

Identities of Mathematics Teacher Educators in a “Hybrid” Mathematics and Mathematics Education Department

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Prospective secondary mathematics teachers in Australia are typically taught by mathematics educators and mathematicians who work in different faculties and seldom collaborate—a situation that can lead to conflicting views about how to teach mathematics. This paper reports on findings from semi-structured interviews with three mathematics teacher educators in a hybrid mathematics/mathematics education department. Valsiner’s zone theory is used to analyse how their beliefs, institutional context and professional learning opportunities shape their identities as MTEs. Findings reveal that the MTEs had developed similar beliefs within supportive institutional context but drew on different sources of professional learning.

There is increasing scrutiny and regulatory pressures on initial teacher education (ITE) in Australia, evidenced by two recent reviews (Australian Government, 2022; TEMAG, 2015). Additionally, the declining performance of Australian students in mathematical literacy in the Programme for International Student Assessment (PISA) has raised concerns about the apparent failure of education systems to produce mathematically capable students (Thomson et al., 2019). Rather than taking a deficit view of mathematics teacher educators (MTEs) who prepare future teachers of mathematics as being part of the *problem* (Dinham, 2013), we regard these developments as acknowledging the vital role of teacher educators in preparing high quality teachers.

In Australian secondary ITE programs, MTEs include both mathematics educators and mathematicians who typically work in different faculties or departments and seldom collaborate. Within mathematics departments, all Australian students learn mathematics together with no acknowledgement of the differing needs of future teachers, mathematicians, or engineers. Leikin et al. (2017) found that in this situation mathematicians focussed on preparing professional mathematicians rather than teachers. Pre-service teachers (PSTs) have described the tensions between how they were taught mathematics at university and how they were taught to teach mathematics (Marshman, 2021), saying that mathematicians tended to present them with the mathematics, and it was up to them how they engaged with it. This presents the challenge of how to facilitate integration of mathematical content knowledge and pedagogical content knowledge when preparing future secondary mathematics teachers.

Innovative models of ITE exist in Australia that not only foster collaborations between mathematicians and mathematics educators but also productively integrate the content and pedagogical knowledge needed by PSTs (Anderson & Tully, 2020; Goos & Bennison, 2018). These collaborations can also lead to the formation of hybrid, *in-between* identities in some MTEs who cross the boundary between these disciplines. Hybrid MTEs might possess qualifications and experience in both disciplines, or change careers to move between disciplinary communities. Thus, hybrid MTEs can feel like they belong to both communities, or neither (Goos & Marshman, 2021). Analysing MTEs’ identity trajectories will increase our understanding of how crossing disciplinary boundaries supports or hinders their learning.

This study is part of our larger research program investigating the professional formation of MTEs in terms of identity development (Goos & Bennison, 2019). We view identity as “a

performative process of *becoming* that addresses social interactions and institutional contexts, while acknowledging that an individual’s knowledge, beliefs, and attitudes can influence identity enactment” (p. 405). The present study responds to an opportunity that originated following our presentation at the 14th International Congress on Mathematics Education (ICME-14) on hybrid MTEs and explored models of ITE in terms of collaboration between MTEs who are mathematics educators or mathematicians (Goos & Marshman, 2021). In universities in some countries, including Bohemia University (pseudonym) in the Czech Republic, prospective mathematics teachers are taught both mathematics and pedagogy within a *hybrid* mathematics/mathematics education department. The Head of this department invited us to extend our research to this new international context thereby providing an opportunity to explore the identities of MTEs in a very different international context from Australia. This paper reports on preliminary findings from interviews conducted with MTEs at Bohemia University. The research question addressed in this paper is: How do beliefs, institutional contexts and professional learning opportunities interact to shape the identities of MTEs in a *hybrid* mathematics/mathematics education department? MTE names are pseudonyms.

Theoretical Perspectives

Wenger (1998) described identity as the “pivot between the social and the individual” (p. 145). He claimed that identity development involves a temporal learning trajectory that connects the past, present, and future. Lately, identity has been found to be a useful concept to understand the learning and change of school mathematics teachers (Lutovac & Kaasila, 2018). Yet this kind of research is limited when it comes to the learning and development of MTEs – research that is needed if we are to improve ITE for future secondary mathematics teachers.

In our previous research we have used Valsiner’s (1997) zone theory to analyse teacher and MTE identity (Goos & Bennison, 2019). Valsiner extended Vygotsky’s concept of the Zone of Proximal Development (ZPD) to incorporate the social setting and goals and actions of participants. He redefined the ZPD and introduced two additional zones – Zone of Free Movement (ZFM), and Zone of Promoted Action (ZPA). The ZFM represents what the environment allows. The ZPA, on the other hand, represents what is promoted. According to Valsiner, the ZFM and ZPA are dynamic and work together as a ZFM/ZPA complex to direct development along a set of possible pathways. Individuals, however, are active participants in this process and are free to change their environment or interactions with the people in it. These features of Valsiner’s zone theory make it suitable for capturing the complexity and dynamic nature of identity.

The analytical approach we have developed (Goos & Bennison, 2019) interprets Valsiner’s (1997) ZPD, ZFM, and ZPA in nuanced ways that are aligned with the influences on teacher learning and development. This approach has enabled us to trace identity trajectories of mathematics teachers, teachers who were embedding numeracy into subjects other than mathematics and, more recently, Australian MTEs participating in the Inspiring Mathematics and Science in Teacher Education (IMSITE) project (<http://www.imsite.edu.au>). This approach has allowed us to foreground the temporal character of identity-as-becoming. For MTEs, the three zones are interpreted in a way that is aligned with the factors that influence MTEs’ learning and development. According to our adaptation of Valsiner’s zone theory, the ZPD represents the possibilities for development of new knowledge, goals, beliefs, and practices. For MTEs, this zone includes their mathematical knowledge, pedagogical content knowledge, knowledge of how new teaching practices are learned, and beliefs about mathematics teaching and learning. The ZFM represents environmental constraints that structure access to particular areas or resources or ways of acting with resources. For MTEs, this zone includes characteristics of teacher education students, structural characteristics of teacher education programs, organisational structures, and university cultures. The ZPA includes activities,

objects or areas in the environment where actions are promoted. For MTEs, this zone includes reflections on practice, research with teachers, professional development, and informal interactions with colleagues.

Research Design

The study is being conducted in 2021/2022 and is funded by a School of Education and Tertiary Access Pilot Research Grant. Ethics approval was granted for the study by the university's Ethics Committee (A211656) and all participants gave informed consent. Data collection involved an online survey and interviews. Some preliminary findings from interview data are reported in this paper.

Context

The hybrid mathematics/mathematics education department at Bohemia University has 18 academic staff and sits within the Faculty of Education. The department is only involved in ITE, preparing PSTs to teach in primary and secondary schools. Prospective secondary mathematics teachers initially study a three-year Bachelor of Mathematics for Education which includes 34% pure mathematics, 4% mathematics education and 2% of credit is assigned to mathematics time in schools. To become fully qualified mathematics teachers, they then study a two-year Masters degree, with 12% mathematics content, 13% is mathematics education and 12% of credit is assigned to mathematics time in schools. A shortage of mathematics teachers means that approximately 60% of PSTs are teaching in schools whilst studying their Masters.

Methodology

Participants. All MTEs in the hybrid department at Bohemia University in the Czech Republic were invited to participate in the study by completing an online survey about their beliefs about teaching and learning mathematics and their teaching practices. The final question invited respondents to indicate their willingness to participate in a semi-structured interview by providing their name and contact details. Of the 18 MTEs who commenced the survey, 15 provided responses to all items and five consented to be interviewed. Interviews were conducted in December 2021 and January 2022.

Data collection. Semi-structured interviews were conducted via Zoom. Participants were asked about their backgrounds (e.g., qualifications, teaching experience and their pathway to becoming a MTE), beliefs about teaching and learning mathematics (e.g., beliefs about the way mathematics should be taught and their role as a MTE), their professional context (nature of students, delivery methods, forms of collaboration with colleagues, accreditation requirements) and their opportunities for professional learning (e.g., the influence of colleagues and their research on their teaching). Interviews lasted between 25 and 49 minutes, were recorded and transcribed using Otter.ai. Transcripts were checked manually and edited to ensure accuracy.

Data analysis. Interview data were analysed thematically using our adaptation of Valsiner's (1997) zone theory described earlier. Transcripts were annotated to identify responses that provided information about a participant's ZPD, ZFM and ZPA. For example, a question about a participant's beliefs about their role as a MTE might provide insights into their beliefs about mathematics teaching and learning (ZPD); a question about accreditation requirements could uncover constraints on their teaching practice (ZFM) and a question about the influence of their research on their teaching might reveal an avenue of professional learning (ZPA). For each participant, relevant excerpts of text were then electronically copied and pasted into table with columns for the ZPD, ZPA and ZFM to represent the MTE's zone configuration. The final step in the analysis was to develop a case study for each participant.

Findings

Case studies of Josef (a mathematician) Eva (whom we have classified as a hybrid) and Marie (a mathematics educator) are presented in this paper. These MTEs illustrate the identities of three types of MTEs working within a hybrid mathematics/mathematics education department.

Josef—a Mathematician

Josef is a mathematician and mid-career academic. He has a PhD in mathematics, and predominantly researches in the field of mathematics. He mainly teaches geometry “from elementary to like really high school and university geometry” to future teachers.

Since Josef began teaching in the hybrid department, his beliefs about how to teach mathematics have changed. He described how he had begun teaching in a mathematics department using a very traditional approach: “because I started from mathematical faculty, I also had some other views on how it should be taught. I was more concentrated on many exercises. ... we couldn't like miss anything”. Now, however, he is more concerned that his students understand the mathematics they are learning because of their future role as teachers:

But I guess they should do much more for mathematicians while they need these methods in their head as they need to learn this for their future research But mathematical teachers, they don't need these results. They don't need so many methods - they don't need so many techniques. But they need to understand them really well. So, it must be taught in different way.

As a result of this change in beliefs, he promotes mathematical understanding through question driven discussions:

I usually lead discussions with them. I ask them too many questions, why and why and why until they know why is why. I simply always ask them, and I don't believe anything. I said they don't believe me if I say anything until they have proof, which comes from mathematics.

Josef felt that his approach was aligned with a desire within the hybrid department to “teach them more abstract competencies, like how to think, how to discuss, how to work”. He had found support for developing his pedagogy within the department:

I went the department with so many people who do research in education ... From my perspective, what I know about the didactics - I'm sitting here with so many people who discuss how they teach and how should, you should do it. So, I attended many conferences. ... I learn from these things. ... I also read sometimes something that is interesting in geometry. In geometry, I think the education or in didactics, there are not so many materials for what I do actually.

While Josef had support within the department in which he worked, constantly changing accreditation requirements and high administrative load impacted on him:

Also, the problem should not be, but it is, is the government here has changed accreditation so many times while I'm teaching. I'm teaching maybe six or seven years here, at Faculty of Education, but I'm in the fourth accreditation system here ... So, this is really what makes it hard - bureaucracy here makes it hard. Very much administrative also.

More recently, the COVID-19 pandemic with the resultant change to online teaching had placed constraints around his use of discussion when teaching mathematics:

No one, from what I know, likes totally online teaching for mathematics. It's hard to discuss with people and also my way of teaching doesn't fit too much with only online teaching. So, if it is possible, we do hybrid or face to face teaching.

Josef's identity has changed since he joined the hybrid department. Changes to his beliefs about how mathematics should be taught (ZPD) were prompted by the needs of his students who were preparing to become mathematics teachers (“they need to understand”). He now uses questioning and discussions, focusing on mathematical thinking and understanding. This shift

in practice, reflective of a change in identity, has been guided by a ZFM/ZPA complex that includes his interactions with colleagues which outweigh constraints imposed by university systems and the COVID-19 pandemic.

Eva—a Hybrid

Eva is a very experienced hybrid MTE and senior academic. She has a PhD in pure mathematics and originally worked as a pure mathematician. When she commenced in the hybrid department, she made a career choice to move into mathematics education: “I did not manage to do both [research in pure mathematics and mathematics education] at a high level. Therefore, I had to decide what I will continue with as a main research area”. She teaches mathematics content (algebra) and mathematics education courses.

For Eva, becoming a MTE meant changing from using mathematics to promoting understanding of mathematics:

I think that when I worked as mathematician, I used a lot of formulas and mathematically precise expressions and ideas. Now I am much more constructivist than I was before ... when students develop something. And then we are working on it together to find out if it's correct how to use it, but what to do with it, to be able to use it, and so on. So, I think that this is the main difference.

She described her teaching as “very interactive. I do not like to present something without interaction from students”. This is evident in how she uses discussions so that her students can build their own understandings:

I am moderator of the discussions. I am somebody who opens the door for them to see what they can do. ... Sometimes I'm in the judge - I'm evaluating their discussions - their ideas in the discussions, and when they do not have the same opinion, I have to say sometimes as somebody who says, who helps them to see the you know, for example, sometimes they accept something which is not correct, which is wrong, the very same mistake and they do not see it, because their knowledge is not developed enough. You know, I have to take care of these difficulties and help them to see. So, I am I am more guide through the topic, than the real presenter.

There have been many people who have influenced the way Eva teaches, including a teacher from her final years of secondary schooling, her colleagues and two internationally renowned mathematics educators whom she named (David Clarke and Guy Brousseau):

I was partly influenced by one of my professors at the upper secondary level, who taught me in the last year of upper secondary and he was excellent. And he used the method that I am copying very often, even now, and then the practice and my colleagues whom I could follow.

She also draws on her research to inform her teaching: “I am using it mainly in the courses in maths education ... I do not do something in research and something else in teaching. I'm linking it together.” Eva works collaboratively with other MTEs in the hybrid department in a mentoring role:

We work together and we are preparing all this together. And during my professorial life and at the faculty I already worked with similar young colleagues who start, either my former PhD students or those coming from other paths who are very often pure mathematicians, who for example, take some seminars to my lecture, and they learn from what I show them, and I give them and ask them to do, how to work with students later. So, I hope when I finish there will be enough young colleagues to take over.

The collegial atmosphere within the department works to alleviate constraints imposed by accreditation requirements: “For accreditation there are very, very many decisions, which are taken by the faculty and at the level of the university, which must be followed”.

Eva's identity has changed since she joined the hybrid department from being a mathematician to becoming a hybrid MTE. Her beliefs about mathematics (ZPD) were initially associated with practicing mathematics (“formulas, mathematically precise expressions and ideas”) and are now centred on promoting her students' understanding of mathematics (“help

them to see”). The ZFM/ZPA complex that has guided this change is dominated by influential people, including her colleagues, and her research. Constraints imposed within her professional context (e.g., accreditation requirements) seem to have limited influence on her practice.

Marie—a Mathematics Educator

Marie is a mathematics educator and Head of the hybrid department. She originally studied teaching mathematics, geography and English and taught in a primary school before commencing her PhD in mathematics education. She initially taught mathematics, analytical geometry, and problem-solving courses but now teaches only mathematics education courses.

Marie’s beliefs about teaching mathematics were formed through her initial experiences in the hybrid department:

When I was starting teaching and, in the faculty, Professor H came to the university, and he completely redesigned some courses for future teachers and analytic geometry was one of them. And it was organised in such a way that there are hardly any lectures, but students got some problems to solve, and by solving the problems, they can, they should discover some something. So, it was like a discovery teaching. And during the seminars and lectures, we discussed it and put it together. So, it was really, for me, it was like a miracle.

According to Marie, “there is a consensus that, of course, definitions and theorems are still important and must be covered, but I believe that in the department nowadays there is a consensus that it could be done in a more inductive way”. Now her focus is not on teaching mathematics but on teaching her students how to teach mathematics for understanding: “My main role, I believe, is to get my students to think hard about how to teach because they know what to teach”. She does this by getting her students to analyse and discuss textbooks, readings and videos of lessons: “It’s usually observing a part of the lesson, for example, a video lesson, or it’s the analysis of a textbook from some point of view, or some reading. And then during the seminar, we speak about it”. Marie wants her students to adopt constructivist approaches and teach for understanding:

We [the hybrid department] advocate constructivist approaches to teaching. When they go to school, I always tell them, you can use any method, but all the methods should be aimed at their understanding. So, if you if your pupils learn by heart, then you failed. So, you must do it differently. And so that’s the message I try to tell them during all my courses.

Mathematics is culturally valued in the Czech Republic and this is evident in the way Marie feels the hybrid department is viewed within the Faculty of Education:

A big advantage of my place is that we are in the Faculty of Education. ... So, we have both mathematicians, and math educators. But the mathematicians who come to our department are the ones who are also active mathematicians, but their approach to math education is positive. ... mathematics is regarded as like, I don’t know, the Queen. We are important. So, we feel important.

Within the hybrid department, there is a culture of collaboration, evidenced by the way in which Marie described how programs were developed collaboratively for accreditation:

So, when we were preparing the content of this study, so we got together as a department, and we spoke about individual courses. So, we agreed on the content as a group, ... And then they teach, and it’s their responsibility that they teach to the syllabus.

Marie’s research is intimately connected with her teaching. Her students are her research participants, and her data analysis informs her teaching:

My research is very much informing my teaching. ... my second stream of research is in future teachers and teachers’ noticing, and knowledge-based reasoning. So, this is really connected, because I use my students also as research subjects. So, they analyse their work and then they use it to inform my teaching. For example, if I give them some video recordings to see, and then when I analyse the reflections I can see well, they didn’t notice what I wanted them to notice. So, this very basic thing next time I am I am

preparing a task for them, which would focus their attention on it. So, I think I am lucky, because my research is really very tightly connected to my math education courses.

In addition to her research, Marie's teaching is informed by her reading: "I try to read a lot. So, I tried to incorporate some research results" and, prior to the COVID-19 pandemic, visitors to the hybrid department: "when we had some visitors from abroad coming, I always invited them to my courses with a view to get some inspiration from their teaching".

Marie's identity as a MTE began developing while she was teaching with Professor H. Her beliefs about how to teach mathematics (ZPD) were shaped by the "miraculous" constructivist methods he introduced her to that have led her to focus on pedagogy that promotes mathematical understanding ("get my students to think hard about how to teach"). The ZPA/ZFM complex that guides her development includes the privileged position of mathematics, the collaborative nature of the department, and her opportunities to link her research and teaching.

Identities of MTEs in the Hybrid Mathematics/Mathematics Education Department

The case studies of Josef (a mathematician), Eva (a hybrid MTE) and Marie (a mathematics educator) illustrate how the identities of these MTEs have been shaped by their environment and the people in it (i.e., the hybrid mathematics department.). The hybrid department has a privileged position in a faculty that values mathematics and brings together three types of MTEs who have a common goal – preparing future mathematics teachers. This common goal has contributed to shaping the *beliefs* (ZPD) about how mathematics should be taught of Josef, Eva, and Marie. Their beliefs have developed under the influence of their *institutional context* (ZFM) and *professional learning opportunities* (ZPA) that canalise their identity development.

Within the hybrid department, the MTEs have developed similar beliefs about the value of constructivist teaching methods and the importance of discussions to build mathematical understanding. Unlike the mathematicians in Leikin et al.'s (2018) study, Josef believed that mathematics needs to be taught differently to prospective teachers so that they developed a deep understanding of the mathematics ("it must be taught in a different way"). Eva, who taught both content and pedagogy courses, described herself as a "moderator of discussions" who helped her students "to see". As a mathematics educator, Marie impressed on her students the importance of thinking about how they will teach for understanding ("if your pupils learn by heart, then you failed"). While beliefs are only one factor that contributes to an individual's ZPD, it seems that this component of the ZPDs of all three MTEs is aligned when it comes to their goal what is required to prepare future mathematics teachers.

The institutional context provided by the hybrid department is one where mathematics is valued and there is a professional culture of collaboration. All three MTEs described instances of collegiality. Josef described regular conversations about teaching ("people who discuss how they teach and how you should do it"), Eva described how she mentored less experienced colleagues ("they learn from what I show them") and Marie described how courses were planned collaboratively for accreditation ("we agreed on the content as a group"). None of the MTEs mentioned insurmountable constraints imposed by their professional context. Thus, the ZPA offered affordances rather than constraints. While there seem to be no formalised professional learning opportunities for the MTEs, there is a culture of learning from colleagues within the hybrid department and beyond. Josef's teaching, for example, is informed by his colleagues within the hybrid department whereas Eva and Marie seem to draw mainly on their research and international mathematics educators. This difference is perhaps due to Josef's research being in mathematics rather than mathematics education. Nevertheless, the ZPA promotes interdisciplinary collaboration. Thus, the ZFM/ZPA complex experienced by the three MTEs is similar but differences exist in the professional learning opportunities each MTE draws on.

Concluding Remarks

The MTEs who teach prospective secondary mathematics teachers in Australia are usually mathematics educators and mathematicians who work in different faculties and seldom collaborate – a situation that can lead to conflicting views about how to teach mathematics. Interdisciplinary collaboration between mathematicians and mathematics educators in ITE in Australia sometimes leads to *hybrid* MTEs who cross the boundaries between the disciplines (Goos & Marshman, 2021). This study extends our earlier research on the identities of MTEs (e.g., Goos & Bennison, 2019) involving interdisciplinary collaboration *across* the boundaries that separate disciplines by providing insights into interdisciplinary collaboration *within* an organisational unit (the hybrid mathematics/mathematics education department). Although we cannot make strong claims because we are drawing on the preliminary analysis of interviews with only three MTEs, our findings suggest possible insights that can be drawn from investigating the identities of MTEs in an ITE context that is different from Australia. We identified three types of MTE—mathematician, hybrid, and mathematics educator—in a hybrid department that seems to afford collaboration between mathematicians and mathematics educators. This collaboration appears to have resulted in similar beliefs that guide the MTEs’ teaching practices. We hope to gain further insights following analysis of data from the online surveys and remaining interviews.

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