solids before it was formally introduced, which is currently located in the Year 5 *Australian Curriculum: Mathematics*. Consequently, introducing iconic representations too early was shown to cause confusion and frustration. The children's Emotional Tendency was not the only dimension of attitude impacted. Children's Mathematical Mindset and Self-concept were also negatively impacted.

Conclusion

The Modified Three-dimensional of Attitude was a flexible framework that moved beyond the identification and classification of children's attitudes to analyse factors that influenced attitudes. The children in this study used many types of representations, with manipulatives emerging as a predominant representation. Investigating how students use and create with manipulatives went beyond gaining insights into their mathematical thinking. Rather, insights were gained about how children viewed manipulatives, how children felt about and their confidence in using these materials. Several themes emerged when investigating the confluence of attitudes towards mathematics and the use of manipulatives. When used effectively and timely, manipulatives provide a solid basis for developing flexible external and internal representations of mathematics and contribute to the formation of Positive Attitudes towards mathematics. Other variables, however, influenced how children viewed and used manipulatives. Disengagement was documented in several cases, where children used manipulatives as a façade for doing mathematics and were a means to actively and passively disengage. A third theme related to the transition between enactive, iconic and symbolic representations was noted and how this can contribute to Negative Attitudes towards mathematics. These contextual variables and the cues and signs children provided need to be considered when planning mathematical learning experiences.

Acknowledgements. I would like to acknowledge Professor Mohan Chinnappan and Dr Sven Trenholm for their supervision of this research.

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Problem-solving Proficiency: Prioritising the Development of Strategic Competence

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This study explored the problem-solving proficiency of Australian Indigenous students in one community school. Diagnostic tests and Newman's problem-solving interviews were utilised to identify common barriers to primary students' problem-solving success. Students' responses to problem-solving tasks were analysed using the five strands of mathematical proficiency and Newman's problem-solving hierarchy. The findings demonstrated that most students were working towards developing their strategic competence, rather than procedural aspects of the problems. This study is significant for the practical insights gained pertaining to the importance of strategic competence in developing students' problem-solving proficiency.

Problem solving in mathematics has received attention in curriculum reforms over many decades, resulting in its emphasis in international curricula (e.g., Anderson, 2014; Australian Curriculum, Assessment and Reporting Authority [ACARA], 2018; Liljedahl et al., 2016; National Council of Teachers of Mathematics [NCTM], 1989). Today, the importance of problem solving is recognised as a central part of students' mathematical literacy and 21st century skills (Organisation for Economic Co-operation and Development [OECD], 2022). Whilst the push for developing students' 21st century skills such as creativity and critical thinking has occurred recently, a focus on improving students' problem-solving proficiency is not new (e.g., Hiebert et al., 1996). The Trends in International Mathematics and Science Study (TIMSS) results continue to demonstrate that problem solving proves to be a challenge for students in mathematics (Mullis et al., 2021). In the Australian context, supporting the problem-solving proficiency for Indigenous learners is particularly significant given inequity in outcomes (Howard et al., 2010). Currently, the capability of Indigenous students (Miller & Armour, 2021; Sarra & Ewing, 2014) is not reflected in the proportion of Indigenous students performing at or above the expected standard for problem solving on international standardised tests in mathematics (Thomson et al., 2020). Though focused on Indigenous learners, the study findings have broad implications for quality primary mathematics education practices that support problem-solving proficiency. Therefore, this paper sets out to discuss generalisable findings for all students in the primary years. The implications of findings for Indigenous learners specifically are focused on and discussed in further detail in Reid O'Connor and Norton (2020). Interviews in conjunction with diagnostic tests were utilised to answer two research questions for primary students in one Indigenous community school:

(1) What is the current state of students' problem-solving?

(2) What are the barriers to students' problem-solving proficiency?

Theoretical Framework

The guiding theoretical framework supporting both the research design and data analysis of the study drew on Kilpatrick et al.'s (2001) seminal work describing mathematical proficiency as five interrelated strands, and the stages of problem solving detailed in Newman's problem-solving hierarchy (Newman, 1983). The five strands of mathematical proficiency encompass conceptual understanding, procedural fluency, adaptative reasoning, strategic competence, and productive dispositions towards mathematics (Kilpatrick et al., 2001). Conceptual understanding and procedural fluency are often central in Australian and

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 474–481. Launceston: MERGA.

international mathematics curricula (e.g., ACARA, 2018; NCTM, 2014). In addition to conceptual understanding and procedural fluency, Kilpatrick et al. (2001) defines strategic competence as "the ability to formulate mathematical problems, represent them, and solve them" (p. 124), and notes that it is often associated with problem solving. Despite the central role of strategic competence in problem solving, little attention has been given to such competence in mathematics education research. Adaptive reasoning refers to a student's ability to justify and reflect on their understanding of concepts (Kilpatrick et al., 2001). Productive dispositions encapsulate a students' ability to perceive themselves as a capable learner of mathematics, as well as their perception of mathematics as useful and worthwhile (Woodward et al., 2017). As the only affective proficiency strand, productive dispositions are not included when describing mathematical proficiency in Australian mathematics curricula, however it is a critical strand given the development of the other four strands are dependent on its presence (Woodward et al., 2017). The five strands of mathematical proficiency framework is useful when evaluating students' mathematical problem-solving proficiency, particularly as all but adaptive reasoning align with Newman's problem-solving hierarchy.

Newman's hierarchy considers the distinct stages that a student must work through to successfully complete a problem-solving task (Newman, 1983). Newman's seminal 1977 work on mathematical error analysis determined that a student must work through the five steps outlined in Figure 1 to successfully solve a worded mathematics problem. These five steps are considered hierarchical and linear, in that failure in any preceding step prevents a student from successfully moving forward in solving the problem, and ultimately results in an incorrect answer. For Indigenous learners', reading is an important stage to consider given that the 2018 PISA findings noted a two-and-a-third year gap between Indigenous learners and non-Indigenous peers in reading (Thomson et al., 2020).

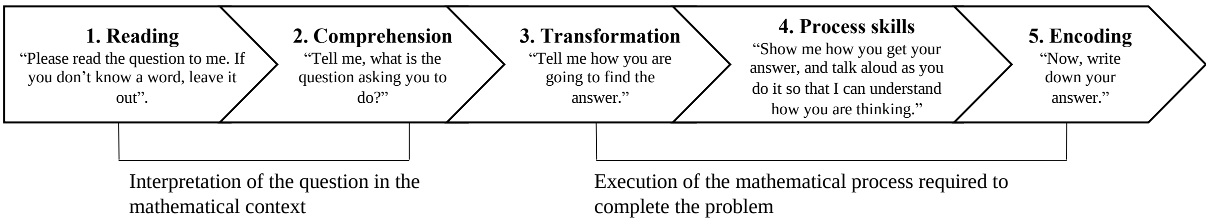


Figure 1: Newman's hierarchy of solving a worded mathematics problem. Figure adapted from Reid O'Connor and Norton (2020) and questions adapted from White (2005).

Following Newman's hierarchy to conduct an interview with a student reveals their mathematical proficiency. A students' conceptual understanding can be evaluated by considering their responses to the comprehension, and transformation questions. Strategic competence can also be observed as a student responds to the transformation question as they formulate and represent the mathematical problem. As a student answers the process skills question, procedural fluency can also be observed. Whilst productive dispositions are not directly assessed in a Newman interview, students' engagement in the questions can be observed. The framework of classifying mathematical proficiency as five strands is useful in identifying students' strengths in mathematics, as well as areas in which further support might be needed. The five strands of mathematics proficiency and Newman's problem-solving stages were utilised in this study as methods of classifying and identifying students' problem-solving barriers. The framework was critical in analysing students' mathematical proficiency prior the beginning of the initiative as these findings then informed the development of the initiative, allowing for practices to be tailored to and focused on students' needs.

Research Design

Overview of Research Design

The research design for this study was mixed methods, which included the collection and interpretation of both quantitative and qualitative data to provide breadth and depth of data to explain the complex phenomenon (Cohen et al., 2011). The mixing of methods supports an understanding of Indigenous ways of knowing that transcends what qualitative methods alone can provide (Botha, 2011). As part of the larger study, a mathematics initiative was conducted over a seven-month period from March to October of the school year. The initiative focused on increasing teachers' use of effective pedagogies in primary mathematics education, as identified in a review of literature (see Reid O'Connor [2020] for further details of the recommended practices that guided the initiative).

The research design of this study addresses a gap in mathematics education research evident for the past 30 years. In a systematic review of 28 studies by Miller and Armour (2021), only three studies gave space to students' voices by including interviews with Indigenous students. This research is significant because the research design opens the opportunity for Indigenous student voices to be heard and showcased.

Study Sample and Context

The findings from three composite age classes, spanning from Year 3 to 6 (ages 8 to 12) with a sample of 37 students, are reported in this paper. The students in the study were of Aboriginal and Torres Strait Islander heritage, encompassing a great diversity of people. The local context was an urban capital city in Australia. The study school was a P–12 community school (under the banner of an Australian Independent school, which is government funded, but run by an Indigenous community identified school board) for Indigenous students. The researcher was a non-Indigenous mathematics teacher who had taught at the school for several years prior to the study, meaning that they had established working relationships with all teachers and students prior to the beginning of the initiative. The prolonged engagement of the researcher in the school potentially increased the credibility and validity of findings.

Data Sources and Data Analysis

Diagnostic testing has often been utilised in studies with Indigenous students (e.g., Miller, 2015; Warren & Miller, 2013). A diagnostic data source provides information on students' achievement levels (what students know), but also makes visible students' understanding of the concept (how students know) by facilitating error analysis. Error analysis of students' scripts from diagnostic assessment reveals students' strengths in the cognitively focused proficiency strands. The diagnostic test utilised in this study was developed by Booker (2011). The tests were conducted in small groups by the researcher. In this paper, the findings from the problem-solving component of the diagnostic tests for addition, subtraction, and multiplication will be reported. Three questions for each operation were administered (see Table 1 for example questions). The mean achievement for each cohort was determined, as well as the frequency of particular errors. Errors were coded following an emergent design (Cohen et al., 2011). Categories of errors were then linked to the three cognitively focused strands of mathematical proficiency that could be observed on the written test (conceptual understanding, procedural fluency, strategic competence). Following the diagnostic tests, Newman interviews were conducted with individual students by the researcher. The interviews were carried out for each operation with each student. The procedure of Newman interviews involved providing students with a new copy of a problem-solving question that they had previously answered incorrectly on the diagnostic test, and sequentially asking the five questions outlined in Figure

1 (White, 2005). Students' attempts were coded by recording the first interview question answered incorrectly (the stopping point), and this determined what stage of the problem-solving process students were currently working towards. Detailed examples of each of the five Newman error types are outlined in Reid O'Connor and Norton (2020).

Table 1.
Addition, Subtraction, and Multiplication Problem-solving Question Examples

Addition 1.	Jackie had 138 marbles. She played with Cathy and Kylie and won 65 more marbles. How many marbles does she have now?
Addition 2.	Gabriel's parrot lost 7 black feathers and 6 blue feathers. How many feathers did his parrot lose?
Addition 3.	At the zoo, the giraffes each had 768 bales of hay. The seals eat 392 buckets of fish. The elephants eat 695 bales of hay. How many bales of hay were eaten?
Subtraction 1.	On Saturday, Ruby and Dora sold ice-creams. They sold 47 chocolate, 32 vanilla and 19 strawberry ice-creams. How many more chocolate than strawberry ice-creams did they sell?
Subtraction 2.	Sam sells milkshakes. He has a 12-litre milk container. He put 5 litres of milk into it. How many more litres of milk does he need to fill his container?
Subtraction 3.	Russell and Jason sold 356 cans of lemonade at the swimming club by 12:00 PM. They also sold 167 bottles of water by this time. The club had 480 cans of lemonade and 360 bottles of water available to sell. How many more cans of lemonade can they sell before it is all gone?
Multiplication 1.	Simone planted 4 rows of strawberries in the morning and 3 rows in the afternoon. Each row had 48 plants. How many strawberry plants did she put in her garden?
Multiplication 2.	A tap leaks 3 litres of water each day. How much water will it leak in one week?
Multiplication 3.	Ali worked at a nursey. During the morning he added fertiliser to 46 pot plants and watered 34 rows of plants. In the afternoon, he watered 52 rows of plants and fertilised 27 pot plants. If there are 68 plants in each row, how many plants did he water?

Findings

Analysis of each cohort's mean score on the diagnostic test items found that students in all cohorts were still developing their problem-solving proficiency for each of the computations (see Table 2). To evaluate how students' problem-solving proficiency could be supported throughout the initiative, error analysis revealed trends in common errors across the sample (see Table 3). Errors that occurred two or more times only are reported for brevity. Students' not attempting the question was the most frequent error observed. A non-attempt can either occur because a student does not know how to begin to attempt a question (conceptual understanding or strategic competence) or does not want to attempt a question (productive dispositions). That is, the error can be a result of a student who is still developing conceptual understanding of the tested concept or skill, strategic competence in formulating the problem, or productive dispositions towards the task. Other than non-attempts, the most frequent errors were associated with students' strategic competence. This was observed in students' attempts indicating that they were still developing their capacity to select the correct operation required, or the correct values in questions where additional information was included. Examples of these errors can be seen in Figure 2.

Table 2.
Mean Scores for Year 3/4, 4/5, and 5/6 for the Problem-solving Diagnostic Test Questions

	Mean scores %		
	Year 3/4, *n* = 12	Year 4/5, *n* = 11	Year 5/6, *n* = 14
Addition	22.3	27.3	52.3
Subtraction	14.0	9.0	35.7
Multiplication	5.7	15.0	19.0
Total for all questions	14.0	17.1	35.7

Table 3.
Frequency of Errors for Year 3/4 (n = 12), Year 4/5 (n = 11), and Year 5/6 (n = 14) for Addition, Subtraction, and Multiplication Problem Solving

Error	Error Type	Error frequency
No attempt	Unknown	107
Did not isolate correct values	Strategic competence	54
Carried out question using incorrect operation	Strategic competence	37
Unsure of error cause, no working shown	-	32
Question only partially complete	-	7
Number fact error	Procedural fluency	7
Did not rename values when subtracting, just found the difference	Conceptual understanding	4
Copied values from the question incorrectly	Procedural fluency	3

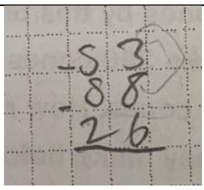
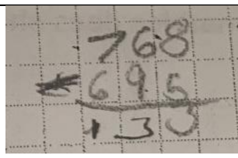
Question: On Friday, Russell and Jason sold drinks at their swimming club. They sold 53 lemon drinks, 58 bottles of water and 26 orange drinks. How many more lemon than orange drinks did they sell? 	Question: At the zoo, the giraffes eat 768 bales of hay. The seals eat 392 buckets of fish. The elephants each 695 bales of hay. How many bales of hay were eaten? 

Figure 2: Examples of student problem-solving errors. Left: A Year 4 student's error associated with selecting the correct values. Right: A Year 4 student's error associated with choosing correct operation.

Difficulties in selecting the correct operation and correct values relate to a student's strategic competence in formulating the problem. When a student was still developing proficiency in formulating problems, "number grabbing" techniques manifested as selecting and operating on all values in the question. It was notable that errors associated with procedural fluency, that is carrying out the actual computations required to complete the question, were not observed frequently compared to other error types.

Given that non-attempts were prevalent on the diagnostic tests, analysis of student errors from written scripts alone did not completely reveal students' problem-solving proficiency, and Newman interviews were able to provide further insight. Table 4 details the frequency of each stopping point during the Newman interview. The findings indicated that the comprehension question (i.e., "tell me, what is the question asking you to do?") was the most frequent stopping point for students. This is the first hurdle for students when problem solving after reading the question successfully.

Table 4.
Frequency of Year 3/4, 4/5, and 5/6 Students’ Stopping Points During Newman Interviews for Addition, Subtraction, and multiplication (*n = 36*).

	Frequency of stopping point
Reading	2
Comprehension	31
Transformation	18
Problem solving	7
Encoding	0
Correct	23

Students’ verbal responses to the question during interviews revealed that being unable to identify what the question was asking them to do (i.e., comprehension) was often the cause of students choosing the incorrect operation or selecting the incorrect values. In such circumstances, students resorted to number grabbing strategies when they could not comprehend what the question was asking them to do. An example of a student drawing on number grabbing strategies as their proficiency in comprehending the question was still developing is outlined in the transcript in Figure 3.

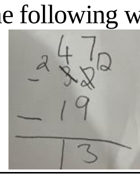
Question: On Saturday, Ruby and Dora sold ice-creams. They sold 47 chocolate, 32 vanilla and 19 strawberry ice-creams. How many more chocolate than strawberry ice-creams did they sell?	
1. Reading: Please read the question to me. If you don’t know a word, leave it out.	Student correctly read the question.
2. Comprehension: Tell me, what is the question asking you to do?	“I don’t get this. Take 47, 32, and 19?”
3. Transformation: Tell me how you are going to find the answer	Student wrote sum 47-32-19.
4. Process skills: Show me how you get your answer, and “talk aloud” as you do it, so that I can understand how you are thinking.	Student would not verbally express thinking and recorded the following working. 
5. Encoding: Now, write down your actual answer.	“13”

Figure 3: Interview script from Year 4 student demonstrating number grabbing strategies.

Interview responses such as this indicated that when comprehension was still developing, difficulties in transformation (forming the correct mathematical computation to solve the problem) also occurred. However, in some instances students who indicated comprehension of the tasks, but manifested subsequent transformation difficulties, also resorted to number grabbing strategies. An example of this is outlined in Figure 4. Overall, these findings indicated that when students were still developing their skills in in comprehending and transforming the question into the correct mathematical operation, the common errors that occurred included selection of the incorrect operation or selection of the incorrect values from the question. These errors are associated with the strand of mathematical proficiency relating to strategic competence. The findings concur with the findings from the analysis of the written problem-solving test scripts.

Question: On Saturday, Ruby and Dora sold ice-creams. They sold 47 chocolate, 32 vanilla and 19 strawberry ice-creams. How many more chocolate than strawberry ice-creams did they sell?	
1. Reading: Please read the question to me. If you don't know a word, leave it out.	The student correctly read the question.
2. Comprehension: Tell me, what is the question asking you to do?	"How many more…" [*Student groaned in frustration*] "I don't get that." [*Student re-read the question aloud*] "So it's asking me how many more ice-creams from chocolate than strawberry ice-creams are they going to sell?"
3. Transformation: Tell me how you are going to find the answer	"I'm going to get all the numbers" [*Student wrote down all the values from the question*] "So I have to figure out how many more they're going to sell? Could they sell 19? I think they're going to sell 32 … the same amount as that [*student pointed to chocolate value – 47*] so they get the same amount of chocolate."

Figure 4: Interview script from Year 4 student demonstrating number grabbing strategies.

Discussion and Conclusion

In relation to students' problem-solving proficiency, it is important that instruction focuses on developing students' strategic competence by increasing their competency in understanding problem structures. Firstly, strategic competence needs to be developed as a skill distinct to computational proficiency within the context of the problem-solving tasks. Students across each cohort had various difficulties with problem-solving tasks which were related to strategic competence (i.e., comprehending and transforming the task into an appropriate computation) as measured by diagnostic tests and Newman interviews. The procedural elements of the problem (e.g., carrying out the computations) were not the primary difficulty for students. Rather, comprehending and transforming the worded task into the appropriate mathematical computation was the critical difficulty. A priority on developing strategic competence is supported by the hierarchy of steps proposed in Newman's error analysis (White, 2005), which places comprehension and transformation prior to process skills (i.e., procedural fluency).

Prevalent errors pertaining to strategic competence that were impacting students when problem solving were associated with the selection of the incorrect operation or the selection of the incorrect values on which to operate. Students' comprehension difficulties manifested as students' employing "number grabbing" techniques in an attempt to solve the problem without understanding the task, or with students seemingly selecting a random incorrect operation in an attempt to complete the task (as observed in analysis of the diagnostic test scripts). Students' key problem-solving difficulties surrounding strategic competence found in this study highlight the importance of exploring and discussing problem structures with students. For example, working with students to identify what within the problem signifies that it is subtraction, and exploring strategies to correctly identify the values on which to operate.

These findings are contextually important for Indigenous learners considering literacy difficulties, as measured by standardised tests (e.g., PISA, Thomson et al., 2020), are commonly cited as impacting on learning. However, this study sheds further light on this matter and goes beyond identifying broad reading or literacy difficulties. The findings from Newman interviews found that it was not the mechanics of reading the problems that was the difficulty for learners, but it was the ability to comprehend. This finding has implications for directing teaching and intervention focuses for learners with problem-solving difficulties. Beyond the context of this study, understanding the critical role of strategic competence highlights the value in targeting students' ability to comprehend and transform mathematical tasks for all learners. In conclusion, this study highlights the value of Newman interviews in making visible students' problem-solving proficiency. In future research, the ways in which the theoretical framework could be expanded or revised to incorporate Indigenous perspectives would be beneficial to explore further and could work to centre students rather than the problem-solving processes. Overall, the theoretical framework drawing on Newman's problem-solving hierarchy and the five strands of mathematical proficiency was shown to be a practical and effective framework to analyse students' problem-solving processes.

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Using Enabling and Extending Prompts in the Early Primary Years When Teaching with Sequences of Challenging Mathematical Tasks

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The current study explores how teachers report using enabling and extending prompts when teaching with sequences of challenging mathematical tasks. Twenty-nine early years primary school teachers completed a questionnaire following their participation in a professional learning project. Findings suggest that teachers': view prompts as important when teaching with challenging tasks; generally prepare prompts in advance of the lesson; consistently allow students to engage with the core task before making prompts available; and consider prompts equally valuable for augmenting learning across all content areas.

Catering to the diversity of learners in their classroom is one of the most significant issues teachers report facing in their practise (Shernoff et al., 2011). Gervasoni and Peter-Koop (2020) noted that, "teachers at all levels struggle to meet the challenge of providing a high-quality inclusive mathematics education that enables all students to thrive" (p. 1). One approach to meeting this challenge that has been in focus over the past decade in Australia and New Zealand is to teach mathematics through challenging tasks, and use enabling and extending prompts to differentiate instruction (Davidson et al., 2019; Ingram et al., 2020; Sawatzki & Goos, 2018; Sullivan et al., 2016).

Enabling prompts are intended to provide students with an additional learning experience that is carefully connected to the main problem-solving task that has been offered to the class. This additional learning experience involves: reducing the number of steps; simplifying modes of representation; making the task more concrete and/or reducing the complexity of the numbers involved (Sullivan et al., 2006). Extending prompts, on the other hand, are prepared for those students who have completed the main problem-solving task. They expose these students to an additional task that has a higher level of cognitive demand, but involves the use of similar reasoning, conceptualisations and representations as the original task (Sullivan et al., 2006). See Table 1 for an example of a challenging task, with an associated enabling and extending prompt.

Table 1
Example of a Challenging Task, and Associated Prompts (Year 1-3)

Challenging Task	Enabling Prompt	Extending Prompt
I visited a farm and counted 8 heads. How many legs did I see?	Draw a picture of your family. How many heads and legs are in your family altogether?	Imagine you only saw one type of animal on the farm when you counted the 8 heads. What are the different possibilities for how many legs there might have been? Explain how you have found all possible solutions.

There is evidence from studies undertaken in a primary school context that teachers who have taught with challenging tasks across multiple lessons both use prompts consistently and view prompts as useful for differentiating learning. Sullivan et al. (2016) supported 30 Year

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 482–489. Launceston: MERGA.

3/4 teachers to implement ten lessons constructed around challenging tasks. An element of the research project involved these teachers completing a proforma that documented their facilitation of the lesson, including how enabling and extending prompts were incorporated. The median number of students who accessed an enabling prompt across the five lessons described by Sullivan et al. (2016) was: 4 (Lesson 1), 4 (Lesson 2), 4 (Lesson 3), 5 (Lesson 4) and 10 (Lesson 5), and almost all teachers provided enabling prompts to at least some students in each lesson. Moreover, the mean time until teachers made prompts available was in the range of 6 to 7 minutes for each of the tasks reported on. By contrast, the median number of students who accessed the extending prompts was: 5 (Lesson 1), 6 (Lesson 2), 6.5 (Lesson 3), 6.5 (Lesson 4) and 1 (Lesson 5). Collating the data from the Sullivan et al. (2016) study, and assuming class sizes of 25 students, it appears that the "typical" (median) teacher in a "typical" (median) lesson involving challenging tasks might administer enabling prompts to around one-fifth of students in the class and extending prompts to around one-quarter of students; although there was substantial variation across both teachers and tasks.

Reporting on a related project, Cheeseman et al. (2017) used grounded theory to analyse 37 Year 3 to 6 primary teachers' responses to an open-ended questionnaire item to ascertain what factors these teachers considered when choosing to use enabling and extending prompts with their students during a lesson involving a challenging task. They found that teachers used enabling prompts for several interrelated reasons, including: to assist the thinking of students who were struggling with the main task, to make the main task more accessible and to support student understanding more generally. They also noted that some teachers reported using prompts to encourage students who lacked confidence, and to facilitate students experiencing success. By contrast, teachers used extending prompts to: maintain the level of mathematical challenge and extend student thinking; to challenge particular students; to match tasks to student thinking; and to invite students to apply their knowledge.

Providing further evidence that prompts are an important aspect of teaching with challenging tasks, Clarke et al. (2014) reported on the responses of 36 Victorian upper primary school teachers involved in a professional learning initiative in terms of strategies that they believed supported student persistence on challenging tasks. It was revealed that attending to differentiation, which included developing and identifying prompts, was the most often-used strategy reported by teachers when planning a lesson to encourage student persistence. By contrast, differentiation was the second most frequently described strategy during the lesson itself for encouraging student persistence, after questioning students and supporting students to reason mathematically. This suggests that preparing enabling and extending prompts for students during planning is likely to be valuable for supporting differentiation, independent of whether students actually use prompts during the lesson.

There is also evidence that students themselves find prompts valuable, at least in the case of enabling prompts. Russo et al. (2020) invited 132 Year 3–6 students to complete a questionnaire describing their attitudes towards enabling prompts in classroom environments where they were expected to access prompts themselves. Students consistently reported that having access to enabling prompts allowed them to be successful with, and take control of, their mathematics learning. Importantly, there was little evidence of any stigma or embarrassment associated with accessing enabling prompts, with the authors concluding that classroom teachers can rapidly establish a culture where students access such supports themselves to support learning mathematics through problem solving.

Much of the existing research into how teachers use and view enabling and extending prompts has focussed on the middle and upper primary years. Consequently, the current study endeavours to examine teacher perspectives on using enabling and extending prompts when teaching with sequences of challenging tasks in the early primary years. In addition to exploring how prompts are used with younger students, we will endeavour to shed some light on several unresolved issues concerning prompts. First, although often it is assumed that the teacher is responsible for disseminating prompts to students (Cheeseman et al., 2017; Sullivan et al., 2016), more recently it has been demonstrated that students in Years 3–6 can benefit from accessing prompts of their own volition (Russo et al., 2020). In the current project, we have deliberately kept the decision as to whether the enabling prompts should be teacher or student initiated ambiguous, in order to probe how project teachers choose to use enabling prompts in their classrooms. Second, although project teachers are generally encouraged to view enabling and extending prompts as an aspect of the lesson planning process (Sullivan et al., 2016), it might be that teachers also generate enabling and extending prompts "on the spot" as they attempt to find ways to optimally differentiate the lesson for students. We will ask project teachers the extent to which prompts were pre-planned or created in vivo. Third, prior research has not inquired into whether teachers view enabling and extending prompts as being equally useful for supporting all mathematical content areas. In fact, our project team has assumed that prompts are perhaps less useful in areas such as measurement and geometry, where the tasks are often inherently low-floor and high-ceiling. Whether or not this view is held by project teachers will be empirically examined. Finally, other than the Sullivan et al. (2016) study, other research in this area has not considered either the proportion of students who access enabling and extending prompts during work on a challenging mathematical task, nor the typical "wait time" before teachers make such prompts available. This will also be a topic of inquiry for the current study. Our research question is:

> *How do teachers utilise prompts to support student mathematical work in a sequence of learning involving challenging tasks in the early years of primary school?*

Method

Our project has involved supporting two school systems in Australia (Melbourne Archdiocese Catholic Schools and Catholic Education Diocese of Parramatta) integrating an approach into their mathematics instruction that can be described as student-centred structured inquiry (Sullivan et al., 2021). The project has targeted generalist, early years (Foundation-Year 2; 5–8-year-olds) primary school teachers. We have provided participating schools with access to up to 14 sequences of problem-solving tasks intended to be connected, cumulative and challenging which are introduced to teachers on the first project day. In addition, schools received some professional learning support around planning mathematics instruction, and how to use the tasks from the sequences, both from the project team, and in-house, system-level instructional experts. The project began formally in 2019, although has been disrupted considerably over the past two years because of COVID-19.

The final project day for Melbourne Archdiocese Catholic Schools was held remotely during November 2021. Attending teachers were invited to complete a questionnaire describing their experience of implementing the sequences in 2021. Twenty-nine teachers from eight schools completed the questionnaire. Participants taught Foundation (45%), Year 1 (45%), and/or Year 2 students (34%), with several teaching composite grades. On average, participating teachers had 9.8 years teaching experience (Range 1 to 30), and almost all were female (n = 27; male, n = 2). Most participating teachers (62%) who completed the questionnaire had also been involved in the project the previous year.

Several of the items related to participants' experience of using enabling and extending prompts to support students to engage with the sequences of challenging tasks in the classroom (as opposed to during remote learning). Reporting on this data will be the focus of the current paper. Data were organised and analysed using SPSS Version 25. Qualitative data from the two open-ended responses were used to supplement the quantitative analysis through illustrating and fleshing out trends apparent in the quantitative data.

Results and Discussion

Prompt Preparation and Administration

When asked whether they have their enabling prompts prepared ahead of time before the lesson begins, over three-quarters of teachers noted they did so frequently; this is all, or most, of the time (*n* = 23; 79%). By contrast, less than one-quarter of teachers (*n* = 6; 21%) indicated that they frequently came up with enabling prompts ‘on the spot’ during the lesson (see Table 2). One teacher described the process of developing prompts during the lesson planning phase: “During planning, we discuss the task and talk about where students might have misconceptions or get stuck. We then use this to plan enabling prompts.” (T16). An even more dramatic difference was evident when comparing the numbers of teachers who reported frequently preparing extending prompts ahead of time (*n* = 25; 86%), compared with those who frequently came up with them ‘on the spot’ (*n* = 3; 10%). A teacher described how they used pre-planned tasks as extending prompts: “If I noticed students finding the task easy or have found all the solutions or solved it systematically, I give them the extension task” (T7). The idea that prompts were generally prepared in advance is consistent with how such prompts were intended to be used in the project (Sullivan et al., 2021).

Table 2
Enabling and Extending Prompt Preparation and Administration (percentage frequency)

Item	All of the time	Most of the time	Some of the time	Never
I have my enabling prompts prepared ahead of time before the lesson begins	41%	38%	21%	0%
I come up with my enabling prompts ‘on the spot’ during the lesson	3%	17%	72%	7%
I have my extending prompts prepared ahead of time before the lesson begins	38%	48%	14%	0%
I come up with my extending prompts ‘on the spot’ during the lesson	3%	7%	83%	7%
I give enabling prompts to students who are struggling on the main task	24%	45%	28%	3%
My students access enabling prompts themselves if they are stuck on the main task	0%	24%	59%	17%

As noted in the introduction, we left it deliberately ambiguous in the current project as to whether teachers might encourage students to access enabling prompts themselves, or instead disseminate prompts directly to students when needed, given contrasting recommendations in the literature (Russo et al., 2020; Sullivan et al., 2006). Whilst over two-thirds of teachers frequently gave enabling prompts to students who were struggling (*n* = 20; 69%), only around one-quarter of teachers (*n* = 7; 24%) observed that students frequently accessed enabling prompts themselves if they were struggling. This may have reflected the limited cognitive maturity of these young students, as implied by one teacher:

> Some students would be willing to take the enabling prompts on their own, however others would sit there struggling and become overwhelmed and I would need to give the enabling prompt to those students as they wouldn't take the initiative themselves. (T15)

This suggests that encouraging students to access enabling prompts of their own volition was relatively unusual, and that, consistent with Sullivan et al.’s (2016) findings, most teachers in the current study viewed disseminating prompts as an active pedagogical action they could take when students were exploring challenging tasks and encountered difficulty.

Prompt Utility and Classroom Culture

The data in Table 3 and Table 4 reveal that almost all teachers agreed, or strongly agreed, that both enabling prompts (*n* = 28; 97%) and extending prompts (*n* = 29; 100%) are an important component of teaching with challenging tasks. This suggests that project teachers found such prompts valuable for differentiating instruction, a finding that has been noted elsewhere in the literature (Clarke et al., 2014). One teacher highlighted the importance of enabling prompts for providing a pathway into the main task for some students:

> After the suggestion is launched and students are given time to come up with ideas, if students have not begun, and look like they are unable to begin, then an enabling prompt may be suggested; if that is successful then I would encourage them to have a go at the original suggestion. (T17)

Interestingly, more teachers strongly agreed that extending prompts (*n* = 13; 45%) were important in supporting such lessons than was the case for enabling prompts (*n* = 8; 28%). As one teacher indicated: "I enjoy using extending prompts for those children that need to be challenged." (T5).

Table 3
Enabling Prompt Utility and Classroom Culture (percentage agreeing/disagreeing)

Item	SA	A	N	D	SD
Enabling prompts are an important component of teaching with sequences of challenging tasks	28%	69%	3%	0%	0%
There is a negative stigma associated with accessing enabling prompts in my classroom	0%	10%	31%	38%	21%
Enabling prompts are useful for supporting students when working on challenging number tasks	24%	72%	3%	0%	0%
Enabling prompts are useful for supporting students when working on challenging measurement tasks	28%	69%	3%	0%	0%
Enabling prompts are useful for supporting students when working on challenging geometry tasks	28%	62%	10%	0%	0%

Table 4
Extending Prompt Utility and Classroom Culture (percentage agreeing/disagreeing)

Item	SA	A	N	D	SD
Extending prompts are an important component of teaching with sequences of challenging tasks	45%	55%	0%	0%	0%
There is pride associated with accessing extending prompts in my classroom	24%	34%	38%	0%	3%
Extending prompts are useful for stretching students on challenging number tasks	38%	59%	3%	0%	0%
Extending prompts are useful for stretching students on challenging measurement tasks	41%	55%	3%	0%	0%
Extending prompts are useful for stretching students on challenging geometry tasks	38%	55%	7%	0%	0%

Only a relatively small number of teachers agreed that there was a negative stigma associated with accessing enabling prompts in their classroom (n = 3; 10%). This resonates with student data presented by Russo et al. (2020), which revealed that very few of the 132 Year 3-6 student respondents agreed that they were embarrassed to access an enabling prompt when working on a challenging task (2%), or that accessing enabling prompts meant that a student was bad at mathematics (3%).

Finally, despite our assumption that project teachers may find enabling and extending prompts more useful for number-related tasks compared with measurements tasks, or geometry tasks, we found no evidence that this was the case. Almost all teachers agreed that enabling prompts were useful for supporting students, and extending prompts useful for stretching students, regardless of the mathematical content area being considered. This may be in part due to the fact that enabling prompts in particular were broadly construed by teachers, and, in addition to an enabling task, might have also included other enablers, such as access to manipulatives or "spotlighting" of other student work. Spotlighting involves the teacher pausing exploration of the task to draw the class's attention to a particular student work sample in order to illustrate something of mathematical importance (e.g., revealing how the student overcame a misconception; sharing a solution strategy; describing a novel insight into the task). For example,

> It [the enabling prompt] varies. Some [students] are asked to come to the floor to work with materials, while others are given a verbal prompt, while others can read from a card. T22.

> After 5 minutes I spotlight or share someone's work- this would be the first enabling prompt I would use. I would then ask the students to try the task again having looked/ listened to other strategies. If I find after another 2 minutes this student is not grasping the concept, I would tap their shoulder and say they are invited to use an enabling prompt/ walk around and look at other people's ideas. T14.

Prompt Wait Time and Propensity to Access

Figure 1 indicates that almost all teachers generally waited at least some length of time before making enabling and extending prompts available to students. In fact, a large majority of teachers typically waited at least five minutes to make an enabling prompt available to students (83%) or an extending prompt available to students (93%). The typical median wait time across participants before providing prompts was 5 or 6 minutes for enabling prompts, and 7 or 8 minutes for extending prompts. One teacher described how they waited several minutes before making an enabling prompt available:

> I would wait for students to have seen a spotlight after the 5-minute mark. If they are still struggling after seeing another student's strategy, I would wait 5 minutes and then give them an enabling prompt. If they can do it successfully, I would ask them to revisit their original task. (T19)

Another teacher indicated the importance of a student exhausting the mathematical possibilities of the original task before making the extending prompt available:

> When students have explored all possibilities and are sure they have found them all and can explain how they can prove that they've found them all, then I would pose the extending prompt. (T17)

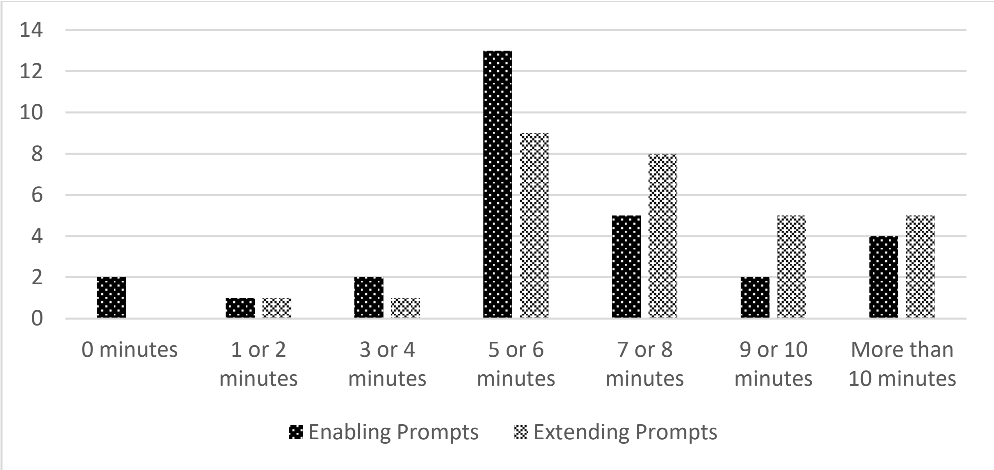

Figure 1. Typical wait time before teachers made prompts available during a typical number task by frequency of response.

The average duration teachers in the current study reported waiting before providing students with enabling prompts was very similar to what teachers described in the Sullivan et al. (2016) study for the typical task (6 to 7 minutes). This is particularly notable because teacher participants in the Sullivan et al. (2016) taught middle years primary school students (Year 3 and Year 4) rather than early years primary school students (Foundation to Year 2), suggesting that the age of the students is a relatively unimportant consideration in terms of when in the lesson teachers choose to make prompts available. In addition, it is interesting to note that for teachers in our study, typical "wait time" for enabling prompts and extending prompts were correlated, suggesting that those teachers who tended to wait longer periods before making the enabling prompt available also waited longer before making the extending prompt available ($r_s = .43$, $p < 0.05$, $N = 29$).

Teachers were also asked to report on the percentage of students that typically accessed an enabling and extending prompt when working on a typical task in the number domain. On average, teachers reported that 17% of students accessed an enabling prompt (range 4% to 80%; median 10%), and 22% of students accessed an extending prompt (range 4% to 55%; median 20%). In comparison to teachers in the Sullivan et al. (2016) study, it appeared that our study teachers reported that their students were slightly less likely to use either type of prompt, although the differences were relatively marginal and not of clear practical significance. Again, it is interesting to note that teacher reports of the percentage of students who accessed enabling and extending prompts were correlated, such that teachers who were more likely to report higher usage of enabling prompts also reported higher usage of extending prompts, and vice versa ($r_s = .51$, $p < 0.01$, $N = 29$).

Conclusions and Future Research Directions

Overall, our results suggest that project teachers perceived enabling and extending prompts to be an important aspect of teaching mathematics through sequences of challenging tasks in the early primary years. In general, teachers prepared prompts ahead of time during lesson planning, and took primary responsibility for administering prompts to students during the lesson. Prompts were perceived as equally valuable for supporting all mathematics content areas, and it was unusual for teachers to report negative stigma associated with students accessing enabling prompts. Furthermore, teachers generally allowed students to grapple with the core task before immediately supporting them with an enabling prompt or offering an extending prompt. Collectively, these findings are consistent with how we intended teachers to use prompts in our project (Sullivan et al., 2021), and resonate with previous research into teachers' use of prompts with middle and upper primary students (Clarke et al., 2014; Cheeseman et al., 2017; Sullivan et al., 2016).

One notable limitation of the current study is that it relies on retrospective self-report data. We intended to use lesson observations and post lesson interviews with teachers to shed further light on how prompts were used, however our data collection efforts were significantly curtailed due to difficulties accessing schools as a result of COVID-19. Future studies might attempt to further validate and illuminate the current findings through systematic lesson observations of several teachers who report different prompt usage patterns (e.g., "high prompt usage", "moderate prompt usage", "low prompt usage"). This would enable further insight into how enabling and extending prompts are used alongside other pedagogical considerations (e.g., access to mathematical manipulatives, use of teacher questioning, encouraging collaborative student work) to facilitate differentiated instruction.

Acknowledgements. The authors are engaged in a project funded by the Australian Research Council, Catholic Education Diocese of Parramatta and Melbourne Archdiocese Catholic Schools (LP 180100611). The views expressed are opinions of the authors who take full responsibility for the ethical conduct of the research and preparation of the article.

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Solving Multistep Problems: What Will It Take?

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Problem solving and reasoning are two key components of becoming numerate. Reports obtained from international assessments show that Australian students' problem solving ability is in a long-term decline. There is little evidence that teachers are embracing problem solving as part of the classroom routine. In this study, we analyse 598 Year 7 to 10 students' responses to a measurement task using Sfard's commognition framework. Four implications lead to recommendations on how to support curriculum, assessment and pedagogical alignment.

Problem solving has always been a key feature of mathematics learning. With the advent of STEM education, globalisation, and the continuing uncertainty caused by the pandemic, it has become central to many educational reform agendas. Weber and Leikin (2016) distinguish four broad traditions of problem solving based research: (i) problem solving as a research tool to investigate other constructs such as understanding of a concept; (ii) as an object of study in terms of the *resources* (knowledge and processes), *heuristics* (strategies), *metacognition*, and *beliefs* (about mathematics); (iii) the activity of problem posing; and (iv) as a didactical tool to teach conceptual understanding. Over the past two decades, problem solving research in Australasia evolved from being hidden by other research, to classroom practice in curricular reform and in the development of more general theoretical conceptions of problem solving as an activity, which is commonly termed as *working mathematically* (Clarke et al., 2007). It was hoped that subsuming problem solving to a general proficiency domain may help emphasise curricular, assessment and pedagogical alignment. The result of taking such a stand may explain the absence of a focus publication on problem solving as an object of study in Australasia since 2008 (Makar et al., 2020). Instead, problem solving appears peripherally in other studies such as STEM education, rich tasks, and learning progression research. Makar et al. also reported a 16-year absence of domain focus research in Algebra and Geometry and Measurement albeit a renewed interest in the latter appeared recently in the form of spatial reasoning.

The scarcity of focus research has dire consequences as there is little evidence that teachers are embracing problem solving effectively as part of their classroom routine. International assessments alert us about Australian students' weakness in geometry and algebra and that their problem solving ability is in a long-term decline (Thomson et al., 2016, 2017). It is estimated that 53% of young Australians did not possess the numeracy skills essential to work effectively in a modern economy (Lamb et al., 2020). Sfard (2021) rightly pointed out, the devil is in the detail. If problem solving is to be nurtured within a general label, attention must be placed in the inter-discursive gaps in the teaching-learning process.

In this paper, we analyse data collected from the *Reframing Mathematical Futures II* (RMF II) project (Siemon et al., 2018). Our purpose is to elicit evidence of students' problem solving and reasoning to highlight gaps when *working mathematically*. This leads to practical implications on how to support curriculum, assessment, and pedagogical alignment.

Theoretical Framework

The *Australian Curriculum: Mathematics* defines problem solving as the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively (Australian Curriculum Assessment and Reporting Authority (ACARA),

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 490–497. Launceston: MERGA.

n.d). Together with understanding, fluency, and reasoning, problem solving is seen as an integral part of becoming proficient in mathematics across the three content strands.

The teaching of problem solving can be traced back to Polya's (1945) *How to solve it*, where students are taught to: 1) understand the problem; 2) make a plan; 3) carry out the plan; and 4) evaluate its effectiveness. Jonassen (2011) cautions on the fallacy of treating problem solving as a reproducible, algorithmic process, assuming that all problems are solved in pretty much the same way, and that generalisable processes can be applied in different contexts with different types of problems in order to yield similar results. He maintains that such views underestimate the role of domain knowledge and pattern recognition (analogical reasoning), resulting in the misrepresentation of knowledge and inhibiting transfer of skills learned. Successful problem solving needs two critical attributes, mental representation of the problem and manipulation and testing of the mental model of the problem to generate a solution.

For Polya, a problem worth solving is one where the solution is unknown to the solver. Consider the Drink Bottles task (coded as GSODA to mean Geometry Soda) in Figure 1 where the context should be familiar to most students. The solution for Part a is fairly straightforward, involving the application of decimal fraction knowledge and proportional reasoning. We acknowledge the difficulties students face when learning fraction and decimal arithmetic (Lortie-Forgues et al., 2015). Part b has several possible solutions and requires an understanding of array, dimensionality, units, and multiplication. In Part c, while the conversion between capacity and mass is given, understandings of magnitudes of measures, that 1L is equivalent to 1000g, and the processes needed to obtain a correct result are needed. To effectively address this task, a solver needs a robust understanding of mathematical concepts and the ability to carry out the processes flexibly, accurately, efficiently, and appropriately. Further, since it is possible to solve the problem by stating the answer without explanation, the task requires students to explain their reasoning. Reasoning is observed when:

> ... students explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, and when they compare and contrast related ideas and explain their choices (Australian Curriculum Assessment and Reporting Authority [ACARA], n.d).

Drink bottles

A 1.25-liter bottle of soda water is 285 millimeters high and has a diameter of 85 millimeters.

a. [SODA1]
How many bottles would be needed to fill a 10-liter container with soda water? Explain your reasoning.

b. [SODA2]
What are the dimensions of a carton that would firmly hold 12 bottles of soda water? Explain your reasoning.

c. [SODA3]
One milliliter of water weighs 1 gram and each empty bottle weighs 80 grams. The cardboard in the box weighs 750 grams. How heavy would the full carton of 12 bottles of soda water be?

Figure 1. Drink bottles task.

Following Sfard's (2021) commognition, reasoning about the process of solving a problem is a form of discourse that requires an understanding of the syntactic and sematic structure of its schemas and is characterised by four components. First, a mathematics discourse is *endorsed narratives* accepted by its participants as faithful accounts of the situation using communicational tools that make the discourse distinguishable from others. In the drink bottles task, the endorsed narratives are about measurement concepts, units, and dimensionality. The communication tools that distinguish the endorsed narratives from other stories hinge on the *keywords* (e.g., litre, millimetres, gram, dimensions) used to explain the focal objects and actions of the discourse. By nature of its abstraction, the third characteristics of a mathematical discourse is the use of *visual mediators* (e.g., symbols, diagrams, and words) to support effective communications. In this instant, a student may use a combination of linguistic, symbolic/algebraic, and diagrammatic tools to explain their reasoning. Indeed, while not necessary, a drawing may help the student to comprehend and clarify their solution for the GSODA task. Lastly, mathematics discourse is made distinct by the *routines*, the recurrent ways of performing different kinds of tasks in obtaining solutions. Routines guide our response to an expectation. They are task specific and depend on their interpreters. As learning is a process of routinisation of our actions, exposing the discursive gaps that threaten the process of learning is a critical step in turning obstacles into opportunities for learning. Since it is not possible to conduct large-scale observations of students' problem solving abilities, the GSODA task served to provide a practical context to help students see the mathematical relevance, and to determine how student coordinate and connect various information that is the hallmark of solving real-life problems faced by the world today. Analysing students' solutions can further help to infer the discursive routines enacted in problem solving situations.

Methodology

The data analysed here are taken from the RMF II project. Using a design-based research method, we applied an iterative cycle of designing, testing, and re-designing assessment tasks and scoring rubrics. Tasks were compiled into multiple assessment forms, both to validate the forms and to test the Learning Progressions, which was the aim of the project. Figure 2 shows the marking rubric for the GSODA task. Note that GSODA3 was included on some forms and not others, thus accounting for the difference in the total sample collected (see Table 1).

SCORE	DESCRIPTION for GSODA1
0	No response or irrelevant response
1	incorrect with no clear reasoning or working
2	Incorrect but with clear attempt to calculate, may use addition and make an error
3	Correct (8 or 8 bottles) but no reasoning or calculations shown
4	Correct, reasoning or working to justify (e.g., 8 × 1.25 = 10 litres)
SCORE	**DESCRIPTION for GSODA2**
0	No response or irrelevant response
1	All dimensions incorrect
2	Height dimensions correct, others not correct
3	Dimensions recognised for array chosen (e.g., 3 × 4; 2 × 6; or 1 × 12) but calculation error in one dimension (e.g., for a 3 × 4 array correctly calculates 4 × 85 mm but incorrectly calculates 3 by 85 mm
4	Correct for array chosen (e.g., 340 mm by 255 mm by 285 mm for a 3 by 4 array). Also correct if a small amount added for width of cardboard
SCORE	**DESCRIPTION for GSODA3**
0	No response or irrelevant response
1	Partially correct working with at least one correct component
2	Error in calculating liquid mass or one component missing
3	Correct (16.71 kg or 16710 g), that is 12 × 1250 g + 12 × 80 g + 750 g = 16710 g or 16.71 kg

Figure 2. GSODA task marking rubric.

The participants were Year 7 to 10 students from across Australia States and Territories. Two groups of cohorts were involved. The first set of data – the trial data, was taken from 214 students from five high schools across social strata in New South Wales, Queensland, and Western Australia. The teachers were asked to administer the assessment tasks and return the student work. Some teachers used the rubrics to mark the responses. All the results were further marked by two markers and validated by a team of researchers to ascertain the usefulness of the scoring rubric and the accuracy of the data entry. The second set of data – the project data, was taken from 377 students from seven high schools situated in lower socioeconomic regions with diverse populations across New South Wales, South Australia, Victoria, and Western Australia. The project schoolteachers were asked to mark and return the raw score instead of individual forms to the researchers. The project schools received two 3-day face-to-face professional learning sessions on developing mathematical reasoning. They also had access to a bank of teaching resources and four on-site visits to support their teaching effort.

Findings

Table 1 shows the overall percentage breakdown of student responses for GSODA. Similar to the data obtained for other tasks (e.g., see Seah & Horne, 2020), there were many no responses especially for GSODA2 and GSODA3. GSODA2 appears to be slightly more difficult to solve than the other two items. Students in Year 8 and 10 project schools performed slightly better than those in trial schools. The Year 7 project students' performance was weaker for all three items in comparison to their counterparts. Unlike the project school data, which show a gradual improvement for each item across the year levels, the trial school cohort's performance was erratic, with the Year 7 outperforming the other year levels in GSODA1 and GSODA2. Note that the Year 9 and 10 trial data were collected from three different States. Small samples may have influenced these results, but other factors may be at play. Furthermore, only four students (2%) from the trial schools, one in Year 7 and 8, and two in Year 9, answered all items correctly, while in the project schools this was 5.6% (4.2% Year 8 and 1.3% Year 10).

Table 1
Breakdown of Student Responses for GSODA

Score	Trial Data					Project Data			
	Year 7	Year 8	Year 9	Year 10	Total	Year 7	Year 8	Year 10	Total
GSODA1	*n = 82*	*n = 69*	*n = 50*	*n = 20*	*n = 221*	*n = 171*	*n = 204*	*n = 37*	*n = 377*
0	40.2	37.7	34	45	38.6	28.6	37.4	47.1	36.3
1	4.9	21.7	22	15	15	27.7	10.5	0	14.1
2	9.8	1.5	8	0	5.9	8.4	7.9	4.4	7.4
3	4.9	5.8	6	0	5	5.9	5.3	4.4	5.3
4	40.2	33.3	30	40	35.9	29.4	39	44.1	36.9
GSODA2									
0	62.2	84.1	56	75	69.1	83.2	63.7	54.4	68.1
1	9.8	4.4	10	10	8.2	11.8	9	13.2	10.6
2	14.6	7.3	24	5	13.6	1.7	6.3	4.4	4.5
3	4.9	2.9	6	10	5	3.4	5.3	10.3	5.6
4	8.5	1.5	4	0	4.6	0	15.8	17.7	11.1
GSODA3					*n=161*				
0	46.2	87	65	60	49.6	77.3	63.2	63.2	67.6
1	30.8	5.8	5	15	10.9	13.5	12.6	4.4	11.4
2	17.3	0	20	20	7.7	7.6	11.1	7.4	9.3
3	5.8	7.3	10	5	5	1.7	13.2	25	11.7

Choice of Strategy for GSODA1

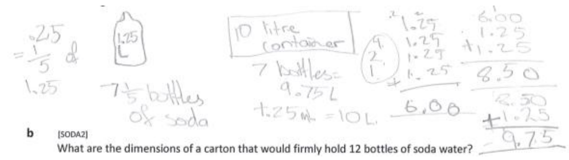
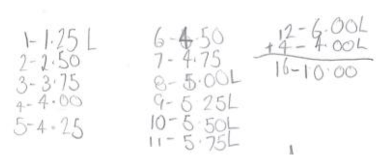
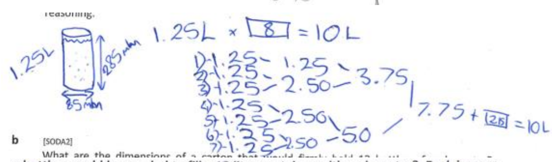
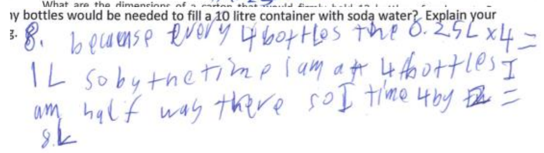
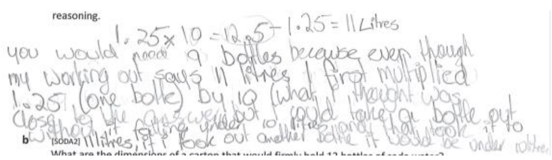
In the trial schools, 143 out of 221 students wrote a response. The most frequent solution was multiplying 1.25 by 8, followed by repeated addition (23.8%) and writing ‘8’ with no reason given (22.4%) (Table 2). Seven of the eight students who divided 10 by 1.25 answered correctly.

Table 2
Types and Percentages of Responses for GSODA1

1.25×8	Repeated addition	No reason	Grouping	Use all numbers	$10 \div 1.25$	1.25×10	Combination
24.5	23.8	22.4	9.1	6.3	5.6	4.2	3.5

Depending on the reasonableness of the answer, different scores were assigned to students who wrote a response without reason. A “0” was scored for 600 ml, 12.5 L or 637, and “1” if students wrote: 4, 6, 9 or 11 bottles. Seven students said 8 bottles with no reasoning and were scored 3. One student altered the capacity to a whole number and stated that “each bottle of coke [sic] is 500 ml, 500 + 500 = 1 L so 20 × 500 ml bottles are needed.” As the response did not address the question concerned, it is scored as 0.

Of the 34 students who used a repeated addition strategy, ten were not successful. Their errors included incorrect addition (see student A in Figure 3), change of increment (student B), or correct answer but incorrect calculation (student C). Twelve students were able to use a grouping strategy (combine addition and multiplication) in their solution (student D). Conversely, seven students chose to multiply 1.25 by 10 (Student E) and produced an incorrect answer. Nine students failed to ignore the irrelevant information. This combined with a lack of understanding of unit, led student F to generate an answer that could match 10,000 and student G to use all the numbers in some sort of procedures.

Student solutions	Interpretation of responses
	Student A: *7ML29112003 (score 2) Added four 1.25 incorrectly, combined this with two lots of 1.25 and then added another 1.25 to get 9.75. Since 0.25 is 1fifth of 1.25L, concluded that $7^1/_5$ bottles of soda must be needed.
	Student B: 7KK1922004 (score 2) Use repeated addition strategy. However, lost track of the increment and instead of increasing by 1.25, increased by 0.25 from the third lot instead.
	Student C: 8BM2112003 (score 3) Listed 7 lots of 1.25. worked out that two lots of 1.25 is 2.5, two lots of 2.5 is 5. Incorrectly added 3.75 and 5 to produce 7.75. Since an extra 2.15 is needed to make 10L, it must need 8 bottles.
	Student D: 7KR852003 (score 4) 8. because every 4 bottles the 0.25Lx4=1L so by the time I am 4 bottles I am halfway there so I time 4 by 2=8L.
	Student E: 7RD1972003 (score 2) You would need 9 bottles because even though my working out says 11 litres. I first multiplied 1.25 (one bottle) by 10 (what I thought was close to the answer but I could take a bottle out without it falling under. 10 could take a bottle and that took it to 11 times, if I took out another bottle it would be under 10 litres.

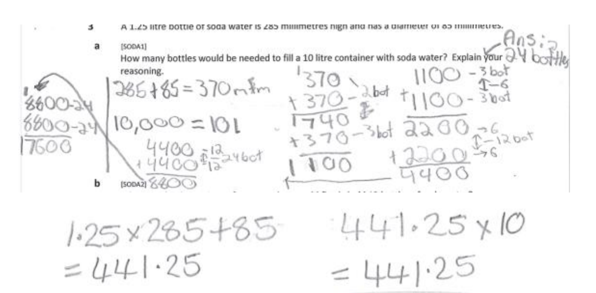
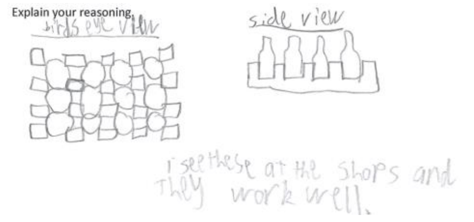
	Student F: 7XK17112003 (score 2) Added 285 and 85 to get 370. Assumed that 10L = 10,000. Proceeded to double 370 and one more gave 1100, 6 lots of 370 gave 2200. Double that gave 4400, which is 12 lots of 370. Double that gave 8800. Attempted to repeat doubling but realised the answer exceeded 10,000, so settle for 24 bottles. Student G: 10ST632001 (score 0) Multiplied 1.25 by the height and added the diameter. Multiplied the total by 10 to obtain 441.25.

Figure 3. Analysis of student solutions for GSODA1 (* The first number indicates the year level).

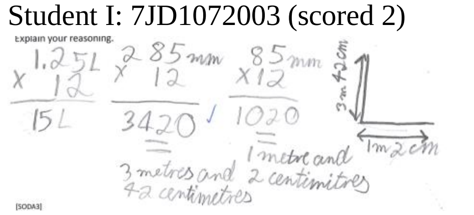
Solving Problem Requiring Multiplication

As shown in Table 1, GSODA2 received the least correct responses when compared with GSODA1and GSODA3. Of the 96 students who wrote a response, 28 gave an irrelevant response such as ‘a big rectangle’, ‘12.5L’ or ‘15L’. On average, 38.5% of the trial school cohort produced some form of drawing (43.6% Year 7, 37.5% Year 9, and 33.3% Year 8 and 10). Some drew an array with no explanation while others drew what they would see from the top and side view but did not give the dimensions (Student H in Figure 4). Around 14.6% of the students multiplied each dimension by 12 (Student I). Often, their efforts were hindered by poor computational skills (Student J) and lack of checking the reasonableness of their answer.

Student H: 7JL13102003 (scored 0)



Student I: 7JD1072003 (scored 2)



Student J: 7SM3172003 (scored 3)

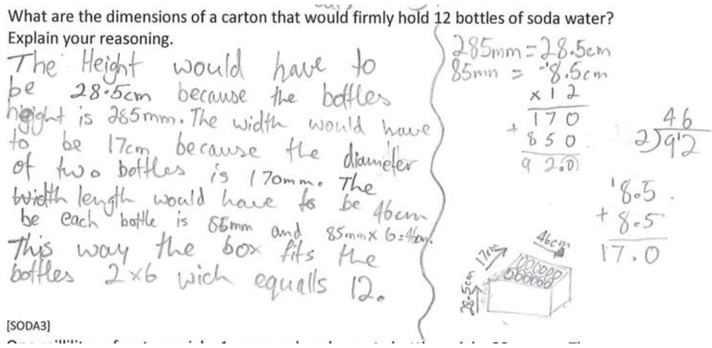


Figure 4. Analysis of student solutions for GSODA2.

Solving GSODA3 involves coordinating three components: 1) recognising the unit conversion, 2) working out the number of litres and hence the number of grams of liquid in 12 bottles, plus adding the mass of the bottles themselves, and 3) adding the weight of the box in the final calculation. Of the 59 students who wrote a response, seven gave an irrelevant response. Of the 52 students who scored at least a “1”, a quarter assumed that 1.25 L is 125 grams, 125 kg, or 1025 ml (see Student K and L in Figure 5). Another 9.6% multiplied 750 grams by 12 or used the height dimension as part of the calculation respectively. The most common error was neglecting to multiply either the soda water or empty bottles by 12 (Student M). These students appeared to have difficulty coordinating all the different aspects of the tasks in a clear plan to find the solution.

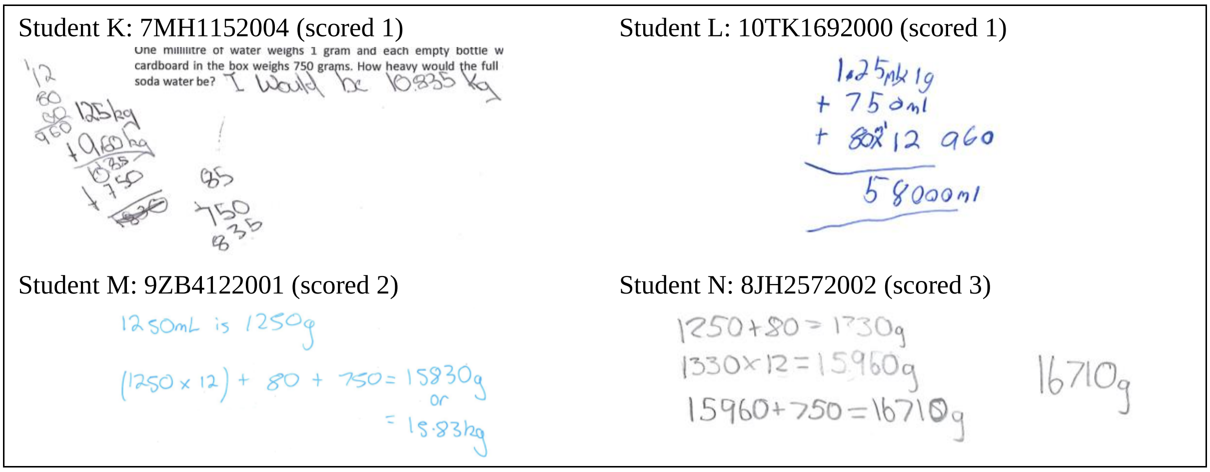


Figure 5. Analysis of student solutions for GSODA3.

Discussion

A problem is only a problem when the solution is not straightforward, as in the GSODA task. Most students did not already know the solution or how to obtain a solution. To successfully solve this problem, the students needed to coordinate several information and communicate their solution. We can see evidence of Sfard's (2021) four components (*endorsed narratives, keywords, visual mediators*, and *routines*) or the lack of them at play. To begin, the students' solutions reported here show a narrative that, while understood by the researchers, were in many instances not endorsed. For example, we found student G in Figure 3 who did not use any *keywords* in their working but added all the numbers, thus showing a lack of understanding of measurement concepts and dimensionality. Conversely, student I in Figure 4 used the measurement terms but multiplied 85 mm by 12 obtaining an answer of 1 m and 2 cm with the drawing of the final solution showing over 3 m height, thus demonstrating a lack of understanding of the relationship between these terms.

In an endorsed narrative, linguistic, symbolic/algebraic, and diagrammatic tools serve as *visual mediators* to facilitate and support effective communication. We observed a convoluted explanation of why 9 bottles are needed by student E and a succinct linguistic explanation of why it should be 8 bottles instead by student D in Figure 3. Others such as student B, who relied on a symbolic tool were often unable to multiply decimal numbers. Some students, such as student J in Figure 4 used a diagram to effectively show the final solution. While student H's diagrams clearly demonstrated how the bottles were visualised in the real situation, they did not help to obtain a correct solution.

The large numbers of no responses and low numbers of correct responses for all three items clearly show that many students did not have an established *routines* upon which they could call when trying to solve such problems. The drawing of diagrams, representing the information from the question on those diagrams and identifying the actual nature of the question are routines in problem solving that would have assisted many. Even routines of simple calculations were lacking as seen in many of the solutions in Figure 3. These combined with a lack of understanding of measurement concepts and dimensionality resulted in students unable to determine the dimensions of a desired carton or to assume that 1.25 L equal 125 kg or to multiply 1.25ml by 1 g (see student K and student L in Figure 5).

The implications of this for curriculum, assessment and pedagogy are fourfold. First, the teaching of measurement concepts needs to emphasise conceptual understanding and the connection between the concepts. Rather than memorising the conversion of units and arithmetic processes, greater emphasis must be placed on the concept of volume (which requires understanding and visualisation of three-dimensional objects) and its relationship to capacity and mass. Second, these concepts need to be presented in problems set in the real contexts where students are encouraged to visualise the problem mentally and on paper, and to manipulate and test the mental model of the problem to generate a solution. Third, routines need to be established, which encouraged the use of a range of communication tools such as diagram, symbols and language associated with the context. The language of explanation, argument and justification needs to be taught specifically and used regularly in classrooms to help students learn to explain their reasoning and justify their solutions (Seah & Horne, 2021). In this, assessment tasks should focus on the reasoning process rather than finding the right answer as shown in many multiple-choice questions. Finally, there should be frequent opportunities for students to solve multi-step problems. Students should be encouraged to discuss in pairs and in groups to fully comprehend the problems, develop all types of communication tools and deciding which type of communication tools work best for different situations. Only by the inculturation of problem solving as part of a socio-mathematical norm and mathematical classroom practices (Cobb & Yackel, 1996) can true change be realised.

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Building Understanding of Algebraic Symbols with an Online Card Game

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The transition between arithmetic and algebraic thinking is challenging for students. One notable difficulty for students is understanding algebraic symbols—pronumerals. Researchers are exploring pedagogical approaches in seeking to address this issue. The current paper is contributing to this body of literature by illustrating how an online card matching game-based learning activity supports students' understanding of pronumerals.

The importance of eliciting students' algebraic thinking is widely highlighted in the literature (e.g., Kieran et al., 2016). Students without solid algebraic understanding potentially experience difficulties in later mathematics learning (Kieran et al., 2016). Fostering students' algebraic thinking at the transition stage between arithmetic and algebra (upper primary or lower secondary level) is generally considered a way of overcoming difficulties and helping to build a promising algebraic foundation (Carraher & Schliemann, 2018).

Previous research has listed some essential conceptual shifts during the transition between arithmetic and algebra, such as thinking of relations among quantities instead of calculation, and focusing on letters (pronumerals) and numbers instead of numbers only (Kieran, 2004). Kieran (2004) stresses that understanding of pronumerals is particularly important when students start learning algebra. According to Radford (2018), understanding of letters involves two things: understanding letters as indeterminate quantities, and understanding that letters can be operated upon in an analytical way. "Analytical way" means letters can be arithmetically operated as if they are known numbers (Radford, 2018). However, a body of literature has documented students' misconception of pronumerals (Kieran et al., 2016). For instance, students treat letters as representing specific objects rather than indeterminate quantities, such as "a" is for "apples" and "b" is for "bananas" (McNeil et al., 2010). Students also possibly assign numeric values to letters based on the alphabetical order (e.g., considering "a" is 1 since it is the first letter). Furthermore, some students are challenged by the coefficients of letters (e.g., students mistakenly consider "2h" is "2+h") (MacGregor & Stacey, 1997). The incomplete understanding of the coefficient system might contribute to students' difficulty around the manipulation of algebraic expressions (e.g., collecting like-terms).

A body of research on algebra education has been contributing to pedagogical approaches in building students' understanding of pronumerals, but more empirical research is still needed (McNeil et al., 2010). Therefore, this research aims to investigate how a card matching game-based learning activity supports students' understanding of pronumerals. The research reported upon here is a part of larger study about designing and applying a game-based pedagogy on early algebra.

Literature Review

Literature has shown that the development of understanding of pronumerals heavily depends on the pedagogical approach taken to introduce letters, particularly, the context involved (MacGregor & Stacey, 1997). One prevalent context in introducing pronumerals that may contribute to students' misconception is using *mnemonic* literal symbols (McNeil et al., 2010). This means that when presenting pronumerals, teachers use sentences such as "a" can refer to "apples" and "p" can represent "pears" (McNeil et al., 2010). Many textbooks also use

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 498–505. Launceston: MERGA.

mnemonic letters, but in a more precise way by highlighting the letter as representing quantities. For example, wording in textbooks often takes the form “a” is for the number of apples and “b” is for the number of bananas (McNeil et al., 2010). Literature shows that the context of mnemonic letters might mislead students’ to consider that letters stand for specific objects rather than quantities (e.g., considering “a” is for apples rather than the quantity of apples) (MacGregor & Stacey, 1997; McNeil et al., 2010).

Furthermore, in addition to the context, building students’ understanding of algebraic symbols requires overcoming results-oriented thinking from their prior arithmetic experience. Results-oriented thinking means considering an expression must have a calculated result (Malara & Navarra, 2018). Students with results-oriented thinking have difficulty accepting “lack of closure” (Kieran et al., 2016). This is to say, students consider the expressions with letters as not valid since they believe the calculation of these expressions are incomplete. In this sense, research has argued that it is important to develop students’ relational view towards mathematical structures, meaning they should focus on the relation among terms in an expression rather than calculating results (Malara & Navarra, 2018).

Algebra education researchers are seeking effective approaches to teach pronumerals. For instance, Fujii and Stephens (2008) showed that students could invent informal non-literal symbols to express relations among quantities, and the formal literal symbol (letters) could emerge based on these informal symbols. Similarly, in Hunter (2010), students used everyday language to discuss the numeric relationship in arithmetic operations, progressively stepping toward using formal literal symbols in representing these relationships. Alternatively, McNeil et al. (2010) deliberately applied the counter-mnemonic strategy where instead of using “c” and “b”, Ψ or Φ were used to represent the quantities of cake and brownies. McNeil et al. (2010) identified that this approach suspended students’ thinking of letters as specific objects and the students with the counter-mnemonic approach outperformed their mnemonic approach counterparts in terms of understanding pronumerals as indeterminate quantities.

As mentioned, understanding of pronumerals could also include understanding that letters can be operated upon in an analytical way (Radford, 2018). This means students are able to carry out formal syntax of operation of letters (e.g., coefficient system, collecting like-terms). McNeil et al. (2010) showed that the mnemonic approach mentioned above could facilitate students’ intuitive understanding of the coefficient system which hastens their idea of collecting like-terms. For instance, when a teacher uses the words “three apples and two bananas” as an analogy to $a + a + a + b + b = 3a + 2b$, students then understand the number in front of the letter is to indicate the quantity of this letter. In this sense, it is noted that the mnemonic approach could hinder students’ conception of letters as representing indeterminate quantities, but it might intuitively trigger students’ understanding of syntax of operating letters.

Currently, digital technology is increasingly applied to mathematics education, and so there is a call for more research about the affordance of digital technology on early algebra instruction (Kieran et al., 2016). In response to this call, this study seeks to investigate how an online card game-based activity supports students’ development of the conception of pronumerals.

Methodology

This research aimed to explore how students learn with a designed game in an everyday classroom setting. The study employed a qualitative case study, which has the affordance to accommodate the complexities of natural classroom learning (Hamilton & Corbett-Whittier, 2012). Case study allows the researcher to have an in-depth understanding of what happens in students’ learning processes by providing fine-grained details of students’ learning (Hamilton & Corbett-Whittier, 2012).

According to Clarke (1997), to better understand students' learning in an everyday classroom context, data sources that reflect different perspectives should be complemented and triangulated against each other. Hence, three data sources were used for data collection and analysis in this study. The first one is the real-time activity recording of students' play including students' actions and conversations during the game. This data were transcribed to text for the researcher's interpretation of students' thinking. The second data source was a video-stimulated post-activity interview, which is a widely applied tool to probe students' thinking from their own perspective. The third data source was teachers' interview, which are used to infer students' learning from the teachers' perspective. The data reported here are from two grade seven students in Australia, and one grade five student in China, who are at the transition stage between arithmetic and algebra as per each country's curriculum, and all students were attending co-educational government schools. The game-based activity was conducted during a double lesson/period (about 80 minutes), as a part of students' everyday mathematics learning.

Game Design

At the macro level, the design of the game in this research was guided by a mathematics learning theory: Realistic Mathematics Education [RME] (Gravemeijer, 1994) and Gee's (2008) principle of game-based learning. RME adopts a constructivist approach and suggests that mathematics learning should start with the content which is experientially realistic to students. Here "experientially realistic" means something that is related to students' prior experiences. Students then progressively build formal mathematical understanding based on these experiences, with experts' facilitation (e.g., teachers, peers). The learning trajectory suggested by RME is in line with Gee's game-based learning principle, which argues that the game supports learning since it provides learners with a "bottom up" learning process, starting from an accessible starting point in which learners develop "performance before competence", and with ongoing experiences gradually step towards the formal abstract knowledge (Gee, 2008). Similarly, mathematics education, researchers have shown that compared to traditional paper format mathematical work, digital games are more likely to provide students with a "bottom up" process in which they gain concrete experience first, and move towards more complex mathematical concepts by reflecting and building upon these experiences (e.g., Jorgensen & Lowrie, 2012). Informed by RME and Gee, this game-based learning activity was designed as having different levels with increasing complexity, with the intention the beginning level would be readily accessible by all students.

At the micro level, the pedagogical approaches in algebra education suggested by the literature were considered. A range of research has shown that the expression/number sentence matching activity effectively supports students' algebraic thinking. For instance, Carpenter et al. (2003) used a number sentence matching activity to foster students' relational understanding of the equal sign. In the matching activity, students were more likely to pay attention to the structural relations in number sentences instead of doing calculation (Carpenter et al., 2003). Fujii and Stephens (2008) applied a similar activity to trigger students' relational view towards mathematical expressions and conception of pronumerals. In addition, exploring arithmetic regulatory (e.g., commutative law) is a good starting point to lead students to think algebraically (Malara & Navarra, 2018). Furthermore, a card matching games (like UNO®) are popular. In sum, this study designed a learning activity that has a card matching game context. The activity was designed to include four levels. At Level 1, students match simple arithmetic number sentences (e.g., $14 \times 5 - 2 - 1$ and $5 \times 14 - 3$). Level 2 included arithmetic operations but with larger numbers (e.g., $73 \times 29 + 3 - 3$ and 29×73). Level 3 mixed numbers with letters (e.g., "$793 \times 21 + 2a - a$", and "$21 \times 793 + 6a - 5a$"), and Level 4 included expressions with only letters (e.g., "a × b + 3c – c" and "b × a+ 5c – 3c"). A snapshot of the game is shown in Figure 1.

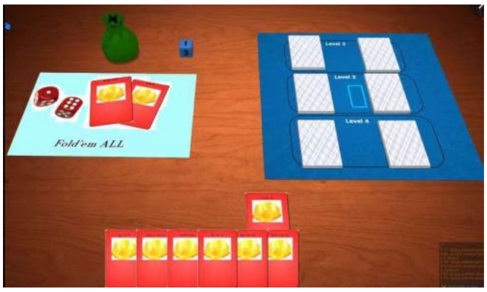

Figure 1. A game snapshot.

According to RME, Level 1 and Level 2 provide students with an "experientially realistic" starting point, since participating students should be very familiar with the arithmetic context. At Level 1, students can simply calculate answers to compare cards, but at Level 2, the calculation becomes more complicated since numbers are larger. Within this setting, it is hoped that this level could push students to start considering the structural relations between number sentences on cards. It is expected during these two levels students gain experience in focusing on considering the relation among quantities in mathematical structures rather than calculating results, so the results-oriented thinking could be suspended. As mentioned, overcoming results-oriented thinking is essential to understand the expressions with letters. At Level 3 and Level 4, the participating students, who have no prior formal algebra experience, need to compare and match expressions with letters. Here it is hoped that equipped with the experiences gained during previous levels, students are likely to possess a disposition to look at the algebraic expressions with a relational view instead of as sequences of calculations. By doing so, it is expected that even without the formal learning of expressions with letters, students, at least, could accept these "uncalculatable" mathematical structures as legitimate, and start comparing the structural similarities. Furthermore, as mentioned, an inappropriate context such as using the mnemonic literal symbols to introduce pronumerals tends to mislead students. In this sense, this game is designed to create a context in which students see letters come out with numbers together in the first instance, and they will be immersed in a context of mixing letters and numbers through engaging in the entire game. It is expected that in this pure numeric/algebraic context, students could intuitively draw a connection between letters and numbers.

Results and Discussion

Due to space restrictions, only one student's (Fiona) data will be reported and analysed in detail. Fiona's data illustrates the typical findings of this study. In addition, two other students' (Michael and Hanwei) data will also be mentioned as a supplement, to demonstrate diversity in findings. Based on their teachers' comments, all students were considered of average mathematical ability according to their level of schooling, and they had no formal algebra experience before the activity. The data reported here are episodes from Level 3 and Level 4 of the card game in which three students were exposed for the first time to expressions with letters.

Fiona

When entering Level 3, Fiona had gained extensive experience in comparing number sentences with considering relation among the numbers rather than doing calculations. For example, when comparing 29 × 73 – 3 and 73 × 29 – 2 – 1, she recognised the multiplication parts were the same without calculation but due to the commutative law, so she only calculated

"– 3" and "– 2 – 1" then accepted two number sentences as the same. When Fiona saw algebraic expressions at Level 3, she spontaneously compared cards without asking what the letters meant. For instance, Fiona matched a × (793 + a) and a × (a + 793), and her explanation is shown,

> Because they had the same start, like "a" times and the bracket. And then they got the same number in the bracket, just switched around.

In the interview, Fiona was asked whether she worried about the value of "a", and she answered that she did not worry about the value of "a", because both sides had the same "a". This episode tends to show that Fiona was able to compare them without knowing the value of "a" because she did the comparison by considering relations among terms. The words such as "same" and "switched around" further indicated Fiona focused on the structural similarities of expressions rather than calculating the results (Malara & Navarra, 2018). Fiona's explanation highlighted that she compared the expressions part by part, suggesting Fiona treated these expressions as standalone entities instead of sequences of calculations. This could be evidence of Fiona's acceptance of lack of closure. As mentioned, acceptance of lack of closure is a precursor to the understanding of pronumerals as indeterminate quantities, as it indicates students are beyond the results-oriented thinking. It could be argued that Fiona had extended her disposition to look at number sentences with a relational view developed during the previous levels to Level 3. Also, Fiona achieved this without any formal algebra instruction. This tends to suggest that Fiona's experiences gained at Level 1 and Level 2 took effect at Level 3.

However, two expressions in the episode above did not require collecting like-terms. In a later episode, Fiona found confronting when for the first time she encountered more complex expressions that collecting like-terms was needed, which were for example, 791 + 2 + 3a – a+ b and 793 + 2a + b. Fiona completed the comparison with the teacher's prompt. The excerpt is shown below,

> Teacher: How many "a" here, you have three "a" then you take away one "a", How many "a" here in total?
> Fiona: Em [pause] two
> Teacher: So how many "b" here?
> Michael [Fiona's teammate]: One
> Teacher: So which card you are looking for now?
> Students seek cards, and point to 793 + 2a + b
> Fiona: Yes, here it is.

With the teacher's prompt, Fiona, who had no previous experiences with formal algebra, could operate letters in an analytical way. When the teacher asked how many "a" were left, Fiona was able to recognise there will be two "a". In the interview, Fiona further explained why she selected 793 + 2a + b to match 791 + 2 + 3a – a + b, as shown below,

> Because at the start, it's seven nine one plus two and the seven nine three that is the same, and then plus two "a" and this is plus three "a" minus "a" that's two a, and plus "b", plus "b".

Fiona's explanation clearly illustrated her conception, which was that she was able to evaluate the expressions with letters part by part and operate the letters in an analytical manner. In this episode, Fiona's teacher Mr I played a role to facilitate students' comparison. The language used by Mr I was similar to the early mentioned mnemonic approach, which is, for example, analogising "a + a" as adding two "apples" in total. This study showed this kind of language was effective to make students understand how these letters could be operated. This kind of language constitutes the notion of "algebraic babbling", coined by Malara and Navarra (2018), which is used to describe a situation in which algebraic ideas are built upon using natural language. According to Malara and Navarra (2018), learning formal algebra can be bridged by using natural language in which underlying algebraic ideas are possibly situated. Here, when using language “three a” then you take away “one a” to describe the expression “3a – a”, the teacher was trying to convey the message to students that the number in front of “a” refer to the quantity of “a”. Then the formal pronumerals syntax of “3a – a” can be portrayed as “three lots of something take away one lots of something.” It appeared that Fiona grasped this idea. In later episodes, Fiona was able to consistently apply this strategy to simplify the expressions to do the comparison.

It appears that when attention is paid to this kind of language, students may grasp the idea of operation upon letters, but it is not sufficient to support students in understanding letters as indeterminate quantities. A student might be able to operate collecting like-terms without understanding the meaning of pronumerals. For instance, as mentioned above, a student who considers “a” as “apple” is still able to do “a + a = 2a” (see McNeil et al., 2010). To this end, in this research, the teacher avoided to use mnemonic words such as “apple” and “banana”, instead, the teacher used the letters as they were presented, not denoting a specific object as such. By doing this, this research tried not to mislead students to consider the letters as objects. However, it is noted that in the language used by the teacher, the conception of letters as indeterminate quantities did not explicitly stand out. In this sense, as mentioned, this activity was designed to let students see that letters came together with numbers as they were immersed in this context at Level 3 and Level 4 of the card matching game, so it was hoped that students would intuitively draw the connection between letters and numbers. In the interview, Fiona was asked about the meaning of the letters:

Interviewer: What do you think these letters represent for?
Fiona: Amounts.
Interview: Any mounts or only can be a fixed number?
Fiona: Any number.
Interviewer: Okay, you see the expressions have the letter “b”. May I use “d” or “e” any other letters to replace “b” here?
Fiona: Yeah, I think it would be the same.
Interviewer: Why?
Fiona: Because it wouldn't matter what letter of it is. It just the first number that matters.

This except shows that Fiona perceived letters as “any number”, indicating she understood the meaning of letters as representing indeterminate quantities. Furthermore, Fiona showed that she understood that it was not important which letter is to be used but the coefficient (“the first number”) of the letter did matter, without being told by the teacher. In the interview, Fiona also indicated that when she was playing Level 3 and Level 4, she continuously worked with numbers and letters together, so she “felt” a letter might stand for a number as well. Also, Fiona said since she did not need to “worry about” the value of letters, she thought the letters could be any value. Fiona’s teacher Mr I revealed that when his class started learning formal algebra after this activity, Fiona could easily understand the formal definition of pronumerals. In fact, many other participants (12 out of 16 participants) in this study had similar responses as Fiona, commenting that letters can be “random numbers” or “any numbers.” This appears to indicate that when students are immersed in a context mixing the numbers and letters, it is possible that they intuitively draw the connection between letters and numbers, and they are likely to perceive letters as quantities. This tends to suggest that in this study, the context to introduce pronumerals, which is mixing letters with numbers, had some effect.

This study also found out that the three students were able to notice letters as quantities because they considered the letters as specific numbers such as “1”. However, as MacGregor and Stacey (1997) suggested, the context to introduce pronumerals needs to be appropriate as it plays a key role in building students’ understanding of pronumerals. In this study, the context to introduce the pronumerals appeared to be an “appropriate” context, in which students, at least, could appreciate letters as quantities.

Hanwei

Hanwei had a similar progress in the game as Fiona. Hanwei was able to spontaneously match the expressions with letters by comparing the structural similarity. With peer support, Hanwei was able to analytically operate letters as if they are known numbers, to simplify expressions. However, when Hanwei was required to explain the meaning of letters, he answered "these letters represent some numbers, for example, "1", and "the letters cannot be any numbers, just for one number." This suggests while Hanwei perceived the letters as numbers, he did not realise they stand for indeterminate numbers. Interestingly, the following excerpt revealed some further insights of Hanwei's conception of pronumerals.

> Interviewer: If you think these letters represent specific values, why you did not substitute these values into the expressions to calculate the answer then do the matching?
>
> Hanwei: I don't have to do this, because it does not matter what values are, it is always quicker to just add or minus the number in front of these letters if both sides have the same letters.

Hanwei's response reflected that while he considered these letters as specific values, he still appreciated some generalities and relations of letters. He noticed that these letters could be directly operated on analytically and it did not matter what values the letters represented. Hanwei's data showed that he appeared to have some understanding of letters whilst he could not fully appreciate the meaning of pronumerals. This is in keeping with what Malara and Navarra (2018) have shown that when students are first introduced to algebra, they might be developing a certain level of understanding, but, it could still be naïve and tentative, meaning that errors and misconceptions might still occur, so teachers' further support is needed. Hanwei's teacher, Ms Q revealed, that after the game, this class was learning the topic of perimeter of square and rectangle, and Hanwei easily understood the meaning of the letters in the formula $(a + b) \times 2$, as "a" could be any length and "b" could be any width of a rectangle. In this sense, this study argues that Hanwei's partial understanding of pronumerals gained in the activity facilitated his later complete understanding. As Ms Q commented, she believed the game in this study built a foundation for Hanwei's latter formal understanding of pronumerals. As mentioned above, Gee suggests the importance of "performance before competence" in learning, and RME argues understanding formal abstract mathematics knowledge can be built on students' realistic prior experience. Hanwei's case shows that the designed game has a potential to provide students with some basic sense of pronumerals which could be a base for their full understanding of letters.

Michael

Michael exhibited similar progress in the game as Fiona. However, Michael was the only student who did not recognise letters as a quantity despite being able to operate analytically with the letters. In the interview, Michael revealed that he did not know what the letters stood for, and only operated them as they were. Like the case of Hanwei, Michael's case showed that a student can collect like-terms to simplify algebraic expressions but not necessarily understand the meaning of pronumerals. Nevertheless, in the interview, Mr I revealed that Michael's relational sense that emerged in the activity still benefited Michael in understanding pronumerals when formally learning algebra later. Mr I's words confirmed that the game, supported students in overcoming results-oriented thinking when developing their understanding of the pronumerals.

Conclusion and Limitations

This study contributes a pedagogical approach for understanding formal algebraic symbols (pronumerals) by arguing the designed game is supportive for students' conception of pronumerals in several ways. First, with a "bottom up" learning trajectory suggested by RME and Gee, students were gaining experience in viewing mathematical structures in a relational way beyond merely doing the calculation at Level 1 and Level 2 which is the accessible starting points for students. Equipped with these experiences, students possessed a disposition to look at expressions with letters at Level 3 and Level 4 as relational structures rather than sequential calculations, therefore they accepted of lack of closure and compared these expressions with structural similarities. Second, the data suggests that the context mixing numbers and letters is effective in facilitating students to build an intuitive impression that letters represent quantities. Third, students' understanding of coefficient system of pronumerals could be triggered by teachers' natural language that constitutes the notion of "algebraic babbling". Finally, this study finds that some students' conception of pronumerals was still naïve or incorrect despite that they were able to analytically operate these letters. However, it appeared that the experiences gained in the game provided these students with a basic sense of pronumerals, so had foundations on which to build when formally introduced to algebraic symbols. The effectiveness of this pedagogical approach needs to be further investigated in a large-scale study, where perhaps quantitative research is desirable.

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Primary Teachers' Mathematical Self-concept and its Relationship with Classroom Practice

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Mathematical self-concept refers to the perceived ability that one has in being able to do mathematics. While it has been shown to be a significant predictor for how students learn and apply mathematics, little research has been conducted into the relationship between the mathematical self-concept of teachers and their pedagogical practices in the mathematics classroom. This paper reports on a section of the findings from a small mixed methods study that sought to ascertain the nature of primary teachers' mathematical self-concept and how it is related to their teaching practices. Findings suggest that a teachers' mathematical self-concept does not necessarily reflect the mathematics practices evident in their classroom.

Negative attitudes towards mathematics often stem from school experiences and are exacerbated by the acceptance of an "I'm no good at maths" attitude in Australia's society (Wilkins, 2016). These experiences are often the result of teachers not having the pedagogical content knowledge or specialised subject knowledge in mathematics to design mathematical opportunities for students that are substantively engaging, purposeful, relevant and reflective of the individual student (Ball et al., 2008; Attard, 2013). Consequently, for some students, the results can be devastating (Gemici et al., 2014). Sustained student engagement, academic achievement, and future career choices are all factors that have the potential to be impacted due to negative experiences when learning mathematics (Gemici et al., 2014; Bourgeois & Boberg, 2016).

For students to successfully learn mathematics and develop an appreciation for what mathematics can bring to their lives, they need teachers who have the knowledge, skills and pedagogical understandings to be able to effectively plan, implement and evaluate mathematics teaching and learning.

Background

Previous research suggests that a teacher's belief in their mathematics ability plays a vital role in the way in which they teach mathematics (Stipek et al., 2000). Teachers who identify as being 'good' and 'able' in the subject often tend to take greater risks in challenging their students mathematically. Conversely, teachers who consider themselves not to be 'good' at mathematics often tend to be much more cautious or reserved in how they teach and engage in the subject with their students (Stipek et al., 2000).

The notion of self-concept has been widely defined. Seaton et al. (2014) state that "individuals embrace self-concepts about themselves that correspond to various aspects of their lives" (p.50). That is, self-concept is a learned trait that is developed as a result of experience. Marsh and Shavelson (1985), view the idea of self-concept as a form of social comparison whereby an individual perceives a self-impression of themselves as a result of comparing themselves to another person. Consequently, it is this direct comparison that often leads to negative associations on an individual's confidence, self-esteem, achievement and motivation (Tenisheva & Alexandrov, 2013). Kinch (1963) identifies self-concept as "the organisation of qualities that the individual attributes to himself in varying situations" (p. 481). Quite often

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 506–513. Launceston: MERGA.

these attributes stem from experiences both negative and positive that the person has encountered and endured over the course of time.

Although there has been an abundance of research conducted around the notions of "self," there is still not a definitive consensus about "self" as an individualised construct (Guay et al., 2015). Guay et al. (2015) expressed, however, that certain aspects of a person's "self" are crucial for understanding and driving human behaviour" (p. 6). Harter (1985) as cited in Cheng-Yu (2014) expresses the idea that an individual's self-concept is derived from their personal description and evaluation of their strengths and weaknesses. A person's mathematical self-concept is a significant predictor for several characteristics, including achievement in the subject, as well as motivation and attitude relating to the learning and application of mathematics (Abu-Hilal, 2000).

The attitudinal beliefs that adults have regarding their mathematical ability often stem from their experiences with mathematics when they were at school (Whitten, 2013). Unfortunately, often it is these experiences that set the trajectory for adults to adopt the mindset of not being good at or being able to 'do' mathematics. This idea is supported by Boaler (2016) who states that "when students get the idea they cannot do math, they often maintain a negative relationship with mathematics throughout the rest of their lives" (p. 10). The consequence for a teacher who has a negative disposition towards the subject, often, is their students also feel the same way. Seaton et al. (2014) raise the question, "should teachers focus solely on improving academic skills or is it also necessary for them to help students develop positive perceptions of their abilities?" (p. 51).

Research suggests that there is a reciprocal relationship between academic achievement and self-concept (Marsh & Craven, 2006). Teachers play a vital role in not just the academic achievement of their students, but in the development of their students' beliefs and perceptions of the subject. It is imperative then when considering the above-mentioned question, to also consider how a teacher who has a negative self-concept of mathematics themselves can develop their students' mathematical self-concept in a way that is going to have a positive effect on their overall engagement and achievement in mathematics.

Engagement and Mathematics

The concept of engagement can be defined as active involvement in learning and includes the mental (cognitive), physical (operative) and emotional (affective) elements of learning (Munns & Martin, 2005). Students who are engaged at all three levels value and enjoy mathematics and see the relevance and purpose that mathematics has to their lives both within and beyond the classroom (Attard, 2013). A student who has a low mathematical self-concept typically exhibits low levels of engagement and motivation. A number of research studies (Erdogan & Sengul, 2014; Bonne, 2016) acknowledge observed declines in a student's mathematical self-concept, their perception of the subject as being important and useful, as well as a significant decline in their motivation and interest in the subject. Watson et al. (2019) acknowledge that this decline, around the middle years of schooling typically correlates with students developing a sense of identity and an awareness of others, resulting in peer-comparison. Ma and Cartwright (2003) acknowledge that these declines quite often parallel with increased feelings of mathematics anxiety. For some students who have low achievement levels in mathematics during high school, this can influence and limit the career choices they make as adults (Gemici et al., 2014). Additionally, this type of student could potentially enter society without having the numeracy skills needed to "meet the complex demands of everyday life and work" (Seaton et al., 2014, p. 49).

Framework for Engagement with Mathematics

Engaging students with learning in the mathematics classroom is crucial if teachers want to support students in not only achieving academically, but also as learners who are able to "put forth effort, persist, self-regulate their behaviour towards goals, challenge themselves to succeed and enjoy challenges and learning" (Christenson et al., 2012, p. 5). The Framework for Engagement with Mathematics (FEM) (see Table 2) was the result of a longitudinal study that examined the influence of student engagement during the middle years of schooling (Attard, 2014). The FEM was selected to be used as an analytical and observational tool in this study as it was generated from both existing literature and research that included student perspectives (Attard, 2012).

Attard's (2014) FEM was used when exploring teacher practices, in particular their pedagogical practices when teaching mathematics. It is these pedagogical repertoires (the day-to-day practices of teaching) and the pedagogical relationships (interpersonal and interconnectedness between teacher and student and teaching and learning) that positively affect engagement and learning in mathematics (Attard, 2012).

Methodology

The purpose of this study was to ascertain the nature of primary teachers' mathematical self-concept and how it is related to their teaching practices. The central research question of this study was:

1. *What is the nature of primary teachers' mathematical self-concept and its relation to their practices in the mathematics classroom?*

The following sub-questions were used to inform the central question:

a. *What is the relationship between a teachers' mathematical self-concept and their pedagogical relationships?*
b. *What is the relationship between a teachers' mathematical self-concept and the pedagogical repertoires of their practice?*

To address the research questions a mixed methods approach was used. The study was organised into two phases. In phase one, three case study participants were identified after completing an online survey/questionnaire that identified their current level of mathematical self-concept, using Marsh and O'Neill's (1984) Self-Description Questionnaire III (SDQIII). In phase two, individual teacher interviews and two classroom observations per case study participant were conducted. The researchers' field notes and transcribed interviews were interrogated and analysed using the FEM (Attard, 2014), identifying the elements of each aspect of the Framework evident in the teachers' lessons. The data collected in phase two of this study was used to build on the findings from phase one.

In total, there were 31 respondents (25 female and 6 male) to the initial survey/questionnaire. Fifteen of the 31 participants indicated that they would be willing to take part in phase two of the study. Originally, case study participants were selected based upon their scores (low, middle, high) on the SDQIII. However, as a result of identified participants being unavailable, the researcher was required to select participants based upon those who were available.

Case Study Participants

Annie. At the time of the study, Annie had been teaching for eight years and was teaching at a government primary school in South-Western Sydney. The observational data collected for Annie came from a year six class of 26 students. It was observed during both visits that there were a number of negative student behaviours apparent in the class (e.g., students calling out, students provoking other students, students back answering the teacher).

In terms of her mathematical self-concept, Annie stated that she remembers enjoying mathematics more when she was younger than when she did in high school. She said during high school, she was placed into the lowest mathematics class and was “acing every test and assignment”. Annie was moved up a level for years eleven and twelve and remembers when the mathematics became challenging, “rather than pushing myself, I got a bit lazy”.

Bree. At the time of the study, Bree had been teaching for five years and was teaching at a government primary school in South-Western Sydney. The observational data collected for Bree came from a year two class of 22 students. This class was located in an open plan classroom space that in total consisted of 44 students and two classroom teachers.

In terms of her mathematical self-concept, Bree referred to herself as a low achieving student and at one point stated that she “hated maths” when she was a child. Bree expressed in her interview that the thought of needing to teach mathematics almost put her off becoming a teacher.

Catherine. At the time of the study, Catherine had been teaching for fifteen years and was teaching at a government primary school in South-Western Sydney. The observational data collected for Catherine came from a year three class of 31 students. During the first observation fifteen students were absent.

In terms of her mathematical self-concept, Catherine acknowledged that during primary and secondary school, she remembers never really knowing what she was doing in mathematics or why she was doing it. She said she remembers this made her feel like she wasn’t “as good” at the subject as her peers. Catherine stated that although she really enjoys teaching mathematics, she doesn’t feel that she is very good at it.

Findings and Discussion

Table 1 illustrates the overall mean score for the whole sample (n = 31) as well as for each case study participant, for each scale of the SDQIII measured. Table 2 demonstrates the depth of evidence that existed for each element of the FEM (Attard, 2014) for each case study participant.

Table 1
Mean Scores of Each Scale

Scale	Mathematics	Academic	Problem Solving	General Esteem
Overall Mean	5.3	5.9	5.5	5.6
Case Study Participants				
Annie	4.9	4.1	5.1	5.3
Bree	3.7	4.9	5.9	5.3
Catherine	4.1	5.7	3.9	5.1

All three case study participants scored below the mean for the mathematics scale in the Self-Description Questionnaire III (Marsh & O’Neill’s, 1984), indicating relatively low mathematical self-concept. However, comparison of the three case study participants indicated that Annie’s mathematical self-concept score (mean = 4.9) was the highest when compared to Bree (mean = 3.7) and Catherine’s (mean = 4.1) scores. Interestingly, analysis of Annie’s pedagogical practices as per the FEM (Attard, 2014) indicate minimal evidence to suggest that her mathematics teaching reflects evidence of engaging mathematics teaching.

Table 2
Framework for Engagement with Mathematics, Illustrating Evidence of Practice

Aspect	Element	Annie	Bree	Catherine
Pedagogical Relationships	*Pre-Existing Knowledge:* Student's background and pre-existing knowledge are acknowledged and contribute to the learning of others	LE	HE	HE
	Continuous Interaction: Interaction amongst students and between teacher and student is continuous	LE	HE	ME
	Pedagogical Content Knowledge: The teacher models enthusiasm and an enjoyment of mathematics and has strong Pedagogical Content Knowledge	NE	HE	HE
	Teacher Awareness: The teacher is aware of each student's mathematical abilities and learning needs	LE	HE	HE
	Constructive Feedback: Feedback to students is constructive, purposeful and timely	NE	ME	ME
Pedagogical Repertoires	*Substantive Conversation:* There is substantive conversation about mathematical concepts and their applications to life	LE	HE	HE
	Challenging Tasks: Tasks are positive, provide opportunity for all students to achieve a level of success and are challenging for all	LE	HE	HE
	Provision of Choice: Students are provided an element of choice	NE	LE	ME
	Student-Centred Technology: Technology is embedded and used to enhance mathematical understanding through a student-centred approach to learning	NE	NE	NE
	Relevant Tasks: The relevance of the mathematics curriculum is explicitly linked to students' lives outside the classroom and empowers students with the capacity to transform and reform their lives	NE	HE	HE
	Variety of Tasks: Mathematics lessons regularly include a variety of tasks that cater to the diverse needs of learners	NE	ME	ME

No Evidence (NE) *Low Evidence (LE)* *Medium Evidence (ME)* *High Evidence (HE)*

Subsequently, this was reflected by way of her students seeming to "comply" with doing the mathematics as opposed to enjoying and valuing the mathematics being taught. Comparatively, Bree and Catherine's pedagogical practices as per the FEM (Attard, 2014) suggest that they are in fact demonstrating evidence of engaging mathematics teaching, reflective of best practice. Subsequently, this was reflected by way of their students seeming to be authentically enjoying mathematics and valuing what they were learning because of the relevant links that the mathematics had to their lives.

Pedagogical Relationships

Annie, although scoring the highest mathematical self-concept score out of the three case study participants, demonstrated the weakest pedagogical relationships. Annie's weak pedagogical relationships were paralleled in her weak pedagogical content knowledge, the minimal opportunities she provided her students to interact with each other as well as her inability to provide her students with constructive and purposeful feedback throughout the learning process. Bree and Catherine demonstrated that they had strong pedagogical relationships, evidenced in their classrooms by their interactions with their students, their understanding and knowledge of individual students' abilities and learning needs, their enthusiasm for teaching mathematics, and their strong pedagogical content knowledge.

Pedagogical Repertoires

As she did with pedagogical relationships, Annie demonstrated the weakest pedagogical repertoires. Annie's weak pedagogical repertoires were reflected by the prescriptive nature of conversations that took place between her and her students as well as her lessons seeming to be stand-alone lessons that provided students with the opportunity to practice content, as opposed to being something that was purposeful and meaningful to their lives. Comparatively, Bree and Catherine demonstrated that they had somewhat strong pedagogical repertoires, evidenced in their classrooms by the opportunities they provided their students to work collaboratively whilst problem solving and investigating. Additionally, there was evidence of purposeful planning in Bree and Catherine's lessons to ensure their students were challenged and that the mathematics being taught was done so in a way that provided relevance to their students' lives.

Mathematical Self-concept and Teacher Practice

Contrary to previous research (Stipek et al., 2000), this study found that a teacher's self-concept score in relation to mathematics does not necessarily reflect the mathematics practices evident in their classroom. It emerged through case study interviews that personal experiences with mathematics in childhood appear to have the potential to affects one's self-concept. This reflects the understanding in the literature that self-concept is often attributed to the negative and positive experiences a person has encountered and endured over the course of time (see for example, Tenisheva and Alexandrov, 2013; Seaton et al., 2014). During interviews, all participants identified negative experiences with mathematics when they were children. It was noted in this study that the three case study participants appear to have responded in different ways to these experiences. Annie, for example appears to have maintained a negative association with the subject, now resulting in her own teaching of the subject in a way that does not reflect effective practice. However, Bree and Catherine appear to have channelled their negative experiences with the subject into an opportunity to ensure that they are teaching mathematics to their students in a more positive way, conducive of practices that embrace quality mathematics teaching.

Conclusion and Limitations

This study has revealed that a teachers' mathematical self-concept does not always reflect their pedagogical practices. Individual school contexts, the proactive engagement in the attainment of professional learning as well as a teacher's personal engagement with the profession, all appear to be factors that influence this finding.

The study had some limitations that should be noted. The first limitation was the small sample size (n = 31) of participants (those who responded to the survey/questionnaire in Phase 1). According to Cohen et al. (2011) for quantitative research "the larger the sample the better, as this not only gives greater reliability but also enables more sophisticated statistics to be used" (p. 144). That being said, it is important to acknowledge that the findings of this study are not able to be generalised across all contexts. Another limitation of this study was that the analysis of observed teacher practice was based on only two observed lessons. A more sustained study with a greater number of observations may have revealed deeper insights.

Additionally, as this study was concerned with the relationship between mathematical self-concept and teacher practices, further research that investigates the impacts that the mathematical self-concept of primary teacher's has on student's mathematical self-concept would be beneficial in adding to the existing research around student engagement in mathematics during the primary years of schooling.

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Teacher Views of Parent Roles in Continued Mathematics Home Learning

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This paper emerges from a broader study that investigated the strategies employed by teachers to continue mathematics teaching and learning during South Africa's COVID-19 lockdowns and through subsequent phased and partial re-opening of schools. In this paper, we focus on teacher views of the role of parents in these efforts gathered through two questionnaires administered from 2020 to 2021. Twenty-five Grade 4–7 mathematics teachers from schools in the Eastern Cape province took part in the study. We address the question: What were teacher perspectives on the role of parents in the continuation of mathematics teaching and learning during COVID-19 and the gradual reopening of schools? We explore the teacher responses and show how the pandemic context provided a stimulus to forge stronger teacher-parent relationships and opened opportunities for productive ways of extending mathematics learning almost exclusively undertaken in the classroom into homes.

Introduction and Context

The COVID-19 pandemic necessitated a rapid re-imagining of how to continue mathematics teaching and learning through a period of initial total lockdown and school closures and then through the extended gradual reopening of schools. In South Africa schools were closed from 27 March 2020. For five weeks citizens were strictly confined to their homes and for a further five weeks schools remained closed (Vale & Graven, 2021). There was a gradual reopening of schools for certain grades from 8 June 2020, but another school shutdown was implemented from 23 July 2020. In total, Grades 5 and 8 and Grades 4 and 9 lost 42% and 39% of their school days, respectively (Hoadley, 2020). A rotational model of attendance in which learners attended school on alternating days was introduced and only officially ended early in 2022. Many school days were lost, and the home became a critical site of learning. This was not unique to South Africa, and thus the research presented here has relevance beyond South Africa. We draw on literature from the South African and the Australasian context to make this relevance explicit.

This paper emerges from a broader study that investigated the strategies employed by teachers to continue mathematics teaching and learning during South Africa's COVID-19 lockdowns and through subsequent phased and partial reopening of schools. In this paper we focus on teacher views of the role of parents in these efforts from two questionnaires gathered from 2020 to 2021. The goal was to understand the school-home relationship in the continuation of mathematics teaching and learning during COVID-19 lockdowns.

Literature Review

Muir (2011) writes that there is relatively little that is published about the role of parents in the mathematics education of their children. This is despite the acknowledgment that parents are influential in the success or otherwise of their children in mathematics (Muir, 2011). Of the literature that is available, there is "widespread agreement … that students' learning is maximised when strong educational partnerships between home and school exist" (p. 1). Weerasinghe (2019) writes that "it is the involvement of parents with their children's education at home that is most likely to result in a positive difference to academic outcomes" (p. 755). Muir (2012) notes while parents have been previously viewed as impeding reform in

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 514–521. Launceston: MERGA.

mathematics education, parents also bring expertise and a knowledge of their children to contribute to their mathematical development.

Wadham et al. (2020) argued that achieving effective mathematics partnerships between the school and the home was difficult. Their study focused on teacher perspectives of parental involvement in mathematics as well as parent reports of their involvement in their children's mathematics work. The study revealed tensions between parent and teacher perspectives showing that "teacher assumptions that parents would not be confident with mathematics were not supported by the data" (p. 18). They concluded that it was important for schools to pursue gaining parent input and better communicate with parents about their children's mathematics learning to improve the flow of information between school and home.

A primary way in which mathematics learning extends into the home is through homework. Graven (2018) found that, in the area where this research is based, very few teachers set homework. At a national level Spaull (2013) found that about half of South African Grade 6 learners were not given regular homework thus limiting extension of learning and independent activity beyond the classroom. Graven (2018) reported that some of the reasons given by teachers for not giving homework included "problems with parents ranging from parents being unable to support homework or parents doing the homework for learners" (p. 41). However, after implementing a homework-drive intervention, Graven (2018) noted that participating teachers shifted towards more positive comments about parental involvement. In this study we seek to explore teachers' views about parental involvement through the pandemic.

Darragh and Franke (2021) report that pre-pandemic research revealed that "mathematics homework is often unsuccessful or stressful for both parents and children and that tension exists between home and school in the learning of mathematics" (p. 1). It is important to realise these challenges when considering the shift to the home becoming an essential site of learning during the COVID-19 school closures. Their findings included that there was a range of parent experiences during lockdown home learning from those who felt very little support from schools and teachers to those who felt supported and had the resources and motivation to help their children (Darragh & Franke, 2021). They suggested that "teacher support is essential for home-learning success" (p. 20). It is therefore of interest to explore teacher perspectives on the role of parents in their children's mathematics education during the pandemic.

Theoretical Perspective

The theoretical perspective taken in this study is that of Cultural Historical Activity Theory (CHAT) (Engeström, 2001). The origins of CHAT lie in Vygotsky's theorising of the mediation of behaviour (Vygotsky, 2012) as a triangular model linking the subject, object and mediating artefact. As Engeström points out, however, this model fails to "fully explicate the societal and collaborative nature of actions" (Engeström, 1999, p. 30). In his model of activity, Engeström (2014) de-centres activity as being focused on individuals but positions it as involving joint activity by people working in interaction with one another. There are six elements to Engeström's model of an activity system: subject, object, mediating artefacts, community, division of labour, and rules. A classroom could be considered an activity system, in which the subject (the teacher) makes use of mediating artefacts in pursuit of the object of the activity system, which would be the learner doing mathematics and thereby the teaching and learning of mathematics. That classroom would comprise of "multiple individuals … who share the same general object" (Engeström, 1993, p. 67), which forms the community. Within that classroom there would be a particular division of labour in the sharing of tasks between the teacher and the learners; rules would guide this activity. These would be the "explicit and implicit regulations, norms and conventions that constrain actions and interactions" (Engeström, 1993, p. 67). This is what is known to be second-generation activity theory.

During COVID-19, however, the classroom activity system was severely disrupted with teachers and learners no longer able to access the classroom. The home became an additional site of learning and teachers needed to reach out to the home activity system to ensure continued mathematics teaching and learning. This implies the need to consider the home to be a second, interacting, activity system. Third generation activity system theorises the interaction of activity systems. Figure 2 provides a visual representation of the interaction of the home and school activity system:

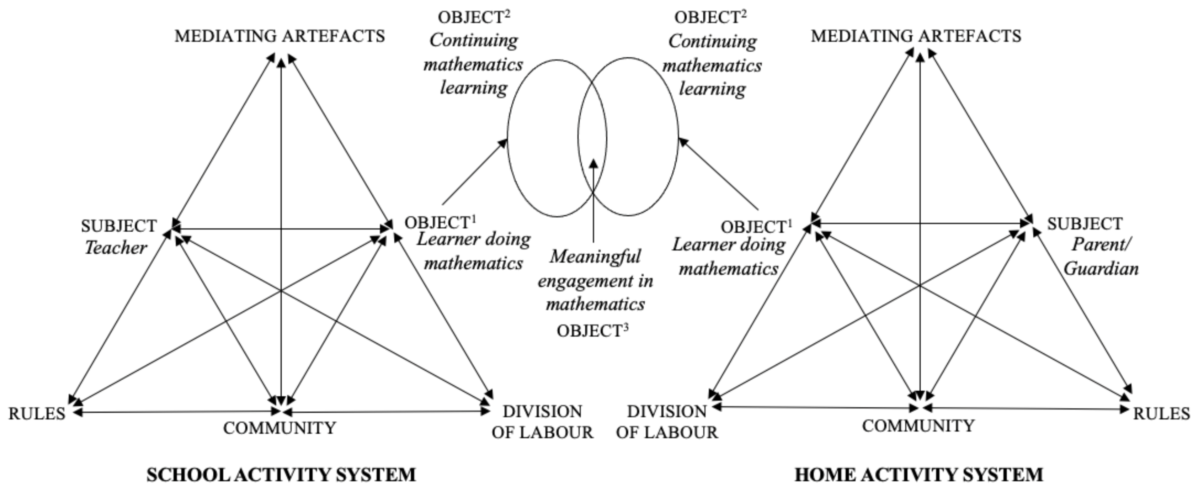

Figure 1: School and home activity systems (Based on Engeström, 2014).

For the periods when learners were learning exclusively from home, and later when learners were only attending school on alternate days, the home activity system was crucial to consider. Both the home and school activity systems needed to become oriented to the shared object of the learner doing mathematics, continuing mathematics learning, and ultimately the goal of meaningful engagement in mathematics. Wadham et al. (2020) reported that "the school and the home are both influential contexts in which a child learns mathematics and therefore schools and families should work collaboratively to achieve shared goals for children's mathematics learning" (p. 1). This involved teacher innovation and resourcefulness and required them to reach out to the parents and caregivers in the home to enable this. Here we explore how teachers referred to parents when reporting on the strategies they employed to continue mathematics education. It is important to note that there were various stages of lockdown, and thus the strategies for connecting with home activity systems shifted as the lockdown regulations shifted.

Methodology

This research is a qualitative, interpretive case study in which we adopt a sociocultural perspective aligned to Vygotsky's (1978) notion of knowledge as being socially constructed. In the broader research project, we focused on seeking teachers' experiences and strategies of continuing mathematics teaching and learning through the pandemic. Through the thematic analysis of those responses, we noted emerging themes in relation to the role of parents in applying these strategies. For this study, we have revisited that data with the research question: What were teacher perspectives on the role of parents in the continuation of mathematics teaching and learning during COVID-19 school closures and the phased and partial reopening of schools?

The first questionnaire was conducted in November 2020, with the aim of eliciting the strategies teachers employed to continue mathematics learning through the shifting phases of lockdown and gradual school re-opening during 2020. The second questionnaire was conducted in September 2021 and included items asking teachers what strategies they were continuing to use in their efforts to ensure mathematics teaching and learning was happening. While the hard lockdowns of 2020 did not occur in 2021, many schools continued to use a rotational model of attendance throughout 2021, and thus it was still relevant to be examining what these strategies might be. It was only in early 2022 that schools returned to full attendance.

There were 25 Grade 4–7 mathematics teacher participants from eight schools. The teachers were all participants in the South African Numeracy Chair's teacher professional development Mathematics Inquiry Community of Leader Educators (MICLE) and were recruited from that group. Schools represented a range of socioeconomic circumstances, from those that were severely resource-constrained (5 schools) to those serving less resource-constrained communities (3 schools). Ethical clearance was granted by the Rhodes University Education Faculty Research Ethics Committee (ref. 2020-2732-4713). Alpha-numeric codes were assigned to participants for reporting purposes to maintain anonymity.

We undertook a thematic analysis to analyse the data. There were 53 responses from 19 teachers that directly referenced parents as members of the home activity system. The first author coded all 53 responses according to these categories. The second author reviewed these codes with strong agreement across all except two of the coded responses. These were discussed and consensus reached about the category of the responses. Questionnaires 1 and 2 (Q1 and Q2) were coded separately to allow for the noticing of any shifts or differences in the type of response at these two distinct time periods.

The questionnaires had a range of questions that looked to establish what strategies the teachers had employed during the COVID-19 school closures and the gradual re-opening of schools. The questions that generated the most responses with reference to parents were:

> What strategies, if any, did your school implement during the school shutdown periods (Questionnaire 1, 8 responses)
>
> Did you use technology to support you I managing the challenge of continuing education during the pandemic? How did you use this technology? (Questionnaire 1, 8 responses)
>
> At the start of learner rotation (in 2020) what strategies, if any, did you use to manage teaching and learning in the classroom and at home? (Questionnaire 2, 5 responses)
>
> Are there strategies that you have developed as a result of COVID disruptions that you might continue using even after schooling returns to 'normal'? (Questionnaire 2, 5 responses)

Results

Overall, there were 53 responses in the data that directly referred to parents as a key member of the home activity system. These responses came from 14 of the teachers in Questionnaire 1 and 9 of the teachers in Questionnaire 2. Five of the teachers wrote about parents in both questionnaires. There were broadly three categories of responses that included the mention of parents. The first was mention of parents, or the circumstances of the home activity system, as a source of hinderance to the continuation of teaching and learning. The second was an instrumental view of the parent as the collector of resources from school and the conveyor of messages between the teacher and the learner, without any reference to any additional role with respect to teaching and learning. The third category comprised those responses that reflected a view of the parent as a partner in the continuation of mathematics teaching and learning, and thus responsible in part for the progress that their children were able to make during that time. Included in this category are those responses that mention sending extra materials aimed at clarifying the content to the parent. The parent was in those cases not merely viewed as the conveyor of messages, but as needing to explain the work to their children. For example, “I used videos and voice records to parents on WhatsApp … involving parents more in the lessons” (GD2). Table 1 below summarises the number of responses per questionnaire and category.

Table 1

Frequency of Responses Per Category Across the Two Questionnaires

	Questionnaire 1	Questionnaire 2
Category 1: Parent/home challenges	8 from 4 teachers	1 from 1 teacher
Category 2: Parents as message/ resource conveyers	18 from 8 teachers	3 from 3 teachers
Category 3: Parents as partners	10 from 7 teachers	13 from 6 teachers

Although there was a drop in the total number of comments about parents from Q1 to Q2 there was a pleasing shift from Q1 to Q2 away from foregrounding challenges of parents and home situations to productive communication (8 in Q1 to only 1 in Q2) matched by a shift towards more comments about parents as partners in the teaching and learning process (from 10 in Q1 to 13 in Q2) along with less mention of parents as message conveyors. This reduction could be linked to the easing of lockdown restrictions which meant that learners were attending school on a rotational basis, and thus there was less need for parents to pass information on to their children. However, even with that shift it was encouraging to see the increase in mention of parents as partners over this time.

Only nine responses from four teachers were made of the first category (with 9 of these being from Q1. Illustrative examples are listed below:

> “Some parents decided to keep their children home…without contacting the school and without reason” (SM1)
>
> “Most parents don’t have cellphones so it was impossible to keep learners busy.” (BH3)
>
> “It was difficult to keep learners, teachers and parents motivated.” (BH3)
>
> “Learners who are [usually] struggling with no support from home are struggling more.” (BH3)
>
> “Parents that lost their job due to COVID couldn’t keep up with WhatsApp lessons on a daily basis.” (OL2)
>
> “Syllabus/curriculum and the maths concepts taught are a lot different compared to parents’ education. This could create a challenge where some children live with grannies and they can’t be helped.” (OL2)

These responses reveal teacher perceptions that the home activity system for some lacked the resources for effective continuation of learning (e.g., “Most parents don’t have cellphones”), or that the parents as the subjects of the home activity system were prolonging the isolation of the learner from the school activity system. In these cases, it appeared there was not productive interaction between the school and home activity systems and the object of continued learning was frustrating.

The second category of responses viewed the role of the parent quite instrumentally as a conveyor of information to the learners and as the person to collect work from the school for the children to complete. For example, “The educators were asked to prepare work for the students to do at home. The parents had to pick it up from school on certain days for certain grades” (SM1). There was a clear indication of the division of labour in this response: the teacher designed the materials, and the parents collected the work and in turn gave it to their children. Other teachers mentioned, “via WhatsApp parents were given the work to be done”

(AP4), “We asked parents to collect the homework packs during certain dates and times” (SM2), and “Parents could fetch booklets/work done and the learners could do it at home” (OL2). Overall, there were 21 responses of this instrumental nature.

There were a further 23 responses in which teachers mentioned a more substantial role of the parent, that of a co-educator, and occupying a position in the home activity system of actively working towards the continuing learning of their child. Some of these responses were similar to the “message conveyor” responses but included mention by the teachers of extra material designed to assist the parents to understand the content. For example, “[I] sent links to relevant lessons to help parents understand what/how they can help teach their kids, and [I] did videos of myself teaching lessons and sent to the parents’ WhatsApp group” (PA1). One teacher reported that he “gave out homework in packs so that learners could get help from their parents [and] I used videos and voice records to parents on WhatsApp” (GD2). Another teacher also noted that “parents contacted me privately with queries or questions” (OL1). One school encouraged parents by sharing “on the school’s Facebook page what they can do to keep their children busy … [and] parents were also encouraged to look for maths activities in the newspapers” (SM3). A teacher noted that “parents were in charge calling and asking about the problem and how to do it” (SN3). Another teacher also noted that “parental support played a big role in making sure homework was done” (OL2). Similarly, it was reported that “the parents were involved and explained to their learners the work and check[ed] their books” (SN3) and “parents are very helpful and concerned about the education of their kids” (GD2). From these quotes we see that the home and school activity system were, in certain cases, productively engaged together in the object of continuing meaningful mathematical learning.

Only one teacher made exclusively Category 1 responses (4 responses) regarding parents in both the questionnaires. Eight teachers made exclusively Category 3 responses in describing the role of the parents (2 in Q1 only, 5 in Q2 only, 1 in both). Across the other teachers there was a mixture of responses across the categories. One teacher who made three Category 1 responses in the first questionnaire went on to comment in the second questionnaire that “parental support played a big part in making sure booklet ‘homework’ was done” (OL2). For the whole group (Table 1) we saw that the responses were predominantly Category 2 for the first questionnaire, and this shifted to being predominantly Category 3 in the second questionnaire. This indicated a potential shift in the teachers’ perspectives of the role of parents in the mathematics education of their children from 2020 to 2021.

Furthermore, it is interesting to note that in response to the question, “Are there strategies that you have developed as a result of COVID disruptions that you might continue using even after schooling returns to ‘normal’?” teachers mentioned the relationship with parents as being one they hoped to continue. Three such responses included:

> “I will continue using WhatsApp and involving parents and use videos and voice records” (GD2)
>
> “Will be more interactive with parents, reminding parents of the importance of homework and self-study. Keep contact with parents to see how they and their children and doing/coping” (PA1)
>
> “Teacher-parent interaction got more important, hoping to keep it like that” (PA2)

Discussion and Conclusion

It is important to note the source of the tensions evident in the Category 1 responses. Many centred on parental decisions to keep children away from school due to anxiety about COVID-19, a factor only mentioned in Questionnaire 1. They also focused on the limited resources in the home activity system, for example, no cellphones or data. This featured in both questionnaires and is likely to continue to be a challenge. It was also mentioned that the parents themselves were educated under a different curriculum and that this caused challenges.

Wadham et al. (2020) similarly found that there were tensions evident "between the contrasting mathematical pedagogies that parents use and those that teachers are directed to teach" (p. 15).

The increased interaction between teachers and parents, which is evident throughout the data, could mitigate this source of tension. Teachers reported creating "notes to explain concepts in detail [and] videos of explanations" (OL1) and "sending worksheets, instructions, pictures, messages and short videos to parents" (OL2). All of these measures would have functioned to support the parents in aligning their assistance of their children with the approach that the teacher would have taken and thus resulting in an effective interaction between the home and school activity systems to produce meaningful mathematical activity.

As with Darragh and Franke's (2021) finding that there was a range of parent experiences during lockdown home learning: from those who felt supported to those who felt unsupported, we note in this data a range of levels of support that teachers reported giving to parents. Some parents received extensive notes and explanations to support their efforts in the home activity system, whereas others were presented with instructions to collect work and requests to convey messages to the learners as evident in the Category 2 responses. These responses indicated that the home activity system was recognised as an important site of learning during COVID-19, but the support of the parents in that activity system was not always evident.

It is encouraging to note that just over half of the responses (23 of the 53) reference parents in a way that recognised them as partners in the continuation of mathematics learning of their children. This points to a possible shift away from earlier research findings (discussed above) that pointed to deficit views of parent support for learning. While there were instances in this data that reveal some of Wadham et al.'s (2020) noted tensions between teachers and parents, these were outnumbered by responses indicating partnering with parents. The stated teacher intentions to continue to engage closely with parents beyond the pandemic were also encouraging. There was recognition by some of the importance of connecting productively with the home activity system through engaging with the parents and a commitment expressed to continuing that practice (e.g., "teacher-parent interaction got more important, hoping to keep it like that [PA2]"). The teacher who made three Category 1 responses in Q1 commented in Q2 that "parental support played a big part in making sure booklet 'homework' was done" (OL2). We also see this in the shift in the balance of comments from eight Category 1 responses in the first questionnaire down to just one in the second, and an increase in Category 3 responses from 10 in Q1 to 13 in Q2. It seems likely that the pandemic had been the stimulus for this shift in perspectives on the role of parents. Parents were increasingly recognised as partners in their child's mathematics education and teachers were reporting on the measures they were taking to support the parents in orienting the home activity system to the object of continued mathematics education.

Despite our acknowledgement of the limitations of our small scale and localised study findings, our data presents a positive opportunity for moving forward. The pandemic has stimulated increased parental involvement in their children's education. As Muir (2012) noted, "students' learning is maximised when strong educational partnerships between home and school exist" (p. 1). We see in the data a shift to teachers viewing the parents as partners in the mathematical learning of their children and have a commitment expressed to continuing pandemic necessitated parent engagement moving forward. This points to the possibility of sustained productive interaction between the school and home activity system that could operate to strengthen the teaching and learning of mathematics.

Practical Implications

The responses of the teachers about what was effective in terms of engaging parents in the home activity system have implications for school communities and education departments who support these schools. Teachers presented practical ideas on how they effectively interacted with parents in a situation of distance, as COVID-19 presented. WhatsApp presented a cheap and effective means of quickly contacting parents, mentioned by 21 of the 25 participating teachers. This platform enabled teachers to share notes, instructions, videos, and voice explanations of work. While this was not possible for all families, as noted in the responses indicating parents had no cell phones or data, it was a simple technology that was within reach for the majority. Furthermore, given that this is a relatively cheap and effective form of communication, departments of education in South Africa could consider providing family data bundles to enable communication or a national policy could look to free internet connectivity, particularly in poorer areas. Teachers' effective use of booklets for 'home'-work also points to opportunities for schools and education departments to supply such workbooks for home use in future. Our data shared in this paper, along with the literature reviewed highlights that partnering with parents is *essential* for continued learning in contexts where learner access to schools is restricted and presents a powerful opportunity for strengthening mathematics learning beyond the classroom and beyond the pandemic.

Acknowledgment. This work is based on research supported by the South African Research Chairs Initiative of the Department of Science and Technology and the National Research Foundation (Grant No. 74658).

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Supporting Pāsifika Students in Mathematics Learning

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In this paper, we report on data gathered through Talanoa on how Pāsifika students can be supported to learn mathematics. The perspectives of five teachers were analysed, highlighting three themes. Firstly, the importance for Pāsifika students to feel a sense of belonging at school. Secondly, how the core cultural values of Pāsifika students can be drawn on to develop effective social and sociomathematical norms. Finally, the significance of presenting learning and mathematical activity in authentic contextual frames to support engagement and participation in learning. These findings offer potential for implementing pedagogical approaches that are meaningful, relevant, and can support the mathematics learning of many students currently marginalised in New Zealand schools.

Pāsifika is an overarching term used to describe a diverse group of people who originate, or identify in terms of ancestry or heritage, from the Pacific Islands of Tonga, Samoa, Cook Islands, Niue, Tokelau, Tuvalu, and Fiji. Each group is unique in terms of how they identify with specific ways of knowing, being, and viewing the world. However, there is a set of common values which Pāsifika peoples share, and some of these are family, respect, spirituality, service, humility, relationships, leadership, love, and reciprocity. All Pāsifika students bring these rich cultural values to school. When teachers understand and recognize the important role these values play in mathematics teaching and learning, they open up opportunities for Pāsifika students to connect important mathematical concepts to their cultural and everyday experiences (Bills et al., 2015). Supporting Pāsifika students to link their cultural values with their education offers potential for greater access and success in learning (Hill et al., 2019; Hunter et al., 2018). Drawing on core cultural Pāsifika values can also support teachers to co-construct important social and participatory norms with students who are often marginalised in mathematics classrooms in New Zealand. To draw on these core values, teachers need to understand what these values are and how they may be used to support mathematics learning.

In this paper, we report on findings from a small inquiry exploring how teachers can support the mathematics learning of diverse (in this case, Pāsifika) students. Specifically, we explored the following research questions:

What do teachers consider important to know about the students they teach?

How can teachers use what they know about their students to enhance mathematics teaching and learning?

Literature Review

Even though it is acknowledged that Pāsifika students enter school with rich cultural values, language, and experiences, there is often a detachment of the school system with the background these students bring to school, which in turn, can affect their academic achievement (Bills et al., 2015; Hill et al., 2019; Hunter et al., 2016).

Several New Zealand studies (Anthony et al., 2015, 2019; Bills & Hunter, 2015; Hunter et al., 2020) have drawn attention to the importance of establishing positive relationship with Pāsifika students to support their engagement in mathematics learning. In interviews, many Pāsifika students have stated that good teachers take time to get to know them as holistic learners, have high expectations, allow them to take risks and value students working collaboratively as a family to solve mathematics problems (Bills & Hunter, 2015; Hill et al.,

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 522–529. Launceston: MERGA.

2019; Hunter, 2014). When these kinds of positive relationships are established, an environment that embraces students' cultural values as resources for learning can be established (Hunter et al., 2018). To create these communities of learning it is essential to develop positive social norms. Social norms help set expectations and guidance for effective student collaboration. In addition, establishing social norms can guide students to work together in respectful and mutually accepted ways.

Establishing social norms can be static and rigid, or inclusive and vibrant. Initially, classroom norms associated with expectations regarding how to work as an individual within a group need to be established. This action supports student engagement with peers as they work together on mathematical activity. Importantly, social norms should be co-constructed with students and aligned with realistic cultural contexts such as working together as a whānau (family) or helping (supporting) each other in the group (Hunter & Miller, 2022). Teachers who use family/whānau as a metaphor for ways of working together allow Pāsifika students to draw on the concept of family to support their mathematics learning. For Pāsifika learners, the concept of family is a vital factor affecting their engagement in learning (Hill et al., 2019). Moreover, the value of respect for family members can be drawn on to support effective student collaboration. Cultural respect is understood as an accepted means of working collectively. When Pāsifika students learn mathematics through this cultural lens, they understand how to ask questions for clarification and to challenge their peers' mathematical claims or content in respectful ways (Hunter et al., 2018). Asking questions supports diverse learners to take ownership of their mathematics learning and become knowers and doers of mathematics. Establishing and maintaining effective social norms drawn from core cultural values are significant to Pāsifika students' learning because these are mutually understood and expected ways of interacting with others. Furthermore, Pāsifika students have been seen to engage more deeply with mathematics learning when teachers regard their cultural values as strengths and use them to establish effective participation and interaction patterns (Bills et al., 2015; Hunter et al., 2016; Hunter et al., 2018). While core cultural values can be drawn on to establish positive and effective norms, these can also be used to support the design of culturally relevant mathematics activity (tasks) that Pāsifika students can relate to.

Meaningful contextually relevant mathematical tasks can support diverse students' learning of mathematics in several ways. Firstly, when mathematical activity or tasks are contextually or culturally relevant, Pāsifika students can begin to connect mathematics they encounter at school with their cultural and everyday experiences (Hunter & Miller, 2022). These kinds of tasks also provide access for students to engage in robust discussion with their peers, as the familiarity and relevance of context provide an entry to the mathematics inherent in the task (Hill et al., 2019). When Pāsifika students can see their lived realities within mathematical activity, they are given opportunities to explore their mathematical understanding through multiple pathways which allow equitable access for all. In such ways, these students are recognized and acknowledged as intellectually capable knowers and doers of mathematics (Lotan, 2003). Contextually relevant mathematics tasks also provide students with multiple entry and exit points to mathematical concepts, as well as numerous opportunities to explore and show intellectual competence (Hunter et al., 2018). In such ways, Pāsifika students are given access to showing their mathematical smartness in many creative ways. In turn, they can openly accept and appreciate their peers' contribution and reasoning

When Pāsifika students' core cultural values are drawn on to support their mathematics learning, they develop a sense of belonging and can engage meaningfully in mathematical discourse, thus experiencing more equitable outcomes and success.

Research Methods

This study was grounded in a sociocultural perspective and drew on a qualitative interpretive framework. This small inquiry was set in three urban primary schools in New Zealand. These schools are situated in very low socio-economic areas, where most of the students are identified as Pāsifika. Five teachers participated in this study. Four of the teachers were experienced, and one teacher was in their first year of teaching. Three of the teachers were of Pāsifika heritage, while two were of European descent.

Data were collected through Talanoa. Talanoa is a talking tool that is used by Pāsifika people to address any concerns within family, extended family, church, or within any collective setting (Havea et al., 2021). Talanoa is a tool utilised for discussing important issues through a process of starting a conversation that can lead to innovative ideas, or a discussion of political views (Vaioleti, 2006). Talanoa can be done collectively where everyone in any gathering, a family, or church can share their feelings and thoughts on certain matters. In essence, Talanoa is viewed as a time for Pāsifika people to share their concerns or ideas for their communities' growth, or to discuss a problem affecting the wellbeing of everyone (Havea et al., 2021). The justification for the use of Talanoa in this inquiry is multilayered. Firstly, the main researcher is of Pāsifika descent and Talanoa is a culturally appropriate way to discuss issues. Secondly, most of the teacher participants were of Pāsifika heritage. The use of Talanoa allowed the main researcher and the participants to develop a positive relationship that enabled the participants to share their ideas and perspectives openly. The main researcher used Talanoa to create a safe Vā (space) for the participants to share their perspectives truthfully and encourage conversation. The use of Talanoa to create a safe Vā (space) allowed both the main researcher and the participants to respect each other's views, but at the same time keep the focus of the talk on ways to support Pāsifika students' mathematics learning.

As this small inquiry took place during a time of mandated lockdown across New Zealand due to Covid 19, Talanoa was conducted online. The first step of this process involved inviting the five participants to participate in an online (Zoom Platform) Talanoa. As Talanoa is a talking tool that Pāsifika people can use for communicating formal or non-formal issues and it can be done via Zoom platform, as long as the participant and the researcher are talking to each other on the matter of concern. Upon agreement to participate, the teachers were sent a set of questions to reflect on, prior to the Talanoa. Each participant agreed to participate in individual Talanoa. The decision to Talanoa individually is justified as it served to provide all participants a safe space and time to discuss their perspectives and ideas. Each Talanoa began with the main researcher greeting each participant respectfully and checking that they were comfortable. These were important elements to ensure that the participants understood that their perspectives were valued. The researcher invited each participant to talk about their work or family, and as the Talanoa progressed, the researcher would participate and share common experiences. When the researcher felt the participant was at ease, Talanoa turned toward the teacher reflections on the set of questions. Talanoa continued until both the researcher and the participant felt they had fully discussed all that was to be shared. The mālie (joy) of the Talanoa was approaching the end of the conversation, whereupon the main researcher acknowledged and thanked that participant. Each Talanoa lasted for 45 minutes to an hour. While two participants were of European descent, both had been working with Pāsifika students in South Auckland, New Zealand for several years. Both participants were familiar with Talanoa and had used Talanoa with their students to resolve matters in their classes. One of the Pāsifika participants was of Tongan descent and the main researcher Talanoa or spoke with her in Tongan. The other Pāsifika teachers spoke in English with the researcher during Talanoa.

The Talanoa provided insight into what the teachers identified as being important to know about their Pāsifika students; and how they could use these insights to support Pāsifika students' mathematics learning. Data analyses consisted of thematic analyses of the transcribed Talanoa.

Findings and Discussion

The findings of this investigation are organised around specific themes evident from the data analyses. The first theme illustrates the importance of Pāsifika students feeling a sense of belonging at school. The second theme highlights how drawing on the cultural values of Pāsifika students can support the establishment of effective participatory norms. The final theme argues for the use of authentic learning contexts to engage and enhance Pāsifika students' mathematics learning. Extracts from Talanoa are provided to illustrate the teachers' insights within these themes.

Establishing a Sense of Belonging

Through Talanoa, the teachers were asked to identify and describe what they thought was important to know about their Pāsifika students. All the teachers' initial responses emphasised the importance of Pāsifika students developing a sense of belonging at school. An extract from their discussion is presented below:

> Lani: We need to get to know our children because we know that our children learn best in environments of learning, which they can identify with.
>
> Maria: Yes, our children learn better when they are in a familiar place.
>
> Susan: It is important to develop learning situations for my learners which they could relate to-culturally and ethnically.

These responses highlighted that these teachers understood how essential it is for Pāsifika students to hold a sense of belonging and familiarity at school and in their classrooms. A sense of belonging is pivotal for Pāsifika students' participation and engagement with their learning. Previous studies (e.g., Hunter, 2014; Hunter & Miller, 2022) have illustrated that Pāsifika students develop respect towards teachers who set high expectations for their studies. These teachers' statements emphasised the importance of supporting Pāsifika students to connect mathematics at school to their cultural and everyday experiences. In addition, the teachers have recognised the value of connecting with their students and getting to know who they are. Susan has stated the importance of knowing her children's cultural backgrounds as well as their ethnicity. Knowing the differences between a Samoan child and a Tongan child means that assumptions that every Pāsifika child's cultural experiences is the same or similar can be avoided. These concepts link to previous research (e.g., Anthony et al., 2019; Bills et al., 2015; Hunter et al., 2020) highlighting the signficance of teachers developing positive relationships with their students to support their engagement in mathematics learning. Other research (e.g., Hunter et al., 2018) has demonstrated that when students develop a sense of belonging in places where their cultural values are acknowledged, their learning can be enhanced.

As the Talanoa progressed, the teachers shared more about how knowing who their Pāsifika students were could support the establishment of an effective learning environment. Specifically, the Talanoa centred on the ways that Pāsifika students cultural values could be drawn on to support the development of effective participatory and interaction patterns in mathematics classrooms.

Drawing on Cultural Values to Develop Effective Social Norms for Participation

Deepening focus in the Talanoa highlighted the teachers' agreement that developing effective learning environments could be enhanced by developing classroom norms that Pāsifika students could identify with. The following excerpt illustrates the teachers' thoughts on how students could be supported to work together in effective ways:

> Judy: By discussing these norms with the students.

> Lani: By discussing the norms with the students and reinforcing the values of the schools and bilingual unit, so that all have an understanding of what the expectations are in our class.
>
> Luisa: I use what they would use in their homes. The norms in the family-to listen when someone is speaking, or to pay attention. These things are taken from our idea of respect and family. Teachers are encouraged to go back to our cultural background on how we behave when we are on the mat, or when we work in our groups. It is important to create a connection to the environment of our children similar to how they would behave at home. Our norms carry the way the lesson will run.

These teachers have identified how norms and expectations for learning mathematics need to be co-constructed and explicitely connected to taken-as-shared ways of interacting in Pāsifika students' home and community environments. These responses highlight the importance of establishing a safe learning environment where all learners' experiences and ideas are valued and everyone has a responsibility to contribute. In a previous study, Hunter (2014) illustrated the importance of students taking responsibility for their own learning and that of the group. This responsibility can be recognised as students ensuring their explanations are clear, asking questions when they are unsure, contributing their ideas, and reasoning with their peers conjectures.

The teachers in this inquiry have also highlighted the significance of aligning social norms with specific cultural contexts, such as how students would interact with family at home; including expectations about how to speak, to listen, and to focus on important matters. Norms in the Pāsifika family can also refer to one's role or roles within the family, such as the older children being expected to look after their younger siblings and cousins. Older siblings are taught from a young age that caring for their younger siblings and cousins is their responsibility. These taken-as-shared expectations mean that students understand that their role in a group is to support their peers. An important connection between the Pāsifika notion of family and the value of respect has also been emphasised. Respect in Pāsifika contexts is demonstrated by people working together, listening to one another respectfully, knowing one's role or place within the family, and being of service. Being of service within Pāsifika culture means helping and supporting each other for the greater good. The teachers have explained that if they utilise what their students are familiar with to co-construct taken-as-shared expectations for participation and interaction, these students would understand what they were expected to do and how they were expected to participate (behave).

As the Talanoa progressed, discussions centred on the idea that if Pāsifika students core cultural values could be drawn on to establish and maintain effective participatory norms, they could also be used to develop sociomathematical norms. Sociomathematical norms encompass actions such as, making conjectures, explaining, and justifying mathematical reasoning to enhance mathematical understanding. The following excerpt illustrates how specific Pāsifika cultural values could be utilised to support students to ask questions, be involved in mathematical discussions, and represent their mathematical reasoning:

> Lani: We need to teach them how to ask questions to clarify; how to agree and disagree respectfully; how to articulate what they want to say; and how to know when they understand.
>
> Judy: Yes, give them time to talk, not push them too much.
>
> Maria: Look at how they respond when they do not know or when they understand. Do they question, do they clarify? Do they know how to articulate their responses and questions?
>
> Luisa: Provide encouragement. Students can also show their learning through their drawings.

The teachers have noted that explicit instructional actions need to be taken to support students to participate meaningfully in group discussions. One of these actions is for teachers to be mindful of their students' cultural identities and support them in appropriate ways to engage in collective discussions. Another action is to give students Vā (space). Vā in a Pāsifka world is a time to think and consider what has already been claimed, explained, or justified. Vā is also a safe space and place for Pāsifika children to Talanoa with their teachers about their feelings and ideas regarding their mathematics learning. Talanoa within Vā between teachers and students can also extend to their peers. In addition, the teachers have acknowledged there is more than one way for students to represent their mathematical reasoning, and that students can demonstrate their understanding in multiple ways.

At this point in the Talanoa, one teacher called for caution when setting expectations for productive discourse. An excerpt follows below:

> Maria: Encouraging students to share or justify their thinking in a small group can only be done if the group norms are set and the environment is one that is safe for our kids. It also has to be noted that in some cultures it is ok for a child not to share. For example, in a Tongan Talanoa, it is not mandatory for a child to share their thoughts-they have the right to pass. It is hoped that one day that child will participate when confident in their own thinking.

Maria has highlighted how care must be taken when drawing on Pāsifika students' cultural values, as, at times, expectations to share thinking or to ask questions may not be expected at home. In these instances, teachers require deep understanding of how some students interpret school expectations for participation, as for some students, participation (at home) means sitting quietly and listening, and having the right to take time to conisder what has been discussed. As the talk continued, attention turned to the learning contexts appropriate to provide support for Pāsifika students. Emphasis on authenticity became evident.

Authentic Learning Contexts

Authenticity emerged as an important theme of the evolving Talanoa, and the teachers began discussing the importance of designing learning content that was meaningful and relevant for their students. Once again, the teachers looked for ways to connect learning at school with the experiences and contexts that Pāsifika students would find familiar or relate to in authentic ways. The following excerpt highlights the teachers' perspectives:

> Maria: I believe it is very important for students to relate to and have authentic learning contexts.
>
> Luisa: Yes, contextual tasks need to be at the forefront when thinking about mathematical context, as this will automatically, hopefully connect them instantly to the problem.
>
> Lani: Yes, relevance and connection to their learning is vital to retaining new information.
>
> Luisa: They are able to reflect culturally on problems they have experienced in their daily lives. They are able to bring in experiences from their culture where it has no limitations for them to solve problems efficiently. There is nothing nicer than seeing the importance of a child when a task is relevant or includes them.
>
> Judy: We are trying to connect to it by thinking when this problem is real life, reality.
>
> Susan: I will set a scene the children are familiar with using culture or home.

These responses highlighted the teachers' understanding of the importance of supporting diverse students to connect with their learning. Previous research (e.g., Hunter, 2014; Hunter & Miller, 2022) has shown that when students can connect to their learning they are more likely to engage actively with learning new concepts, or seeing relevance to what they are learning in school with their everyday experiences outside the classroom. These teachers have recognised that an authentic contextual task is a platform for Pāsifika students to take ownership of their own learning, as the task allows them to Talanoa about their own realities. As has been highlighted in previous study (Lotan, 2003), the use of authentic contextual tasks strengthens diverse students' mathematical knowledge and reasoning while at the same time acknowledging their cultural heritage and experiences. These teachers have demonstrated their awareness of the significance of presenting learning in meaningful ways. Learning that is relevant to students provides multiple access points for students to connect, engage, and reason with important mathematical concepts. When all students can access relevant mathematics content they are more apt to demonstrate a wider range of capabilities and success. In such ways, there is greater potential for Pāsifika students to experience more equitable outcomes and success.

Conclusions

This paper has reported on key elements identified through Talanoa of what teachers believe is important to know about Pāsifika students, and how to use what they know to support these students in learning mathematics. Several themes were presented and discussed. The first theme explored the importance of establishing a sense of belonging for Pāsifika students and how by doing so, these students could be provided with opportunities to recognise connections between what is important at home and what is valued at school. Establishing a strong sense of belonging offers potential for Pāsifika students to develop positive relationships within the school environment. The second theme explored how drawing on Pāsifika cultural values could support the co-construction of positive and taken-as-shared social norms essential for establishing and maintaining effective collaborative learning environments. Highlighted within this theme was the significance of understanding Pāsifika notions of family and respect. The final theme focused on the essential role authentic learning contexts play in supporting Pāsifika students to access, engage, and be successful in learning mathematic. Woven throughout these themes was the emphasis for teachers to take deliberate action to build meaningful support structures for Pāsifika students.

Although not the main focus of this inquiry, the use of Talanoa as a data collection tool was an important cultural artefact in this research. Utilising this data collection method drew on the cultural practices of the main researcher and most of the teacher participants. In this way, the cultural identities of the teachers were acknowledged and positioned as significant. We acknowledge that the research reported here was drawn from a small inquiry. However, the findings highlight the complexities of teaching mathematics in ways that provide equitable access for students who are often marginalised in New Zealand classrooms. Drawing on the rich cultural backgrounds, experiences, and ways of Pāsifika students to create authentic, relevant learning environments is an important pedagogical approach that provides opportunities for these students to feel a sense of belonging and the right to be there. In addition, when the lived experiences of the students are used as realistic contexts for mathematical activity, these students are able to make connections to important mathematical concepts.The instructional actions highlighted by the teachers in this research illustrate potential opportunities for Pāsifika students to achieve success in learning mathematics.

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Exploring the Potential for Student Development of the Big Ideas of Statistics with Random Trials: The Case of the Mystery Spinner

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This paper reports on the potential for engaging students in an activity that considers the interconnectedness of the five Big Ideas of Statistics in the context of conducting random trials. In the study, two classes of Year 6 students (aged 11-12 years) used *TinkerPlots* to determine the sample space of a "Mystery Spinner." Analysed for this paper were data collected from entries made in completed workbooks while engaged in the learning activity and responses to relevant questions in an end-of-year questionnaire from 27 students. The results indicated using an activity that included a probability model contributed to students working mathematically with percentages and frequencies and supported the development of intuitions about randomness and informal inference. This was promoted by students analysing the variation in the distribution of data and describing their expectations about an unknown sample space.

It has been common for primary students to use hands-on spinners when investigating probability in the primary years of schooling (e.g., Chick & Baker, 2005; Torok, 2000). In most situations the frequency and relative frequency of events occurring, the confirmation of the outcomes according to the known sample space (actually seen on the spinner), and fairness/randomness of the devices are the focal points of such investigations. These investigations tend to emphasise that many trials will ensure the experimental results reflect the known sample space (theoretical expectations). In this case, variation and expectation are fundamental concepts associated with randomness. Developed from early experiences of probability that usually involve tossing dice or coins, the students appreciate that the experimental results can *vary* from the theoretical *expectations* when only a few trials have been undertaken but tend to be confident that conducting more trials will ensure "success" in getting the outcomes expected, with little acknowledgement of the variation that can still occur. There are benefits of using hands-on materials to develop and extend mathematical knowledge and problem-solving strategies (Chick, 2018) but the scope for artefacts such as dice, coins and spinners is limited. Traditionally, success in these situations is measured by getting what would be considered "the correct answer." Taking the traditional approach in the classroom constrains students' opportunities to engage in the productive struggle required to develop higher order thinking, such as critical thinking and reasoning skills.

Probability and statistics are often taught as two discrete subjects within the mathematics curriculum. This does not assist in connecting the areas of probability and statistics, which is required when questioning data and making inferences from statistical information (Batanero et al., 2016; Watson et al., 2018). With the goal of probability activities to compare experimental results to theoretical outcomes determined by the known sample space (Lee et al., 2010), the opportunity of developing understanding of probability concepts encountered in statistics later in students' studies and future working situations (e.g., *p*-values) is scant.

Probability activities in the early years of schooling are essentially devoid of the exploratory nature of statistical enquiries that meet goals of student development of higher-order thinking. There is, however, scope to deepen students' learning about probability concepts by introducing elements of cognitive conflict into learning opportunities. Cognitive conflict is the disagreement between cognitive structures, such as knowledge and mental representations, and experience (Waxer & Morton, 2012). The challenge is to develop activities that build on established knowledge and take advantage of what students would consider familiar in order to promote engagement in deeper and more complex problem solving and

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 530–537. Launceston: MERGA.

reasoning. Learning activities need to be not only cognitively accessible for students but also cognitively challenging to ensure opportunities for growth in learning are maximised. Therefore, it is important to develop learning sequences that establish prior knowledge, consolidate learning, and extend that initial learning in anticipation of targeting long-term goals of critical thinking. Important research problems in this regard are:

> (a) clarifying the way in which probabilistic thinking could contribute to improving mathematical competencies of students, (b) analysing how different probability models and their applications can be presented to the students, (c) finding ways in which it is possible to engage students in questions related to how to obtain knowledge from data and why a probability model is suitable, and (d) how to help students develop valid intuitions in this field. (Batanero et al., 2016, p. 25)

Hence, the research question for the study reported in this paper is:

In terms of the Big Ideas of Statistics, what is the learning potential of an activity that requires students to use randomly generated data to make conjectures (hypotheses) about the sample space of a hidden spinner?

Related Literature

Five Big Ideas underpin statistics education at school and beyond (Watson et al., 2018). In relation to probability, carrying out *Random* trials, students experience *Variation*, which is viewed through a summary *Distribution*, resulting in an *Expectation* about the underlying sample space. As trials increase, more *Variation* occurs and *Expectation* about the ultimate outcome is likely to change. The goal is to make an *Informal Inference* about the underlying probability of the elements of the sample space. The interconnectedness of the five underpinning concepts is shown in Figure 1.

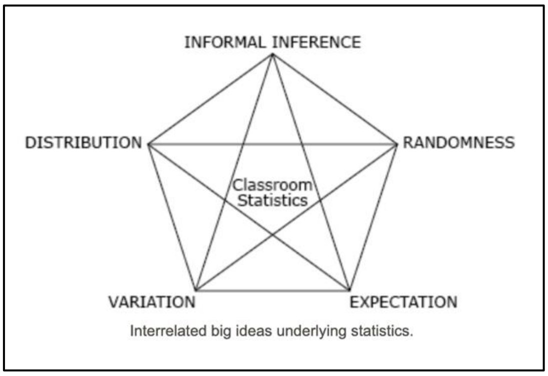

Figure 1. The Big Ideas of Statistics (Watson et al., 2018, p. 126).

Randomness and informal inference are the more complex of the Big Ideas, as each requires two components for understanding. For randomness, there is more than the individual outcome of being uncertain; there also needs to be a progression towards a fixed outcome with many trials (Lee et al., 2010). For informal inference, there needs to be both the expression of an expectation and a level of confidence expressed in the expectation because the result is not known for certain (Watson et al., 2018). Classroom experiences demonstrate that "random" is not an easy principle for students to understand fully (Chick, 2018; Lee et al., 2010; Watson & Fitzallen, 2019). Part of this may be associated with the activities employed. Chick discussed the pros and cons of many of the main activities that were used in linking probability and randomness, including one based on two spinners, and pointed out the importance of recognising the learning opportunities afforded by activities used to foster intuitions about randomness. This imperative concurred with suggestions made by Batanero et al. (2016) and Lee et al. (2010).

Probability, as taught in school, is basically the endpoint of random phenomena, based on the “sample spaces” that are observable on the sides of a die or coin that is tossed. In this case, activities do not produce challenges for two of the big ideas: the expectation is *seen* on the device and there is no informal inference to be made because the *answer* is known at the beginning. Specific approaches to the learning and teaching of probability are needed to enhance the learning outcomes of traditional probability activities (Batanero et al., 2016; Chick, 2018; Chick & Baker, 2005). As suggested in the *Australian: Mathematics Curriculum* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2021), frequency and sample space tend to focus on the Proficiency Strands of understanding and fluency. It is interesting that although ‘random number’ and ‘random sample’ were defined in the Australian Curriculum Glossary for Mathematics (ACARA, 2021), the term ‘random’ itself was not defined. Random is associated directly with the relationship between expectation and variation.

Research Approach

The methodology employed was exploratory, which warranted using qualitative research and data analysis strategies to look for possibilities and opportunities in a pragmatic fashion (Mackenzie & Knipe, 2006). The learning sequence in the classroom included establishing prior knowledge, consolidating foundational learning, recognising issues arising, and carrying out the main investigation of random trials with a mystery spinner to extend learning. Data were collected from student workbooks and, as part of measuring adoption of the intended outcomes, responses to questions in the end-of-year questionnaire.

Context of the Study

As part of a classroom intervention with Year 6 students (aged 11-12 years) near the end of a 4-year project (Fitzallen & Watson, 2020), 56 students in two classes took part in two extended lessons, taught by the classroom teacher, addressing the Chance curriculum for Year 6 (ACARA, 2021):

> Conduct chance experiments with both small and large numbers of trials using appropriate digital technologies (ACMSP145)
>
> Compare observed frequencies across experiments with expected frequencies (ACMSP146)

The students had been involved in data handling activities to answer statistical questions, but the activity reported in this paper was their first encounter as part of the project with the Chance part of the curriculum. The activity was hence planned to establish students’ previous knowledge and explore an extension encompassing an unknown sample space, allowing for an informal inference to be made (Batanero et al., 2016). The activity reported in this paper was designed to allow for the focus to be on the random nature of trials, the variation encountered, the expectation created, the distribution of results to assist with the expectation, and the inference made about the underlying sample space being examined with spinners (cf. Figure 1). Although advances in technology have made it possible for students to complete many trials of a spinner (e.g., Chick, 2018; Lee et al., 2010), often these activities still display the final goal of the trials as trials are taking place. The *TinkerPlots* software (Konold & Miller, 2015), however, offered the option of a Mystery Spinner, where the proportions of the spinner were not visible as trials were performed. The purpose of the blank spinner was to introduce cognitive conflict into the learning experience to provide a situation where students were required to determine the sample space from the data generated. As Batanero et al. (2016) suggested,

> These “black-box” types of simulations may assist students in thinking about probability from a subjective or frequentist perspective where they can only use data generated from a simulation to make estimates of probabilities that they can use in inference or decision-making situations. (p. 19)

Learning Sequence

Establishing prior knowledge in terms of Expectation. The goal was to establish that students could relate to the term expectation and appreciate what it meant in relation to their learning and real-life contexts. Could they make reasonable predictions and be creative and imaginative in describing what would be considered realistic expectations? The lessons began with a review of likelihood in terms of expectation, with students in groups of four, writing down on an A3 sheet of paper divided into quarters, and discussing:

1. Something I expect will **definitely** happen this weekend …
2. Something I expect **might** happen this weekend …
3. Something I expect **may** happen this weekend but I do not think it is very likely …
4. Something I definitely **do not** expect to happen this weekend …

Figure 2 illustrates the typical responses of students in these categories. The responses established that the students could relate to the term expectation and could appreciate what it meant in relation to their school and real-life contexts. The students made reasonable predictions and were creative and imaginative in describing what would be considered impossible expectations. A few of the apparently less appropriate responses were discussed within the groups and with the class. This was then combined with a review of the general language of likelihood with a clothesline labelled from 0 to 1. Students drew words/phrases out of a bag and attached them to the clothesline with discussion about their placement in terms of the probability represented.

1. The sun will come up … Won't go to school … Humans will exist … Baby will be born (every 19 secs) … Eat food … Speak to my family.	2. Someone might watch a Marvel movie … People will be on their phones … Going to church … Homework … Go rock climbing.
3. I will clean my room … WW3 … MacDonald's will shut down … Donald Trump gets killed … Go to the park … Might have a 4-day weekend.	4. The sun to fall out of the sky… The moon will rain water … Go to Mars … Meet an alien … To play tag around the park … Go to school.

Figure 2. Examples of students' expressions of expectation.

Consolidating foundational learning. The initial data collection activity included using conventional spinners. The aim was to establish mathematical expectations in terms of the outcomes of spinners according to the sample space with the benefit of many, many trials. After the teacher reviewed the expression of probabilities as fractions and did trials of a 50-50 black and white spinner, students conducted trials with hands-on spinners as seen in Figure 3, recognising $Pr(\text{red}) = Pr(\text{yellow}) = Pr(\text{green}) = Pr(\text{blue}) = \frac{1}{4}$. A four-colour spinner was chosen as a hands-on beginning of the data collection because of the possible extensions using the technology, *TinkerPlots*, to extend the number of trails as implied in the curriculum statement.

The students worked in groups of four, individually conducting 20 spins of the spinner, recording the results in their workbooks, and commenting on how close they were to each other and to the probabilities built into the spinner. They then combined their results in pairs, reporting whether their results were closer to the probabilities than before or further away. Finally, they combined results for the four students in the group, again commenting on their results and the expected probability for 80 spins. Figure 3 shows the outcomes for the four-part spinner from two students in different groups. Obvious from this, and many cases, the results of the approximations for some groups got worse over the trials but most got closer. For some students it took until the data from the whole class were combined before they were convinced that the desired percentages for the probabilities were approached.

Number of trials	Red		Yellow		Green		Blue		Comment
	n	%	n	%	n	%	n	%	
20	6	30%	4	20%	5	25%	5	25%	Very close
40	16	40%	8	20%	8	20%	8	20%	No (not closer). Much variation
80	31	38.75%	20	25%	16	20%	13	16.25%	More variation

Number of trials	Red		Yellow		Green		Blue		Comment
	n	%	n	%	n	%	n	%	
20	9	45%	4	20%	1	5%	6	30%	Spinner doesn't like me.
40	14	35%	11	27.5%	5	12.5%	10	25%	Closer
80	23	28.75%	19	23.75%	15	18.75%	23	28.75%	More closer

Figure 3. Results from four-part spinner trials.

Recognising issues arising. There are two issues, however, that arise here: after an introduction to these devices, assuming they are fair, (1) students *know* the answer they are trying to approach: it is just a matter of how long it takes to get there! Also, (2) it can get tedious doing many trials by hand and mistakes can creep in using a calculator to make calculations.

For Issue (2), the technology employed was *TinkerPlots* (Konold & Miller, 2015) to save the tedium of performing hundreds of trials. Students had used *TinkerPlots* in other activities as part of the project (e.g., Watson et al., 2022) and were keen to use the program for performing the spins of the spinner. *TinkerPlots* had a "Sampler", which could repeat the trials (pseudo-randomly) for a model of the spinner, producing increasingly large numbers of trials quickly and displaying the results in a table or plot. Once using the technology, it was not necessary for the spinner to be separated into four equal parts. The classroom teacher then did a demonstration to show the possibility of using other divisions of the spinner, as seen in Figure 4. As well *TinkerPlots* displayed the distribution of the results in a format different from the tables that the students had used in their workbooks with the hands-on spinners (see Figure 3). The teacher then increased the number of trials, eventually reaching several hundred, with discussion of how close the percentages in the Results of Sampler 1 distribution were getting to the values that could be seen on the spinner.

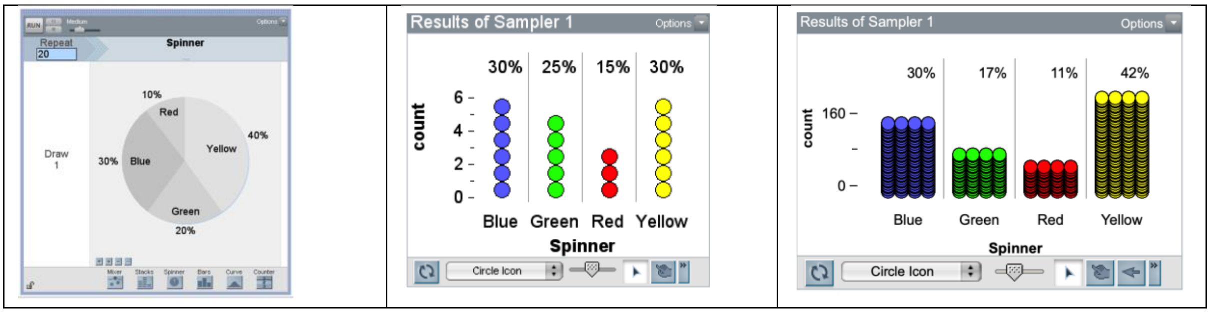

Figure 4. Teacher demonstration with 20 spins and then with 500 spins.

Extending learning. In relation to Issue (1) of knowing the makeup (sample space) of the spinner at the beginning, a blank spinner was introduced. *TinkerPlots* had the option of being able to hide the proportions of each colour. The aim was to introduce cognitive conflict as variation and expectation were juxtaposed in inferring the construction of the spinner. Students were given a "Mystery Spinner" with unequal sections, with the goal to work out the proportion of each colour. Six different Mystery Spinners (appearing in *TinkerPlots* as Figure 5, left) were prepared for the students to explore individually. The spinners were divided into three segments, labelled Apple, Banana, and Cherry and the Sampler was labelled "Fruit_Spinner".

In their workbooks, students started by recording the outcomes for 20 spins, then drawing a spinner with their estimates of the percentage of each fruit as their expectations of its construction (Figure 5, centre). They were then asked to mark their level of confidence on the arrow (Figure 5, right) and explain their reasoning. The number of trials was then increased to 50, 100, and 500, with similar data recorded and diagrams drawn. Student also answered the question, “Has your prediction or level of confidence changed? Why/why not?” This activity exhibited the five Big Ideas of Statistics (Figure 1). It was a (pseudo)*random* process taking place with the technology; there was *variation* as the trials progressed; after each trial students expressed their *expectations* by sketching in the divisions of the spinner in Figure 5; this was based on the *distribution* of *TinkerPlots* results similar to that seen in Figure 4; and each time an *informal inference* (prediction) was made, the students marked their level of confidence on the accompanying arrow image. The level of confidence was an indicator of the degree the students expected the results at each set of spins reflected what they expected to be the makeup of the actual spinner. The students’ confidence for one set of spins was influenced by how much the result varied from the previous set.

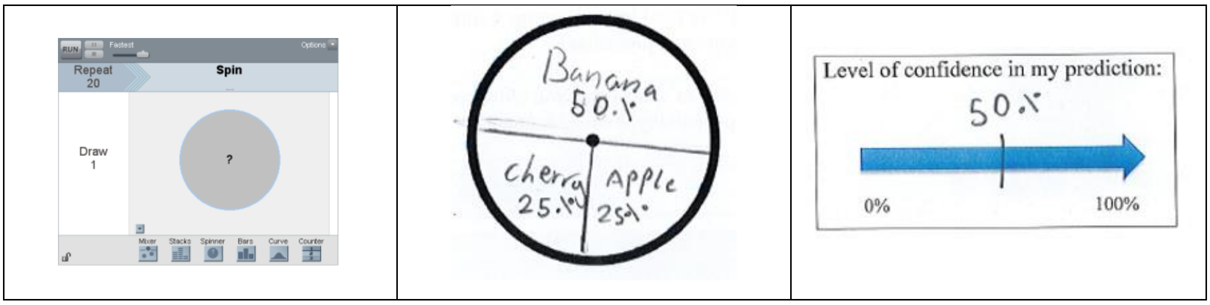

Figure 5. Trials with an unknown spinner.

Table 1.
Student confidence levels and comments

ID	Confidence: 50 spins	Confidence: 100 spins	Confidence: 500 spins
ID143	MC: There’s more spins.	LC: Because they are changing a lot. So I’m not as confident.	MC: I’m confident because the spinners in the last 2 examples have been very close.
ID129	LC: Yes and no, I’m not very sure.	MC: Yes, because I have really thought about them and I’m a bit more confident.	LC: Cherry has done better. We are confident that Cherry is really low but we are not confident what % to give it!
ID122	NC: It’s gone down a bit but my results were quite similar still.	NC: It hasn’t changed because I think that my prediction is correct or very close because the results are all similar.	NC: It still hasn’t changed because I think I’m close.
ID175	NC: Because I think there is more fruit [only two different fruits appeared on the first 20 spins]	MC: Yes, because I think there might only be 3 fruits.	MC: Yes, I think there is definitely only 3 fruits.

The results reported here are from a subset of the workbook data. The data were included if students had completed and reported all spins and written responses ($n = 27$). Based on their marked confidence arrows and/or their comments, after 50 spins eight students had no change

(NC) in confidence in their expected result from the first 20 spins, seven had less confidence (LC), and 12 had more confidence (MC). After 100 spins there were four with no change in confidence, 18 with more confidence, and five with less confidence than before. After 500 spins results were similar with three students indicating no change in confidence, 19 more confident, and five less confident. The reasons for these judgments varied across the trials. Typical responses are presented in Table 1.

Students could then choose as many trials as they liked and record their results making a final prediction of the percentage of each fruit, answering the question, "What makes you confident that your final prediction is correct? Not many students took up this option but all who did recognised that the often-huge number of trials produced the required percentages.

Measuring adoption of intended outcomes. On the end-of-year questionnaire, approximately eight weeks later, students were presented with the distribution of outcomes for two Mystery Spinners of the type they had investigated in the classroom, one from 30 spins and the other from 600 spins (Figure 6), asked to make a conjecture—hypothesis about the theoretical probability—about the sizes of the three parts of the spinner each time, and to explain why they made the decision each time.

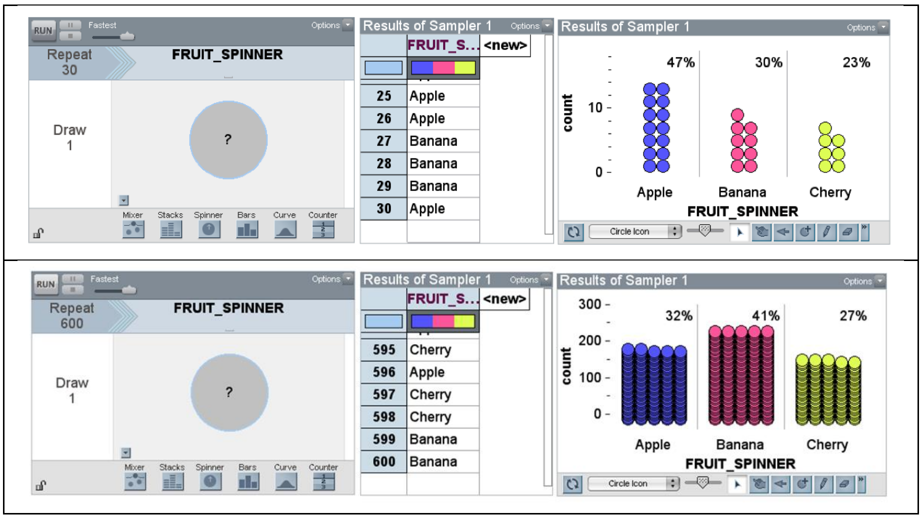

Figure 6. Distribution of data for a mystery spinner used in the end-of-year questionnaire.

For the 30 spins trial, 30% of responses reflected the type of rounding to multiples of 5% or 10%, with 20% of reasons being less precise rounding or "adding to 100%", and 52% saying "that's what the plot tells me." Similar results occurred for 600 spins, with 20% of responses reflecting appropriate approximations and 46% remaining exact values from the figure or suggesting more extreme values. When then asked in which results they had more confidence, 61% said 600 spins and 39% said 30 spins. For the reasons for the choice of the number of spins, 43% specifically noted "more data", "more spins" or "more chance" for choosing the 600 spins. The other 57% either chose 30, with reasons like, "cause it's quicker" or "because it's hardly any spins", or chose 600, with nebulous reasons like, "I just feel more confident". Hence, although 61% of students chose the appropriate number of spins only 70% of them also justified the choice based on the sample size.

Discussion and Conclusion

The learning potential embedded within the Mystery Spinner activity was associated with all five Big Ideas of Statistics (Figure 1). This allowed for students to express expectations during trialling and to revise them as variation was seen in successive random trials, which increased in size progressively throughout the activity. An informal inference was then made, accompanied by a degree of confidence. In this case the confidence level expressed was an indicator of student expectation. The opportunity to shift learning to a technological environment demonstrated the potential of using technology to expose students to extended learning opportunities not possible within the constraints of using physical manipulates (Batanero et al., 2016; Lee et al., 2010). The Mystery Spinner activity provided a rich learning opportunity that included outcomes in terms of frequency and percentage through attempting to balance variation and expectation to make judgments about the sample space of the mystery spinner. From a research perspective, this study illustrated that the students were able to make connections between the relative frequency of data and the theoretical probability, but some students only reiterated the raw data—expressed as counts or percentages—when making conjectures about the makeup of the spinner. This suggested that the learning potential of the activity was not totally realised for these students, who, in many cases, had unstable ideas about making inferences from data and did not yet appreciate that the theoretical model embedded in their mystery spinner could potentially vary from the results of the trials. More research is needed that focuses on student understanding of the relationships among the Big Ideas of Statistics to inform effective learning and teaching of statistical concepts, thereby addressing outcomes for the Proficiency Strands: problem solving and reasoning (ACARA, 2021).

Acknowledgements. This study was funded as part of an Australian Research Council (ARC) Discovery Grant (DP150100120). Jane and Noleine acknowledge the assistance of Helen Chick, Ben Kelly, and Suzie Wright in the implementation of the activity.

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RESEARCH PAPER

A Call for Translational Research in Embodied Learning in Early Mathematics and Science Education: The ELEMS Project

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Previous psychological, neuroscientific, and educational research indicates that a focus on individual haptic modes of learning (touch, body movement, gesture, tracing), and on the development of emerging mathematical and scientific drawing, can enhance children's learning in mathematics and science. However, most of these studies have focused on one embodied mode and one piece of content, and many have been laboratory studies with limited ecological validity Though less numerous, educational studies in classroom settings have confirmed the potential of particular haptic-mode strategies to improve learning outcomes. Missing from the body of research are studies that take an integrated approach to developing embodiment-rich pedagogy with teachers in authentic classroom settings. This paper argues that translational research is needed to develop evidence-based, curriculum-connected advice for teachers.

Finding ways to improve engagement and outcomes in the STEM disciplines is an ongoing goal in education (Office of the Chief Scientisit, 2014), though less attention has been given to developing educational strategies for the early years than to later stages of schooling. In recent years, research from the fields of neuroscience and psychology has been accumulating knowledge of the brain and the body's roles in children's processing of mathematics and science concepts (Ibrahim-Didi et al., 2017; Kinnear et al., 2019). Although some advances have been made in arts-based embodied learning (e.g., Garrett et al., 2018), little of the embodied learning research has been translated into pedagogical knowledge through STEM-oriented educational research.

The aim of this paper is to present some background to a call for research and development projects that collaborate with teachers in naturalistic classroom settings to translate previous research findings into curriculum-connected pedagogy to enhance children's learning and cater for diversity in learning needs. The focus is on the age range of four to eight years (spanning Pre-school to Year 2), a developmental period when foundational concepts, skills and dispositions in mathematics and science are emerging. The paper begins by presenting a theoretical stance that connects representation theory and embodied learning theory in the context of mathematics and science education. The second part of the paper draws on an analysis of research literature about each haptic mode to identify some of the key findings of potential relevance to classroom practice for the early years. Space limitations necessitate an indicative rather than comprehensive approach to the review of available literature. The paper concludes with a proposed approach for translational research that has the potential to build an evidence base for effective classroom pedagogy utilising embodied learning techniques.

Theoretical Stance

The growth in the field of embodied cognition research has, to some extent, led to a broader acceptance of closer relationships between the roles of mind and body in learning (Hutto et al., 2015; Kersting et al., 2021). In the disciplines of mathematics and science, a representational view of knowledge and learning allows researchers to describe variation in children's current achievement levels in terms of the development of each child's 'cognitive architecture' or sets of interrelated *internal* representational systems, such as language, imagery, and symbols and touch/kinaesthetic (Goldin & Shteingold, 2001). To some extent researchers (and teachers) can infer the development of particular *internal* representation systems through a child's self-

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created *external* representations (e.g., spoken words, drawings, modelling and actions) and their interactions with more formal representations created by others (e.g., symbols, diagrams) (Bobis & Way, 2018). Under-developed representational systems, or mismatches between internal representations and externally imposed representations, potentially create barriers to learning (Goldin & Shteingold, 2001). Incomplete representational systems are particularly noticeable in the disciplines of mathematics and science which place high demand on the formation and connection of multiple representation systems. The interplay between internal and external representations should therefore be of primary concern to teachers (Goldin & Shteingold, 2001). This view of representational theory acknowledges that the internal-external 'interplay' is dynamic and two-way, meaning that external representations are more than *communication tools,* but are also *thinking tools* which can cause cognitive change (Goldin & Kaput, 1996; Prain & Tytler, 2013; Tytler et al., 2013). Hence the need to consider 'representation' as an act or process, not just a product or reflection of existing internal mental images.

Sensorimotor learning is accepted as an essential form of development for infants and toddlers (Gerber et al., 2010; Piaget & Inhelder, 1969), but it has been largely ignored as an important ingredient in the learning of school-age children in the disciplines of mathematics and science. Embodied cognition theories capture enactive types of knowing and learning that cannot be well explained in terms of the 'within brain' processing of sensory inputs as information, as advocated by traditional Cognitivism. In the field of education, the term 'embodied learning' is often preferred, rather than 'embodied cognition', as it better reflects the learning intention behind applying embodied approaches in educational settings (Skulmowski & Rey, 2018). Embodied learning in mathematics and science places emphasis on representation through the haptic mode which includes tactile and kinaesthetic actions (gesture, pointing, touching, tracing, and larger body movements). A critical point is that the origins of embodied representation lay in the movement of the body itself and/or in the interactions between the body and the external environment (Hutto et al., 2015). The active engagement with a variety of movements assists the child to notice and attend to the essential properties, structures and relationships of the mathematical and scientific ideas, beyond what 'looking', 'talking' and retrieving mental images can achieve alone (Alibali & Nathan, 2012; Ginns et al., 2016).

Although most research has found that embodied learning approaches enhance learning, there is less agreement on how much engagement with the haptic modes is needed to achieve learning gains. Although classroom-based research is still relatively limited, positive learning effects have been found with low, moderate and high levels of engagement with embodied-based learning, apparently depending on the type of embodiment and the type of engagement (Skulmowski & Rey, 2018). Different types of embodiment influence learning in different ways. The next section of the paper groups together key research findings into two broad categories of embodiment: the hands used in gesture and tracing, and larger body movement. A third category of 'drawing' is proposed as a further area of embodied learning that is not often considered in the context of embodied-cognition literature. The review is intended to demonstrate how various modes of embodiment can potentially support learning in different ways, rather than provide a comprehensive review of literature.

How Modes of Embodiment Can Enhance Learning

Gesture and Tracing

Humans are genetically predisposed to attend to nonverbal behaviours including gestures and so gesture can be used to focus student attention and enhance engagement (Paas & Sweller, 2012). While basic gestures come naturally and do not need to be taught, science and mathematics knowledge that requires explicit instruction can be supported by using accompanying gestures (Martinez-Lincoln et al., 2019).

Co-speech gestures can be deliberately planned to help connect concepts and mental images to the words that are spoken by the teacher (Alibali et al., 2013), and can take the form of non-representational gestures (beat gestures and pointing gestures) or representational gestures (iconic or metaphoric). Non-representational gestures do not actually depict a concept, object or process, but are used to direct attention while talking. Representational gestures depict a tangible object or dynamic scenario (Alibali & Nathan, 2012). Students will often mirror the teacher's gestures when talking about the topic themselves (Elia et al., 2014). The use of gestures in a *pantomime* way can assist young learners of second-language vocabulary (Mavilidi et al., 2015), and so may support acquisition of key terms and concepts from the science and mathematics syllabi and meaning-making during learning experiences.

While children's learning can be enhanced by observing various types of teacher's gestures, they also learn through their own spontaneous or prompted gesturing. Children's own use of gesture can create more robust memories because gesturing while communicating simultaneously activates two different regions of the brain and increases the likelihood of retrieval (Skipper et al., 2009). Gesture is not always directly connected to speech or words. *Co-thought gestures* are produced while silently thinking, visualising and problem-solving and can reflect strategies and enhance performance. Co-thought gesture may enhance learning because of its ability to represent imagistic, spatial and/or motor information that is difficult or even impossible to represent in speech (Alibali, 2005) and can lead to new problem-solving strategies. Therefore, encouraging children to 'think with their hands' and attending carefully to their gestures may be beneficial. Interestingly, mismatch between a spontaneous gesture and the spoken words may signal that a cognitive change is imminent (Goldin-Meadow, 2015). The gesture may be representing an emerging idea or additional thinking, that the words are not expressing. Noticing such a mismatch can alert a teacher to a 'teachable moment' so they can assist the child in further development of the idea.

Touch-pointing (our term) or *tracing*, with the finger contacting a surface, differs from 'in-air' gesturing as it activates further sensorimotor learning opportunities (Gerber et al., 2010; Hu et al., 2015; Montessori, 1912). Therefore, the child's own pointing-tracing actions are likely to be more powerful than only seeing the teacher perform such actions. Unlike the physical manipulation of concrete materials, this form of touching usually involves interacting with graphics such as symbols, photos, drawings or diagrams. Touch-pointing and tracing, when used by either the teacher or the child, serves to focus attention on specific features of the graphic, such as objects to be counted (Alibali & DiRusso, 1999), the parts of an equation (Ginns et al., 2016; Wang et al., 2022), or the structure of a two-dimensional geometric figure or diagram (Ginns et al., 2016; Ginns & King, 2021; Tang et al., 2019). This seems to be particularly useful when the quantity and/or complexity of the information presented in the graphic is difficult to process. The touch actions also appear to support 'chunking' of elements within the information, thus avoiding information-overload and supporting comprehension.

Black et al. (2012) argue an additional stage of *imagination* following embodied learning experiences based on action and gesture can further enhance learning. Glenberg et al. (2004) found 2nd grade students who imagined acted-out stories about farms by manipulating representative toys learned more than students re-reading the equivalent story. More recently, Wang et al. (2022) found students who traced elements of mathematics worked examples first with eyes open, then with eyes closed to encourage imagination, solved more practice problems than students who traced an equivalent number of worked examples with eyes open. The "eyes closed" students were reported to have lower intrinsic cognitive load and to solve more similar test questions than the "eyes open" students.

Body Movement

Conceptual body-movement (movement of limbs or the whole-body in task-specific actions) by learners reflects the dynamic nature of mathematical and scientific phenomena and can enhance learning (Mavilidi et al., 2018). Learning about aspects of the physical environment such as perspective, structures, measurable attributes, sequence, position or geometric properties may be more effectively achieved through experiences that facilitate exploration in relation to the young child's own body; that is, building egocentric spatial frames of reference (Dackermann et al., 2017). Experiences that require making sense of relationships between objects external to the child or from the visual perspectives of others (allocentric frames of reference) are more challenging. For example, a 5-year-old may gain a better understanding of the function of wheel through the sensation of rolling themselves down a hill that just observing a toy car on a ramp.

When exploring physical phenomena, learning can also be enhanced through co-operative action and re-enactment using their bodies, often in conjunction with the materials that model the phenomenon (Shoval, 2011). Such research highlights the social affordances of groups of children in school settings. The socio-kinaesthetic interaction of performing movements with others, joint contact with the physical objects and watching re-enactments by others, have been found to be important factors in maximising the learning benefits by supporting memory of movements and making connections between concepts processes and movements (Shoval, 2011).

Focusing on the notion of *movement memory* leads us to an emerging area of investigation in mathematics and science: touch and hand movement as a form of 'thinking' in exploratory situations that require the creation of new knowledge. For example, feeling an object to explore its characteristics, particularly without seeing the object, triggers a different learning mechanism than visual or verbal stimuli (Roth, 2014). The sense-making and image-building of the object under investigation resides in the hands (Roth, 2014). Exploratory hand movement is different to gesture which often has its origin in the brain, whereas the 'ideation' with touching originates in the hands. The focus instead is on the sense of touch and the *movement memory* of the hands, rather than concrete and visual representation. The possible intersection of this research with the work on the role of imagination following touch experiences (Black et al., 2012; Glenberg et al., 2004) may be a productive avenue for future research.

Drawing

The close relationship between drawing and gesture warrants the inclusion of drawing in discussions of embodied learning (de Freitas & Sinclair, 2012; Robbins, 2009). Like gesture, drawing typically involves the hand, is dynamic, and comes naturally to young humans, but is also culturally contextualised (Pinto et al., 2011). *Natural drawing* is a form of graphic narrative play, often used in combination with gesture and movement, speech, dramatization and expressive sound effects as part of meaning-making. Young children's drawing develops in its purpose and form over several years, moving in stages from playful scribble and exploration of movement and forms, to pictorial and iconic representations of visualisations and real-world objects (Machón, 2013). Until children reach a representational level in their natural drawing development it is unreasonable to expect them to draw actual objects and scenes (Way, 2018) and care is needed when interpreting their personally contextualised drawings (MacDonald, 2013).

The representational mode of *drawing* is significant in the STEM disciplines because of the prevalence of diagrams to depict essential properties, structures and relationships (Mulligan & Mitchelmore, 2009). Drawing is more accurately viewed as a multi-representational mode because the act of drawing captures haptic, visual-spatial and symbolic modes as well as the complexities of transforming three-dimensional phenomena into a two-dimensional representation (Preston et al., 2021). Drawings often represent dynamic events rather than static situations, so children invent, share and borrow techniques and symbols (such as arrows) to express movement or the passage of time, and peer dialogue has been shown to support the development of drawing (Hopperstad, 2008). However, the act of drawing is part of thinking and of exploring emerging ideas, so the process is important, not just a finished representation (Thom & McGarvey, 2015).

Of concern in education, is the conceptual and representational distance between children's own self-created drawings and the conventions and rules of mathematical and scientific diagrams. Frequent use of drawing as a form of assessment of student learning in mathematics and science, without explicit teaching of drawing skills, falsely assumes that all children can naturally use drawing as an effective communication tool (Preston, 2016; Way, 2018). Recent research suggests that valuing drawing as a *thinking tool* in conjunction with haptic modes can support the development of children's mathematical and scientific drawing skills and enhance learning in those disciplines (Brooks, 2009; Preston et al., 2021; Tytler et al., 2013).

Summation

Though brief, the above review of literature relevant to early years mathematics and science has illustrated the variety of ways in which young children's learning can be enhanced through increasing the use of embodied learning modes in their educational experiences. Opportunity to utilise primary biological or instinctive behaviours, for children to physically *be the representation* and to ideate and create representations by dynamically interacting with their environment and with their internal representation systems, has been shown to help focus attention on key concepts and promote understanding of content.

The Need for Translational Research

Previous international research has established that specific embodied learning approaches can enhance children's learning in specific tasks. Much of the published research has been experimental in nature and appears in psychology journals in formats not readily accessible to teachers. Although a growing number of classroom-based studies have produced valuable insights into potential impacts on student engagement and learning, these studies have tended to focus on one mode of embodiment (such as gesture), deal with one content topic (e.g., the number line or the water cycle), and have typically been researcher-directed enquiries. With the accumulation of research findings now pointing towards the learning benefits of embodied learning approaches in education, we argue that the time is right for translational research projects to transform the findings into pedagogical knowledge and practice. To our knowledge, no studies have worked collaboratively with teachers to produce a set of teacher-friendly embodied learning principles supported by evidence of children's progress towards achieving a range of syllabus outcomes in mathematics and science in the early years of schooling.

The *Embodied Learning in Early Mathematics and Science* (ELEMS) project has been designed as step towards addressing the current lack of translational research. The aim of the project is to develop an evidence-based, classroom-ready professional learning resource for teachers that empowers them to implement pedagogy that uses a child's full repertoire of representational modes - emphasising the under-utilised haptic and drawing modes.

Our key objectives are to:

a) translate the findings of embodied cognition research from psychology, neuroscience and education fields into general learning-design principles that can be applied by teachers of early-years mathematics and science.

b) test the impact of these learning design principles on student learning outcomes, developing an evidence base and building on theory in this area of research.
c) produce and evaluate a professional learning package to support teachers to develop pedagogy that includes attention to the haptic and drawing modes in children's learning.

The project seeks to work with early-years teachers (Preschool to Year 2) to collaboratively develop and test teaching approaches and task types that effectively utilise haptic modes, including drawing, to support children's achievement of outcomes. The work will support teachers in curriculum enactment bridging from the national Early Years Learning Framework to the NSW Mathematics, and Science and Technology syllabi (implementing the Australian Curriculum).

Design-based research provides the ideal methodological framework for the project, with its emphasis on improving learning outcomes through authentic collaboration with practising teachers, and the dual goals of theory building and practical products (Reimann, 2010). The iterative nature of design-based research supports a three-phase project structure, with each phase informing the next, over three years. Phase 1 begins in 2022 with intensive work in one school with a diverse student population, before expanding to a larger number of schools in Phases 2 and 3. The initial research schools will be selected from the pool of 100 NSW schools with attached pre-schools. As well as teacher and whole class studies, the selection of focus children will allow case studies of children with a variety of learning needs.

In addition to the psychology-education orientated knowledge-building, we anticipate gaining insights into social-educational perspectives such as improving transition between pre-school and school, and the potential benefits of embodied learning approaches for disadvantaged groups of children. While the ELEMS project is likely to advance our understanding of how embodied learning approaches can benefit mathematics and science education in the early years, we call on other researchers to also consider classroom focused studies to strengthen the evidence base in this field.

Acknowledgments. The ELEMS project is funded by the NSW Department of Education, Strategic Research Fund (2021–2024).

The University of Sydney research team: Dr Jennifer Way (Mathematics), Dr Paul Ginns (Educational Psychology), Dr Christine Preston (Science), Dr Amanda Niland (Early Childhood), Dr Jonnell Upton (TESOL), Dr Katherin Cartwright (Mathematics).

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The Nature of Research on Pre-service Teachers' Mental Mathematics: A Brief Systematic Review

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For pre-service teachers to develop learners' number sense, they need have a broad repertoire of non-standard calculation strategies. In this paper, we present an analysis of selected research literature from 2000 to 2022 on pre-service teachers' mental mathematics published in peer-reviewed journals. The results of this brief review indicate that more research on pre-service teachers' mental mathematics would be beneficial. Of the 12 studies examined, five promoted a deficit view of pre-service teachers' knowledge. The remaining seven differed on the starting point in developing pre-service teachers' knowledge of non-standard mental calculation strategies.

Introduction

Mental mathematics is regarded as central to developing learners' and pre-service teachers' [PSTs] number sense (e.g., Courtney-Clarke & Wessels, 2014). Number sense is regarded as elusive and contested with various authors describing it as "an intuitive feel for numbers" (e.g., Howden, 1989), a set of characteristics (e.g., Berch, 2005) or more broadly a framework for guiding the development of number sense (e.g., McIntosh et al., 1992). Whitacre et al. (2020) maintain that there are three different number sense constructs, each with its own traditions and characteristics. These three constructs include core number sense (e.g., Spelke, 2000), early number sense (e.g., Andrews & Sayer, 2015) and mature number sense (e.g., McIntosh et al., 1992). The focus of this article is on mature number sense. Mature number sense "refers to a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations" (McIntosh et al., 1992, p. 3). It includes "knowledge and facility with numbers", "knowledge and facility with operations" and the competence in "applying knowledge of and facility with numbers and operations to computational settings" (McIntosh et al., 1992, p. 4). Central to the development of mature number sense is the increasing use of various strategies in flexible ways when calculating (Graven et al., 2013).

Whitacre and Rumsey (2018) argued that for PSTs to develop learners' number sense, they need to be able to work flexibly with calculation strategies. It is through the process of calculating mentally that PSTs develop a range of strategies and are able to select strategies appropriate to the particular computational situation (Pourdavood et al., 2020). Verschaffel et al. (2007) distinguished between mental mathematics done *in one's head* and mental mathematics done *with one's head*. Mental mathematics done in one's head focuses on the memorisation of basic facts, whereas mental mathematics with one's head involves the flexible use of appropriate and efficient strategies for calculating. When PSTs are required to solve calculations in writing, they tend to draw on the standard algorithm rather than identifying which strategy is most efficient (Whitacre & Rumsey, 2018). Standard algorithms privilege knowledge of basic facts and implementation of taught procedures whereas mental calculations focus on the structure of number operations and their relationships (Rathgeb-Schnierer & Green, 2019).

For PSTs to develop learners' number sense, they need to be familiar with a variety of strategies for calculating, and these strategies should be "unpacked" in teacher education courses (Westaway & Vale, 2021). A focus on mental mathematics in teacher education

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 546–553. Launceston: MERGA.

mathematics courses should enable PSTs to notice, attend to, and respond to learners' strategies when calculating (Westaway & Vale, 2021). This study is based on a brief systematic review of research literature. The question asked is:

> *What insights can be gained from research on pre-service teachers' mental mathematics?*

Due to the constraints of writing a conference paper, a comprehensive review was not conducted. The aim of conducting a brief review was to identify the nature of research conducted with PSTs that could be explored in more depth through the literature in the future.

Methodology

The Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) framework guided the study. The PRISMA framework incudes four stages in the research process: identification, screening, eligibility and inclusion (Moher et al., 2015). A copy of the PRISMA framework generated from the review described in this section is included as Figure 1.

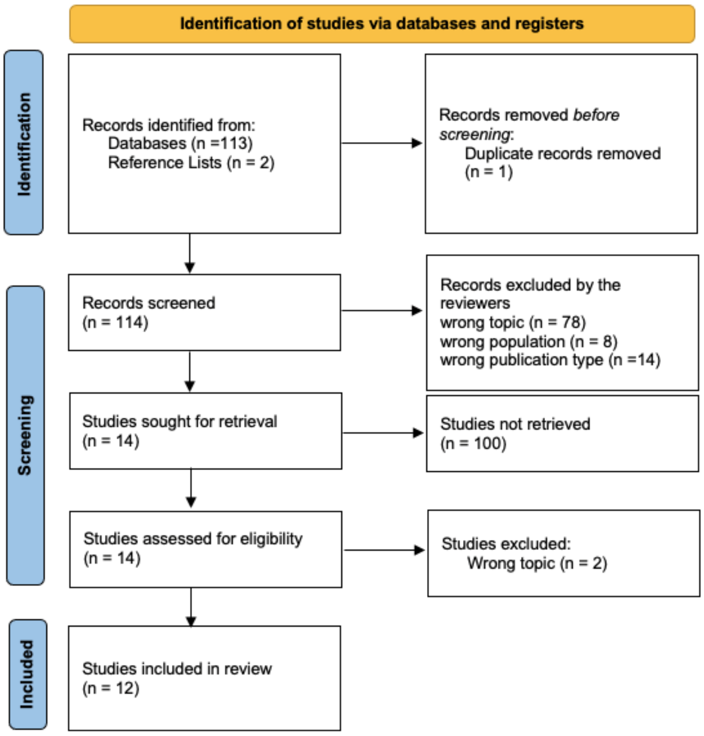

Figure 1. PRISMA Framework (Moher et al., 2015).

To initiate the process, the first author conducted a review on the EbscoHost database (Academic Search Premier, APA PsychInfo, APA PsychArticles, Eric and SocIndex) to identify articles that would relate to the research question. The keywords for the review included: "mental mathematics OR mental calculation OR mental computation OR mental arithmetic" AND "pre-service teachers OR preservice teachers OR prospective teachers OR initial teacher education OR student teachers". We specifically sought articles that were peer-reviewed and published between 2000 and 2020. The search identified 113 results. These were imported into Rayyan, an open-source software specifically designed for systematic reviews (rayyan.ai).

The authors read independently through the titles, abstracts, and keywords of each of the 113 papers and decided which of the articles should be retrieved. This screening process required that we develop a set of inclusion and exclusion criteria. The inclusion criteria focused on PSTs' mental calculations with whole numbers. The exclusion criteria included: wrong topic, wrong population, and wrong publication type. For the most part, the literature excluded focused on practicing teachers and/or students or mathematics topics other than whole number Examples of the wrong publication type were conference papers, books and professional articles. These publications were excluded because they do not always follow rigorous blind peer review processes. Through this process twelve of the initial 113 articles were deemed relevant to the study and were retrieved. A further review of the reference lists of the twelve articles identified another two articles. This brought the total number of articles retrieved to 14. These were read to assess their eligibility for inclusion in the review. Two articles were discarded at this point as they focused on rational numbers. The final number of articles included is twelve.

Results

Research on PSTs' mental mathematics between 2000 and 2022 is descriptive and illuminating. Table 1 shows the years in which each of the identified articles was published. Most prolific in authorship was Whitacre, who contributed to Whitacre (2015, 2018) and Whitacre and Rumsey (2018), three of the twelve articles reviewed.

Table 1
Research on PSTs' Mental Mathematics

2004	2008	2009	2013	2014	2015	2018	2019
2	1	2	1	1	1	2	2

Non-intervention Studies

A large proportion of the papers reviewed (5 out of the 12) presented case studies in which PSTs' ability to apply mental calculation strategies was assessed, with no intervention. These papers presented a deficit-focused perspective on PSTs' knowledge and ability without revealing any possible strategies to address the knowledge-gaps identified.

Tsao (2004) assessed the number sense, mental computation performance and written computation performance of 155 participants and concluded that the three were significantly correlated. They reported that the mean scores on the mental computation test and the written computation test were low. Courtney-Clarke and Wessels (2014) similarly used a number sense assessment with a written computation and a mental computation assessment to assess 47 final-year primary PSTs. Their study aimed to explore whether teacher knowledge was a possible factor explaining the poor performance of Namibian primary school children. They found "very poor performance on the mental calculations questionnaire" (p. 6) and concluded that the participants were not proficient in number sense and claimed that this "lack of a sound foundation in the domain of numbers and operations may be the root cause of the low standards of performance of Namibian learners" (p. 8).

Yang et al. (2009) assessed the number sense strategies used by PSTs through a written test in which the participants were instructed to estimate or mentally compute the answer. Despite instructing the participants not to use a written algorithm, their findings indicated that a low percentage used number sense-based methods and most preferred to use written, rule-based methods to compute the answer. Şengül (2013) also sought to understand the strategies used by PSTs in solving problems. In that study, 133 PSTs were given a "Number Sense Test" and it was also found that the participants "preferred using rule-based methods instead of number sense" (p. 1965). Similarly, Lemonidis et al. (2004) investigated the strategies used by PSTs when solving two-digit multiplication problems and their flexibility in applying these strategies. Their findings showed that the participants were "not flexible in two-digit multiplications and they mostly used the written algorithms mentally in order to calculate" (p. 110).

Teaching a Variety of Strategies

There were seven articles that focused on interventions: Baranyai et al. (2009); Ineson (2008); Mutawah (2016); Son et al. (2019); Whitacre (2015, 2018); and Whitacre & Rumsey, (2018). Three of the articles were based on research that had a pre-test, intervention, post-test design (Baranyai et al., 2009; Son et al., 2019; Whitacre, 2015) and one conducted a post-test after an intervention (Ineson, 2008).

Ineson's (2008) research sought to assess the development of PSTs' connected and relational knowledge (i.e., "use of multiple strategies, estimation and justification of solutions" (p. 51)). The intervention ran for 33 weeks over 2 years. One hundred and seventy PST were explicitly taught different non-standard strategies. In the first year, the pre-service teachers were introduced to addition, subtraction, multiplication, and division strategies sequentially using mental calculations prior to written calculations. The results of the post-test at the end of the first year suggested that the PSTs showed "little evidence of relational or connected thinking" (p. 47). In second year the emphasis was on the PSTs' strategy choices and the justifications for their choices. The results demonstrated that most students in the second year of the intervention used "a variety of informal checking strategies and recognised that justifying results mathematically was a crucial part of the process" (p. 51), thus demonstrating evidence of connected and relational thinking.

Baranyai et al.'s (2009) research examined the efficacy of three different types of games (didactical games, board games and mobile games) on the development of 85 preschool and primary school PSTs. The PSTs were divided into three experimental groups, with each group playing a different type of game. The intervention consisted of eight 20–25-minute sessions conducted over a period of 8 weeks. The results of the research showed that the didactic games, where calculations were asked orally, were most beneficial in developing the PSTs' mental mathematics. The researchers made no comments as to the implications of their research for teacher education.

Fifty-eight PSTs participated in the research of Son et al. (2019). The PSTs wrote pre-and post-tests that consisted of an estimation task, a computational test and a belief survey. The intervention was designed to develop PSTs' use of various strategies for computational estimation. The intervention concentrated on understanding the value of computational estimation and developing and practising computational estimation strategies. The results of the study showed that PSTs performed better on the computation test than the estimation test, and PSTs with a positive perception of their mathematics abilities achieved higher scores in mental estimation.

Three of the articles in this review were either authored or co-authored by Whitacre (Whitacre, 2015; 2018; Whitacre & Rumsey, 2018). All three articles were based on data from a single study in which 39 undergraduate PSTs participated. The learning goal for the research was for PSTs "to move from dependence on standard algorithms to reasoning flexibly about numbers and operations" through participation in collaborative activities and engaging in meaningful mathematical discussions. The intervention that sought to achieve these learning goals was based on five classroom mathematics practices presented sequentially: (1) understanding the standard algorithm, (2) making sense of place value, (3) applying knowledge of place value to make sense of the standard algorithm and transitional strategies (e.g., calculating from right to left), (4) reasoning flexibly about addition calculations, and (5) reasoning flexibly about subtraction algorithms. The intervention aimed to shift the PSTs from a dependence on the *Mental Analog of the Standard Algorithm* [MASA] to working more flexibly with a variety of non-standard strategies. The strategy framework of Markovits and Sowder (1994) and flexible mental computation framework of Heirdsfield and Cooper (2004) guided the analysis across the three articles. Markovits and Sowder's (1994) strategy framework is a continuum that was used by Whitacre (2015, 2018) to identify PST strategy ranges and the changes that occurred during the intervention. The continuum includes *MASA, transition* (the PST is still dependent on the standard algorithm but gives attention to the numbers in the calculation rather than simply performing the procedure), *non-standard with no reformulation* (calculating from left-to-right) and *non-standard with reformulation* (non-standard strategies).

Whitacre's 2015 article was based on data generated through pre- and post-instruction interviews of whole number mental calculation tasks with seven PSTs. The purpose of the interviews was to elicit the strategy ranges of the PSTs before and after the intervention to identify if there were any changes to the choice of strategies used. The participants were presented with several word problems and were asked to give the answer and explain the strategy they used to obtain the answer.

The data were coded using Markovits and Sowder's (1994) strategy framework and the flexible mental computation framework of Heirdsfield and Cooper (2004). The data revealed a greater variety of categories than the Markovits and Sowder (1994) strategy framework, which reflected changes in PSTs' strategy ranges. These included (1) MASA-bound strategies, (2) Polarised strategies (a combination of MASA and non-standard strategies), (3) Transitional strategies (MASA and versions thereof, e.g., calculating from right to left), (4) Spread (MASA and two non-standard strategies), (5) Transition strategies (transitional or non-standard strategies) and (6) Independent strategies (non-standard strategies). The profile of strategy ranges assisted in identifying the flexibility of the PSTs' reasoning in both the pre-and post-interviews. The results provided evidence of increased flexibility and a shift to non-standard strategies. Whitacre's research generated an analytic framework that views the progression of flexible reasoning as a process of development from MASA-bound strategies to non-standard strategies.

The articles written by Whitacre (2018) and Whitacre and Rumsey (2018) focused on the intervention. Whitacre (2018) "presents a viable learning trajectory … with a focus on whole-number place value, addition and subtraction" (p. 56). Drawing on the five classroom mathematics practices (mentioned above), Whitacre (2018) showed the shift in PST flexibility in thinking and a change to using more non-standard strategies for mental calculations. Like Rasmussen and Stephan (2008, as cited in Whitacre, 2018), he drew on an anatomy of argument framework to show this shift by focusing on the *claims* made by the PSTs, the *warrants* (i.e., the evidence to support the claim), and *backing* (i.e., justification of the warrant) as the PST engaged in collaborative activities. The results showed: (1) number sense development as a "cumulative process" (p. 76) that involved a shift from MASA to the use of more non-standard strategies for calculating; (2) The development of PSTs' number sense should start with their knowledge of the standard algorithm. Unlike learners whose number sense development starts with the use of informal strategies, PSTs "approach their learning from a fundamentally different starting place because they have long since learned the standard algorithm and often grown dependent on it" (p. 77); (3) The use of accountable arguments as PSTs engaged in collaborative, meaningful, problem-solving activities and discussions that highlighted that

> taken-as-shared ideas do not have to always be viewed as an element of only one classroom mathematical practice—they may contribute to the emergence of other practices and form a network of practices instead of a sequential chain of practices with distinct taken-as-shared ideas. (Stephan & Rasmussen, 2002, as cited in Whitacre, 2018, p. 62)

Put differently, the classroom mathematics practices in the intervention enabled "as-if shared ideas" (e.g., the standard algorithm) to produce more "as-if shared ideas" (non-standard strategies).

Whitacre and Rumsey's (2018) article focused on a single participant (Brandy) and demonstrated how the scaffolding of the strategy ranges framework from standard to non-standard (Whitacre, 2015) influenced the development of an understanding of non-standard strategies and the ability to reason more flexibly when calculating. Specifically, the research examined how the socio-mathematical norms established during collaborative activity supported Brandy in her transition from her initial reliance on the standard algorithm to calculating in more flexible ways. The authors coded the argument log that formed part of the previous paper (Whitacre, 2018) according to type and frequency of mental calculation strategies used over time. The argument logs were also coded for socio-mathematical norms. The researchers drew on Fukawa-Connelly (2012, as cited in Whitacre, 2018) to identify the socio-mathematical norms. Three socio-mathematical norms were identified relating to discussions of addition and subtraction strategies: *desirable characteristics of strategies*, *distinguishing and communicating the details of strategies* and *strategy naming conventions* (p. 342). Brandy's strategy range showed improvement between the pre- and post-interviews. As Brandy participated in the intervention, the researchers noted that her unscaffolded strategy range (i.e., the strategy range she used during pre-interview) differed from her scaffolded strategy range (i.e., her strategy range when asked to use an alternative strategy for calculating mentally). In the first interview, Brandy was bound to the standard algorithm. However, in the scaffolded activity tasks she showed some indication of semi-flexibility in her reasoning. In the second interview, Brandy showed more flexibility in her thinking when presented with unscaffolded tasks. While not proficient in some of the non-standard strategies, she demonstrated she was no longer bound to MASA and chose rather to use transition and non-standard strategies. She was able to reason and justify her strategies, a change consistent with the three socio-mathematical norms. The results of Whitacre and Rumsey's (2018) research suggest that PSTs may be more flexible in their use of strategies when presented with scaffolded tasks and that the development of socio-mathematical norms as the PSTs engaged in collective activities possibly lead to greater flexibility in the choice of non-standard strategies used. The latter is an important finding in that it challenges the dichotomous view that PSTs use either the standard algorithm or non-standard strategies (viewed as number sense) when calculating mentally. Rather, the research suggests that the process of engaging collectively in classroom mathematical practices that require PSTs to attend more closely to different non-standard strategies promotes greater flexibility. With the development of flexible thinking, Brandy's initial scaffolded strategy ranges became her unscaffolded strategy range.

Discussion and Conclusion

All of the papers advocated that PSTs require training in order to develop a broad repertoire of mental calculation strategies. The five non-intervention studies provided no indication of what the nature of those potential interventions should entail. There exists a wealth of research that reports PSTs tend to rely on the standard algorithms for mental calculations (MASA), and seemingly lack flexibility in their mathematical thinking when calculating mentally. Five of the studies in this review appear to confirm what is already known. These five articles thus promote a deficit view of PSTs ability to reason and calculate in flexible ways.

All seven articles on interventionist studies were based on the view that interventions are required to develop PSTs' knowledge of non-standard strategies for calculating mentally. Ineson (2008), Son et al. (2019), Whitacre (2015, 2018) and Whitacre and Rumsey (2018) commented on the types of the non-standard strategies that PSTs develop as they engage in various interventions.

Three of the authors whose research focused on intervention studies prioritised the teaching of non-standard methods only, ignoring what PSTs are already familiar with, namely the standard algorithm (Ineson, 2008; Mutawah, 2016; Son et al., 2019). By contrast, Whitacre (2018) proposed that a learning trajectory that starts with what is already known to the PST is required prior to introducing non-standard strategies. His use of an adaptation of the strategy range framework developed by Markovits and Sowder (1994), challenges the dichotomous view that PST either use standard algorithms or non-standard strategies.

Whitacre (2018) argued that interventions should be based on classroom practices that move PSTs from the known to unknown. He advocated that developing an understanding of the standard algorithm, place-value and transitional strategies are ultimately necessary to enable PSTs to reason flexibly with a variety of non-standard strategies. Whitacre and Rumsey (2018) extended this argument suggesting that classroom practices that encourage collaboration and shared sense-making develop social-mathematical norms that encourage PSTs to identify desirable characteristics of strategies, distinguishing and communicating the details of strategies and strategy naming conventions (p. 342). These socio-mathematical norms also promote greater flexibility in using a wide repertoire of mental calculation strategies.

There is little doubt that PSTs need to be familiar with a range of strategies in order to support the development of learners' number sense. As Shulman (1987) noted "to teach is first to understand" (p. 14). Given the importance of developing learners' number sense, it is surprising that there are seemingly few studies that have been published in peer-reviewed journals on how to develop PSTs' knowledge and promote use of various mental calculation strategies.

The research reviewed in this paper comprises non-interventionist studies that focused on identifying PST strategy ranges through pre-test and post-test, interventionist studies that started with the development of a wide repertoire of non-standard strategies, and interventions that began with developing an understanding of the standard algorithm and place value prior to supporting PSTs development of a range of strategies. The implication of this is that there is no consensus on how best to enable pre-service teachers to develop learners' number sense. We suggest that research that does not take cognisance of PSTs' use of the standard algorithm for mental computation assumes that PSTs understand the standard algorithm. This is not our experience as teachers of PSTs. Interventions that ignore the standard algorithm assume that it is not a strategy that should be regarded as one of various strategies in the development of number sense, particularly when calculating in writing. Notably, absent from the research reviewed is the transition from the lecture room to the classroom and the extent to which PSTs are able to translate their knowledge and understanding of non-standard strategies during their teaching practice. We suggest that transferability and sustainability of practice may be worth researching in the future.

The review presented in this paper provides indicators of practice that could be explored in more depth in a more comprehensive review. Future reviews should include a broadening of the search terms. For example, inclusion of "number sense", "number talks", "number flexibility", and variations of the terminology associated with mental mathematics, such as "mental math" and "mental maths".

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The Role of Mathematics Education in Developing Students' 21st Century Skills, Competencies and STEM Capabilities

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In order to discuss the role of mathematics education in developing students' STEM Capabilities and 21st Century skills, it is necessary to define what is meant by these terms given the extent to which they are broadly used in global contexts. A literature review aimed at providing clarity, through affording a concise interpretation of each term for the Australian context, enabled the development of a working framework for defining 21st Century skills and STEM capabilities. This paper provides working definitions and reports on initial findings from a larger three-phase study aimed at exploring secondary mathematics teachers' beliefs, attitudes and practices, towards the role mathematics education plays in developing students' STEM capabilities and 21st Century skills.

Industry and education policy makers, politicians, environmentalists, economists and futurists are all in agreement that in order for Australia to maintain its global prosperity and remain innovative and resourceful, we need to build and sustain a STEM capable workforce (Australian Academy of Science [AAS], 2016; Australian Industry Group [AIG], 2015; CSIRO, 2020) which is able to readily employ 21st Century ways of thinking and working (Business Council of Australia [BCA], 2017; Torii & O'Connell, 2017). To support the development of Australia's future STEM workforce and provide opportunities for all Australian students to develop the sound STEM skills necessary to be successful in their future career pathways (AIG, 2015; BCA, 2017), a number of National and State initiatives have been enacted, aimed at supporting STEM education in Australian schools (e.g., Australian Curriculum, Assessment and Reporting Authority [ACARA], 2016; Department of Education Skills and Employment [DESE], 2021; Education Council, 2015). Hatisaru et al. (2019) shared that although it is widely supported that STEM education is crucial for students' future success, there exists no universally accepted definition of STEM, and as a consequence, STEM skills and STEM capabilities are conceptualised, interpreted and defined in diverse ways (Anderson et al., 2020) dependent upon the audience and context through which it is used. The Organisation for Economic Cooperation and Development (OECD), in the Education 2030 project, identified three broad global challenges for a 21st Century society, which include environmental, economic and social challenges (OECD, 2018). To address these challenges students will need to:

> … apply their knowledge in unknown and evolving circumstances. For this, they will need a broad range of skills, including cognitive and meta-cognitive skills (e.g., critical thinking, creative thinking, learning to learn and self-regulation); social and emotional skills (e.g., empathy, self-efficacy and collaboration); and practical and physical skills (e.g., using new information and communication technology devices). (OECD, 2018, p. 5)

STEM Capabilities

STEM evolved as an acronym for Science, Technology, Engineering and Mathematics (Anderson et al., 2020) and as an initiative in the United States towards the end of the 20th century (Myers & Berkowicz, 2015). For many educators, STEM education remains a

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 554–561. Launceston: MERGA.

relatively new idea (English, 2016; Hatisaru et al., 2019; Myers & Berkowicz, 2015) and common definitions of what are STEM capabilities are not well established (Anderson et al., 2020; Hatisaru et al., 2019). In 2014, the Office of the Chief Scientist (OCS) published a report highlighting the importance of having a STEM capable workforce and the shortfalls predicted if initiatives were not put in place to improve the STEM capabilities of Australian students (OCS, 2014). This led to the Australian Education Council (AEC) publishing the National STEM School Strategy in 2015 (AEC, 2015). As a direct result of these strategies, STEM education policy initiatives, resources, national projects and grants emerged, aimed at promoting the development of students' STEM capabilities (DESE, 2021).

21st Century Skills and Competencies

More than two decades into the 21st Century, the identification of a universally agreed list of skills that encapsulate 21st Century skills and competencies has not come to fruition; however, a number of global organisations have attempted to define and establish frameworks through which to describe 21st Century skills and competencies (Bialik et al., 2015; Greenstein, 2013; Griffin et al., 2012; OECD, 2021). The OECD Learning Compass 2030, a learning framework co-created by a number of countries participating in the OECD Future of Education and Skills Project 2030 (OECD, 2019), built upon the concept of 21st Century skills to provide the competencies necessary for students to thrive in 2030 and beyond. The framework included reference to seven elements that comprise, "core foundations", an agreed set of future orientated "knowledge, skills, attitudes and values", along with "student agency/co-agency", "anticipation-action-reflection" and "transformative competencies" (OECD, 2019, p. 16). In reviewing the literature on 21st Century skills across multiple frameworks, the overarching skills associated with creativity, critical thinking, communication and collaboration are consistently used when describing 21st Century skills (e.g., English, 2016; Griffin et al., 2012).

Organisations such as the OECD and the Center for Curriculum Redesign (CCR) assert that certain learning areas are more aligned to particular 21st Century competencies than others (Bialik et al., 2015; OECD, 2019). For example, through the learning area of mathematics, it is possible to implement pedagogies that provide opportunity for students to collaboratively utilise their strategic competence, and that mathematical reasoning skills can foster students' 21st Century skills (Bailik et al., 2015; English, 2016; Griffin, et al., 2012).

Framework

This section of the paper will provide a framework that can be used to define 21st skills, competencies and STEM capabilities for an Australian context. The three-dimensional framework of the *Australian Curriculum* (AC) aims to prepare students to be successful in the twenty-first century, and states that the *Australian Curriculum: General Capabilities* (AC: GC) play a crucial role in this preparation (ACARA, 2017b). In conducting a comparison and contrast of existing international 21st Century frameworks, drawing on work by the Center of Curriculum Redesign (CCR) and the Assessment and Teaching of 21st Century Skills (ATC21S), the various skills and competencies detailed in each framework can be clustered and aligned to the three dimensions of the AC (see Table 1). For example, the OECD 2030 learning framework's competencies of "core foundations, knowledge and skills" (OECD, 2019, p. 16) and CCR's "traditional and interdisciplinary knowledge" (Fadel et al., 2015, p. 43) can be interpreted in the Australian context as the eight Learning Areas (AC: LA) combined with the Literacy and Numeracy general capabilities (ACARA, 2017a, 2022).

Table 1

Comparison and Contrast of 21st Century Skills and Competencies Frameworks

AC	OECD	CCR	ATC21S
8 x AC: LA, Literacy & Numeracy	Core foundations, knowledge and skills, literacy, numeracy, financial literacy, programming, physical health & sustainable development literacy	*Knowledge*: traditional, interdisciplinary, environmental literacy	Life and career skills, Communication
Critical & creative thinking	Critical thinking & problem solving, learning to learn, anticipation, reflection, creating new value	*Skills:* critical thinking, creativity *Meta-learning:* metacognition, growth mindset, meta-learning, *Character:* curiosity	Critical thinking, problem solving & decision making, Learning to learn, Metacognition, Creativity & innovation, Flexibility
Digital literacy Literacy Numeracy	ICT literacy, digital literacy, data literacy, media literacy, computational thinking	*Knowledge:* digital literacy, systems thinking, design thinking	Information literacy, ICT, Communication
Personal & social capability	Cooperation, collaboration, empathy, respect, student agency, co-agency, reconciling tensions & dilemmas	*Skills:* communication & collaboration, *Character*: mindfulness, courage, resilience, leadership	Personal & social responsibility, Citizenship, Communication and Collaboration
Intercultural understanding	Global competency, respect		Global understanding
Ethical understanding	Literacy for sustainable development	*Character:* ethics	Citizenship

The ACARA STEM Connections report (2016) stated that STEM in the Australian Curriculum is addressed through the learning areas of Science, Technologies and Mathematics and through the General Capabilities, particularly Numeracy, Information and Communication Technology (ICT) capability, and Critical and Creative Thinking. The Australian Curriculum defines capability (ACARA, 2017b);

> …capability encompasses knowledge, skills, behaviours and dispositions. Students develop capability when they apply knowledge and skills confidently, effectively and appropriately in complex and changing circumstances, in their learning at school and in their lives outside school. (para. 2)

Applying the ACARA definition of capability, students' STEM capability (see Table 2) encompasses the discipline specific knowledge of the three learning areas Science, Technologies and Mathematics and the ability to apply this knowledge and the various analytical thinking, reasoning, inquiry and problem-solving skills both within and across the learning areas to complex situations, all coupled with positive dispositions towards the STEM learning areas and future STEM pathways (ACARA, 2016, 2022; OCS, 2014). Applying this working definition, the role of mathematics in developing students' STEM capabilities is more easily conceptualised.

Table 2
Students' STEM Capabilities

Science	Technologies	Design & Technologies	Mathematics
Science Understanding: Biological, chemical, earth & space, physical sciences	Knowledge & Understanding	Knowledge & Understanding	Content knowledge: Number, Algebra, Measurement, Space, Statistics & Probability
Science inquiry skills: questioning & predicting, planning & conducting, processing, modelling & analysing, evaluating & communicating	Process & Production Skills: the skills needed to create digital solutions. Systems, design & computational thinking	Process & Production Skills: the skills needed to create designed solutions. Systems, design & computational thinking	Proficiencies: understanding, fluency, reasoning & problem-solving Processes: modelling, computational thinking, statistical investigations, probability experiments & simulations
Science as a human endeavour	Privacy & security	Technologies and society	Productive disposition towards mathematics

Note: Created from AC version 9.0 (ACARA, 2022)

The National STEM School Education Strategy 2016—2026 focusses on raising the foundational skills of each of these STEM disciplines and "promotes the development of the 21st century skills of problem solving, critical analysis and creative thinking" (Education Council, 2015, p. 5). The strategy includes a suite of seven "guiding principles for schools to support STEM education" (Education Council, 2015, p. 11*)*, the third being:

> Build on students' curiosity and connect STEM learning to solving real world problems, including through collaborative and individual learning experiences that are hands-on and inquiry-based and support the achievement of deep knowledge.

The Study

There exists broad support for the notion that teacher beliefs influence their pedagogical choices (e.g., Beswick, 2012; Goos et al., 2017). Although teacher training and professional learning programs have focused on the adoption of pedagogies that enhance students' problem-solving capabilities and improve student thinking and reasoning in mathematics, a significant number of teachers have not adopted these strategies in their classrooms (Bailey, 2018). Beswick (2012) suggested an inherent disparity between teachers' perceptions of best practice and their portrayed classroom practices. There also exists an incongruity regarding the pedagogies teachers profess to use, compared to what they implement (Clarke & Lomas, 2016).

Although assumptions can be made as to why this incongruity exists, there is value in exploring the beliefs and practices of secondary mathematics teachers towards the role mathematics plays in developing students' STEM capabilities and 21st Century skills. Such an exploration is necessary to ascertain the factors that influence teachers' pedagogical choices. The influencing factors impacting upon teachers' pedagogical decisions need to initially be identified in the general context, in order to explore them in the specific.

Methodology

This study has adopted an explanatory sequential mixed method design (Creswell & Plano Clark, 2011) allowing the researcher to draw upon the findings of a large scale, statistically representative sample to inform and provide contextual support for the qualitative study. Findings from the qualitative study will be used to draw inferences to help explain the quantitative results (Creswell, 2015). The three-phase design aims to explore the phenomena from the general to the specific within a philosophical framework. Each phase will inform the next, using complementary methods with both random and purposive sampling in a Quantitative (Phase 1) → Qualitative model (Phase 2 & 3) (Creswell, 2015).

Given the interpretative nature of this research approach, emphasis is placed on interpretation constructed collaboratively by the participants and the researcher. The following research questions are being explored in the extended study, but the focus on this paper is on the initial findings around Research Questions 1, 2 and 3:

RQ1: What pedagogical approaches are extant in Western Australian Years 7-10 mathematics classrooms?

RQ2: What beliefs and attitudes do Western Australian Years 7-10 teachers of mathematics hold concerning the role of Years 7-10 mathematics in developing students' STEM capabilities?

RQ3: What beliefs and attitudes do Western Australian Years 7-10 teachers of mathematics hold concerning how Years 7-10 mathematics contributes to the development of students' 21st century skills?

RQ4: According to Western Australian Year 7-10 teachers of mathematics, what factors might afford and constrain them in the implementation of pedagogies that foster student STEM capabilities and 21st century skills?

Phase One (quantitative phase)

Instrument. Phase One involved data collection via a web-based survey aimed at providing insight into common themes. The survey instrument was validated by academic peers and consisted of 19 questions. The initial seven questions gathered background information about the participants' location, experience and training. The remaining questions gathered information on teacher pedagogical content knowledge, beliefs, attitudes and practices, comprised of six five-point Likert-type items and 5 yes/no response items, and an extended response section to provide explanation for responses. The purpose of the survey instrument was to provide insight into participants' pedagogical beliefs, practices and attitudes towards adopting pedagogies that support students' STEM capabilities and 21st Century skills and identify any common themes.

Procedure. The intended participants were Western Australian teachers of mathematics in Years 7–10 from all WA educational sectors. The survey procedure involved a recruitment email to all WA secondary school principals, that included an email link to an anonymous questionnaire using an online commercial platform. Due to COVID-19 lockdown restrictions in place at the time, it was agreed that any further contact with schools should be delayed. Further participants were then recruited later in the year through a email campaign to members of the Mathematical Association of Western Australia (MAWA). The sample ($n = 60$), although smaller than intended, was representative of the population both in location ($n = 45$ Metropolitan, $n = 15$ Regional & Remote) and teaching experience. All participants were teaching at least one lower secondary mathematics class and identified as classroom teachers (75%) or in leadership roles (25%).

Preliminary Results and Discussion

Phase 1 data analysis found that the majority of respondents agreed that the ability of their students (83%), and the curriculum year level they were teaching (77%), had influence over their choice of teaching methods. In responding to the level of influence factors of; time, pedagogical knowledge and content knowledge have over their pedagogical choices, the majority selected large or extreme (see Table 3).

Table 3
Question 11: What Impact do the Following Factors Have on Your Pedagogical Choices?

Factors	Nil or no response	Small	Medium	Large	Extreme
Academic level	1 (2%)	3 (5%)	17 (28%)	27 (45%)	12 (20%)
Curriculum level	1 (2%)	9 (15%)	19 (32%)	23 (38%)	8 (13%)
Time constraints	1 (2%)	1 (2%)	16 (27%)	26 (43%)	16 (27%)
Content knowledge	4 (7%)	2 (3%)	12 (20%)	28 (47%)	14 (23%)
Pedagogical knowledge	2 (3%)	3 (5%)	13 (22%)	28 (47%)	14 (23%)

The majority of respondents agreed with statements such as; “The role of mathematics is crucial to STEM fields”, “Mathematics should form an integral part of STEM learning in schools”, and “Mathematics supports students in developing STEM capabilities.” This contrasted with the majority responding that they never, rarely or sometimes use STEM projects (98%), authentic problem solving (78%), integrated learning tasks (93%), or inquiry tasks (81%) when teaching mathematics (see Table 4).

Very few participants indicated that they regularly provide integrated STEM and other cross curricula learning opportunities in their teaching programs. The majority responded that they either never, or only once or twice a year, used collaborative group work, problem-based learning, project-based learning or inquiry-based learning pedagogies with their Year 7–10 classes, all of which are considered to be student centred pedagogical approaches that support student development of 21st Century skills and STEM capabilities (ACARA, 2016; Anderson et al., 2020; Griffin & Care, 2015; Myers & Berkowicz, 2015).

Table 4

Question 13: How Often Do You Use the Following Tasks/tools or Resources in Your Lessons?

	Never/no response	Rarely	Sometimes	Regularly	Always
Investigations	1 (2%)	2 (3%)	32 (53%)	24 (40%)	1 (2%)
Inquiry tasks	4 (7%)	15 (25%)	30 (50%)	11 (18%)	0 (0%)
Modelling	3 (5%)	11 (18%)	25 (42%)	16 (27%)	5 (8%)
Computer simulation	10 (17%)	16 (27%)	25 (42%)	7 (12%)	1 (2%)
STEM projects	20 (33%)	22 (37%)	17 (28%)	1 (2%)	0 (0%)
Integrated learning tasks	10 (17%)	20 (33%)	26 (43%)	4 (7%)	0 (0%)
Real world contexts	2 (3%)	3 (5%)	27 (45%)	24 (40%)	4 (7%)
Authentic problem-solving	4 (7%)	16 (27%)	27 (45%)	13 (22%)	0 (0%)

Conclusion

This paper has articulated a framework through which to define what is meant by 21^{st} Century Skills and STEM capabilities in the Australian context. It presents a snapshot of initial findings of a study aimed at investigating secondary teachers' beliefs, attitudes and practices towards the role mathematics education plays in developing students 21^{st} Century skills and STEM capabilities. The data collected in the initial phase of this study found that teachers of secondary mathematics generally agree with the importance of students developing sound STEM skills and capabilities and address the General Capabilities in their mathematics teaching programs. Interestingly more than a third of the 60 respondents (37%) shared that they did not teach mathematical content through the proficiency strands with all of their classes. Half of the respondents reported using real world contexts or applications; however, when asked whether they agreed that the use of real world and authentic contexts are important in mathematics education, the majority responded with agree or strongly agree (88%). Further, most participants responded that the curriculum year level (77%) and the mathematical ability (85%) of their students influence their choice of pedagogy. The ongoing study aims in Phases 2 and 3 to provide a qualitative analysis of the common themes emerging from Phase 1 to contribute to the literature on teachers' attitudes, beliefs and practices towards the role mathematics plays in the development of students STEM capabilities and 21^{st} Century skills and the factors that might afford or constrain teachers adopting pedagogies that foster these skills and capabilities.

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RESEARCH PAPER

Using Mathematics Curriculum Materials When Planning on Practicum: A Case Study of One Primary Year Three Pre-service Teacher

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This paper describes how one third and final year pre-service (PST) used curriculum materials when planning for primary mathematics teaching on practicum. The findings were drawn from a semi-structured focus group interview, where four PSTs recalled how they planned for primary teaching during a previous practicum. This case study shows how one PST used curriculum planning documents from the practicum setting, and a teacher's guide when planning mathematics lessons. Planning processes are also identified, and implications for associate teachers (ATs), PSTs, and mathematics Initial Teacher Educators are discussed.

There is widespread agreement that curriculum materials play a key role in supporting teachers, especially pre-service teachers (PSTs) to plan for teaching (Mutton et al., 2011; Remillard, 2000). In the field of mathematics education there are a range of curriculum materials for teachers to choose from when planning, for example, national, local, and school based curriculum programmes and plans (Sullivan et al., 2012), and textbooks, teacher guides, and commercially produced hard copy or digital materials (Rezat et al., 2021). PSTs as novice teachers are faced with many decisions when planning for mathematics teaching such as the focus of their lessons, what tasks and equipment, examples, and representations to use, and the structure of their plans (John, 2006). Curriculum materials in whatever form, can provide guidance for PSTs when making these decisions, particularly when, as novice teachers they are only just beginning to develop their repertoire of ideas for mathematics planning and teaching (Ensor, 2001). In the field of mathematics education there is some literature about how more experienced teachers plan for mathematics teaching, for example Sullivan et al. (2013), and more recently Chin et al. (2021), but very little is known about how PSTs plan for mathematics teaching in different practicum settings with varying year levels and for varying mathematics curriculum topics (Mutton et al., 2011; Superfine, 2008).

In New Zealand (NZ), teachers and PSTs are required to use *The New Zealand Curriculum* (NZC, Ministry of Education [MOE], 2007), when planning for mathematics teaching. The mathematics and statistics learning area in NZC includes prescribed achievement objectives that must be used when planning. From this information schools design localised curriculum plans, including long-term plans, weekly, and daily plans, to support their teaching. The MOE provides some published mathematics curriculum materials, for example, teacher guides, student booklets, and a website, called *nzmaths*, but none of these are mandated for use. This means that NZ teachers can use any curriculum materials when planning mathematics lessons.

Initial Teacher Education (ITE) programmes provide opportunities both within course work and practicum experiences for PSTs to explore the vast array of curriculum materials available for mathematics teaching, and how to use these when planning. During practicum PSTs are typically expected to follow the existing classroom programme, adhering to the established long-term and shorter-term curriculum plans. Some must use the curriculum materials selected by their Associate Teachers (ATs), while others are able to choose their own. Mathematics educators have a responsibility to prepare PSTs prior to practicum, and therefore need to understand what curriculum materials PSTs might be expected to use, and how they could use these when planning. This is challenging work, because unlike course work,

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 562–569. Launceston: MERGA.

practicum experiences occur away from ITE settings, making it difficult for ITE mathematics educators to fully understand how these planning processes are conducted (Remillard, 2000).

This paper aims to shed light on how PSTs plan by examining the planning processes of one third year PST Ben (a pseudonym), with a focus on the curriculum materials he selected, and how he used these when planning for mathematics teaching. The data is drawn from a doctoral study which focuses on how PSTs plan for primary mathematics teaching during their final practicum. Prior to this practicum, a focus group interview was held with four of the study participants, where they each recalled what curriculum materials they used, and how they used these during a previous practicum. Ben was selected for this paper because he recalled using school-based curriculum plans, and a MOE teacher's guide when planning. The research question guiding this case study is "what curriculum materials did a year three PST use, and how did he use these when planning for mathematics teaching on practicum?"

Background Literature

Teacher's Use of Curriculum Materials When Planning

Shulman (1987) contended that planning involves a process of "pedagogical reasoning" (p. 16), and an important part of this process is critically interpreting and analysing curriculum materials, selecting, and analysing these to determine which instructional strategies to use. Also critical is modifying activities for learners. Grossman and Thompson (2008) agreed that curriculum materials play an important role in influencing and guiding planning decisions. In their research they found the beginning teachers relied on curriculum materials to inform planning decisions, including using suggested activities and pedagogical approaches, and adapting these when necessary.

In the field of mathematics education several researchers have investigated how teachers use curriculum materials when planning (Sherin & Drake, 2009; Superfine, 2008). In a study that examined how teachers used a new textbook, Sherin and Drake (2009), found the teachers followed three processes, which were: reading lesson outlines in textbooks; evaluating this content for use with students; and adapting these by creating, replacing, or omitting activities or mathematics materials suggested for use in lessons. Superfine (2008) also found that planning involved processes of reading curriculum materials, considering the mathematics content within these, and modifying tasks for learners. She contended that curriculum materials, such as textbooks, play a key role in providing a base for mathematics lessons.

Similarly, Kauffman (2002) found that new secondary mathematics teachers relied heavily on textbooks, using them as a base for planning lessons. They also read and examined lesson suggestions in textbooks and then selected specific mathematics objectives and activities for lessons. One participant described this as "picking and choosing" (p. 10), information from a textbook. Some participants chose to use activities as described in the text, while others adapted activities to meet learner needs. All the teachers felt using the textbook made planning more efficient and allowed them to focus on learners when teaching.

PSTs Use of Mathematics Curriculum Materials When Planning

Earnest and Amador (2019) examined how PSTs used mathematics curriculum materials when planning, as part of an ITE course assignment. Unlike the study reported in this paper, that research was conducted in the university setting, not a practicum setting. These researchers provided a group of PSTs with a commonly used curriculum resource for developing a lesson. The PSTs read these materials, drew from various aspects of these by choosing some aspects and omitting others, similar to the teachers in Kauffman's study. While there was variation across the group of PSTs, they chose activities that related to the important mathematics ideas, and examples they evaluated as being enjoyable for learners. The PSTs noticed aspects of the curriculum to determine what they needed to teach, modified the selected activities, included more mathematics language, and introduced mathematics materials to support learners (Amador & Earnest, 2019).

In a NZ context Wilson and McChesney (2013, 2018) investigated how PSTs plan in the school setting of practicum. In their ongoing study investigating how NZ primary PSTs plan for mathematics teaching, they also found that given the choice, PSTs used a range of curriculum materials when planning for mathematics teaching. This included textbooks such as teacher and student guides, online materials from websites such as *Namath's*, and localised school curriculum materials including long and short-term mathematics planning. At the beginning of practicum this localised planning was important for PSTs, helping them determine what they had to teach on practicum, and by indicating the topic from NZC. The long-term planning information prompted them to explore the contents of the NZC to find information about what they had to teach, specifically the objectives for planning. Their next process was to search for curriculum materials in their setting to guide their planning decisions for their lessons. Some were given textbooks, while others had to source, search for, and find their own hard and online copies of materials. They found and selected non mandated MOE teacher and student guides, and activities on *nzmaths* and other internet sites. Similar to the teachers in Kauffman's study, they reported feeling secure using these because they had national status. At the beginning of their study, this planning process was described as an active process of "navigating" through a vast landscape of curriculum materials for mathematics teaching, and "noticing" aspects from these that were relevant for their lessons (Wilson & McChesney, 2013).

Research Design

This paper draws on data from a doctoral study that investigated how PSTs plan for primary mathematics teaching during their final practicum. Adopting an interpretive methodology, the doctoral study aimed to identify and describe in detail the mathematics planning processes of four cases of primary PSTs (Cohen et al., 2011; Yin, 2014). Each participant was a final year PST in a three-year undergraduate degree for primary teaching. The participants were a purposive sample because their practicum settings were in different schools, and at different primary year levels. The author was their mathematics education lecturer but did not teach them during the time of the study. Ethical consent was granted for the study, and data collection included a focus group interview, three self-recorded "think-alouds" at different times during the practicum, and a post-practicum individual interview supported with planning documentation provided by the participant. The first data collection was a semi-structured focus group interview, where participants reflected on how they had previously planned for mathematics teaching, and where one prompt question related to the kinds of curriculum materials used, how were they used, and reasons for their decisions. The interview was audiotaped, and the transcripts returned to each participant for checking. Each checked transcript was analysed using a thematic analysis approach, which involved several iterations of the author listening to and reading the transcripts, identifying themes, coding, and then looking for patterns (Miles et al., 2018). One analysis theme was how the PSTs used a range of curriculum materials when planning for mathematics teaching. The next section reports the analysis of Ben's reflections on and recall of how he planned for mathematics teaching during the previous practicum that occurred three months before the focus group interview. Extracts from the transcript are used to illustrate his recollections.

Results

Finding Out About the Practicum Setting

Ben's practicum setting was a Year 2 class (seven-year-olds) with approximately fifty learners, his AT and one other teacher. Prior to practicum Ben met with his AT and sought information about what he had to teach for mathematics, which curriculum materials he had to use, and how learners were organised for teaching. His motivation for this was "to get my head around" what he had to teach and how he would be expected to do this in this setting. The AT shared with him her mathematics long term plan, and from this he found out he would be teaching addition, e.g., "doing problems like 2 + □ = 8", and place value. The main curriculum material he was expected to use for his lessons, was a teacher's guide from the numeracy project resources called *Book 5: Teaching addition subtraction and place value* (MOE, 2012). He also had the freedom to choose and use other curriculum materials if he wanted to. He also learned that learners were grouped by ability, which meant he would be planning lessons for small group teaching.

Reading and Selecting Information from NZC and the Teacher's Guide.

Once he had information about his class, and before practicum began, Ben recalled searching through the NZC and corresponding information on *nzmaths*, looking for detailed information about what he had to teach. He explained, "I looked through the levels to find the specific thing that I was looking for," looking for key words like addition and subtraction to guide his decisions. He settled on the *Number strategies* section of NZC, read the relevant achievement objectives for this, and "just picked one" that he decided aligned with what his AT wanted him to teach. Next, he browsed through the *suggested learning activities* section on *nzmaths* that aligned with the achievement objective he selected, to "quickly see what they had". This information provided links to relevant activities on *nzmaths* that he could choose for his lessons. At this stage he did not choose activities for his lessons, but was scoping possibilities, to get a feel for the types of activities he could use.

Moving away from *nzmaths* Ben found his copy of the teacher's guide and carried out a similar process of reading and searching for content to align with the selected achievement objectives. He knew he had to use this teacher's guide, so "spent a lot of time looking through it". Like his search through the NZC, and *nzmaths*, the information given to him from his AT guided his search. He said, "it was good because I knew what to look at, and I needed some tips". He easily found the *lesson experiences* for teaching the addition equations and scoped out suggested activities to help him do this. He made a mental note of these and waited to begin practicum to get more information before beginning planning.

Reading and Analysing a Weekly Curriculum Plan

Ben recalled that on the first day of practicum his AT was absent, so he began the week by analysing her weekly curriculum plan, which was left on her desk. He explained this plan included, "short notes that I could sort of understand," and which gave him "enough information" to teach addition equations that day. The notes outlined the focus of her intended lesson, and indicated the activity from the teacher's guide she had selected. His previous scoping of this material helped him to find it easily in his own guide, which enabled him to teach on the first day of practicum. To further his knowledge for mathematics teaching, he recalled observing another teacher teaching a group using the teacher's guide, describing how he "followed and listened" to what she did, and made notes about this so that he could copy her actions when planning and teaching his own lessons. He was confident that what he observed, was also how his AT taught, saying "it was the same stuff." He used this information to inform the structure of his own lessons, which he began planning early in week one of practicum.

Selecting Objectives and Equation Examples from the Teacher's Guide.

Away from the classroom Ben began planning for one student group (approximately ten learners) by returning to the *lesson experiences* he had earlier identified in the teacher's guide. Each lesson experience contained information about lesson objectives, word problem, equipment that could be used, instructions for teaching, and examples of addition equations. He read these and decided to only use the equations such as, "4 + □ = 6, 7 + □ = 9, 6 + □ = 8" (MOE, 2012, p. 31) in his lessons. He copied these directly onto his lesson plans and used these with learners during teaching. He opted to use these sets of examples because they "saved" him from "making up my own examples". He liked the efficiency of being able to copy someone else's ideas and did not see the need to make up his own. He also trusted them as examples that he "should" be using, because they came from a MOE published resource. He felt confident that the examples he planned aligned with the NZC and were therefore appropriate for his lessons.

Ben also chose to use the readymade examples because within the sets of equations, the range of numbers that students worked with, increased in complexity. The addition sets began by adding numbers in the range zero to ten, increasing in value from ten to twenty, and then twenty to one hundred, e.g., "6 + □ = 8", 12 + □ = 14, and 87 + □ = 89", (MOE, 2012, p. 32). Ben described these as a "a sort of a progression … that can be stepped through." He liked the guidance provided by these examples because they showed him how to extend learning within a lesson, especially when learners needed more complex examples. Having the progressions also gave him options if he had to return to easier examples during teaching. He also used the progressions between lessons, picking up where he left off from a lesson, to develop learning in future lessons. Ben said that this helped him sequence lessons for learners during the week.

During the interview, Ben reflected that he continued to use them because they provided readymade examples which he could refer to during teaching. He described how stopping to create examples distracted him from the lesson, which meant he lost the flow of the lesson, and left opportunities for learners to disengage in the activities. He also admitted that the teacher's guide offered better examples than what he would have made up if he had to do this "on the spot" in lessons. He remarked that when he made up his own examples during lessons, he felt pressured and often "stuck to the numbers 2, 3 and 5" for equations, e.g., "2 + □ = 5". Using the examples from the teacher's guide, meant he could extend this range, and not feel pressured. He commented that near the end of practicum when he "dropped full planning," he continued to use these examples, which shows how he valued these as a key component of his lessons.

Creating Word Problems from Equation Examples

When carrying out his observations early in the practicum, Ben noticed the teacher used word problems for some of the equations, and the teacher's guide also suggested doing this. He initially discarded these but after a few lessons, returned to the examples in the guide for ideas about how to create his own, commenting that they were "so good … really good, to use as a basis for my own." He also described these as being "a good launching platform" for writing his own. He re-read the examples and adapted these choosing a class theme of "pirates" as a "common context", creating pirate names for each learner and using "treasure" as different numeric amounts that could be added. While he was guided to use word problems by the teachers in his setting, and by the teacher's guide, he also acknowledged using them as an important part of his decisions when planning. He believed writing word problems based on contexts learners could relate to, supported them to connect mathematics to "real life." He remembered saying to learners, “we’re not just learning it because we need to learn it, but to use it in our lives.” He also wanted to ensure learners enjoyed mathematics and the subject was not “boring.”

Discussion and Implications

Scoping Curriculum Materials Prior to Practicum

The results show that Ben’s planning process began prior to practicum, and the school based long-term curriculum plan was an important document that helped him begin this process. From this he gained valuable information about what aspects of NZC he had to teach, and this was elaborated on by his AT, who told him to teach the addition equations. This long-term plan, which included key curriculum terms helped him find specific information on NZC and led him to choose the *number strategies* objectives for his planning. This prompted him to focus his searching through *nzmaths* where he investigated possible activities for his lessons. The direction from his AT to use the teacher’s guide, meant that he found his own copy of this text, and as he did with *nzmaths* spent time exploring this, looking for possible tasks for his lessons. Again, the direction he had from the long-term plan and his AT, meant he could search through this text with purpose. This process is similar to the PSTs in the study by Wilson and McChesney (2018), who spent time searching and finding activities for teaching. Once he began practicum this initial scoping helped him to interpret the notes written by his AT on her weekly curriculum plan, which meant he had the confidence to teach on the first day in her absence. It also set him up for carrying out the observation of another teacher using the teacher’s guide. Unfortunately, he did not recall the specific aspects of this, other than how she used word problems as part of her lesson, but this did reinforce to him that the guide could be used as the base for his lessons.

Using the Teacher’s Guide

The results also show that the teacher’s guide, was an important curriculum resource that Ben used throughout practicum to support his lesson planning (Amador & Earnest, 2019). Once he had gained information about what to teach from his AT, the long term and weekly curriculum plans, NZC, and *nzmaths*, he was able to begin planning using this document. He began this process early in week one, and his initial scoping of the guide meant that he could easily find appropriate *lesson experiences*. He read these again, this time making decisions about what to use, and what to omit in his lessons, a similar selection process to the teachers in Kauffman’s study (2002). He consistently used three aspects, the specific objectives, the sets of equation examples, and the word problems.

Ben used the objectives as specific foci for his lessons, copying these directly onto his planning. He did the same with the equation examples, describing how this saved him time, so that he did not have to make up his own for his lessons. He recognised that the readymade equations provided a wider range of numbers than what he would create when feeling pressured during teaching. He also valued the learning progressions, recognising their use during teaching, particularly when responding to learners who needed extending, or to work with easier examples. The equation examples helped Ben keep the flow and pace of the lessons during the teaching, which was important for keeping his attention on learners.

Ben used the word problems by adapting the examples in the guide, keeping the addition focus, but changing the context to suit his learners. He made the decision to adapt the word problems, because he had observed this pedagogical approach, and there were examples in the guide. This prompted him to adapt the examples in the guide, choosing the pirate theme because it was a context the class were already working with. His justification of the importance of using contexts suggests that Ben valued this as an effective pedagogical approach for learning mathematics, because it linked mathematics to contexts that learners could relate to while also helping them enjoy and engage with the lessons (Earnest & Amador, 2019).

While Ben was obliged to use the teacher's guide, he spoke positively about using it. He had the freedom to choose from it what he wanted to use, and the parts he did choose were beneficial for his lessons. There was a sense that because the guide was authored by the MOE, he trusted the objectives, the equation examples (particularly the progressions), and the word problems as representing the NZC content that he need to teach, along with ways to do this. This gave him a sense of confidence that he was planning and teaching what was expected of him in the practicum setting, by both the AT and as directed by NZC. On a more pragmatic level, using the guide made planning efficient and less time consuming than if he had to search for, find and use his own materials (Kauffman, 2002).

The results also highlight that hidden underneath the term "planning," Ben carried out several smaller grain processes (Amador & Earnest, 2019; Boerst et al., 2011; Superfine, 2008), that Ben carried out. These included, gathering information from his AT, reading, analysing, and interpreting school-based curriculum documents, and searching for and scoping curriculum materials (e.g., NZC, related websites, and teacher's guides,) before planning lessons or sequences of lessons. Once in the practicum setting, he interpreted weekly curriculum plans, observed teaching, analysed weekly curriculum plans, read the teacher's guide, and from these made selections about what he would use. These selections were then copied onto plans or adapted for the practicum setting. While Ben was constrained by what he had to teach, and the materials he could use, the identification and naming of these smaller grain processes show that he was an active designer of mathematics lessons for his teaching. Curriculum materials played a significant role in helping him do this (Remillard, 2000; Rezat et al., 2021).

Although a single case is limited there are tentative implications in the findings for both ATs, PSTs, and ITE mathematics educators. It was beneficial for Ben, as a PST to meet with his AT to discuss the mathematics programme, determine what he had to teach, and what materials like the teacher's guide he was expected to use. Receiving copies of mathematics long-term planning, weekly planning, and teacher's guides also help with preparation for teaching. Therefore, it is important for ATs and PSTs to make time to meet with each other prior to practicum to share important information about expectations, documentation, and curriculum materials for mathematics teaching for the duration of practicum. It is also important for PSTs to observe mathematics teaching before beginning planning to gain further information about the pedagogical approaches used in lessons, and how materials such as the teacher's guide could be used. For PSTs there is value in spending time before practicum, reading and scoping out curriculum materials they might use in their lessons. This would help them become ready to plan their own lessons once practicum begins. It also seems that using a text such as the teacher's guide can provide a base when designing these lessons, providing direct examples to use, as well as those that can be adapted.

In ITE courses it would be helpful for PSTs to spend time with mathematics educators practising how to plan for mathematics teaching prior to practicum. This could include planning lessons for a range of mathematics concepts, searching and finding curriculum materials both in hard copy and digital form to align with this, critiquing and analysing their selections, and choosing aspects from these that are most useful for lessons. This could also include adapting activities for specific learner needs and working with contexts that help learners connect with mathematics ideas. It would also be useful to include time in course work to explore how mathematics concepts progress both within and between lessons, so that PSTs like Ben do not have to rely on materials, such as teacher's guides, to learn these particularly important concepts. Rehearsals of these practices carried out in an ITE setting with the support of mathematics educators would positions PSTs as designers of learning, preparing them for future mathematics teaching experiences (Grossman et al., 2009).

Finally, Ben presented as a confident and capable PST who was ready, willing, and able to take on the professional responsibility of planning and teaching mathematics lessons on practicum. He relished the opportunity to work with his AT, and to plan his own mathematics lessons using the curriculum materials available to him. His reflections on his planning processes revealed smaller grain practices related to planning, which will be examined in depth in the larger doctoral study.

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Student Perspectives of Engagement in Mathematics

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Engagement in mathematics education is an important factor in a successful student experience. This paper reports findings from a study of Year 5 student perceptions of engagement during a two-week, inquiry-based learning (IBL), problem-posing investigation. The study triangulated data from semi-structured interviews, video observations and student work samples to understand the student's perspectives of IBL; however, this paper reports the interview data. The findings indicate that most of the students were behaviourally, emotionally, and cognitively engaged during the IBL investigation, and that the investigation provided an opportunity for students to experience levels of competence, relatedness, and autonomy need satisfaction.

Over the past decade, inquiry-based learning (IBL) has become increasingly more prominent in educational policy and curriculum as countries look to increase student engagement and achievement (Artigue & Blomhøj, 2013). More recently, there has also been increased interest in the use of IBL in mathematics education (Dorier & Maaß, 2020). In response to declining achievement and engagement in mathematics and science, the European Union invested heavily in research into, and the development of, IBL. The 4-year international project, Promoting Inquiry in Mathematics and Science Education Across Europe (PRIMAS), examined inquiry-based pedagogy and sought to develop best practices for implementing IBL in mathematics and science (PRIMAS, 2013). Several studies have subsequently indicated the positive effect of IBL on student achievement and student motivation in mathematics (Dorier & Maaß, 2020).

In Australia, the emphasis on IBL in mathematics was encouraged by the Office of the Chief Scientist (2013) and highlighted to improve learning in mathematics. Although some research projects have looked at different aspects of IBL such as: teacher training (Fielding-Wells et al., 2017); learning environments (Brough, 2012); and, student motivation and engagement (Dorier & Maaß, 2020); little has been done to implement IBL in education systems more broadly. Whilst there are criticisms of IBL (Sweller, 2011), there is also a considerable amount of evidence that supports it as an approach that actively engages students in their learning and supports transfer to other learning contexts (Sullivan, 2011). More specifically, inquiry in mathematics has been found to support the development of meaning making and collaborative norms in mathematics classrooms. It has also been recognised for its potential to develop understanding, interest and engagement in mathematics, independence and creativity in solving problems, and student ability to transfer their learning to authentic problems (Laird et al., 2019).

How to engage students meaningfully in mathematics is a long-standing issue and the need to improve their engagement across all age levels remains a concern. Engaging students positively with mathematics early in primary school is a necessary first step, with Attard (2012) suggesting that engagement at this level is "crucial if students are to develop an appreciation for and understanding of the value of mathematics learning" (p. 22). Indeed, active participation is a component of the Framework for Engagement with Mathematics (Attard, 2012). We later refer to this framework in the Discussion to understand better student engagement.

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 570–578. Launceston: MERGA.

Engagement is multifaceted but is commonly conceptualised in three ways; behavioural, emotional, and cognitive (Fredricks et al., 2004).

- Behavioural engagement refers to what the students are doing; participation, effort, persistence, and on-task behaviour. It can be identified by observing student's self-directed behaviours such as asking questions, raising their hands, participating in discussions, and actively engaging with their peers (Fredricks et al., 2004).
- Emotional engagement involves enjoyment and interest, and incorporates student attitudes, interests, and values. It is a consideration of students' willingness to take on challenges, their sense of belonging, and focuses on both positive and negative aspects of student reactions to school, teachers, and activities. It considers whether students are experiencing boredom, happiness, sadness, or anxiety and the impact of these emotions on learning. The idea that emotion can affect the cognitive processes in humans is well developed (Tyng et al., 2017), and it can positively or negatively influence a student's perception, attention, learning, memory, reasoning, and problem-solving ability (Fredricks et al., 2004).
- Cognitive engagement focuses on self-regulated learning and personal investment in learning. It considers intrinsic motivation and how students control and manage tasks, maintain effort despite distractions, display flexibility in problem solving, make connections between ideas, and exert effort to develop complex ideas and develop understanding (Fredricks et al., 2004).

Research regarding engagement is often intertwined with motivation (Grootenboer & Marshman, 2016), with motivation being referred to as "the underlying reasons for a given behaviour" (Fredricks & McColskey, 2012, p. 765). This study used the three dimensions of engagement (behavioural, emotional, cognitive), and the self-system model of motivational development (SSMMD) (Ryan & Deci, 2017), to understand engagement and motivation. The SSMMD focuses on three fundamental motivational needs: competence, autonomy, and relatedness, and assumes that if "schools provide children with opportunities to meet these three needs, students will be more engaged" (Fredricks & McColskey, 2012, p. 765).

The effect of disengagement in mathematics has significant future implications for individual life opportunities and success (Attard & Holmes, 2020), as well as economic implications more broadly (e.g., employment opportunities; contribution of mathematics to the knowledge economy). This paper reports on one aspect of the study which sought to understand student engagement by investigating student perceptions of their levels of engagement as they created their own mathematics investigations, based on a video stimulus provided by the researcher. Two key questions underpinned the research: "How do Year 5 students perceive engagement in mathematics during an IBL mathematical problem-posing investigation?"; and "How do Year 5 students perceive their ability to problem-pose using a video prompt as stimulus during an IBL mathematical problem-posing investigation?" This paper presents the findings on the first question (i.e., the impact problem-posing on student engagement).

Theoretical Framework

The theory which underpinned this study, and supported the paradigmatic choices, is cultural-historical activity theory (CHAT). The origins of CHAT date back to Vygotsky's insights into the effect that social and cultural experiences have on and subsequently developed by Leontiev, and then Engeström, to form a theory that assists researchers to understand social environments, such as school classrooms learning (Koszalka & Wu, 2004). CHAT defines learning as "a process of constant interaction with the environment and others. Knowledge is constructed by individual learners, built on historical experiences, within his or her context, knowledge is not transferred, rather it is constructed differently in all individuals" (Koszalka

& Wu, 2004, p. 494). CHAT aligns with a social-constructivist view that knowledge is socially constructed, with no single reality (Merriam & Tisdell, 2016). CHAT contributes to our understanding of learning in this study as the students are engaged in a social activity, where they are constructing their own understanding of mathematics, rather than having it transferred to them via the teacher or a textbook. From this perspective, learning occurs best when a learner is engaged in the experience. In this way constructivist research aims to develop rich, contextual understandings about the world through the point of view of those living it.

Method

A qualitative, single instrumental case study approach was used, adopting the perspective that there are multiple realities worth representing (Merriam & Tisdell, 2016). Case study methodology provides the researcher a platform to investigate the various perspectives of the participants and identify patterns, relationships, and themes. The use of a single instrumental case study was appropriate in this study as it investigated one class and developed understandings based on the student perspectives from that one 'bounded' class. The researcher led a two-week open, mathematical investigation which required students to develop their own investigation questions based on a video prompt. The video prompt centred on a tennis theme and was created by the researcher to provide students with a wide scope for the creation of appropriate investigation questions. Seventeen students worked in collaborative pairs, or groups of three, to investigate their own questions and present their findings.

Data Collection and Analysis

Data were collected through semi-structured interviews with students (Merriam & Tisdell, 2016), video observations, and student work samples. The students were interviewed to identify their perceptions of engagement in mathematics during the IBL mathematical problem-posing investigation. The students were interviewed in a quiet space in pairs and were grouped with their collaborative partners wherever possible. However, two groups of three were divided to make three interview pairs. Students were interviewed in pairs to ensure they felt comfortable and safe to share their thoughts and to help eliminate any anxiousness or apprehension regarding their participation in an interview. The interviews were audio recorded and transcribed by the researcher. Thematic analysis was used to identify, analyse, and interpret patterns within the data. Thematic analysis "provides accessible and systematic procedures for generating codes and themes from qualitative data" (Clarke & Braun, 2017, p. 297). Following thematic analysis, the themes were then triangulated (Flick, 2018) using student work samples, video observations, and literature related to the phenomenon.

Participants

The research was conducted at an independent school in South-East Queensland, Australia. The Year 5 class (9.5-10.5 years of age) consisted of 17 students: 8 boys and 9 girls, and one participating teacher. 16 out of 17 students participated in a semi-structured interview, one student was absent and was thus unable to be interviewed. The school teaches the Australian curriculum through student-centred, individual pathways and students are engaged in a range of pedagogies throughout each day. These students learn mathematics using a commercial mathematics program, which includes student online learning and a suggested pedagogical approach that incorporates support for the delivery of personalised learning to each student. The program is organised in a modular format, and students work on individual pieces of learning on a computer and worksheets. Students engage in online tutorials, and answer questions based on their readiness, and take fortnightly paper tests to identify growth and areas for improvement. The teacher provides additional support through mini workshops as needed.

Other areas of the curriculum are taught through integrated units, focused language lessons and discovery learning based on personal interests. Students regularly experienced posing questions for personal investigations.

Findings and Discussion

Three themes emerged from the data regarding the student's perceived engagement in mathematics during the IBL mathematical problem-posing investigation: collaborative learning; enjoyment and interest; and cognitive engagement and learning transfer.

Collaborative Learning

The students indicated that working together and actively engaging with their peers was something that made learning more engaging and supported the development of their understanding. The comments suggested they felt a sense of competence and relatedness during the investigation. For example (pseudonyms are used for student names):

> Freya: Having different people in my group that we could all understand, so I can share with them what I wouldn't normally share with them so then it made me personally understand it more.

> Koby: It's been really fun and umm, engaging. Like we're working, we are collaborating, and we are like talking together. And we are figuring out problems, like not just looking at a screen [referring to commercial program], most of the time we were like organising stuff, not organising stuff, but like writing things down, figuring it out on paper.

> John: But I like it in groups to be honest because students who say they know a question, or I know a question that other people don't, I can help them with that, or they could help me with this.

Competence is associated with the perception of academic ability and self-efficacy and posits that all individuals need to experience themselves as effective individuals in their interactions (Ryan & Deci, 2017). Throughout the peer learning environment of the investigation, the students were offering explanations, justifying their thinking, and providing peer support, which assisted their classmates to understand at a deeper level. This provided the opportunity for those students to feel competent, while increasing the sense of belonging for all students. A sense of belonging or relatedness was further evidenced by their choice of words such as, "communicate", "collaborate", "working together", "talking together", "socialise", and "bonded" when describing their experience. Having a sense of belonging in a certain environment, or in a particular activity, has been associated with increased levels of engagement (Skinner et al., 2008). Active participation, which is associated with behavioural engagement, is considered imperative to the achievement of positive academic outcomes and is a component of the Framework for Engagement with Mathematics (Attard, 2012). While experiencing levels of relatedness and competence need satisfaction, the students also explained that the element of choice impacted their engagement, Gemma's comment highlights this:

> Gemma: If the teacher's kind of going, do you want to do this? Or do you want to do this? If you can have lots of options, it keeps me engaged. Like what we did, we had options to make a tournament or build a tennis court or something like that, we had different options.

Learning environments that offer choice and provide students with the opportunity to be creative have been shown to be engaging (Attard, 2012; Attard & Holmes, 2020). Offering choices to students provides them with a sense of autonomy over their learning. The third element of the self-system model of motivational development (Ryan & Deci, 2017) emphasises the need for teachers to allow students to demonstrate autonomy within the classroom environment (Fredricks & McColskey, 2012). Experiencing a sense of autonomy in school settings has been linked to "better academic outcomes such as classroom engagement, persistence, achievement, and learning" (Skinner et al., 2008, p. 768).

When analysing the video observations for behavioural engagement, the researcher looked for on-task behaviour such as the students being engaged in writing, discussions with teachers or peers (listening, explaining), and positive gestures and postures. Out of the 17 students participating in the study, 15 of them were observed being behaviourally engaged almost all the time. During the lessons, the students were observed writing and calculating, having discussions, and seeking help from the teacher when required. Occasionally, one of these students, needed redirecting to the task; however, this was generally towards the end of a session. Two of the 17 students, who were working together, were observed to require continual redirection from the teacher and were rarely focused without direct teacher support. Although the groups were strategically organised by the classroom teacher and Eric (one of the two students) reported that he found the investigation "fun" because they could, "carve our own paths", it is possible that the autonomy offered in the investigation did not support learning for these two students.

Enjoyment and Interest

The second theme that arose from the data was related to emotions associated with the investigation and the idea of having fun. The word "fun" was found fifty-one times in the transcripts. The students reflected on their regular mathematics lessons using emotive language such as, "boring", "bland", "bored" and described the two-week investigation as "fun", "funner [sic]", "interesting", and "awesome". This aspect of engagement is known as affective or emotional engagement. Emotions have been shown to play a significant role in how students lose engagement or become dissatisfied and frustrated in schooling (Skinner et al., 2008). When students feel negative emotions such as boredom, it affects their willingness to actively participate and influences their overall perception of that subject. The students also explained that the investigation offered an element of challenge, helped them to connect mathematics to real-life events, and provided the novelty of doing something different. For example:

> Nick: Well, it was kind of out of the blue when you showed us the tennis video. Also, I thought how can this relate to maths? And when we actually go into it, I realise how like, how much maths is involved in in everything. I would just look at these flowers and say they aren't really maths, but now I can see there is math in them.

> Lucy: Because it was like, it was different, and I like different. And so, it was really fun, really engaging. Yeah, just, it was really different.

> Lara: It was just a really great experience to just enjoy a different way of math. The experience, it was fun, enjoyable and challenging at some points.

The positive emotions that the students experienced may be related to the academic content, available choices, challenge (Attard, 2012), their friends, teacher, or the novelty of doing something different (Fredricks et al., 2004). Recent research on the psychological needs outlined by self-determination theory (Ryan & Deci, 2017) has included the potential need for novelty (Benlahcene et al., 2020). The need for novelty "refers to the innate desire to experience new things that have not been experienced before or that differ from a person's daily routine" (Benlahcene et al., 2020, p. 1291). González-Cutre and Sicilia (2019) found that, beyond the three basic needs of autonomy, competence, and relatedness, novelty was significantly associated with positive outcomes, motivation, and satisfaction of students. Within this study many of the students indicated that they enjoyed the investigation because it was "unique", "different", or "different to what we normally do"; therefore, the novelty factor needs to be considered as one of the factors that may have impacted emotional engagement.

The students experienced a range of positive emotions including interest, enjoyment, fun, and inquisitiveness. Furthermore, the interview data indicated that all students were interested in participating in a similar investigation in the future, reinforcing the earlier claim that this was a positive experience for them. Skinner et al. (2008) explain that emotional and behavioural engagement are closely linked and that theories related to engagement and motivation suggest that "it is engaged emotions, such as interest and enthusiasm, that fuel engaged behaviours, such as effort and persistence" (p. 767). As previously mentioned, the video observations indicated that 15 of the 17 students were observed to be behaviourally engaged; however, it is difficult to observe emotional engagement and thus it must be inferred from demonstrated behaviours (Fredricks & McColskey, 2012). Therefore, the self-reports of emotional engagement collected through the semi-structured interviews are triangulated with the behavioural engagement observations to support the finding that the students were emotionally engaged during the IBL mathematical problem-posing investigation.

Cognitive Engagement and Learning Transfer

During the interviews the students explained that they were able to use the knowledge and skills previously acquired while working on their investigations in new scenarios. The investigation offered the students an opportunity to practically apply previously learnt skills, build confidence, and challenge themselves intellectually. This aspect of engagement is known as cognitive engagement, which emphasises self-regulation and personal investment in learning (Fredricks et al., 2004). The qualitative data from the interviews, video observations, and student work samples together demonstrated that 15 of the 17 students were cognitively engaged. Many students explained that the investigation provided an opportunity to practise learnt skills, which helped them to feel more confident in their mathematical understanding:

> Milly: Maybe [I learnt] a few things like, but mostly just practising the things that we already knew. And like, putting it to the test and using it.
>
> Koby: Practicing stuff that I already knew. But I felt like I had grown, I learned. I had boosted my confidence on that stuff that I was still learning…because I practiced stuff that I don't really practice anymore. So, I feel like that I got a lot better, like at times tables.
>
> Lucy: It was helpful in many ways, but mostly practising and like, trying to know the maths you already know but like knowing it better and understanding it.

The interview data provided evidence that the IBL mathematical problem-posing investigation was an opportunity for the students to develop their conceptual understanding, a key component of cognitive engagement (Fredricks et al., 2004). Additionally, the students practised their skills, reviewed their knowledge, and transferred their understanding through practical application. In his review of Australian mathematics education, Sullivan (2011) outlined six key principles for effective teaching of mathematics with one of the key principles highlighting the importance of transferring learnt skills. Haskell (2001) explains that learning transfer happens when students recognise past learning and apply and extend that learning in a different situation.

An emphasis on cognitive transfer aligns with the expectations of the Learning Continuum of Critical and Creative Thinking (CCT) within the F-10 Australian curriculum, which expects learners to transfer knowledge into new contexts (Australian Curriculum, Assessment and Reporting Authority, 2016). The inability of students to transfer concepts, skills and procedures is one that concerns many educators and managers in work environments (Dixon & Brown, 2012). Students often fail to recognise that their prior learning can be used to solve similar real-life problems because such problems differ from the structured situations often presented in school (Dixon & Brown, 2012). Paige's comments about the investigation, and "usual math" reflect a previous lack of opportunities to transfer mathematical learning to real-world situations or investigations:

> Paige: Um, it was very different, because it's a different world, different math learning, because you'd always have to think about money, and like how much it costs and your budget. So that's very different to the ones that we do on our levels [referring to Maths Pathway]. And you learn like how to, you learn new things, like what some people actually do this for, like to organise tournaments, like in different sports, and they actually have to plan this out every time. So, it's like, wow.

The requirement to transfer knowledge highlights the need for a variety of learning engagements within the classroom, so that students can transfer their understanding, apply and develop their mathematical skills, and connect mathematics to real-life contexts. The IBL mathematical problem-posing investigation provided an opportunity for students to do so.

Conclusion

This paper focusses on student perceptions of engagement in mathematics during an IBL mathematical problem-posing investigation. The findings indicate that all students perceived themselves to be emotionally engaged (n = 17), while almost all (n = 15) were behaviourally and cognitively engaged. The students suggested that the investigation provided them with opportunities to make choices, work autonomously within their group, support peers in their learning, build peer relationships, and challenge themselves. These findings align with Ryan and Deci's (2017) self-system model of motivational development, which is based on self-determination theory and focuses on fundamental motivational needs: competence, relatedness, and autonomy. Novelty was found to be an additional important factor in student engagement.

Although the findings are presented as three separate themes, they are interconnected, affecting each other in various ways. Students who are actively participating, asking questions, engaging in discussions (behaviourally engaged), and self-reporting or demonstrating interest and enjoyment (emotionally engaged) may or may not be cognitively engaged. When one aspect of the environment or learning engagement is altered, all three types of engagement are influenced (Fredricks et al., 2004). Various combinations and levels of engagement may exist, and when designing learning environments, all three dimensions of engagement should be considered holistically with consideration to the multidimensional and complex field of engagement in learning.

There are many factors that may influence engagement or disengagement, such as prior experiences, the teacher, learning needs, and personal interests. However, it was beyond the scope of this project to delve into each aspect that may or may not have been a contributing factor. The use of student voice and the triangulation of video observations and student work samples did, however, provide insights into the ways engagement in mathematics learning can be enriched from the student's perspective.

The limitations of this study include the small case size and timeframe: the study was conducted in one class with 17 students, over a two-week period. The students had not created mathematical investigations or problem-posed before and were limited by the two-week timeframe. Future research directions could include a longitudinal study to provide greater insight into different aspects of engagement during an IBL mathematical problem-posing investigation, and further explore the relationship between engagement and learning during an investigation.

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A Multifaceted Project Design to Understand and Build the Strengths of Out-of-field Secondary Mathematics Teachers

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Teaching out-of-field (OOF) contributes to student underperformance in mathematics (Clotfelter et al., 2010). Australia's largest study on OOF STEM teaching (Shah et al., 2020) showed mathematics was the most frequently OOF subject taught at Year 10 (at 19%). In addition, only 24% of students are continuously taught by suitably qualified teachers in Years 7–10 (Prince & O'Connor, 2018). This is concerning when Australian students' international mathematics test scores have fallen below the OECD average (Thomson et al., 2019); alarming numbers study no mathematics or choose easier courses for NSW Years 11–12 (Jaremus et al., 2019); and mathematics interest, value, and self-concept decline across Years 7–12 (Watt, 2004). This compounds teacher shortages that threaten Australia's development of a capable, innovative STEM sector. Research reveals OOF teaching can also be detrimental to teachers' professional engagement. OOF mathematics teachers hold lower self-concepts, enjoy mathematics less, and experience higher stress/anxiety than in-field teachers; these relate to their reported instructional quality, burnout, and turnover intent.

This roundtable presentation elaborates our NSW Department of Education funded project conceptualisation and multifaceted design, to dialogue with cognate researchers and interested stakeholders. It is anticipated that discussion amongst roundtable participants will assist development of a deeper understanding of the complexities involved in effectively supporting OOF teachers of mathematics; and inform the design of quality PL to strengthen the expertise of non-specialist mathematics teachers to teach junior secondary mathematics.

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Numeracy ≠ Mathematics: Numeracy and the General Public

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Although we may intuitively know what *mathematics* is, you may be surprised to learn that there is no agreed upon definition. The Oxford dictionary definition appeals to me:

> The abstract science of number, quantity, and space, either as abstract concepts (pure mathematics), or as applied to other disciplines such as physics and engineering (applied mathematics). (Lexico, 2022)

Numeracy is something quite different. Within the Australian Curriculum, numeracy is one of seven general capabilities and is defined as follows:

> Numeracy encompasses the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations. It involves students recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully. (Australian Curriculum, Assessment and Reporting Authority, n.d.)

Unfortunately, the terms mathematics and numeracy are all too often used interchangeably in everyday conversation, within Australian schools (particularly at the primary level), as well as amongst some of our own mathematics education colleagues.

In order to have an informed citizenry, I contend that numeracy is an essential pre-requisite. Yet, in my view, the Australian education system appears to have failed in appropriately equipping citizens with basic numeracy capabilities and the capacity to judge whether information they encounter is realistic or can by substantiated.

In recent times, the general public has been bombarded with statistical and other mathematical information with respect to the COVID-19 pandemic, climate change, and a range of other social, economic, political, and geographical issues. Regrettably, journalists, media commentators, and some politicians also appear to have limited statistical and mathematical skills to interpret data accurately, resulting in confusion or misleading information (whether intentionally or not), and fuelling conspiracy theories.

At this round table session, my aim will be to elicit participants' views on aspects of citizenry for which numeracy capabilities are needed. I will be particularly interested in gaining views on how the general public's numeracy capabilities might be researched, and which aspects of contemporary and future societal issues and concerns should be the focus. Participants' insights will be sought on what the findings of such research endeavours might be, what implications there would be for the educational system, what actions might be needed, and how mathematics education and mathematics educators might contribute to challenging the status quo.

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Intended Versus Enacted Curriculum: Teacher Knowledge and Curriculum Change at the Senior Secondary Level

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Shulman (1987) identified curriculum knowledge as part of the knowledge base for teaching. With considerable curriculum changes in mathematics education in Australia in recent years, including the review of the F-10 syllabus and implementation by several states of the new senior secondary subjects, it is vital that teachers are cognisant of any change. It is also important to understand how these changes influence a teacher's practice.

A two-year longitudinal study on student, teacher, and lecturer perspectives on the transition from secondary to tertiary mathematics was conducted across Queensland. Participants included 1000 Year 12 students, 49 teachers and 16 university academics. Part of the study involved students completing two sets of mathematics questions, with teachers and lecturers asked for their perspectives on how students would answer the questions and how difficult the students would find them.

In general, the teachers were overly optimistic and the lecturers pessimistic about the chances of students answering the questions successfully. There were, however, some unexpected findings that highlighted the importance of teachers' knowledge of mathematics curricula. These findings have implications for the enacted curriculum, which Remillard and Heck (2014) assert is the piece of the curriculum framework that impacts student outcomes the most.

At this round table we will use several teacher and lecturer quotes from the two-year study as a starting point for a discussion on teacher knowledge and the intended versus enacted curriculum.

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What Makes Effective Leadership When Implementing Research-based, Equity-driven Professional Learning and Development?

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There is growing awareness of the importance of leadership within schools to lead shifts in pedagogical practices to address equity issues in mathematics teaching and learning. Individuals in middle leadership roles balance complex challenges while undertaking teaching and management roles within their workload. We need school leaders who have a positive influence on student outcomes to use strategies that make a difference to the way teachers teach (Day et al., 2016). The ability to build relational trust between teachers and middle leadership has been found to be critical for sustained improvement in student outcomes (Edwards-Groves & Grootenboer, 2021). Recently, Patuawa et al. (2022) highlighted potential barriers to implementation of new initiatives, including a lack of concurrence among middle leadership beliefs and the principal's vision, which can result in dissonance. Two questions underpin our wonderings. What approaches could be used to ensure middle leadership develop strong pedagogical knowledge and the capabilities to implement and support transformative change? Furthermore, how can mathematics educators work in partnership with middle leaders to support mathematics teachers within their organisations to use inclusive thinking strategies regularly within their daily teaching?

At this round table, we will present an exemplar of effective leadership from one low-socio-economic primary school. Participants will be invited to share models of effective leadership from different settings. We will explore the importance of leadership beliefs and consistency across the whole leadership structure, and how this may influence the development of imbedded and sustainable pedagogical change in mathematics teaching and learning.

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Teacher Practices in the Mathematics Classroom Following Professional Learning and Development: Association with Student Outcomes

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Equity is one of the most complex and pressing issues related to mathematics education both in New Zealand and internationally. In multi-cultural societies, the cultural mismatch of students and the educational environment in which they learn has been suggested as a reason for the reported gaps in student academic outcomes (Stephens et al., 2012). Rather than take a deficit approach, attention on student strengths (Burack et al., 2017) is needed and a has been adopted as a key foundation of Developing Mathematical Inquiry Communities (DMIC) professional learning and development. The programme focuses on ambitious practices and culturally sustaining pedagogy to shift student outcomes (Hunter et al., 2018). We will present findings of a project exploring the frequency of use of DMIC pedagogical practices in the classroom, teacher confidence, and self-reported teaching proportions of mathematical strands. We relate these findings to student outcomes of mathematical achievement, wellbeing, and engagement. A key finding is that both changes in teacher practice and student outcomes were noted after involvement over an extended period of time (4 years) in the DMIC professional learning and development programme.

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Raising Teacher Expectations of Students' Capabilities by Examining Student Work Samples

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Teachers in 28 Catholic Education Diocese Parramatta (CEDP) schools, Australia were involved in the EMC^3 Research Project. Within the project teachers were using an Instructional Model for Student-Centred Inquiry when implementing challenging tasks. An obstacle for teachers is having high expectations for student learning, beyond those associated with the New South Wales syllabus requirements.

This project involved teaching educators employed by CEDP working in project schools to support the implementation of the instructional model to engage students in challenging tasks. After co-teaching in classrooms, we analysed student work samples with the teachers to look for evidence of knowledge transfer between tasks and across a sequence of lessons. We found that K-2 students were able to make the connection between the learning from one task to another. The students were able to show a clear understanding of the big mathematical ideas leading to abstraction.

For example, when 5, 6- and 7-year-old students attempted a sequence of tasks on *Making Things Equal*, they began by moving "cakes" from one plate to another in various ways to make both plates equal. Students were able to imagine many possibilities that were not anticipated by teachers and were able to write matching equations, even though that was not an expectation of the NSW syllabus at this stage. Teachers were surprised by students' mathematical thinking, reasoning and recording, and initially considered this particular sequence to be too difficult for students in their first years of schooling.

We noticed that by raising the expectations of teachers and focussing on what students could do, deeper learning was evidenced in the work samples. We conjecture that if teachers have high expectations and are able to pose challenging tasks without "telling", students will be more open to taking risks with their learning. This would enable teachers to focus on the learning presented in the work samples rather than the performance of individual students (Sullivan et al., 2020), and recognise the learning gains that can be made by all students by engaging in challenging tasks. This is an aspect that we wish to research in the future, with the support of our academic partners.

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Concept Maps as a Resource for Teaching and Learning of Mathematics

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Although conceptual knowledge has been identified as key to students' mathematics understanding, there is limited focus on how to enhance its development at senior secondary level. This mixed methods study underpinned by constructivism explored teachers' perceptions on how concept maps relating junior prior mathematics knowledge (years 7 to 10) to senior mathematical knowledge (year 11 and 12) can enhance the teaching and learning of senior secondary mathematics. Surveys that included Likert scale items and open-ended questions were conducted with sixteen senior secondary mathematics teachers. To gain deeper understanding, eight semi structured interviews were also conducted. Results showed that concept maps relating junior to senior mathematics concepts can be a resource that enhances conceptual knowledge, consolidation, and assessment of students' mathematical knowledge. The role of visual representations in mathematics teaching and learning that is enhanced by concept maps is an area that needs more attention to help improve students' participation and achievement.

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Use and Development of Mathematical Processes During an Online Escape Game

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Mathematical processes, such as problem solving, reasoning and proof, representation, connections, and communication are considered drivers of mathematical thinking (Isoda & Katagiri, 2012) and essential components of mathematical learning (Neyland, 2004). However, as an unintended consequence of curriculum redesign, they have declined in visibility from mathematics learning experiences in New Zealand primary school classrooms (Clune, 2021; McChesney, 2017). Parallel to this, the last decade has seen the education sector inundated with digital tools (applications, programs etc.) that claim to facilitate learning. While many studies support the use of digital tools to enhance engagement (Attard & Holmes, 2020) research evidence to support claims made by the vendors of these tools, regarding learning, is scarce. My study, conducted with 12–15-year-old school students, aimed to explore how a purpose-designed, fully online, digital "escape game" could make mathematical processes central to the student experience. I used Sfard's (2008) theory of commognition to analyse the conversations and interactions of students as they engaged with the game. The preliminary findings suggest that the digital escape game enriched the use and development of core mathematical processes, such as problem-solving, reasoning, and communication, and facilitated making connections within, across and beyond mathematics. However, there are implications regarding the design of digital experiences like the one in this study. For teachers, a core design challenge lies in pre-empting the learning conversations as well as designing learning tasks that will provoke entry and allow for immersion into the intended discourse—how the tasks are designed may mean the difference between whether the students action "ritual" or "explorative" routines (Sfard & Lavie, 2005).

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Repurposing Bronfenbrenner's Ecological Theory to Focus on Very Young Preverbal Children's Mathematical Engagement

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It is important for early childhood educators to 'think differently' about their beliefs regarding their role and their engagement with very young children (Garvis & Prendergast, 2015). This presentation is designed to share one way of addressing this. It is a theoretical consideration of Bronfenbrenner's (1977) ecological theory with the aim of repurposing it to function more as a 'coat hanger' (Tudge et al., 2016)—a frame—on which the early childhood educator can consider the aspects that will impact the opportunities created for very young preverbal children to engage with mathematical thinking. Specifically, this presentation uses the nested structures of Bronfenbrenner's (1977) ecological theory as a frame to help early childhood educators focus on the elements that may impact very young preverbal children's demonstration of and engagement with mathematical thinking.

Bronfenbrenner (1977) positions his ecology of human development as a description of how the growing individual interacts with their environment. He views the environment as both the close context the individual inhabits and the more distant settings within which the individual's context is situated, as well as the interactions that occur between the contexts and settings. These contexts, settings, and interactions incorporate formal and informal elements. Bronfenbrenner's (1977) ecological theory explores the interactions that are possible, framing the relationships between all elements within the environment as not just one-way processes.

The very young preverbal child is an active participant in their world and in their mathematical learning within their world (Franzén, 2015). Blömeke et al. (2020) note that there are many mathematical opportunities in the young child's everyday life and throughout their play. The nested structures of Bronfenbrenner's (1977) ecological theory can assist the early childhood educator in deconstructing the environment in which the child resides. Forcing the elements of this deconstruction into the nested structures, much like hanging clothes on coat hangers in a wardrobe, provides an organisation that can help the early childhood educator to see and attend to each of the elements, to consider their impact on the child's capacity to engage with mathematical thinking, and to identify the affordances offered by the interactions of these elements.

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Challenges to Inclusive Teaching of Mathematics in Aotearoa New Zealand

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Inclusive teaching is an area of strategic importance in Aotearoa New Zealand (Education Council, 2017) and access to education is a human right worldwide (United Nations, 1989). Yet currently, children with disabilities in our schools are reported as being excluded from quality learning opportunities. Approaches such as inclusive pedagogies (Florian & Black-Hawkins, 2011) and Universal Design for Learning (Rose et al., 2014) support teachers to plan for and teach all children and young people in inclusive ways. However, we know little about the extent to which teachers know about and apply these approaches in their classrooms, or how these approaches can support the inclusive teaching of mathematics particularly. In fact, this area is under-explored internationally (Gervasoni & Peter-Koop, 2020); those studies that investigate quality mathematics learning tend to exclude from research learners with disabilities or support needs (Tan et al., 2019).

Although the teaching standards require inclusive practice across all curriculum areas (Education Council, 2017), teachers may lack confidence in their own curriculum knowledge when teaching students with disabilities, and/or have low expectations for their learning generally (MacArthur & Rutherford, 2016), and more specifically for their learning in mathematics. It seems likely that some teachers require considerable support when they transition to a more inclusive approach to teaching mathematics. To facilitate this support, we first need a better understanding of the mathematics-specific challenges, as perceived by schools, teachers, and teacher aides.

In this short communication, we report on the initial results into a study that firstly gains the perspectives of one school principal, two classroom teachers, and a teacher aide, regarding their challenges for inclusive teaching of mathematics. We will also discuss the initial stages of our lesson study research design in working with teachers to develop more inclusive practices in mathematics.

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Investigating High School Students' Understanding of Decomposition Techniques in Mathematics

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In completing arithmetic and algebraic problems during mathematics lessons, students are required to proficiently manipulate numbers and expressions. Decomposition of numbers and algebraic terms is an important skill as part of this manipulation. This presentation will report on a study that involved Year 9 students completing a computational fluency test that investigated their understanding of decomposition techniques such as using the associative, commutative, and distributive properties. Interviews with students assisted in gauging the level of understanding of decomposition techniques.

This study demonstrated that students in high school continue to have difficulty due to lack of development of conceptual understanding that has been noted in primary school students (Downton et al., 2019). The study also looked at the continued teacher influence on a student's choice of computation strategy (Swan & Bana, 2000) and the implications of student arithmetic thinking on the development of algebraic thinking (Warren, 2003).

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Co-teaching Mathematics in Flexible Learning Spaces: What is the Effect on Pedagogy and Achievement?

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The traditional classroom was designed in the 1800s and was characterised by an authoritarian approach, where the "sage on the stage" would impart knowledge, facts and procedures to the masses. In the 1970s, countries began to experiment with open classrooms, incorporating student choice of activity and time frames, more small-group collaboration, and combining of subject areas. Although this transformation faltered, recently there has been a resurgence in the construction of flexible classrooms and use of associated pedagogies. For example, in 2017 the NSW Department of Education invested $6 billion towards public school infrastructure, aimed at remodelling schools' learning spaces to "engage students in ways that reflect 21st century learning" (Kariippanon et al., 2019, p. 572). This was in conjunction with the implementation of the NSW Mathematics K–10 Syllabus (Education Standards Authority, 2012), which had an increased focus on developing "mathematics through inquiry, exploring and connecting mathematical concepts, choosing and applying problem-solving skills." However, as Kariippanon et al. state, the assumption that changing the structure of the classroom will translate into changes in the teaching and learning process is not supported by empirical evidence. In a survey of 822 schools, Imms and Byers (2017) found that non-traditional classrooms account for 25% of classrooms across Australia and up to 50% in some sectors. As flexible learning spaces become more common, researchers and teachers need to determine the impact these spaces have on learning.

At the start of 2022, a study was initiated to focus on one Western Sydney secondary college that had numerous flexible learning spaces. The spaces have no focal point, thus were designed to increase the student-centred nature of classes and the amount of problem solving involved in lessons. As the study progresses, it will consider the Year 8 cohort, all being taught in flexible learning spaces using a co-teaching model and the Year 7 cohort, who are all being taught in traditional classrooms. Comparing the experiences of each group, the study will explore the students' perceptions of what is important when learning mathematics and determine whether there are any differences in achievement. The study foci are:

- Pedagogy: Focusing on whether there is a difference in the amount of time spent on surface level questions compared to deep problems and the amount of teacher talk in each environment. Similarly, whether there is a difference in how often a lesson is launched with a problem compared to starting with an explanation.
- Mathematics learning: Students will rank what is important when learning mathematics, recalling facts, remembering procedures, breaking down questions into steps and understanding why a procedure works.

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A Comparison of Rational Number Word Problem Types Across Three Grade 4 to 6 South African Textbook Series

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Being able to interpret the different fraction constructs within problem solving contexts often provides challenges for learners. This paper is based on a comparative document analysis of three of the most popular textbook series for Grade 4 to 6 learners in South Africa. The focus was on exploring the nature of fraction word problem types. Such an analysis sets out to identify the word problem types and how they are presented across the grades and across the various textbooks.

Poor performance in the conceptualisation of fraction word problems in mathematics in the South African education system results from two common problems. Learners have difficulty learning and understanding rational numbers, more specifically common fractions (Zhang et al., 2015), and they also often find it challenging to solve word problems (Giganti, 2007).

In most South African classrooms, teachers and learners work closely with and rely on textbooks and the Department of Basic Education (DBE) workbooks. This has come to the fore, particularly during the COVID-19 pandemic and ensuing lockdown. This paper sets out to analyse and compare the fraction word problem types within the Department of Basic Education workbooks and two of the most used textbooks in Grades 4 to 6, to establish the nature of fraction word problem types. The research question explored was:

> *What fraction constructs are evident within the fraction word problems in Grades 4 to 6 South African textbooks?*

The results show that the part-whole construct is dominant across all the textbooks and there is little progression from one grade to the next in the presentation of each construct. The findings also show that the total number of fraction word problems appears erratic across the Grades 4 to 6 texts. The most fraction word problems across all the texts were in the Grade 5 workbook followed by the Grade 4 workbook. The number of fraction word problems increased from Grades 4 to 6 in Textbook A. This could suggest that word problems are seen as a means of assessing learners' fraction competence after they have been taught the concept. In other words, learners are not introduced to fractions via problem-solving. Textbook B had very few fraction word problems in Grades 4 and 5. In Textbook B, there were fewer word problems in Grade 5 than Grades 4 and 6.

In conclusion, it was seen throughout the three texts that there was no clear progression from Grade 4 to Grade 6 in terms of mastery of one fraction construct before the introduction and focus on the next fraction construct. A developmental progression is needed to build understanding of one fraction construct before moving to the next. The presentation of word problems with reference to fraction constructs can thus be seen as seen as problematic and may be an influencing factor as to why learners struggle with mastering fraction construct skills in fraction word problems.

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Senior Secondary Probability Assessment Task Support for Development of Thinking Skills

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Mathematical curricula aim to develop thinking skills in students. This paper reports on the analysis of senior secondary probability assessment tasks, using Engstrom's Activity Theory (2001), to disclose the tensions and possibilities for the development of thinking skills. Senior mathematics (Year 12) modelling and problem-solving tasks (Victorian Curriculum and Assessment Authority [VCAA], 2015) on probability were analysed according to a two-tiered thinking framework (Ernst, 2021), based on several thinking frameworks including Bloom's Revised Taxonomy (Krathwohl, 2002) and the SOLO (Structure of the Observed Learning Outcome) Taxonomy (Biggs & Collis, 1982). The rules of implementation of the tasks were triangulated with data from interviews with students ($n = 20$) and teachers ($n = 14$) interrogating their experiences in doing or administering such tasks, respectively.

The findings of this analysis show that school-based modelling and problem-solving assessment tasks (VCAA, 2015) can support the development of thinking skills; however, many contextual factors hinder this. Teachers feel they do not have the skills and time to prepare the tasks, school rules and conventions can limit the effectiveness of tasks, students lack experience with these types of tasks, and textbooks offer limited support. As well, the supporting tools of reference books and calculators are not used to full advantage by students. Thus, many opportunities for the development of thinking skills in students are potentially lost in the implementation of internally assessed modelling and problem-solving tasks. Activity theory proved useful for describing the influences and tensions on the internally assessed tasks and the analysis of the complex elements involved.

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Finding Effective Methods for Mathematics Learning: Concept Mapping as an Assessment Task

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Finding the most effective ways to teach and learn mathematics at university is becoming a critical problem in higher education as rapid advances in related fields demand graduates with advanced mathematics knowledge and skills. In this short communication, we report on a study undertaken in a large university classroom setting (N = 355). An instructional innovation was designed, developed, implemented, and evaluated in a mathematics course involving novel assessment tasks—Knowledge Organisers (Jeong & Evans, 2021). The tasks comprised prompts for students to generate examples and non-examples (Fukawa-Connelly & Newton, 2014; Mason & Watson, 2008) and construct concept maps of the key concepts covered in the course (Novak & Cañas, 2008). The original design of the initiative was based on the current understanding of human cognitive architecture and cognitive science research (Sweller et al., 2019). A concept map is a visualisation of a group of related abstract concepts with their relationships identified by connections using directed arrows, which can be viewed as an externalisation of a schema to be stored in a learner's long-term memory (Schroeder et al., 2018). By utilising a mixed-methods approach and triangulation of the findings from qualitative and quantitative analyses, we were able to discern critical aspects pertaining to the feasibility of implementation and evaluate learners' perceptions. Students' performance on concept mapping is positively correlated with their perceptions of the novel tasks and the time spent to complete them. Qualitative analysis showed that students' perceptions are demonstrably insightful about the key mechanisms that supposedly make the novel tasks beneficial to their learning. Based on the results of the data analyses and their theoretical interpretations, we offer practical recommendations and suggest future research directions.

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Pedagogical Factors Predicting Mathematics Achievement: Analysis of the TIMSS 2019 Large-scale Data

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New Zealand's mathematics achievement at a school level has deteriorated markedly in the last two decades. This is evidenced by substantial declines in achievement indicators across the primary large-scale international studies: Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA). Out of 64 countries investigated in TIMSS in 2019, New Zealand scored significantly lower than all OECD countries, except for Chile and France, and significantly lower than the centre point for 64 countries (Morrow et al., 2022). Over time, the trend is particularly concerning for high school students, with Year 9 average achievement being the lowest recorded since 1995. Of particular note are the substantial declines among 15-year-old students in Australia (33 pts) and New Zealand (29 pts) against the relative stability of the OECD average (5 pts), as recorded by the PISA average performance indicator (Royal Society of New Zealand Expert Advisory Panel, 2021). The reasons for these declines remain largely unclear.

In this short communication, we present the results of the analysis of the TIMSS 2019 data with the aim to identify significant predictors of achievement. In addition to well-known predictors such as socio-economic status (SES), the evidence suggests that certain pedagogical choices teachers make in their day-to-day practice are significant positive predictors of performance, such as a preference for spending more time explaining new mathematics. The influence of pedagogical choices is particularly striking when considering the effect on academically resilient students—those who succeed academically despite the odds associated with economically disadvantaged backgrounds (e.g., low SES). A discussion of the results is offered, bringing in the ever-polarising debate on the merits of student-oriented, inquiry-based learning over the more traditional approaches prioritising explicit instruction (Kirschner et al., 2006), based on the recent advances in cognitive science (Dehaene, 2020).

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Investigating Students' Engagement with Teach-first and Task-first Lesson Structures Incorporating Challenging Mathematical Tasks

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The focus of this study is on how variation in the structure of lessons (Task-first and Teach-first) that incorporate challenging tasks impacts student engagement and learning of mathematics, from the students' perspective. This intervention study will adopt a qualitative, exploratory design with multiple data sources including questionnaires, lesson observations, survey methods and post-lesson semi-structured interviews with two classes of Year 3 and 4 students (approx. 2 groups of 25 students aged 8–10 years) from one primary school in NSW. The practical implication of gauging students' reactions to different lesson structures includes assisting teachers broaden their choice of pedagogy to suit various student characteristics, including learner preferences for enhanced mathematical engagement and achievement. Theoretically, findings from this study will extend existing theories of learning and of instruction by deepening our understanding of how students effectively learn challenging mathematics. In this presentation, we give an overview of the research project and highlight the potential the findings have for deepening our understanding of how students learn mathematics.

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Making the Invisible, Visible: Supporting Numeracy in the Arts

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Improving students' numeracy outcomes continues to be an explicit goal in the secondary school sector (Australian Governments Education Council, 2019); however, the responsibility of taking action to support student numeracy has largely rested with specialist mathematics teachers, or school leaders whose purpose may be focused on standardised assessment targets. This short communication will present the findings of a pilot study in which members of an Arts faculty were provided with targeted professional learning in embedding numeracy. The study's design drew on a theoretical model developed in the context of a doctoral thesis, verifying the relevance of the model to enhance teachers' numeracy practices. This presentation will discuss some of the key factors impacting secondary school teacher uptake of deliberate and purposeful numeracy teaching in their classroom, share vignettes of arts-based numeracy tasks, and communicate insights into the impact of the theoretical model on teacher practice.

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Enabling Students' Critical Mathematical Thinking

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The capacity to use mathematics *critically* is essential for making decisions and forming judgements about challenges facing society including those related to the economy, health and the environment (Geiger et al., 2020). Critical Mathematical Thinking (CMT) involves the use of mathematical techniques and reasoning to address complex real-world problems in a wide range of contexts. The capacity to reflect on the consequences of proposed solutions to real-world problems (e.g., social, ethical) is a key dimension of CMT because of potential impacts on individuals and society at large (Maass et al., 2019). People who are unable to apply CMT in real-world contexts have fewer opportunities for both employment and participation in society (D'Ambrosio & D'Ambrosio, 2013). Developing students' CMT, however, is difficult and there is little evidence that current curriculum and pedagogical responses to this challenge have been effective, which indicates that CMT teaching and learning practices are under researched and theorised.

In this session, we outline a project that aims to generate new insight into teaching practices that can promote or inhibit students' CMT development. This requires attention to what teachers *see* when students work on CMT tasks and how they respond to what they *see* —a complex process described by Sherin et al. (2011) as *teacher noticing*. Student success has also been linked to teachers' beliefs about student capability (Beswick, 2018).

The research team will work with five schools from metropolitan and non-metropolitan areas using an innovative video-based methodology that integrates researcher and teacher perspectives on students' CMT development. The goal of this approach is to support teachers in learning to notice students' CMT within classroom settings. Planned outcomes of the project include new theoretical and practical knowledge as well as resources designed to promote teaching and learning practices that support students' CMT development.

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Difficult Progressions in Multiplicative Thinking for Primary Students

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Many students encounter difficulties with whole number learning at some point in their primary schooling. It is helpful for school leaders and teachers to anticipate these moments in order to plan relevant curriculum and differentiated teaching, including interventions. Our research investigated *Mathematics Assessment Interview* (MAI) longitudinal data across six years for 2052 primary students to identify any difficult progressions in the multiplication and division domain, and the grade levels when these occur. Using the MAI growth point framework for the Multiplications and Division Strategies domain to measure growth, two prolonged progressions were identified at different time-points in primary schooling.

The first difficult progression occurred for students who were moving to partial modelling in the MAI growth point framework, across Grade 1, Grade 2, and Grade 3. This represented 48%, 64%, and 54% of students in these grades, respectively. This first finding suggests that learning experiences for students that focus on partial modelling or that screen concrete models need to happen earlier than teachers may anticipate. The second difficult progression identified was for those students developing basic, derived and intuitive strategies for multiplication, particularly in Grade 5 and Grade 6. This represented 35% of the students in both Grade 5 and Grade 6. This second finding suggests that supporting students' progression to multiplicative strategies in Grade 5 and Grade 6 is an important curriculum focus.

Overall, our findings highlight the need for professional learning for primary teachers to deepen their understanding of the nature of the two prolonged progressions in the multiplication and division strategies domain identified by our research, the time points when these are likely to occur, and the pedagogies and curricula that advance students learning towards these two growth points. Further, it is likely that mathematics intervention approaches for students who experience these prolonged progressions need to focus on classroom-based approaches due to the large proportion of students who experience these difficult progressions.

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Mathematics Teacher Noticing: Adapting Practices in the Online Environment

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During the COVID-19 pandemic teachers and students spent a significant amount of time online in what came to be known as "emergency remote teaching" (Hodges et al., 2020). One of the aspects of online teaching and learning that was unknown was how teachers would adapt their regular classroom interactions with students—in this case primary school students—to an online environment. In a larger study investigating the cues that lead teachers to noticing moments of mathematical significance, interviews were conducted on two separate occasions with eight teacher participants. Interview 1 was conducted early Term 4 in 2020, when teachers were in the last weeks of remote learning before preparing for a return to the classroom. In this short communication, I will share a story of one teacher's experience adapting the practice of noticing and interacting with a student in the online environment. This presentation will illustrate the qualitative data analysis used to identify what was noticed, what led to the moment, how the information was interpreted, and what actions were actions taken. Analyses to be used to contrast these actions with the face-to-face classroom environment will be discussed.

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Application of the Legitimation Code Theory to the Draw a Mathematician and Draw a Mathematics Classroom Research

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Drawings have been widely used to elicit data from school students relating to their views about mathematics, mathematicians, teaching and learning of mathematics, and mathematics classroom experiences. What might be valued and emphasised in drawing-based research in mathematics education, and accordingly, what kind of knowledge is produced through student drawings are less known. This study aims to investigate these questions (Hatisaru, 2022). By using the *Legitimation Code Theory (LCT)* (Maton, 2014), a social realist framework, the study analyses the *codes*, or foci, in drawing-based research. It is situated within two primarily qualitative, drawing-based research: *Draw a Mathematician* and *Draw a Mathematics Classroom*. This study makes an original contribution to the literature by offering a conceptualisation that can be used to critically analyse the contribution of drawing-based research to the mathematics education field.

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Exploring the Incentive to Study a Higher-level Mathematics Course at Secondary School: A Western Australian Perspective

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A review of international literature highlights how mathematics course enrolments in the final years of secondary school are persistently low or declining for some time (Arnoux et al., 2009; Brown et al., 2008; Hogden et al., 2010; O'Meara et al., 2020). In an Australian context, this trend has also been reported and researched to better understand the many factors impacting on the elective study of mathematics. This replicated research project (see Hine, 2019) explored (i) the reasons why senior secondary students elected not to enrol in a higher-level mathematics course, and (ii) the extent to which students feel a bonus points initiative (i.e., a 10% bonus) introduced in 2017 is a sufficient incentive for students to enrol in higher-level courses. For this project, all Year 11 and Year 12 mathematics students within Western Australian schools (aged 17–18 years) were invited to participate in an anonymous, online survey comprised predominantly of qualitative items. 1633 students participated.

For the first aim of the project, students indicated several reasons influencing course enrolment decisions. These reasons included a general dissatisfaction towards mathematics, and the viability or attractiveness of other courses. The "dissatisfied" students described how higher-level courses were too challenging, unenjoyable and uninteresting, and that they generally lacked confidence to success in these courses. Comments concerning the viability or attractiveness of other courses revealed a tendency for students to "play the system" and maximise their final score with lower courses. At the same time, other students taking a lower-level mathematics course highlighted how lower-level courses require less time, effort and stress to complete successfully.

For the second aim, 47% of students felt the 10% bonus was sufficient, with 42% expressing that this incentive was insufficient and 11% were unsure. Consistent with findings from the first aim, students tended to anchor their responses in statements of course complexity, workload, and associated effort, time and stress. However, irrespective of which response students offered for Question 6 (viz., Yes, No, Unsure) they seemed unified in comments about the Mathematics Specialist (MAS) course. That is, students expressed unequivocally that the 10% was an insufficient incentive to attract enrolments for the MAS course, and that it should be greater.

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Understanding Mathematical Identities of Learners Who Chose Mathematical Literacy in High School After Participating in After-school Mathematics Clubs in Primary School

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This short paper presentation reports the mathematical identity of learners who participated in after-school mathematics clubs in primary school and chose to pursue Mathematical Literacy post the compulsory mathematics stage in high school. In South Africa, learners do Mathematics as a subject up to Grade 9 level, at which time they must choose whether or not to continue with Mathematics or to rather select Mathematical Literacy for Grades 10–12. In my doctoral study (Hokonya, 2021) I captured stories of the learners' mathematical journeys from primary to high school, written and narrated when they were in high school, several years after participating in after-school mathematics clubs. In this paper I present an analysis of three of the six learner stories to understand their sense in choosing to pursue Mathematical Literacy despite storying positive mathematical identities in their narratives. The paper is guided by the research question:

> *What are the mathematical learner identities of learners who chose to do Mathematical Literacy?*

It draws on Sfard and Prusak's (2005) conceptualisation of narrative identity as the reified, significant and endorse-able stories people tell about themselves or others tell about them. Furthermore, I use Wenger's (1998) Modes of Belonging framework to analyse the three learners' stories. The participants were purposely drawn from a group of high school learners who participated in after-school mathematics clubs in primary school. The findings show student reasoning about their decision to choose Mathematical Literacy post the compulsory stage, and the way in which their mathematical identities built in after-school clubs play a role in their positive dispositions towards the subject. Additionally, it was their envisaged future career choices and the amount of perceived difficulty of high school mathematics that influenced their decisions.

Acknowledgment. This work is based on research supported by the South African Research Chairs Initiative of the Department of Science and Technology and the National Research Foundation (Grant No. 74658).

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Teaching Demands for Mathematical Explorations

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Mathematical Exploration is a problem situation that enables investigatory work by students, allowing for multiple starting points, multiple trajectories and approaches, and the flexibility to work at different levels of formalisation. Facilitating such explorations in the classroom places multiple demands on teachers. Literature acknowledges the need for teachers to have mathematical content knowledge, knowledge of students and teaching and the sensitivity to respond to contingent moments in the course of a planned activity (Ball et al., 2008; Rowland et al., 2005). The nature of these demands in an exploratory context differs from that in a curricular context. For example, being based in a game or a puzzle, not necessarily rooted in the curriculum, the nature of content knowledge that comes into play is different compared to that in a curricular context. Moreover, mathematical practices and the structure of mathematics take centre stage in an exploratory context. The flexibility promoted by explorations makes it more likely to encounter student ideas that are different from familiar mathematics. Listening, validating, and responding to students can be a challenging task. In my presentation I intend to look at the teaching demands in an exploratory context and illustrate them through an exemplar exploration.

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Enhancing Mathematics Teachers' Pedagogical Content Knowledge in Communities of Practice

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Communities of Practice (CoPs) are defined as groups of people who share a common interest, concern, or a passion about a topic, and who aim to deepen their knowledge and expertise in this area by interacting on an ongoing basis (Wenger et al., 2002). As such, they provide "a promising theme in the professional development of teachers" (p. 352) and a good framework for examining teachers' learning (Patton & Parker, 2017). The application of CoP in educational research has suggested a positive impact for teacher professional learning (Goos, 2014). While there is much interest in the CoP approach to mathematics teacher professional learning (e.g., Jaworski, 2005), less is known about the impact of this approach on mathematics teachers' pedagogical content knowledge (PCK) in a specific content area such as algebra word problems.

In this study, the first author established a CoP initiative where a group of eight junior high school mathematics teachers in Ghana met regularly during six months (once a month) to explore pedagogical strategies intended to support students in solving algebraic word problems. Whilst word problems are applied within most domains of mathematics in the Ghanaian mathematics teaching syllabus (e.g., fractions, integers), it is particularly in the algebra strand that most students find them challenging. A major difficulty is in "translating word problems into mathematical equations" (West African Examinations Council, p. 311).

Within the mentioned CoP, the teachers shared ideas and reflected on their use of various adopted strategies with their students. In this short presentation we provide some examples to illustrate the influence of the CoP on participating teachers' PCK in relation to the use of visual representations such as bar models in solving algebra word problems.

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Spatialising the Pedagogy: Directions for Future Research

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The past ten years has seen a resurgence of research attempting to understand the role of spatial reasoning in mathematics education and learning (Woolcott et al., 2020; Lowrie et al., 2020). In the working paper commissioned by the Organisation for Economic Co-operation and Development (OECD), Newcombe (2017) identified. two possible strategies to harness spatial thinking for improved educational outcomes in the STEM area. The first strategy supported improving spatial reasoning skills through training interventions. This approach has indicated mostly good outcomes for mathematics and STEM learning (Hawes et al., 2022). The second strategy suggested by Newcombe (2017) was termed "spatialising the curriculum." This strategy suggested the use of a range of spatial tools in educational settings, such as spatial language; encoding and decoding spatial representations such as sketching, interpreting maps and diagrams; and notions of embodied cognition such as gesture and movement.

This presentation discusses these two approaches and proposes a new phrasing of "spatializing the pedagogy" instead of the "curriculum", since so much of the mathematics curriculum is already spatial. The word pedagogy is important because it centres the role of teachers and their practice and subsequently empowers teachers to take ownership of how they construct opportunities for engagement with mathematics. Through this approach, we consider what teachers can do to support students in understanding the mathematics curriculum, some of which may already be implicitly spatial.

Future research may consider the following questions: For what mathematics topics do students lack the prerequisite spatial reasoning skills (and what are they), and for what mathematics topics do they need spatial tools to help identify what the spatial content means? This presentation is a call for researchers interested in this intersection to identify pathways contrasting skills versus tools instead of historical approaches of seeing what specific mathematics content is correlated with which specific spatial skill.

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Towards Increasing Interest in Teaching School Mathematics as a Career

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Interest in teaching school mathematics has been waning globally. We report on Stage 1 of an interdisciplinary collaboration between the UK and Australia, with an aspirational aim to increase interest of undergraduate mathematics majors to consider teaching as a career. A small international seed grant tasked us to explore interest in a coordinated UK-Australia exchange or potential for a joint degree program in the future that could enable graduates to work in either country. This could be an inviting opportunity for students in the UK, for example, where some teacher education programs are only offered as a post-graduate degree. It may also facilitate education students in Australia to undertake a coordinated international exchange within restrictive course requirements of their teacher education program.

We quickly realised the importance of establishing a common understanding of the diverse contexts in which we each worked. The broad and exploratory nature of the aim created ambiguities that generated low-stakes dialogue. We report on exploratory boundary crossing activities that helped us to build a shared foundation and generative space in which to progress creatively (Akkerman & Bakker, 2011). Employing student interns in our own disciplines helped us hear students' perspectives and, through their experiences interacting with each other, highlighted boundary objects and challenges in the diverse spaces we were working. We offered seminars and surveyed students about career aspirations in teaching; the student interns generated and researched their own questions, held focus-group discussions with peers, and created narrative cases to illustrate their findings; we investigated opportunities for exchanges and navigated the opportunities and challenges for transferring teacher registrations between countries.

Our experiences have energised us to continue to progress our common interests together, and our experience has encouraged greater engagement within and beyond our home universities to broaden our networks.

Acknowledgements. This work was funded by the QUEX Institute to enable us to explore and report on these preliminary findings.

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Addressing the Learning Gap Through Talk in Mathematics Classrooms

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Effective use of talk in the classroom is "a powerful motor for the development of reasoning and the improvement of academic performance" (Mercer & Howe, 2012, p. 13). Everyday language, however, does not always mirror academic language, and it is the latter that is required to benefit from talk in the classroom. The challenges in academic language for mathematics are distinctive (Halliday, 1978). Students are required to use grammatical forms and phrases with which they may not be familiar (Schleppegrell, 2007). Talk that has clear, formal use of these new meanings and grammatical forms is needed to engage in the learning of mathematical concepts but the adjustment to these forms is particularly difficult for students from low socioeconomic status (SES) backgrounds who are less likely to be exposed to such language at home (Black, 2011). Consequently, low SES students become marginalised from the mathematical talk required to promote mathematical thinking (Prediger, 2019).

This is the premise that underpins our recently funded ARC Discovery project. In this short oral presentation, we set out the proposed cross-case comparison methodology based on a school-based participatory design. The project positions functional language as a key factor in determining the impact of a research-based intervention. Analysis of class discourse is based on the Scheme for Educational Dialogue Analysis (SEDA) (Hennessy et al., 2016) and on Halliday's systemic functional linguistics.

Evidence from the cross-case comparison will indicate relationships between a language-based pedagogy and achievement in mathematics and identify the role of functional language in learning mathematics. The aim is to build a compelling evidence base of the relationship between talk and learning in mathematics with the potential to break the cycle of socio educational disadvantage.

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Mathematics Homework and Intergenerational Reproduction of Confidence

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Teachers assign homework for many purposes including the reinforcement of skills taught in school, the completion of unfinished work, and to encourage children to read outside of school time. However, the complexity of what happens when schoolwork is "sent-home" cannot be overlooked. Clarke (2012) draws on Bourdieu (1993) to highlight the way in which intergenerational reproduction of educational inequities is manifested in approaches such as sending home schoolwork that a child could not do in school. In our current research, we focus on mathematics homework and the role of the mother or female caregiver. Mathematics anxiety, low confidence, and low self-efficacy are still being reported as more common among girls than boys (O'Keeffe et al., 2018). Girls continue to be less likely than their male counterparts to choose from the STEM school subjects and careers (Archer et al., 2013). The gender gap in terms of confidence in mathematical ability remains. Ganely and Lubienski (2016) note that the gender gap, though small, is persistent and warrants further exploration as the gap cannot be explained, the gap increases over time (whereas the literacy gender gap narrows over time), the initial small gap leads to *stark disparities* in mathematics related career pathways, and hence, adds to the issue of the gender pay gap.

Compounding this is the intergenerational reproduction of gender-based low confidence and self-efficacy. For example, O'Bryan et al. (2004) highlight mothers are more likely to communicate mathematics gender stereotypes and as a result, influence their children's self-efficacy**.** In this presentation, we will explore the way in which intergenerational reproduction of low mathematical confidence is supported by the practice of sending school mathematics work home.

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Learners' Affective Field During the COVID-19 Pandemic: Predicting Perceptions of Impact on Learning

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Universities around the world were affected by the COVID-19 pandemic-driven lockdowns. The unprecedented circumstances left educators with little understanding on how this global disruption and shift to online learning environments would impact students' learning. This study investigated how student achievement, achievement-related affect, and perceived wellbeing contributed to predicting how students perceived the lockdown and consequential shift online to have impacted their learning. The overarching theoretical framework of a student's *affective field* (Schindler & Bakker, 2020) was used to combine key psychoeducational variables to be studied simultaneously—an approach that has been advocated for extensively in recent mathematics education literature. For example, Schindler and Bakker (2020) highlighted limitations of research that tended to investigate learners' motivation, anxiety, and other affective characteristics separately by focusing only on one theoretical construct. In this study, we respond to this concern by considering constructs such as self-efficacy, achievement emotions, and student wellbeing simultaneously in conjunction with other variables. The data in this study included survey responses and assessment results from a second-year, tertiary mathematics course (N = 208). The analysis showed that, despite returning to in-person teaching after community-transmission of the virus was eliminated, students reported an increased impact of the effects of the disruption on both their learning and wellbeing at the end of semester than during the lockdown itself. Hierarchical multiple regression demonstrated that gender; prior achievement; performance on frequent, low-stakes assessment; exam-related self-efficacy; and exam-related hope all made independent, significant contributions to explaining students' perceived learning impact. When controlling for student achievement and achievement-related affect, the impact to students' perceived wellbeing still made a significant and substantial contribution to the impact on their learning. The findings provide motivation to investigate whether addressing student achievement-related affect can mitigate the effects of major life disruptions while studying. We also suggest that frequent, low-stakes assessment (Evans et al., 2021; Riegel & Evans, 2021) can be used to help identify students during a semester who are more likely to report a greater negative impact on their learning. Finally, we conclude that student wellbeing is paramount to how students perceive their own learning, even when controlling for actual measures of and about their achievement.

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Working On and With Verbal, Visual and Gestured Confluences in Mathematical Meaning-making

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At the start of our research journey our focus was clear: teachers' use of language in supporting learners' developing mathematical understandings. This was in contexts where English was the official language of learning and teaching, albeit that the majority of the learners we worked with were isiXhosa speakers. isiXhosa is the first language for close on 80% of people in our Eastern Cape Province. Learner talk, as we had observed in the classrooms and after-school mathematics clubs, was limited, highly formulaic, and lacking in the exploratory talk likely to conduce to learners engaging in productive "interthinking" (Mercer, 2000) about mathematical ideas.

Awareness of the severe constraints faced both by teachers and by learners in situations where there is limited proficiency in the language of learning and teaching moved our research journey to the point where we recognised the need to give more explicit and systematic attention to the multimodal aspects of communication of mathematical ideas. While not the focus of this presentation, our work unfolds alongside our commitment to advocating for increased bi-/multi-lingualism for South Africa's mathematics classrooms.

In our presentation we will share research insights from our analyses of multimodality at work in two separate fora. In both fora the second author has been involved in the design of activities and support materials, though the attention to multimodal forms of communication has been somewhat intuitive, rather than an explicit goal. In the presentation we intend focusing on a retrospective analysis of confluences of multimodal forms of communication in supporting mathematical sense-making. We also demonstrate the power, for both teaching and learning, of key representations such as the empty number line. We will begin with brief mention of our analysis of video material from an after-school mathematics club session. We will show how the club facilitator (thinking on her feet and in response to limited verbal responses from learners) scaffolded club members' mathematical understanding through the communicative power of physical objects, visual representations, inscriptions, and gesturing. Insights from this club session analysis led us further on our multimodal journey. Recently we began analysis of the synergistic power of scripted lesson plans alongside video-recordings (accessed through QR codes) for modelling mental mathematics strategies. We will discuss a brief (< 2 minutes) videorecording of a facilitator's hand, drawing and gesturing a "mental maths" problem (35 + 16) on an empty number line using "bridging through ten" and "jump" strategy. These materials form part of the South African Mental Starters Assessment Project currently being rolled out by the Department of Basic Education across public schools, and now being extended also to teacher education institutions (Graven & Venkat, 2021).

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The Use of iPads in the Early Years: Investigating the Effectiveness of Apps in Mathematics Learning

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Touch-screen tablets such as iPads are a type of mobile technology coined for their ability to be portable and are deemed useful to enhance mathematics education. Touch-screen tablets provide the ability for users to interact directly on screens through stylus or pens, or simply with fingers (Baccaglini-Frank, 2018). Calder et al. (2018) explain the many affordances of touch screen tablets to enhance mathematical learning as users can utilise the inbuilt camera, video, or audio tools to capture genuine data. The way in which children learn can also be enhanced when utilising touch-screen technology via digital applications (apps), as they learn by trial-and-error along with the notion of repetition. To date, there has been little research to determine whether mathematics apps add value to mathematical understanding for early years children.

In this short communication, I will discuss my doctoral research that aims to investigate how the use of digital technologies, in the form of iPad apps, impact the learning and teaching of mathematics in children in a Foundation and Year 1 context. A case study approach was utilised involving three classroom cohorts in one primary school. Qualitative data were collected including recordings of students' app use, classroom observations, and interviews with children, teachers, and key administrative personnel.

Preliminary findings from my research indicate that the mathematically specific apps adopted in mathematics lessons were predominantly utilised to consolidate learning, and instances of deeper mathematical learning occurred with the use of generic apps in mathematics lessons. The recording function of the app enabled children to demonstrate their understanding of key concepts by self-recording verbal explanations in conjunction to the use of concrete materials. The utilisation of self-recordings with early years children is particularly innovative and provides new opportunities for children to demonstrate their mathematical understanding at their own pace without handwriting, the typical way children exhibit knowledge.

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Beyond the Arithmetic Operation: How an Equal Sign is Introduced in the Chinese Classroom

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The relational understanding of the equal sign is an essential foundation for algebra (Blanton et al., 2015; Carpenter et al., 2003; Kieran, 2004). In recent decades, researchers (e.g., Carpenter et al., 2003; Kieran, 1981; McNeil, Fyfe, & Dunwiddie, 2015) have documented that many students have narrow conception of the equal sign, which is viewing it as a one-directional "show result" symbol. In contrast, Li et al. (2008) evidenced that Chinese primary students commonly understood the equal sign relationally. This report showcases how an equal sign is introduced in Chinese school. Four features of the Chinese approach will be discussed, which are a) introducing the equal sign before traditional arithmetic operations, b) an instructional sequence which is in line with the RME theory, c) the way of drawing an equal sign, d) the emphasis of "two-sides" sense. This research contributes to providing an effective starting point of developing students' relational understanding of the equal sign, which could be adapted to align with the *Australian Curriculum: Mathematics.*

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Is it "off-task"? Non-game Interaction During Game-based Mathematics Learning

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Using a game-based learning activity to promote students' engagement in mathematics learning is broadly discussed in the literature (e.g., Jorgensen & Lowrie, 2012). Research stresses students' interaction as part of game progression as an engaging factor in student learning (Ke, 2008). However, interactions such as students' idle chatter and banter, which are not game related per se, might be considered as "off-task" behaviours from a learning perspective (Kim & Ho, 2018). Therefore, these non-game interactions might undermine students' engagement in the game-based learning activity, even though, from the game perspective, non-game interaction is an integral part of gameplay (Gee, 2008). This short communication will highlight the importance of the "authentic" game experience in game-based learning and report that the non-game interaction contributes to an "authentic" game experience for students, whereby an optimistic classroom atmosphere could be enhanced. Thus, students are more likely to possess positive emotions towards learning and likely to overcome challenges experienced during the activity. An atmosphere of this kind is likely to flatten the teacher-student relationship towards "co-partnership", smoothing the teachers' facilitation of the game.

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Action Learning as a Tool for Teachers: A Case of Promoting Self-regulated Learning Pedagogy to Secondary Mathematics and Science Teachers

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The majority of existing studies on student self-regulation conclude that students and teachers are unsuccessful in either practising or teaching self-regulation. The problems and gaps identified in the literature were that teachers lacked professional development and clarity in self-regulation pedagogy and had not made a significant contribution to the conversation. This study investigated the features and impact of Action Learning as a professional development tool for secondary mathematics and science teachers to promote self-regulation pedagogy. The inquiry included a creative exploration of teaching methods to encourage self-regulation, the findings of which can help to foster positive changes to its systematic development and implementation.

Action learning was used both as a professional development tool for secondary classroom teachers and as a qualitative data collection method. Action learning was supplemented by individual teacher interviews, student work samples and an anonymous teacher survey.

The principal research question in this study was:

What effect does action learning, as a professional development tool, have on secondary school mathematics and science classroom teachers' promotion of student self-regulation?

Action learning—a sustained and creative group-based research approach—was adopted for its regular provision of optimal opportunities. It was chosen not just for its rigorous analysis but also for the creation, trial, and review of self-regulation teaching methods. Action learning fostered a questioning culture where participants only made statements in response to a question. Action learning also enabled participants to gain a stronger understanding of the challenge at hand before proposing solutions. This contended with the natural desire to quickly devise solutions before the problem was fully understood.

By using action learning mathematics and science teachers successfully designed, tracked and developed systematic in-service methodologies for teaching self-regulation to students. Teachers' analyses resulted in fresh ideas and a novel theoretical framework applicable to the professional development of teachers in student self-regulation methods. The new theoretical framework provided practical and theoretical guidance to implement professional development programs for teachers of science and mathematics.

The two core findings in this study were *teacher learning* and *context-specific pedagogy*. The first finding revealed the effectiveness of action learning as a professional development tool to promote teacher learning. The second finding was the discovery of context-specific pedagogy by incorporating classroom teachings into a reflective method of inquiry to discover solutions relevant to their students.

This study was the first time that action learning was used both as a professional development tool for secondary classroom teachers and as a qualitative data collection method in researching self-regulation pedagogy.

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Designing Inquiry Tasks in Primary Mathematics

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Inquiry tasks are important to implement contemporary mathematics pedagogies aimed at transforming the teaching and learning of mathematics. This presentation reports on a small-scale study that investigated the nature of a unique suite of inquiry tasks designed to promote a spirit of inquiry in primary school mathematics. The tasks were developed as part of the *reSolve: Mathematics by Inquiry* project funded by the Australian government. A qualitative research method was employed to analyse the primary school tasks based on four stages of an instructional design model. The Task Analysis Guide (Stein et al., 1996; Smith et al., 1998) was then used to assess the cognitive demand of each instructional stage. Findings indicated how shifts in the cognitive demands were used in the various stages of the tasks and outlined implications of how teachers could adapt such tasks to engage all students.

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Developing Primary Teachers' Teaching Practices Through Communities of Inquiry

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Mathematical proficiency (Kilpatrick et al., 2011) is widely accepted as a key goal of school mathematics. Teaching for proficiency enables students to engage in the processes of mathematics such as mathematical thinking and problem solving (Takahashi, 2021). *Ambitious teaching* (Anthony et al., 2015; Lampert et al., 2010) consisting of high-quality instructional practices is necessary for students to achieve mathematical proficiency. To meet the challenges and complexities of ambitious teaching, teachers require a vision for high-quality instruction and sustained professional learning (Cobb et al., 2018). In this presentation, we give an overview of a research project that incorporates a vision of high-quality instruction, the pedagogical framework of the *reSolve* approach to conducting mathematical inquires, associated curriculum materials, and sustained professional learning through school-based *Communities of Inquiry* (CoI). The intent of the study is to determine the impact of the *reSolve* approach and curriculum materials on the professional learning, thinking and practices of primary school mathematics teachers.

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Learning Mathematics as a Child of a So-called "Tiger Mother"

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The presentation is about a mother's excessive involvement in mathematics education of her son. The data are collected using semi-structured interviews of a mother and her son. This Asian mother is originally from Vietnam and now resides in Australia. She acts as a tiger-mother involving in her son's mathematics education while the son finds it exasperating. The qualitative analysis of the study shows that both mother and son have a hard time because of the aspirations, expectations, and demands of the immigrant mother.

Parental involvement in their children's education has captivated the attention of the world for some time. Asian parents, for example, are often reported to spend time each day in monitoring the academic activities of their children. In her controversial memoir entitled, *Battle Hymn of the Tiger Mother*, which is one of the 2011 bestselling books, Amy Chua depicted a Chinese model of parenting. Main features of "tiger parenting" are harsh parental control and extreme demands for excellence from children in both behaviour and academic performance. The term "tiger mother" self-proclaimed by Chua is sometimes used to describe an authoritarian parenting style in which parents give their children few choices and seldom ask children for opinions. It is not only Chinese mothers who act as tiger mothers, for example, it seems that some non-Chinese parents from other Asian countries such as Korea, Vietnam, India, Bangladesh, and Sri Lanka have similar mindsets. The well-prepared offspring of these tiger mothers seem to be outperforming non-Asian counterparts at schools where both Asian and non-Asian ethnic background children study together. However, some rigorous methods of Asian parenting contradict with the child-rearing actions of non-Asian parents.

Even though children may listen to advice of their parents, children do not expect ongoing advice. This implies that parents should know their limits in advising children, so that they may maintain a good relationship with them and encourage them effectively. If the outcome seems negative, parents need to step back and consider different approaches to be involved with their adolescent children. Therefore, in comparing Asian and non–Asian students' performances in mathematics with respect to parental involvement, ethnic or cultural differences can be considered as an important factor. Some parents become too involved in their children's education, which seems to put pressure on parents and children, creating negative effects on both groups.

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The Influence of Traditional Mathematics Teaching and Assessment on the Pedagogical Use of Technology

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There has been a traditional way to teach secondary mathematics, called *chalk and talk*. This method, which involves talking aloud to students while writing the mathematical narrative on a display device such as a whiteboard (Artemeva & Fox, 2011), has been used extensively in Mathematics classes. However, in the words of one participant, chalk and talk, "does not leave much room for the integration of technology." In addition, assessment has mostly been summative pen and paper examination style tasks, utilising little digital technology. The present study was completed in the context of a wider doctoral study, that used the TPACK framework and Pedagogical Reasoning and Action to determine factors that influence the pedagogical use of technology. One of the principal factors found to influence technology integration was traditional teaching and examination style assessment.

A case study was conducted for each of the four participants who teach predominantly Mathematics in secondary schools. Each participant was interviewed to ascertain the factors that influenced their pedagogical use of technology. In addition, each participant had several lessons observed and their planning documentation examined to further determine the factors that influenced their pedagogical use of technology. Data were analysed using thematic analysis, which helped determine the factors that most affected technology integration.

Participants utilised the chalk and talk method in 80% of observed lessons and more than 90% of formal assessment items were pen and paper examinations. For these reasons, three participants claimed that students believed that real learning was only taking place when chalk and talk lessons were undertaken, due to the need to complete pen and paper problems in formal assessment items. Therefore, each participant claimed that the chalk and talk method of teaching was highly influential in their pedagogical choices as it reduced required planning time; was expected and welcomed by students and parents; and was encouraged by colleagues. However, during chalk and talk lessons, technology was rarely used and when used, it was only used to support chalk and talk methods, such as viewing the textbook.

Recent developments in Mathematics syllabus documentation have encouraged technology integration, with an aim of the Victorian Further Mathematics syllabus being for students to "use technology effectively as a tool for working mathematically" (Victorian Assessment and Reporting Authority, 2016). However, to allow for increased technology integration, the findings of this study suggest that there is a need to tackle the entrenched methods of teaching and assessment that are prevalent in secondary Mathematics education.

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