RESEARCH PAPER

How Big is a Leaf? Using Cognitive Tuning to Explore a Teacher's Communication Processes to Elicit Children's Emerging Ideas About Data

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The intangible concept of data, as part of statistical literacy, can be complex for young children to grasp. Inquiry as a pedagogy has potential for supporting student development of statistical literacy as the investigation process is driven by the inquiry question. The aim of this paper is to gain insight into how a teacher's communication processes with her students supported their emerging understandings about the abstract concept of data. In this exploratory case study, we present data from a Year 4 classroom (age 9) in a guided mathematical inquiry within the STEM context of agricultural science. The inquiry question the students addressed was, "How big is a leaf?" The inquiry focused on linking data to the real-life context the data represented.

The research question guiding our research was,

> *How can a teacher's communication processes support students' emerging mathematical understandings about measuring area and the abstract concept of data?*

Area as an attribute to measure was a new concept for student exploration. Students' emerging mathematical understandings about data included the abstract idea that numbers used in a statistical investigation represent, data which in this instance, depicted the area of leaves and ultimately, the leaves they investigated.

The study took place in a Year 4 classroom (n = 25) in a metropolitan school in Queensland. Student ideas about data were analysed using the theoretical framework of cognitive tuning to illustrate how one class built common understandings about the concepts of area and data. The focus was on how the teacher supported the processes of *normalisation*, *conformity* and *innovation* (Wit, 2019). In this paper, we look closely at the teacher's communicative processes around the *normalisation* of the problem task, and *conformity* of common understandings about area and data. Teacher dialogue will be used to explore how the processes of cognitive tuning made the connections more visible between the teacher's support and the students' emerging ideas.

Background

In classroom data modelling, data and its attributes are defined and negotiated to suit a purpose or to address the context in which the problem is set (Leavy & Hourigan, 2018). For example, the mathematical concepts of area (as an attribute to measure) and data (for investigation and interpretation) in a biological crop model, are very closely linked. Similar to Meyer and Land's (2005) description of a threshold concept, in a biological model the concepts of area and data are tied together (integrative) and understanding the first would change the way a student looked at the latter (transformative). Threshold concepts would potentially be troublesome for students but once a relationship between the mathematical concepts was established, it would be difficult to unlearn.

Mathematical challenge and problem solving in the classroom can present students with uncertainty and even anxiety (Buckley & Sullivan, 2021). However, the *Australian*

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Curriculum: Mathematics emphasised the proficiency of problem solving and encouraged teachers helping students through "active participation in challenging and engaging experiences" as part of the rationale for the curriculum (Australian Curriculum, Assessment and Reporting Authority. [ACARA], 2014). A socio-cultural perspective is often acknowledged as underpinning inquiry pedagogy and socio-mathematical norms of an inquiry classroom, which differs to the traditional classroom sense of individual success and classroom teacher as "sage", and include the embrace and normalisation of intellectual challenge (Goos, 2004; Hunter & Hunter, 2018; Makar & Fielding-Wells, 2018). An inquiry classroom is characterised by collaborative groupwork and a knowledge-building culture. This in turn, can foster positive mindsets and strengthen intellectual engagement (Australian Government Department of Education, Skills and Employment, 2022; Confrey, 1995; Fielding-Wells et al., 2017; Goos, 2004).

Key in group problem solving situations is the communication between its members. Collaboration is part of the problem-solving approach used in inquiry and centres on communication between students to develop thinking together (Allmond et al., 2010). Research articulates the argumentation processes students encounter in inquiry as they defend their mathematical findings with evidence (Fielding-Wells, 2013). The teacher's role in inquiry settings, to orchestrate classroom conversations, is critical to facilitate problem solving without giving away the solution (Fry & Hillman, 2018).

Theoretical Framework: Cognitive Tuning

Cognitive tuning was used by Wit (2019) to explain the communicative processes involved in group settings when reaching a common understanding, or when making progress towards a socially anchored representation of a task. Although Wit applied the framework to an adult setting, cognitive tuning encompasses modalities, which seem to align with teacher goals more generally of pursuing commonality in their students' thinking when teaching mathematical concepts. The basic modalities of *normalisation*, *conformity* and *innovation*, are offered to build a commonly shared frame of reference in group settings and will be used to gain insight into the ways one teacher scaffolded student thinking in one classroom, to address the research question,

> *How can a teacher's communication processes support students' emerging understandings about measuring area and the abstract concept of data?*

Normalisation describes the moment when the participants (problem solvers in this instance) do not yet have a shared interpretation of the task and provides time for a group to build a normative frame of reference in which to work together, for creating solutions *to* the problem. Wit's (2019) framework acknowledges the influence individual prior experience presents to a group setting and the cognitive conflict participants may encounter. A common understanding of a task can lead to a common response. *Conformity* describes the communicative processes when a less-experienced group member lacks confidence and faces conformity pressures from a majority, or a common belief. In a classroom context, this can be likened to a teacher's efforts to reach conformity about mathematical ideas or learning goals, and the pressures a teacher can exert to guide and scaffold student learning. *Innovation* is when a naïve individual resists pressure from the majority and challenges the group norm. Although consistency of thinking is often a classroom goal for teachers, an individual's line of reasoning (which may be in the minority) may also have innovative impact by introducing a new frame of reference. To have impact, the new frame of reference needs to be publicly supported to build less-threatening environments that encourage innovation.

In this exploratory study, the authors are aware of the importance of building a shared frame of reference about the statistical concept of data specifically, related closely in this instance to the concept of area. In this paper we use cognitive tuning to look closely at the ways students were supported by the teacher, to develop their thinking about the concepts of area and data, framed specifically by the processes of *normalisation* and *conformity*. The cognitive tuning framework may also provide insights into innovations in problem solving, developed by students, that are shared.

Research Design

Setting and Participants

There were 25 students in this classroom, situated in a metropolitan public school in Australia. With slightly more boys than girls, all students had their own learning needs, special needs and achievement levels—typical of a Year 4 class in this setting. The inquiry took place over nine lessons (1–1.5 hours each), across two weeks. This was the first mathematical inquiry the students had encountered although they were familiar with solving problems posed by open questions. The first author, also the joint classroom teacher working in a shared partnership at the time, worked with her classroom teaching partner to plan the inquiry based on conversations with their STEM professional; an agricultural scientist from Commonwealth Scientific and Industrial Research Organisation (CSIRO). The agricultural scientist and the teachers had met three times (once face-to-face and twice over the phone) to discuss the key mathematical concepts involved in the modelling agricultural scientists do to predict crop size.

Context

The mathematical problem of "How big is a leaf?" arose when the classroom teacher and the teacher-researcher (classroom teacher partners) worked with an agricultural scientist to learn how mathematical modelling could be used to predict crop production. The relationship between the length and/or width of a leaf and its area was an important first step in determining how much energy a plant produces, using sunlight. A passionfruit vine on the school grounds provided the model of a biological system to explore (passionfruit) and this presented an authentic STEM context in which Year 4 students could informally calculate the area of leaves and irregular shapes. However, the classroom teacher and teacher-researcher partner felt that the relationship between the length and/or width of a leaf and its area was important and difficult for the students to consider. Surface area was a topic that had not been explored in the classroom prior to the inquiry and it was not initially clear to the students, which part of the leaf might absorb sunlight for photosynthesis. Student strategies for solving the problem were important as the focus for learning, as outlined in the curriculum, compare the areas of regular and irregular shapes by informal means (ACARA, 2014). The mathematical focus also included making comparisons between objects involving familiar metric units. Once area was established to determine size, this information would constitute data for investigating the kinds of relationships (between data points) that would be useful to an agricultural scientist in the field.

The strand devoted to learning statistics in the *Australian Curriculum: Mathematics* (ACARA, 2014), brings focus to data representation and interpretation from an early age (Foundation Year) with students answering yes/no questions to collect information. Understanding that data are information is the basis for considering data, and (part of) a Data Science K–10 Big Idea to Inform Teaching Practice (Bargagliotti et al., 2020; YouCubed, 2022). Little is reported on how primary-aged children perceive the concept of data and it is a wonder that children can accept the answer "information" in response.

Data Collection and Analysis

All nine classroom sessions were video recorded by the teacher-researcher and field notes were taken to record events. Powell et al. (2003) present a model for analysing video data, used by researchers to inquire into students' mathematical activity. The approach supported the process of analysis of the classroom videos in this study, offering a glimpse into the non-linear learning progression of students, anticipated in inquiry settings. First the authors viewed the video sessions to familiarise themselves with the lessons and to briefly describe the content when the term "data" was used in discussion between the teacher and students, and between students. Time-coded descriptions allowed the researchers to review and identify critical events—where discourse demonstrated change in understanding about data. This was crucial to consider in terms of Wit's (2019) cognitive tuning processes of normalisation and conformity. These critical events were transcribed for closer analysis and annotated for themes and patterns, enabling the researchers to make sense of the data and to construct a storyline. The narrative below is composed of the storyline revealed through analysis of the classroom data.

Results

Building Shared Interpretations of the Measurement Attribute of Area

Prior to Year 4, Australian students' experiences with measuring area involve making direct physical comparisons. There were two mathematical aims of this inquiry; (1) to support students to compare areas of regular and irregular shapes by informal means (ACARA, 2014); and (2) students needed to look for, identify and describe patterns in surface area measurements in order to make conjectures or hypotheses about ways to predict leaf size, as part of the mathematical modelling used by agricultural scientists. When asked in the first session to consider how much sunlight a leaf 'gets' (photosynthesis), the students appeared unsure about how to approach this. The teacher needed her students to have a shared interpretation of the task which required knowing what part of the leaf constituted the surface area. She guided her students to consider this through their initial explorations. Students brainstormed ways to measure how *big* their leaves were, and the teacher recorded suggestions on the board: length, width, mass (weight), thickness (depth), how much it grows, how much light it reflects and how much sunlight it takes in. Based on this, the students devised ways to measure their leaves taking out rulers to do so. However, curved edges made this difficult.

> Liam: They're curved.
>
> Kyrie: Well that makes it really hard
>
> Liam: Wait! (he picks up a 30cm ruler)
>
> Kyrie: Have you got something curved in your desk? A piece of paper or something?
>
> (They both shake their heads and rip a small corner from a page in their book. They measure the piece of paper and then line it up with the edge of their leaf.)
>
> Kyrie: One centimetre. One centimetre. One centimetre.

The teacher brought the class together in a Checkpoint, to share plans for measuring leaves but it was clear that no table groups had a plan yet. The students returned to this task.

> Isla: Measure the stem from here to here (She points to the bottom of the stem to the top of the leaf [furthest point]. Her neighbour explains the importance of measuring across the leaf also (the width)).
>
> Liam: Don't we have to measure the whole thing? (Moves his hand across the surface of his tri-lobed leaf.) I think we should measure that, and that, and that (Points from the centre of his leaf along each lobe/leaf section).

Isla rephrased what Liam said and pointed to the sections of her leaf to show her agreement.

Liam: But how can we find around this bit? (Runs his finger along the edge)

Isla: I don't know.

Helena: I've got this idea if we just measure across the middle. We should measure it through the middle.

Isla: Yeah but if you want to know there to there then you have to measure it that way (she points from the bottom of the stem along to the end of a leaf section/lobe). Because then it's not the actual leaf.

The students continued to explore this for a further twenty-five minutes (with a lunch break in between). One student decided to cover her leaf with a round container, whereas all other students struggled with rulers to measure different dimensions of their leaves. This student was the only person to try this approach and although she explained her idea to them, peers at her table group relied on their own approaches. This was an idea the classroom teacher wanted to encourage, and she focused attention on this as a successful approach.

The teacher asked if students in the class had ever tried to cover something to measure area and one student recalled a previous experience about a little cube – putting little cubes on something. Bringing focus to the covering method now validated the approach, and other students started to cover their own leaves in different objects to measure area. Marbles turned out not to be useful to cover a leaf as they did not tessellate, but shape tiles proved popular. Triangles had "pointy bits" like the leaves and combined with a rhombus shape, could cover most of a passionfruit vine leaf.

In the following lesson, the teacher reminded students of the importance of standard units of measurement. Two students had thought overnight about using grid paper to cover their leaves and the teacher built on this idea by introducing students to the 'square centimetre.' Using grid paper to measure surface area became the new focus of the inquiry.

The teacher continued to guide her students through repeated opportunities to measure the sizes of their leaves using grid paper displaying square centimetres. Two students introduced the idea of drawing a box around their leaf and this was pursued by all students in the class. By the fourth lesson, students were able to express the area of their leaf as a fraction depicting the relationship between the area of the leaf and the area of a box drawn around the leaf. Now that the students were aware of the mathematical focus on area and what part of the leaf this related to, the teacher turned the focus to the statistics notion that the measurements they had collected and recorded (on the board) constituted "data."

Building Shared Interpretations of the Statistical Concept of Data

It was important to compare leaves that were very different in size, proportionally. A number of data points were also required before making conjectures about how to accurately predict leaf size. Lesson six began with the first acknowledgment by the teacher, that the solution involved a focus on the statistical concept of data. In fact, their investigation by now had resulted in a "heap of data" to consider. The students were encouraged to look at the data and to share what they noticed (Figure 1).

Teacher: When we look at data we try and make sense of what the data means. We look for patterns. We look for things they have in common or things that are really different, things that really stand out—things that are similar.

The students were encouraged to consider what "this" (Figure 1, the data) actually meant.

Rafael: They're representing how big our leaves are.

Teacher: (agrees) Precisely what is it representing?

Akayla: How much of the box is taken up by the leaf.

One student described the pink fractions as easier to compare because they were "not as big" and the teacher prompted for further response (pictured in Figure 1).

Kyrie: Because the pink ones have the same denominator.

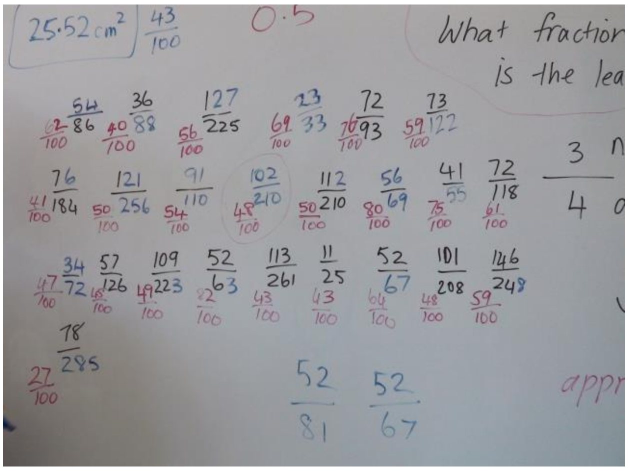

Figure 1. Classroom board depicting "*heaps* of data."

The teacher was pleased with this result as it was a mathematical response – the fractions written in pink (Figure 1) all included hundredths as the denominator and this made them easier to compare than their original fractions, such as 76/184 and 146/248 for example. Two leaves were represented by the fraction 43/100 and the owners of the leaves stood up to show their leaves to the class. Surprisingly, one leaf was very small in size and the other leaf was larger than the teacher's hand! The class continued to explore the data and the teacher made multiple connections between the fractions recorded on the board and the leaves they represented.

By the seventh session, the teacher was still dissatisfied with the messy data on the board and asked the students what they could do. Suggestions by students were recorded on the board: bin it, sort it out, arrange it, organise it, and group it. Students sorted the data in different ways but appeared unsure of how to make a statement about what the data showed them. They started the next lesson looking at one student's workbook—selected by the teacher—projected on the screen in front of the class. The student had made the statement, "This data shows us that a lot of people have their decimal fractions between 40 and 48 [hundredths]" and the class considered this. However, the teacher explained how the statement was not specific nor a comparison. Other students shared what they noticed about the data now it was sorted into columns. The teacher encouraged all students to predict what they would expect to find if they placed all the leaves together, prompting them to think what the leaves might have in common. What might students expect to see?

Sandy: A big fraction?

Teacher: No, I mean if I get all these leaves in a group, what do you think those leaves are going to look like?

Sandy: I think they will be big leaves.

The students were still unsure of what the teacher meant and so she took a leaf that matched a data point in the 40s (hundredths). She pointed to another data points and asked for the leaf it represented to physically compare the two leaves. The students made predictions about what they might see before the leaves were placed in groups that matched the sorted data.

Eventually, students were encouraged to connect the leaves with the data on the board through sorting to reflect ways the data were organised. Students recorded statements to demonstrate their ideas about the relationship between the area of the leaf to the area of the box around it. This is one example from Naomi:

> Most boxes are 40% to 60% taken up by the leaf. Then you can fill the box up 40% to 60% and that could be close to the area of the leaf. But with a mono leaf, it will take up between 70% and 80%.

Discussion

Inquiry presented the pedagogical approach in this classroom, to solving the open-ended and complex problem, "How big is a leaf?" *Normalisation* is the initial phase in cognitive tuning (Wit, 2019) and in this classroom, is used to describe the initial phase of problem-solving when the teacher supported her students to build a shared understanding of the task, or a normative frame of reference about the requirements of the problem: finding ways to determine the size of a leaf in relation to the amount of sunlight it received. This would support the mathematical focus on data and support students to develop understandings about area as an attribute to measure, as a threshold concept. Building a normative frame of reference about the problem being posed, assisted the class with moving forward as a whole group. *Normalisation* encompasses the problem-solving aspects of moving from not knowing to knowing and involves communication and mathematical reasoning between students and the teacher to reach the normative frame of reference, or shared interpretation of the task. The results depicting *Building shared interpretations of the measurement attribute of area* are lengthy and involve much student talk. However, teacher moves such as allowing students time to think and struggle with not knowing, validating particular approaches, introducing measurement conventions (square centimetres), and allowing multiple opportunities to measure, communicated the importance of the shared frame of reference about the mathematical concept of area.

Beliefs that are common to a majority in a group situation, can influence the beliefs of those with less confidence. These pressures constitute the idea of *conformity* (Wit (2019). Although inquiry places students at the centre of the problem-solving process, conformity pressures by the teacher were reflected in the valuing of specific student approaches that related to surface area. The struggle for students to figure out which part of the leaf to measure was important and only one student approached the problem with the idea to cover the surface of the leaf. In the results for *Building shared interpretations of the statistical concept of data*, attention turned to student data illustrating leaf measurements which students were required to interpret to identify patterns. Conformity in a group can be difficult to achieve and teachers are certainly aware of the difficulties faced in supporting all students to demonstrate particular learning goals. Here, the teacher made repeated connections between data points and the real-life objects they represented and emphasised students' particular ways of sorting data. Attention was drawn to patterns or clusters in the students' data that could be interpreted, and students could begin to perceive relationships between the size of a leaf to the size of the box around it.

Cognitive tuning places importance on providing opportunities for student discussion as well as listening to ideas shared. The openness of GMI supported *innovation:* one student showed innovation in their approach to measuring area by covering her leaf with a lunchbox container. This new way of thinking had innovative impact as it proved a way forward in problem solving. The mathematics classroom should be a place which supports innovation and fosters positive dispositions towards mathematics. This responsibility lies with the classroom teacher in establishing a safe environment to do so.

The focus of this paper was on how the classroom teacher supported her students to develop understandings about area and data. Cognitive tuning presented one way to describe the process of supporting students through stages of not knowing to knowing, recognising the role of the teacher to develop a knowledge building classroom culture. This process may prove useful for describing the processes of reaching shared interpretations of a problem-solving task. It acknowledges how knowledgeable others, in this instance the classroom teacher, can apply pressure within a safe learning environment, to conform or guide student solution processes, while allowing for innovation in student approaches.

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