The Role of Mathematics Learning in the Interdisciplinary Mathematics and Science (IMS) Project

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While interdisciplinary approaches in the STEM subjects are widely advocated there are concerns that disciplinary learning can be compromised, especially in mathematics. The Interdisciplinary Mathematics and Science (IMS) project is a three- year longitudinal study in four Victorian primary schools that has developed a pedagogical approach to mathematics and science learning where data modelling and representation are common to each. Investigations include astronomy, ecology, chemistry, fast plant growth, force and motion, water use, heat and temperature, body height, light and microorganisms. The paper describes the role of mathematics in the IMS pedagogical model and design of learning sequences.

The promotion of integrated or interdisciplinary approaches to mathematics and science learning has been characterised by enquiry-based processes that reflect the way these disciplines contribute to problem solving in real-world contexts. There is also increasing interest in the potential of interdisciplinary approaches that integrate mathematics learning with learning in other disciplines and in particular in the context of Science, Technology, Engineering and Mathematics (STEM) (Doig et al., 2019; Maass et al., 2019). One of the concerns of such approaches is preserving the integrity of disciplinary knowledge, especially when opportunities for rich mathematics learning are not realised. Nevertheless, Lehrer (2021) argues that such interdisciplinary work a) opens up possibilities of knowledge transfer between disciplines as science and mathematics constructs interact, b) emphasises disciplinary knowledge as relevant to solving important problems, and c) can build the sort of connected and structured knowledge systems that expert STEM practitioners display. By focusing on the development of mathematics and science disciplinary representational practices students come to understand the distinctions between these practices which in turn supports their reasoning and knowledge-building, and develops their representational competence (diSessa, 2004; English, 2012).

Background Literature

Enquiry-based approaches to mathematics learning have been promoted in a range of studies, for example in problem-solving challenges (Sullivan et al., 2016) and in teacher orchestration of student work (Dorier & Maass, 2020; Pinto & Koichu, 2021). Student-led representations have been central to the problem-solving process as a tool for mathematical thinking. A number of recent studies have focused explicitly on young students' meta-representational competence in solving problems in real-life contexts often integrated with scientific concepts and investigations (English, 2012; Makar, 2016). For example, in a study

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of third graders' predictive reasoning students interpreted the aggregate properties and variability of a "real-world" data set comprising monthly maximum temperatures over time (Oslington et al., 2020). Other studies have described the development of mathematical thinking in constructing and interpreting graphs from data collected in their investigations of ice melting, growth of plants and measures of change in the growth pattern of chickens (Mulligan, 2015). While studies of meta-representational competence often focus on the students' mathematical understanding portrayed through representations, there needs to be complementary studies of how teachers use pedagogical strategies to support students' interpretation of data.

The *Interdisciplinary Mathematics and Science (IMS)* project was conceptualised and designed to explore the principles and possibilities of interdisciplinary alignments in a variety of topics, across a range of grade levels in the primary school (https://imslearning.org/). The principle underpinning the project is that robust learning involves the invention, evaluation, refinement and coordination of representational systems in both science and mathematics, and by focusing on how these science and mathematics systems interrelate, more robust learning of foundational concepts will occur. The key challenge was to generate tasks and learning sequences where science opened up productive possibilities for new mathematics learning, and vice versa. The project operated under several design constraints: a) the challenge of productively aligning scientific and mathematical concepts and practices at the appropriate developmental level, so that they are mutually reinforcing, b) the expectations of teachers regarding appropriate content at that grade level, and c) teachers' disciplinary knowledge and pedagogical capabilities needed to support students. In this paper mathematics learning is described in 12 learning sequences which were implemented through a pedagogical model that promotes the development of mathematical concepts and meta-representational competence.

The IMS Pedagogical Model

The IMS pedagogical model focused on students' investigations across a range of scientific problems with an emphasis on constructing and refining representations (Prain & Tytler, 2021; Tytler et al., 2022). This process enabled students to develop connections between everyday ideas and mathematics and science representational systems. We also drew on the work of Lehrer and Schauble (2020) who describe their approach as establishing the need to create/invent representations, explore what they reveal, make decisions about appropriate representations, and engage with an expanded set of representational tools. In the IMS project the disciplinary focus shifted back and forth between mathematics and science, with each iteration involving new questions and idea refinement (Tytler et al., 2021), and productive knowledge-building in each subject. From these perspectives we developed and refined a pedagogical model which consists of four stages, each with a disciplinary purpose, shown in Figure 1.

Orienting: Teachers pose questions, explore student ideas and guide students to focus their attention on what is worth noticing, asking for predictions, questioning what they have noticed, asking for ideas about what could be measured, and introducing resources for the later stages of the enquiry.

Posing representational challenges: Students are challenged, individually or in groups, to invent/construct representations that reflect a process of claim-making and predictive and causal reasoning or justification. The process involves students in meaningful material exploration, organised by and feeding into the representational practices.

Building consensus: This stage entails teacher guided sharing/display and comparison/evaluation of the comprehensiveness and clarity of the representations. The teacher guides comparative review, feedback, and refinement/revision, drawing strategically on the variation in students' representations to guide an emerging consensus.

Applying and extending conceptual understanding: Students are given new representational challenges to extend their new knowledge and practices in related situations, or further concepts are introduced through representational tasks, to repeat the cycle.
n some sequences an iterative process involved more than one cycle of stages focused on the refinement of the same concept (e.g., motion and force or variability), or developing a sequence of concepts (shadow patterns leading to modelling of earth's rotation).

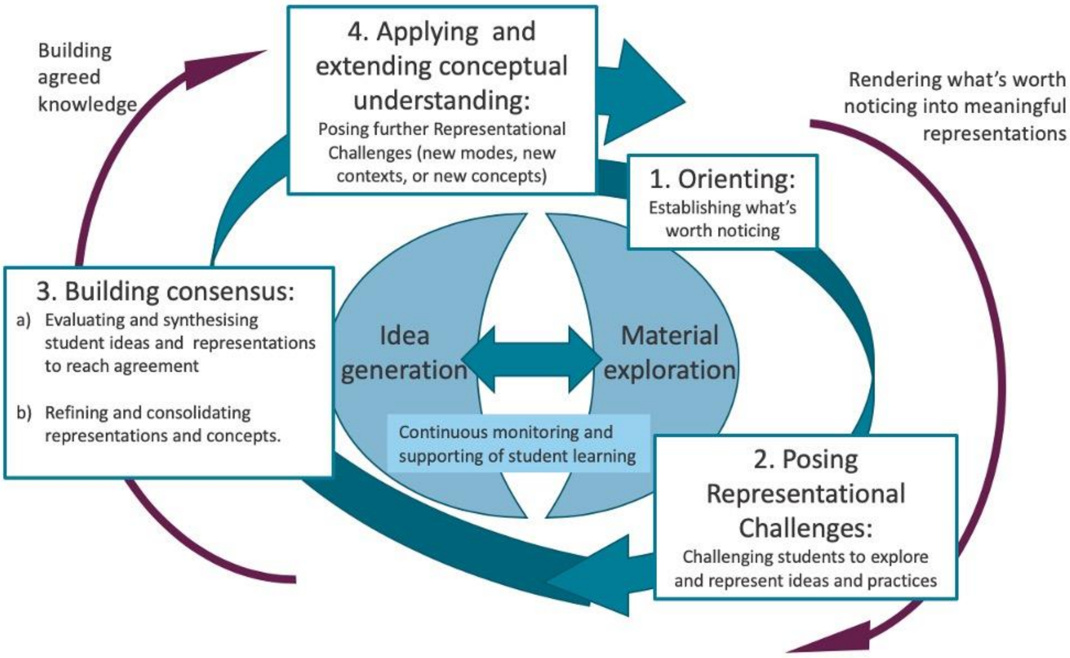

Figure 1: The Interdisciplinary Mathematics and Science (IMS) pedagogy model (from Tytler et al., 2021).

Methodology

The IMS project study was conducted in two metropolitan and two regional primary schools in Victoria, Australia and one regional school in Wisconsin in the US. For each of six grade levels the sample comprised between one and six classes each with 25-30 students, a case-study teacher, and up to 21 case study students representing a range of abilities.

The study adopted a design experiment methodology (Cobb et al., 2003) based on a cycle of planning, trialling, data generation/evaluation, and refinement of 12 learning sequences over a three-year period for Grades 1 through 3 and Grades 4 through 6 (see Tytler et al., 2021). Each cycle involved modifications to teacher support, including changes in framing of the pedagogy and the sequences, workshop design, development of ways of effectively integrating the science and mathematics, and methods of assessing student learning. Professional planning and review meetings were conducted where the learning sequences were refined in consultation with teachers. The workshops afforded opportunities for teachers to engage in the learning experiences and raise questions about the mathematical and scientific content knowledge and statistical ideas inherent in the investigations.

Learning sequences were implemented in each of four school terms for the first two years and one term in the third year, comprising between three and eight weekly lessons of 1-2 hours duration, depending on school timetables and the demands of the topic. At least two members of the research team provided support to the teachers and students during the lessons as well as liaising with the research assistant in collecting data. Data sources comprised records of teacher planning, reflective workshops and review meetings, classroom observations and video capture of lessons conducted by case study teachers, student work samples and class displays, pre- and post-student assessments, and individual and focus group interviews with case study teachers and students.

Analysis of these data focussed on micro-ethnographic discursive analysis of video capture to analyse teacher and student interactions and reasoning during lessons, reflecting the stages of the IMS pedagogical model. The analysis process required independent multiple views of these data from two perspectives: mathematics and science learning respectively, prior to the research team identifying similarities and differences between these perspectives and establishing how data modelling was common to establishing interrelationships.

Development of Learning Sequences

In planning interdisciplinary mathematics and science sequences we focused on concepts that were common to both disciplines and where the mathematics and science contexts productively interacted, and mutually reinforced. Some sequences drew upon the enquiry-based modules of Primary Connections (Australian Academy of Science, 2012). Table 1 provides an overview of the mathematics and science concepts across 12 learning sequences. In all but one, the sequence is identified by the science context, but the mathematics and data modelling is developed strategically and synergistically within the science exploration, and feeds back into representational work.

Table 1:
Interrelationships between science and mathematics concepts in learning sequences (adapted from Tytler et al., 2021).

Grade	Topic	Science	Mathematics
1, 4	Astronomy	Shadows, sun movement, day and night, earth and space perspectives.	Angle as rotation, estimating and measuring, recording and graphing shadow length (formal and informal measures), recording and interpreting time, perspective taking and spatial reasoning.
1, 4	Ecology	Living things, diversity, distribution and adaptive features related to habitat.	Data modelling of living things in sample plots, constructing tables and graphs, variability and sampling, spatial reasoning, mapping, area, coordinates, directionality, constructing and using a scale.
1, 4	Motion	Dynamic concept of motion, Measuring and representing distance/time/speed relations, constant speed, acceleration.	Measuring and representing distance/time/speed relations, using informal and formal units, noticing variation, graphing, and slope of ramp. Using, ordering and recording decimals.
2	Chemistry	Dissolving and mixing, physical change, particle ideas, chemical reactions, change to substances experimental methods.	Measuring and representing time sequences, recording and interpreting timing, measuring substances using formal and informal units, common fractions, and proportional reasoning.

2/4	Fast Plant Growth	Plant growth, structure and function, growth needs and patterns, plant life cycles: germination, flower structures, and fertilisation.	Measures of height, width, leaf and root size, shape and pattern, informal and formal units (cm and mm), time (days, 24-hour time), constructing a scale and graphs.
2	Force and Motion: Helicopters	Flight and air flow, gravity, representing forces, modelling and design, and variable control.	Estimating and describing helicopter parameters, measuring time and timelines, variation and its sources, graphing, and spatial properties.
2	Water Use	Personal use and conservation of water, impact of water use and conservation on the environment.	Estimation and measure of water use, informal and formal measures of volume/capacity, 24-hour time, time line, data collection, organisation and representation.
3	Heat and Temperature	Heat sources and heat flow, temperature measurement, conduction, insulation, and material/design.	Attributes of time and temperature, count and interval, units of time and temperature, relation between informal and formal measures of time and temperature, constructing a scale, and representing data.
4/5/6	Light	Properties of light: vision, reflection, refraction, and image creation.	Angle type and measure, rotation, reflection, symmetry and directionality.
5/6	Measurement: Body Heights	Anatomy, relation between growth and age, estimation and measure of heights, interpreting variability and differences between populations.	Establish need to measure, identifying attributes, comparing, estimating and measuring (informal and formal) ordering, clustering, comparing samples, measures of central tendency and variation, dot plots, timeline, aggregation and predictive reasoning
5/6	Astronomy	Solar system, day and night, planetary features, moon movement and phases.	Ratio of planetary size and distance, angle, compass points, tracking position over time, perspective taking, cosmological distances.
6	Magic Microorganisms	Structure and function of microorganisms, magnification, and growth patterns	Spatial patterns, multiplication, ratio, and proportion, measuring area, sampling and distribution and measurement tools and units.

In each of the learning sequences (except Body Heights), the science context contextualised the mathematical enquiry by creating a need to explore and represent underlying patterns (spatial, numerical) in ways that fed back into questions that directed data representation.

Illustrations of Learning Sequences

Each of the 12 learning sequences were implemented with particular grade levels across a number of cohorts and refined through multiple iterations. Data analyses from each learning sequence enabled different insights into the mathematics–science interrelationships. Three examples described below provide pertinent illustrations of the role of mathematics in the learning sequences: Ecology, Body Heights, and Force/Motion.

At a fundamental level, the Ecology sequence engaged Grade 1 and 2 students in developing effective counting and tallying skills as well as representing data as a simple table, picture or bar graph. Students, in groups, investigated and individually represented the number and location of different living things in different habitats represented by sample plots. But the exploration extended to a wider range and depth of mathematical ideas. Students then evaluated and refined their data representations, tabulated class data, represented the distribution of particular animals across the different plots, and proposed reasons for this. Spatial skills were also involved by visualising and drawing a map of the area and including a key and invented icons (or simple coordinates). Following the groups' refinement of representational work to produce documented counts of living things in the different plots, teachers raised questions about variation in populations across the different sites: Where are particular living things found? Why are the numbers of particular animals different across the plots? Moreover, the mathematical representations and data interpretation supported the students' reasoning about the distribution of living things in various locations leading to the scientific concepts of diversity, habitat and adaptation.

The Body Heights sequence involved a series of investigations conducted in Grades 5 and 6 across six classes in one school (Mulligan et al., 2022). Students were initially challenged to consider whether students in their Grade 5/6 class would meet the 1.4m height requirement of a theme park ride. Students estimated their own height, compared their height to their peers in an iterative process in which they clustered, displayed, graphed and interpreted class data. Comparison of estimates with actual measures then initiated ideas about measures of central tendency. They compared height data from Prep/Grade 1 classes with their own to make inferences about the sampling and to draw conclusions about student growth patterns over six years. These findings indicated that students were more than capable of measuring height—they were able to organise and interpret measures to support the data-modelling process and development of their statistical reasoning.

The Force and Motion sequence provided a coherent example of the interrelationships between the mathematics and science concepts and processes. The investigation involved concepts of forces due to air flow, uplift and gravity in the design of helicopters including variations in wing length, shape and weight, controlled by the number and size of paper clips attached to the body or the wings. The students utilised their measurement skills in making estimates of height of the drop, understood the need for a fine-grained measure of time in seconds and parts of seconds (one or two-place decimals), ordered and interpreted times, related time to the speed of the helicopter, and recorded, organised and made inferences from the data.

The science and mathematics, while they had commonalities, were somewhat distinct, but mutually reinforcing. Students were able to explain the relative affordances of different ways of representing their data sets and recognised and responded to the influence of variability. They discussed different ways of constructing data tables to display multiple trials with different conditions, and displayed these on a timeline with an interval scale. Students were able to recognise and use the median value of a set of numbers as being a fair representation of the data set, and to articulate the inevitability of variation on repeat measures, and suggest sensible reasons for this. That they could do this, through gentle guidance, was surprising to the teachers. The science provided the setting that drove authentic mathematical problem solving and meta-representational competence, and the mathematics in turn contributed to the science in raising questions about, for instance, the need to control for variation, and the effect of weight or wing design on the helicopter flight.

Implications and Conclusions

Further research, that evaluates the efficacy and impact of the IMS approach on teaching and learning is necessary to validate and upscale the approach in a variety of contexts and with a more diverse populations. Attention to professional learning and support might be prioritised: enabling sustained research with teams of mathematics and science education researchers in collaboration with teachers and school systems; and the development and implementation of programs to support professional learning about the synergistic nature of mathematics and science learning.

The generative nature of the mathematics and science interdisciplinary model has significant implications for curriculum review and practice. The IMS learning sequences demonstrated a range of possibilities for mathematics learning supporting the Australian Curriculum–Mathematics, particularly the reasoning and problem-solving aspects of the Proficiencies (Australian Curriculum, Assessment & Reporting Authority [ACARA], 2018). Student learning through purposeful invention, comparison and refinement processes was evidenced by students' explanations and representations of data supported by explicit teacher scaffolding and the building-consensus process. Through situating mathematics in meaningful investigations, the purpose of measurement and data-modelling processes were realised.

The IMS project provided multiple opportunities to develop students' conceptual knowledge: number and pattern, spatial reasoning, measurement, and data-modelling simultaneously—as well as developing statistical concepts and representational processes that are central to mathematics and science investigations. These outcomes were achieved at varying levels of depth and competence for all students, often well beyond curriculum expectations, when appropriate problem contexts were explored systematically over a series of iterations.

Moving forward, one way of prioritising mathematics could be to integrate scientific problems during the mathematics learning space. The science curriculum provides this flexibility in ways that the tightly prescribed practices in mathematics content strands do not often allow. Blending these disciplines in the mathematics learning space might enable teachers to focus more on the mathematical proficiencies of problem solving and reasoning (ACARA, 2018). More flexible learning structures and more time within school curricula would support such as approach as was necessary in the IMS project. The depth of practice achieved by this interdisciplinary approach would compensate for the time invested. A re-conceptualisation of the role of data modelling in mathematics curricula would necessitate a shift in emphasis from a traditional "siloed" approach to one that is more flexible and interrelated. This approach would encompass forms of interdisciplinarity that honour the epistemic processes of both subjects in ways that lead to rich learning in each.

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